$(a^* b^*)^*$ $\quad\quad$ $(a+b)^*$

$\downarrow$ $\quad\quad\quad\quad$ $\downarrow$

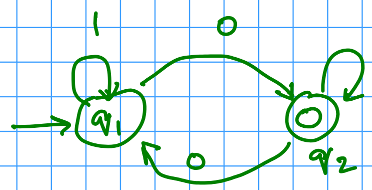
$L(DFA_1) \overset{\checkmark}{=} L(DFA_2)$

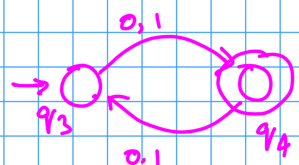
$L_1 \subseteq L_2$

$\supseteq$

$\overbrace{\widetilde{L_1} \cap \overbrace{(\Sigma^* \setminus L_2)}} = \phi$

L_2 is reg. $\iff$ L_2^c or $\overline{L_2}$ is reg.

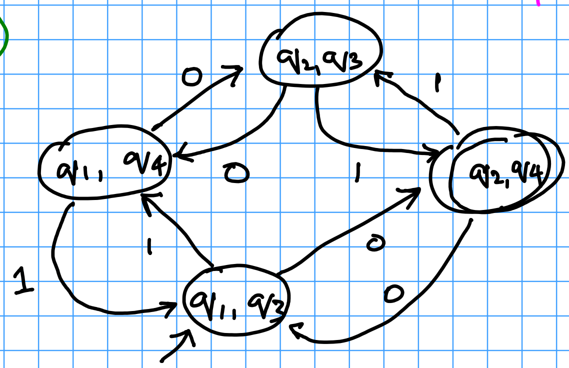




$(Q, \Sigma, q_0, \delta, F)$

$Q \times \Sigma \to Q$

$(Q', \Sigma, q_0', \delta', F')$



$(Q \times Q', \Sigma, (q_0, q_0), \hat{\delta}, \hat{F})$

Product Construction for Automata

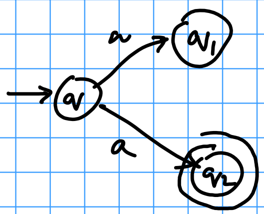
$\hat{\delta} : (Q \times Q') \times \Sigma \to (Q \times Q')$

$\hat{\delta}((q_i, q_j'), a) = (\delta(q_i, a), \delta'(q_j', a))$

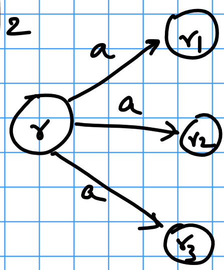
$\in \Sigma$

$L(A_1) \quad \cap \quad \overline{L(A_2)}$

N_1



N_2



$\{a\}$

$\{\epsilon, a\}$

$$\hat{\delta}\left((q, r), a\right) = \{q_1, q_2\} \times \{r_1, r_2, r_3\}$$

$$\delta(q, a) \times \delta'(r, a)$$

Closure Properties of RL :

L_1, L_2 : reg. lang. over $\Sigma_1 = \{a, b\}$

$\overline{L_1}$

$L_1 \cup L_2$, $L_1 \cap L_2$

$\Sigma_2 = \{0, 1, 2\}$

Substitution:

Reg. lang. $L_a \subseteq \Sigma_2^*$ $\quad L_b \subseteq \Sigma_2^*$

$= 0^*(1+2)^*1^*$ $\quad 1^*.(0+2)^*$

Reg. lang. $L_1 \subseteq \Sigma_1^*$. $\quad \underline{\underline{a^* b^*}}$. $\quad aabbb$

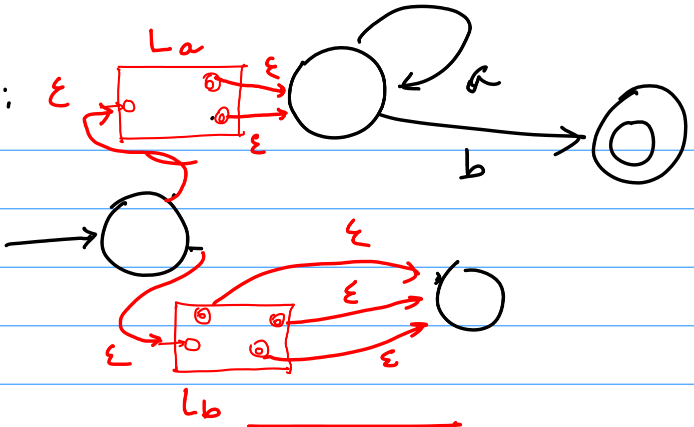
$$\boxed{\begin{aligned} subst(L_1, L_a, L_b) &= \{\omega \in \Sigma_2^* \mid \text{exists } u \in L_1 \text{ s.t. } \omega \in L_{\alpha_1}.L_{\alpha_2}.\cdots L_{\alpha_k}\} \\ &= \bigcup_{u=\alpha_1\ldots\alpha_k \in L_1} L_{\alpha_1}.L_{\alpha_2}.\ldots L_{\alpha_k} \end{aligned}}$$

$u = \alpha_1.\alpha_2.\cdots\alpha_k.$

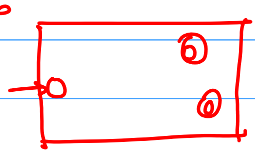
$L_1 = a^* b^*$ $\quad L_a = 0^*(1+2)^*1^*$ $\quad L_b = 1^*.(0+2)^*$

$u = \underset{\alpha_1}{a}\overset{\alpha_2}{a}bb\underset{\alpha_5}{b} \in L_1$ $\quad \omega \overset{?}{\in} L_a.L_a.L_b.L_b.L_b$

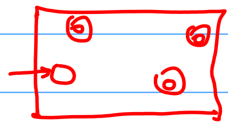
L_1:



L_a



L_b



$L_a \cdot L_b$

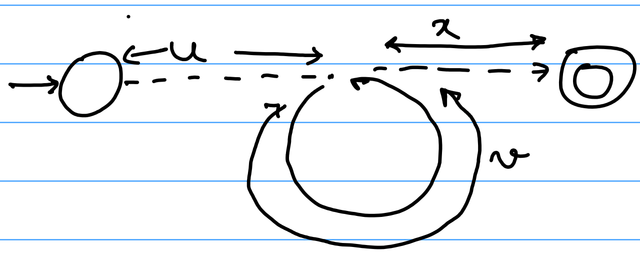
$\Sigma_1 = \{a, b\}$ $\qquad$ $L_1 = \{a^n b^m \mid n = 0 \bmod 2,\ m = 1 \bmod 2\}$

$$L_1 = (a^*(a+b)^*)^* + b^*$$

Pumping Lemma:

$|A| = 20$ states

$w \in L(A)$, $|w| = 100$ $\qquad$ $w = u.v.x$



$u.x \in L$

$u.v.v.x$

$u.v^i.x \in L$

for all $i \geq 0$

Nothing special about |A| = 20 and |w| = 100.

For every reg. lang L, there is a DFA A of p states. Take any w in L, where |w| >= p and the above argument applies.

As we run w over A, we require to visit |w|+1 states, but |w|+1 > p, so we must encounter the same state at least twice along the path traced out in the DFA. This identifies a loop in the path, and allows us to break w as u. v. x, s.t. |uv| <= p and u.v^i.x in L for all i >= 0

What is the length of the shortest string |w| that will encounter such a loop. Basically, prune out all but one loop in the path traced out in the DFA by w, and we will get a string w' of length at most 2p that has a loop in the path traced out by it in the DFA. Therefore, we don't need to look for strings beyond length 2p in L, if we are to find one which can be pumped