NFA with ϵ $\equiv$ NFA w/o ϵ $\equiv$ DFA

Regular Expressions

$\Sigma = \{a, b\}$

Syntax:

$e ::= a \mid b \mid \varepsilon \mid e_1 + e_2 \mid e_1 \cdot e_2 \mid e_1 e_2 \mid e_1^* \mid (e_1) \mid x$

$+ a * b$

$(a+b)^* b$

Semantics:

$[\![a]\!] = \{a\}$ $\quad [\![b]\!] = \{b\}$

$[\![\varepsilon]\!] = \{\varepsilon\}$

$[\![e_1 + e_2]\!] = [\![e_1]\!] \cup [\![e_2]\!]$

precedence: $*$ $\cdot$ $+$

$[\![e_1 \cdot e_2]\!] = [\![e_1]\!] \cdot [\![e_2]\!]$

$= \{u.v \mid u \in [\![e_1]\!], v \in [\![e_2]\!]\}$

$u \in \Sigma^*, v \in \Sigma^*$

$a.b + a$ $\quad [\![(a.b)+a]\!] = \{ab, a\}$

$$[\![e_1^*]\!] = \bigcup_{n \geq 0} [\![e_1^n]\!]$$

$$e_1^n = \underbrace{e_1 \cdot e_1 \cdot \cdots \cdot e_1}_{n \text{ times}} \qquad e_1^0 = \varepsilon$$

$e_1 = a+b$ $\quad [\![e_1]\!] = \{a, b\}$

$[\![e_1^3]\!] \Rightarrow a.a.a$, $a.b.a$, $b.b.a$

$$[\![(\underbrace{a+b}_{e_1})^*]\!] = \{\underbrace{\varepsilon}_{e_1^0}, \underbrace{a, b}_{e_1^1}, \underbrace{aa, ab, ba, bb}_{e_1^2}, \ldots\}$$

$[\![e_1]\!] = [\![a+b]\!] = \{a, b\}$

$[\![a^* + b^*]\!] = \{u \in \Sigma^* \mid u = a^n \text{ or } b^n, n \geq 0\}$

$[\![a^* \cdot b^*]\!]$

$L(a^* \cdot b^*)$

$[\![(a^* \cdot b^*)^*]\!]$

$L(\ \downarrow\ )$

$ba \in [\![(a^* \cdot b^*) \cdot (a^* \cdot b^*)]\!]$

$\varepsilon\ b \quad a\ \varepsilon$

$aba \in a\ b \quad a\ \varepsilon$

ε	✓	✓
a	✓	✓
b	✓	✓
ab	✓	✓
ba	x	✓
abb	✓	✓
aba	x	✓
aaa	✓	✓

a^* $\quad a^* b^*$ $\quad$ Is $L((a^* b^*)^*) = L((a+b)^*)$

$\subseteq$ $\quad$ easy

$\supseteq$

$(a^* b^*)^6$ $\quad$ $ababba$

$(a^* b^*)(a^* b^*)(a^* b^*)(a^* b^*)(a^* b^*)(a^* b^*)$

$a\ \varepsilon \quad \varepsilon\ b \quad a\ \varepsilon \quad \varepsilon\ b \quad \varepsilon\ b \quad a\ \varepsilon$

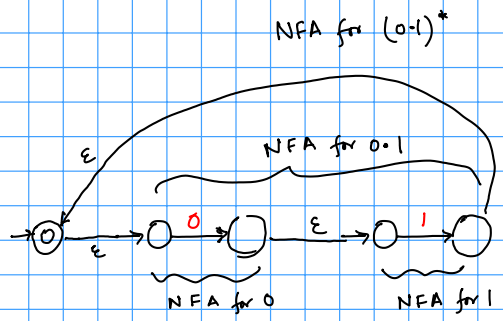
$*$: Kleene closure

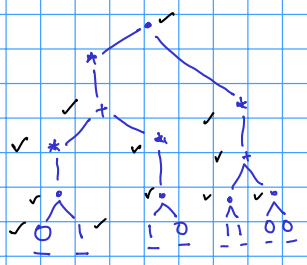
Kleene star

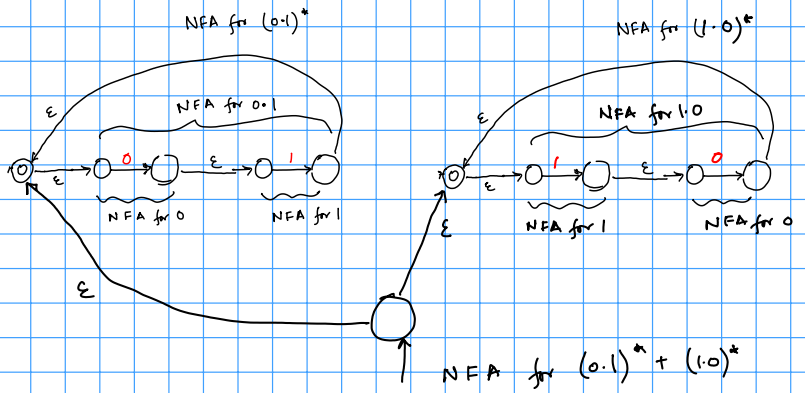
$L(\text{reg. expn}) \subseteq L(\text{NFA with } \epsilon) = L(\text{NFA w/o } \epsilon)$
$= L(\text{DFA})$

$\Sigma = \{0, 1\}$

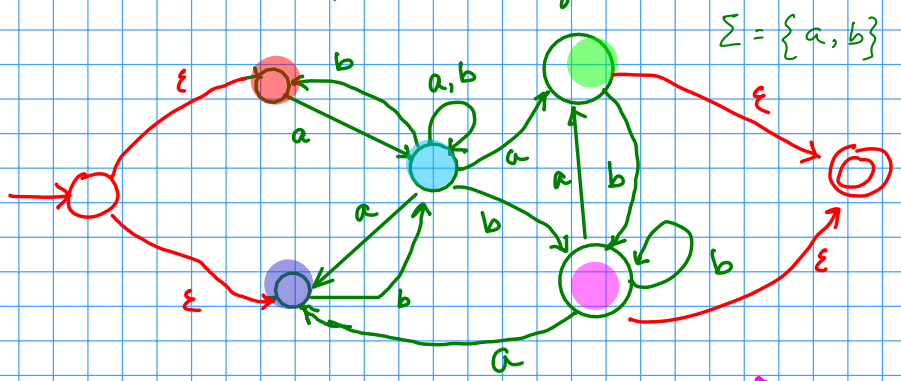
$\left((01)^* + (10)^* \right)^* \cdot (11+00)^*$

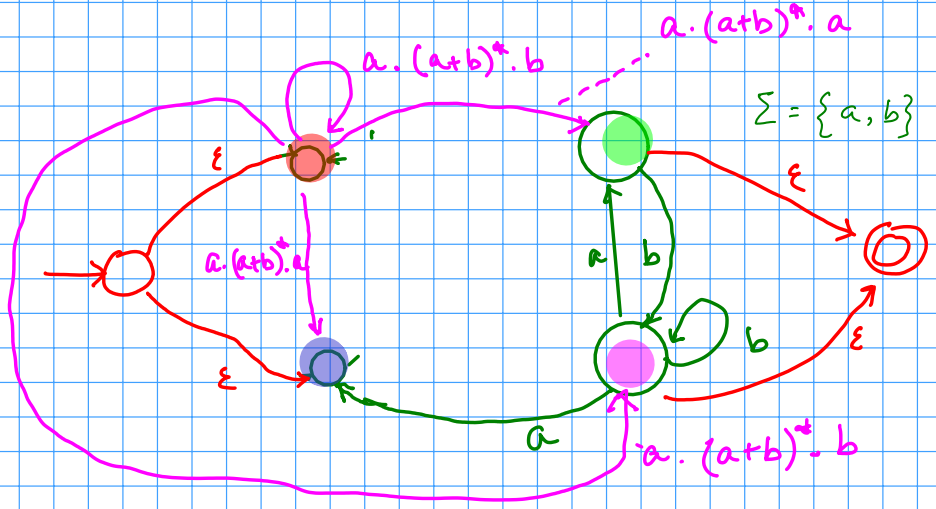


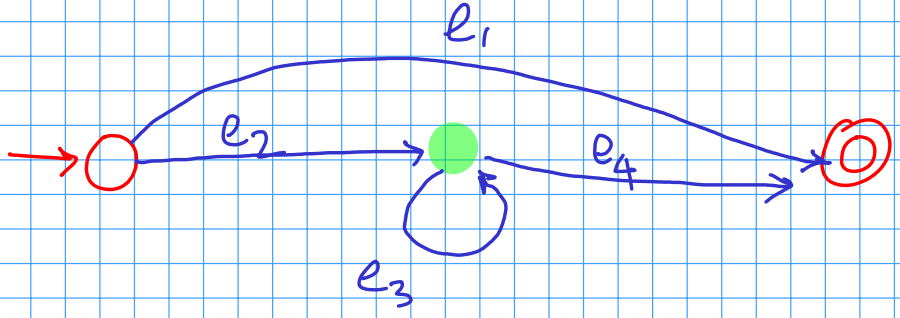


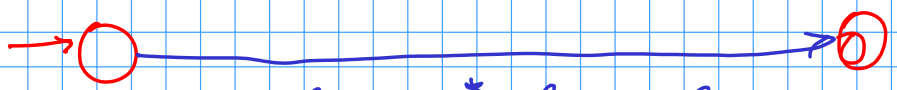


L(NFA without ε) $\subseteq$ L(reg. expn)









$e_2 . e_3^* . e_4 + e_1$

Reg. Expn. ≡ NFA with ε ≡ NFA w/o ε ≡ DFA : Kleene's Thm.

Regular Languages

$(a^*b^*)^*$ $\qquad$ $(a+b)^*$

$L(DFA_1) = L_1 \subseteq L_2 = L(DFA_2)$ ✓

$L_1 \cap (\Sigma^* \setminus L_2) = \phi$

L_2 is reg. $\Leftrightarrow$ L_2^c or $\overline{L_2}$ is reg.