FABIÁN ROMERO

CHARACTERIZATION OF TOPOLOGICAL MODELS FOR DISTRIBUTED COMPUTING WITH LOGICAL SYSTEMS.

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SUPERVISOR: FRANCISCO HERNÁNDEZ QUIROZ

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Synopsis

The objective of this work is to study the topological model for distributed computing from a formal logic perspective by mapping those topological models to Kripke frames and then, by applying model checking and other techniques. Specifically targeting consensus problem as a subject of study.

Introduction

A distributed system ***DS*** can be defined operationally as a collection of processes executing a protocol (algorithm) within a communicating environment.

Studying ***DS*** operationally has shown to be a complex task, even when the number of processes and connections is limited and/or following simple patterns. It's complexity arises from the number of possible interactions between their elements and the dynamics of such interactions.

On it's seminal paper [1] Herlihy introduced a characterization of ***DS***, mapping the specifics of such protocols and environments to a topological structure. In this combinatorial description all possible events on the dynamic of the interactions are represented simultaneously on a single topological structure. Such approach is a deliberated trade off. Interchanging one model with complex dynamics and relatively simple elements for other with complex elements but static.

[1] Maurice Herlihy and Nir Shavit. The topological structure of asynchronous computability. *J. ACM*, 46(6):858–923, November 1999. ISSN 0004-5411. DOI: 10.1145/331524.331529. URL http://doi.acm.org/10.1145/331524.331529

This technique for the analysis of ***DS*** has gained significant traction and has become widely used. Gaining from a large set of topology results that can be immediately applied, and, as he argues. Our minds are much better equipped for analyzing static entities even when they are very complex than for dynamic ones even if they are sparse.

DS have been studied from many areas and multiple angles, in particular, it has been covered on many different logic approaches, epistemic logic [2] is a natural formulation for them, there's a great tradition on the field of artificial intelligence associating several logics to the study of ***DS*** [3], and many other logics as multi modal logic [4], dynamic epistemic logic [5], among many others.

[2] John-Jules Ch Meyer and Wiebe Van Der Hoek. *Epistemic Logic for AI and Computer Science*. Cambridge University Press, New York, NY, USA, 1995. ISBN 052146014X

[3] Dov M. Gabbay, C.J. Hogger, and J.A. Robinson, editors. *Handbook of Logic in Artificial Intelligence and Logic Programming*, volume 1 Logical Foundations. Clarendon Press, Oxford, 1993

[4] Jaakko Hintikka. *Knowledge and Belief*. Ithaca, N.Y.,Cornell University Press, 1962

[5] Hans van Ditmarsch, Wiebe van der Hoek, and Barteld Kooi. *Dynamic Epistemic Logic*. Springer Publishing Company, Incorporated, 1st edition, 2007. ISBN 1402058381, 9781402058387

Those works are intended to comprehend communications, epistemic development and behavior of such systems from an operational point of view.

General Objective

The objective of this work, is to mimic the approach taken by Herlihy but using formal logic as model, first, by mapping topological descriptions of ***DS*** into Kripke frames. Then, by studying a specific problem in the light of logical systems. The problem to study will be the consensus problem on the wait free shared memory model (***WFSM***).

Specific Objectives

- Describe a way to map topological models to Kripke frames.
- Study which properties such frames will have.

Background

Distributed computing, wait free shared memory model ***WFSM***

Very early on the study of distributed systems, Leslie Lamport noticed an important trait of them: There is a partial order on the set of all system events. [6]. He also show how to find a total order for a specific model, but this is not limited to such model, the **order-extension principle** states that a total order can be found as a linear extension for any partial order.

[6] Leslie Lamport. Time, clocks, and the ordering of events in a distributed system. *Commun. ACM*, 21(7):558–565, July 1978. ISSN 0001-0782. DOI: 10.1145/359545.359563. URL http://doi.acm.org/10.1145/359545.359563

Lets consider n identified processes communicating in a shared memory environment running a wait-free protocol. Such as that described on [7].

[7] Maurice Herlihy and Nir Shavit. The topological structure of asynchronous computability. *J. ACM*, 46(6):858–923, November 1999. ISSN 0004-5411. DOI: 10.1145/331524.331529. URL http://doi.acm.org/10.1145/331524.331529

Listing 1: Normal form wait free protocol

```
--code for process i
update(i,a,input_value)    -- a[i] := input_value
for round in 1 .. r do
   local_state:=scan(a)
   update(i,a,local_state) -- a[i] := local_state
return δ(local_state)
```

So, the partial order induced by such algorithm, says if $a \geq b$ in time, meaning b updated the shared memory not latter than a, also can be interpreted as a "knows" the state of b.

Observe that this simple fact will have a deep impact when we translate this model to a logic system, because it means we can derive epistemic results from the adequate accessibility semantics.

It is easy to find a total order in this case, as the time (in the clock of the shared memory) when writings were done.

Partial Orders

Definitions

poset A partially ordered set (a.k.a poset) is a pair $(P, \geq)$ where P is a finite set and $\geq$ is a reflexive, anti symmetric and transitive relation over P.

linear extension Given a poset $(P, \geq)$ let λ be a bijection from P to $\{1, ..., |P|\}$ such that $\forall i, j$ if $|P| \geq j > i \geq 1$ then $\lambda(j) > \lambda(i)$. λ is a linear extension

order polytope The following linear constraints:

$$O(P) = \{X \in \mathbb{R}^{|P|} | 1 \geq X_i \geq 0, X_i > X_j \quad if \quad x_i > x_j \quad in \quad P\}$$

Define a polytope $O(P)$ in $\mathbb{R}^{|P|}$, such polytope is called order polytope of P [8]

[8] RichardP. Stanley. Two poset polytopes. *Discrete and Computational Geometry*, 1(1): 9–23, 1986

Theorem The number of distinct linear extensions of the poset $(P, \geq)$ equals the number of simplices in a maximal size triangulation of the order polytope $O(P)$

The Topological Structure of Asynchronous Computability

Once we have established that fact, if we consider all possible orderings of events under ***WFSM***, we know by symmetry that they are manifolds.

The interpretation is the following:

Every element of P is a process, every simplex where it is contained is the **linear extension** of all process contained on that simplex.

Let see for example, the case of 3 processes and a single round of communication.

All total orders (possible executions):

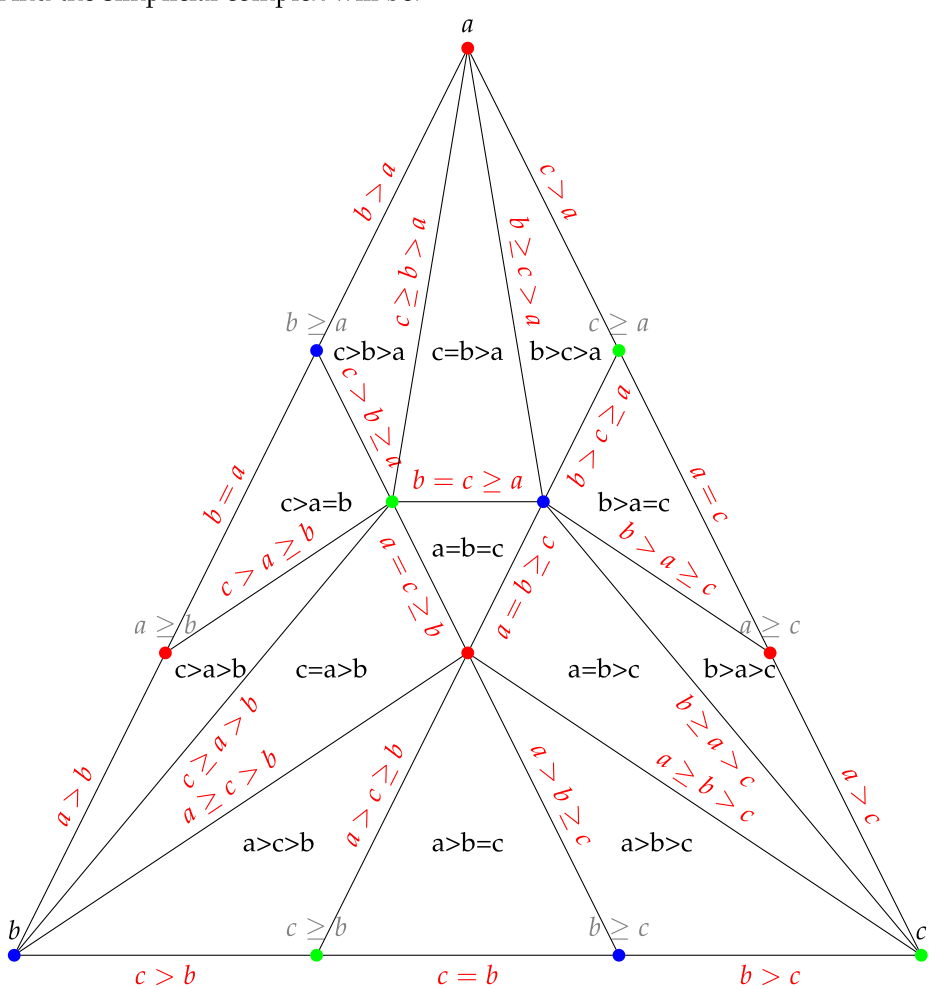
order	$=,=$	$=,<$	$<,=$	$<,<$
abc	$a = b = c$	$a = b < c$	$a < b = c$	$a < b < c$
acb		$a = c < b$		$a < c < b$
bac			$b < a = c$	$b < a < c$
bca		$b = c < a$		$b < c < a$
cab			$c < a = b$	$c < a < b$
cba				$c < b < a$

Those are all (13) possible executions, in this model, they will be faces.

A node, can view all elements that are lesser or equals than itself, and only knows that.

So, all nodes are: $\begin{array}{c} \hline a \\ a \leq b \\ b \leq a \\ b \\ b \leq c \\ c \leq b \\ c \\ c \leq a \\ a \leq c \\ a, b \leq c \\ b, c \leq a \\ c, a \leq b \\ \hline \end{array}$

And the simplicial complex will be:



Remarks

- Every face (triangle) is a possible execution (total order).
- Given the edges of a triangle, the execution is the linear extension of them.
- Two triangles joined by an edge are two executions where 2 processes can't distinguish between (those are the processes in the edge).
- Any edge belonging to two triangles has exactly one $\leq$ sign.
- If two triangles are joined by an edge, their execution is one the case "=" and the other ">" in the $\leq$ sign.
- two triangles are joined by only a vertex, are such that only that process can't distinguish between them.

Kripke structures

Definition

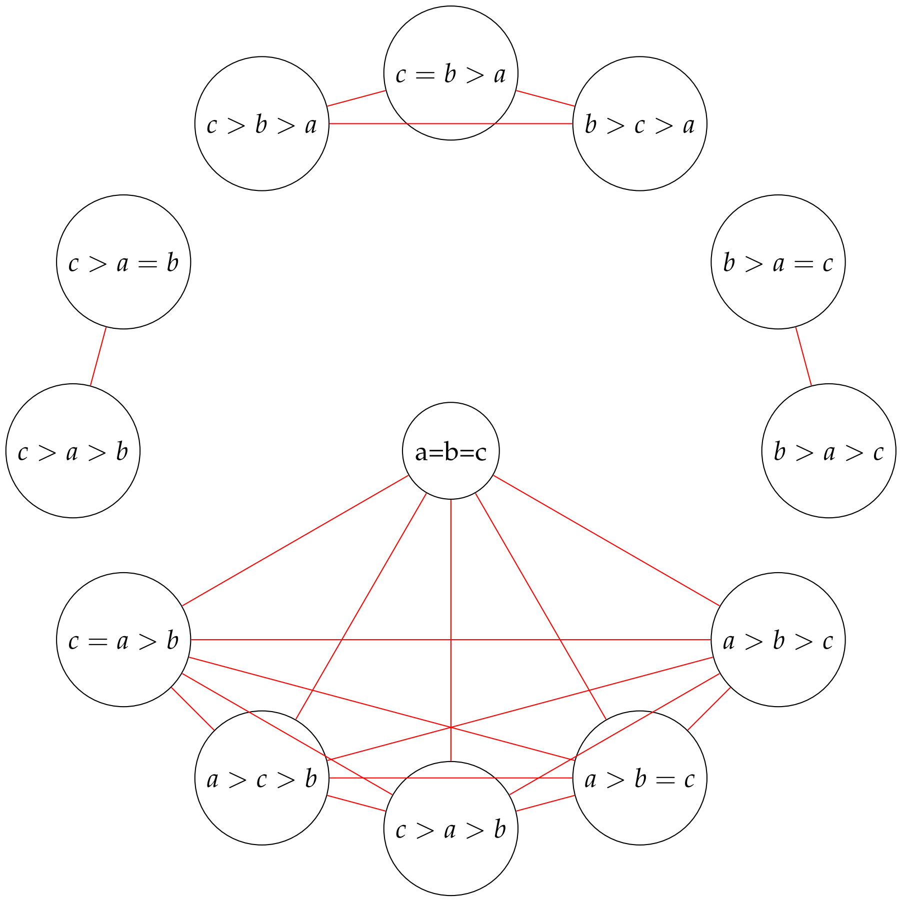
A multi modal Kripke structure is a pair $K = (W, \rightarrow)$ where W is a set of worlds and $\rightarrow$ is a family of relations $\rightarrow_i \subseteq W \times W$ indexed by a fixed set I.

A multi modal Kripke model is a tuple $M = (W, \rightarrow, P, V)$ where $(W, \rightarrow)$ is a Kripke structure, P is the set of propositional constants, and $V : W \rightarrow \mathcal{P}(P)$ is a valuation function

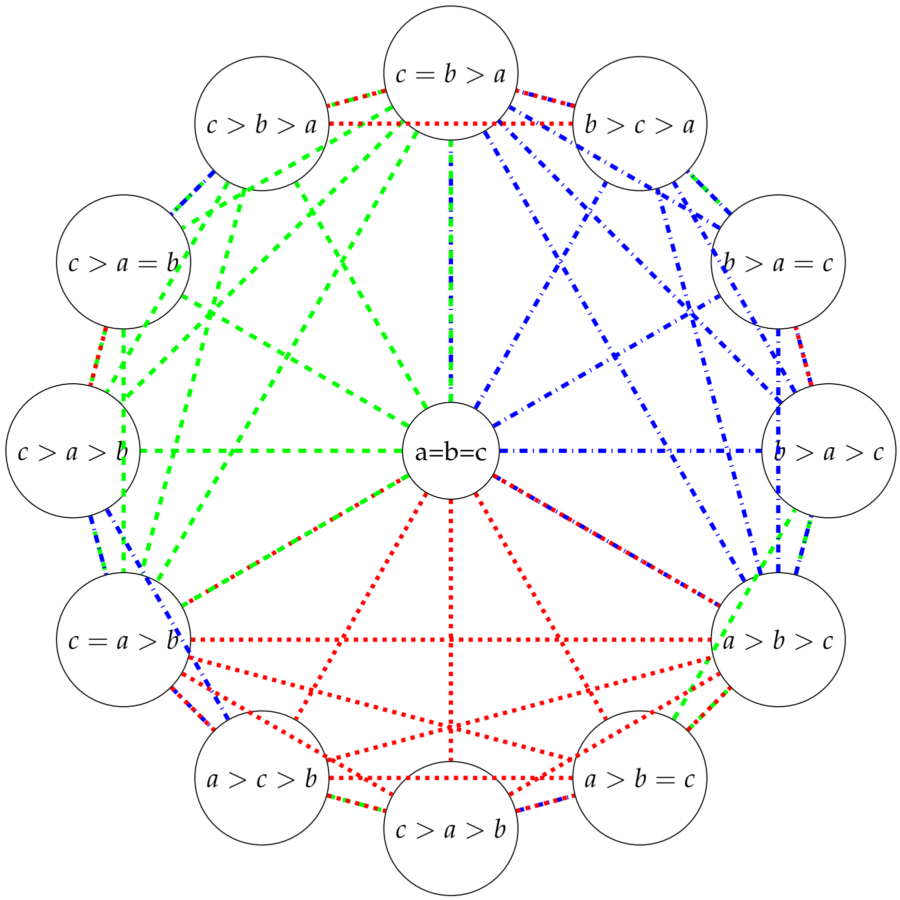
A formula in $\mathcal{L}(P)$ is interpreted in a Kripke model $M = (W, \rightarrow, P, V)$ at some $w \in W$ as follows:

- $(M, w) \models p$ iff $p \in V(w)$
- $(M, w) \models \phi_1 \vee \phi_2$ iff $(M, w) \models \phi_1$ or $(M, w) \models \phi_2$
- $(M, w) \models \neg\phi$ if is not the case that $(M, w) \models \phi$
- $(M, w) \models \langle i \rangle \phi$ iff there exists $w' \in W$ such that: $w \rightarrow_i w'$ and $(M, w') \models \phi$

We will use the total order as propositional constants, for the example we are dealing with, possible executions are here W, let's see accesibility for agent a



We can see all three agents simultaneously:



Remarks

- transitivity.
- reflexivity.
- every world has access to another world for every modality.

Bibliography

Hans van Ditmarsch, Wiebe van der Hoek, and Barteld Kooi. *Dynamic Epistemic Logic*. Springer Publishing Company, Incorporated, 1st edition, 2007. ISBN 1402058381, 9781402058387.

Dov M. Gabbay, C.J. Hogger, and J.A. Robinson, editors. *Handbook of Logic in Artificial Intelligence and Logic Programming*, volume 1 Logical Foundations. Clarendon Press, Oxford, 1993.

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Jaakko Hintikka. *Knowledge and Belief*. Ithaca, N.Y.,Cornell University Press, 1962.

Leslie Lamport. Time, clocks, and the ordering of events in a distributed system. *Commun. ACM*, 21(7):558–565, July 1978. ISSN 0001-0782. DOI: 10.1145/359545.359563. URL http://doi.acm.org/10.1145/359545.359563.

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