

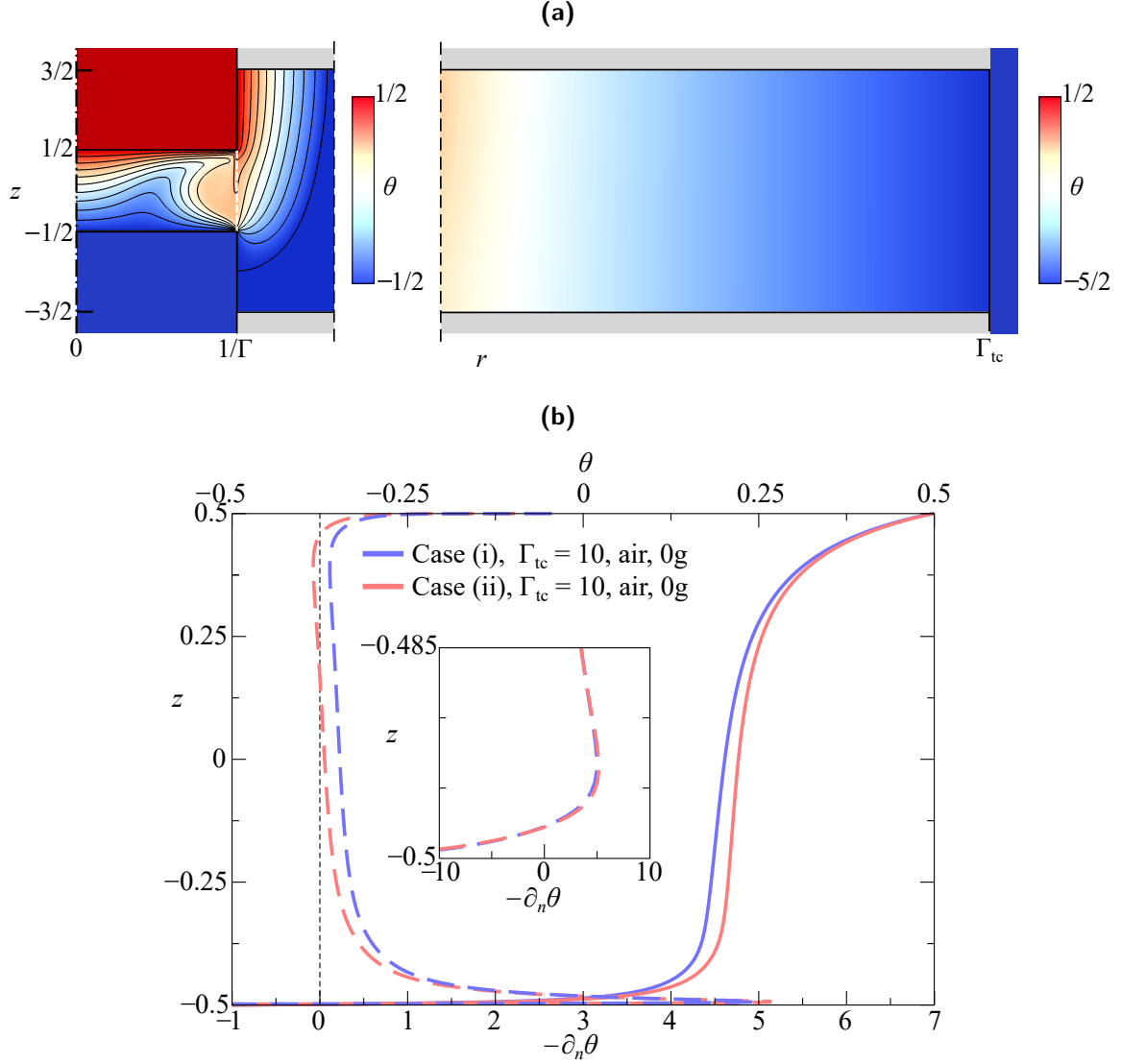
# **Supplementary Material for**

## **“Scaling and modeling of the heat transfer across the free surface of a thermocapillary liquid bridge”**

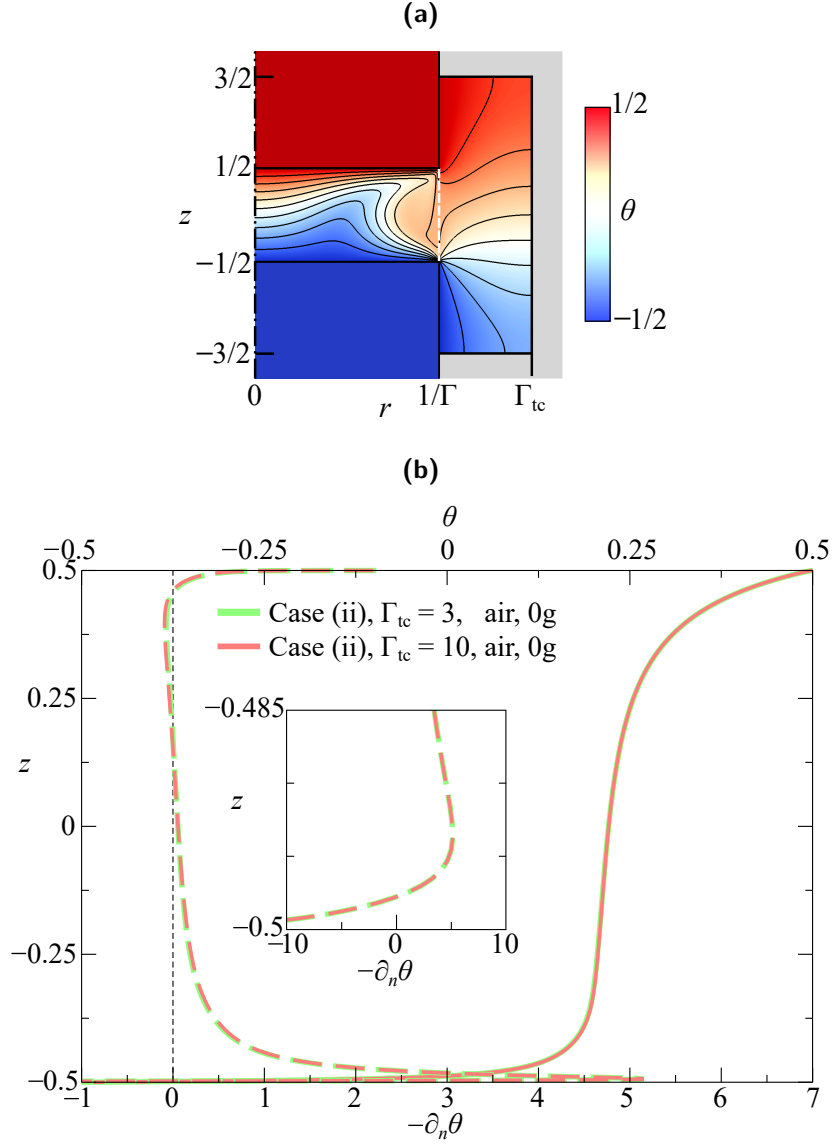
### **1 Bulk temperature maps and thermal quantities at the free surface**

In order to complement the analysis on the temperature field in the bulk and the temperature and heat transfer at the free surface, three comparisons are reported in figs. 1, 2, and 3. In fig. 1, the effect of the thermal conditions on the outer wall is depicted. For a large chamber, passing from a perfectly conducting to a perfectly insulating outer wall produces a shift of the heat transfer and temperature profile at the free surface, consistently with the cooling produced at the cold wall ( $\theta = -5/2$ ) at  $r = \Gamma_{tc}$ .

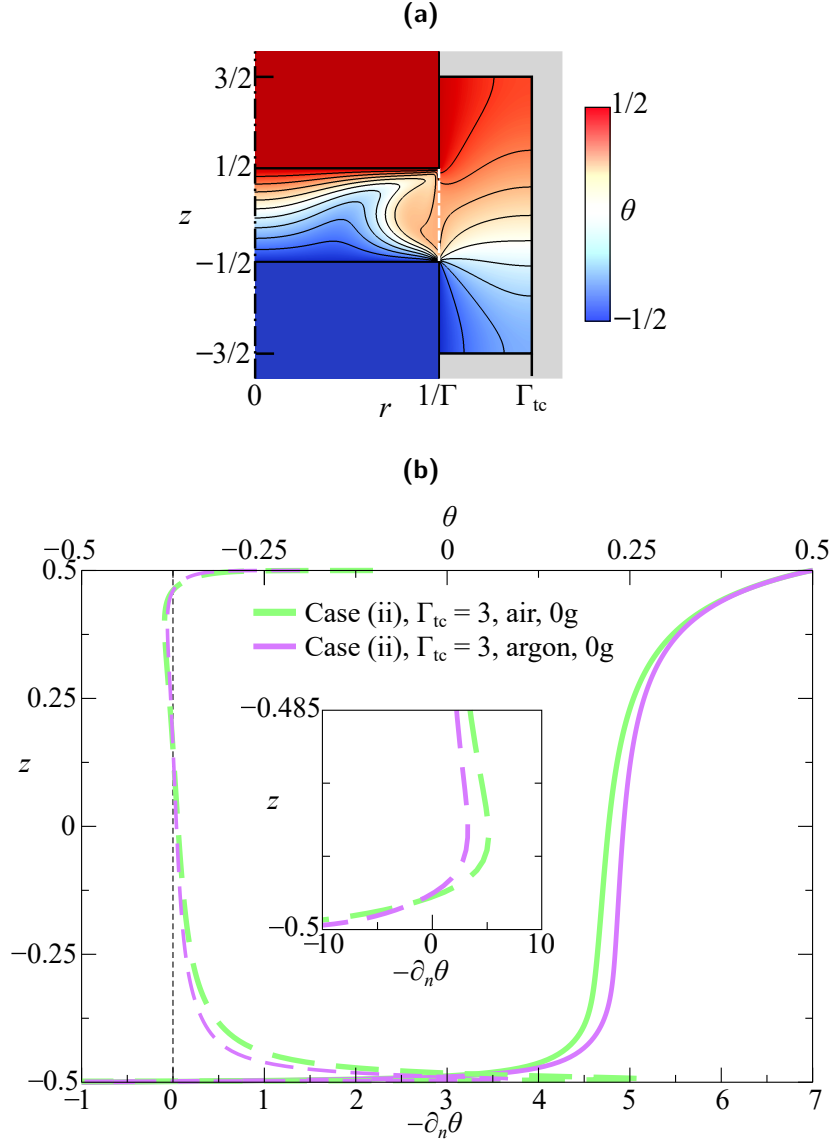
Figures 2 and 3 report on the effect of changing the size of the closed adiabatic chamber and the filling gas, respectively. The former has negligible effects, that compares between  $\Gamma_{tc} = 3$  to 10. On the other hand, employing argon rather than air, hence decreasing the thermal conductivity of the surrounding fluid, increases the mean temperature at the free surface. Owing to the convective effects, it decreases, in turn, the heat transfer near the cold wall.



**Figure 1:** Effect of thermal conditions of the outer chamber wall: Temperature for  $Pr = 28.8$ ,  $Ma = 10^4$ ,  $\Gamma = 0.5$ ,  $\Gamma_{tc} = 10$ ,  $Bo = Bd = 0$ , and air as surrounding gas. (a) Temperature field  $\theta(r, z)$  for Case (i). (b) Surface temperature  $\theta$  (full line) and surface heat flux  $-\partial_n\theta$  (dashed line) as functions of  $z$ . The color-coding is consistent with fig. 7 of the manuscript.



**Figure 2:** Effect of radial aspect ratio of the chamber: Temperature for Case (ii),  $\text{Pr} = 28.8$ ,  $\text{Ma} = 10^4$ ,  $\Gamma = 0.5$ ,  $\text{Bo} = \text{Bd} = 0$ , air as surrounding gas, and  $\Gamma_{tc} = 3$  and  $\Gamma_{tc} = 10$ . (a) Temperature field  $\theta(r, z)$  for  $\Gamma_{tc} = 3$ . (b) Surface temperature  $\theta$  (full line) and surface heat flux  $-\partial_n \theta$  (dashed line) as functions of  $z$ . The color-coding is consistent with fig. 7 of the manuscript.



**Figure 3:** Effect of the surrounding gas: Temperature for Case (ii),  $\text{Pr} = 28.8$ ,  $\text{Ma} = 10^4$ ,  $\Gamma = 0.5$ ,  $\text{Bo} = \text{Bd} = 0$ ,  $\Gamma_{\text{tc}} = 3$ , and air and argon as surrounding gas. (a) Temperature field  $\theta(r, z)$  for argon as surrounding gas. (b) Surface temperature  $\theta$  (full line) and surface heat flux  $-\partial_n \theta$  (dashed line) as functions of  $z$ . The color-coding is consistent with fig. 7 of the manuscript.