Tutorial 1: CS 215, Autumn 2023

2nd September, 2023

Question 1

Suppose I gather some n independent measurements of a quantity. Let us treat the measurements as independent random variables with mean μ and standard deviation σ. If I want to be 99% certain that the average of these measurements is accurate to within $\pm\frac{\sigma}{4}$ units, how many measurements must I take, i.e. what is the value of n?

Question 2

Show that the sum of two independent Poisson random variables with mean λ_1 and λ_2 respectively is another Poisson random variable. What is its mean? Show all steps clearly.

Question 3

Let X and Y be independent random variables on a probability space $(\Omega, \sigma, \texttt{Pr})$. Let g and h be real-valued functions defined on the codomains of X and Y respectively. Show that $g(X)$ and $h(Y)$ are independent random variables.

Question 4

Let X be a continuous random variable with a strictly increasing distribution function $F_X(x)$. Consider random variables $Y_1 = F_X(x)$ and $Y_2 = F_X^{-1}(U)$ where U is a random variable from a $(0,1)$ uniform distribution and F_X^{-1} is the inverse of function F_X. Derive the CDF of Y_1 and the CDF of Y_2. Hence write down a simple procedure to draw a sample from a Gaussian distribution with mean 0 and standard deviation 1 assuming you have access to ready-made algorithm to draw a sample from Uniform(0, 1).

Question 5

Consider that $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}$ is a set of n independent random variables from an underlying distribution $F_X(x)$. Define $X_{(k)}$ to be the k^{th} smallest element in $\mathbf{X}$. Derive CDF of $X_{(k)}$ in terms of $F_X(x)$. Define $N_x = \sum_{i=1} \mathbf{1}(X_i \leq x)$ where $\mathbf{1}(y) = 1$ for $y = true$ and 0 otherwise. Derive the CDF of N_x. How does it relate to the CDF of $X_{(k)}$?