To show: $L(A) = L(A')$: $L(A) \subseteq L(A')$ and $L(A') \subseteq L(A)$

A: NFA

A': DFA obtd by subset construction

We will show:

$$\boxed{\begin{array}{l} \text{for every } n \geq 0, \text{ for every } w \in \Sigma^* \text{ s.t. } |w| = n \\ \quad \text{NFA } A \text{ can reach state } q \in Q \text{ on reading } w \\ \qquad\qquad \text{iff} \\ \quad \text{DFA } A' \text{ reaches state } S \subseteq Q \text{ on reading } w \text{ s.t. } q \in S \end{array}}$$

Induction on n

Base: $n = 0$ from defn. of initial state of A'

Hyp: Claim holds for $0 \leq n < k$

Ind.: Show that claim holds for $n = k$

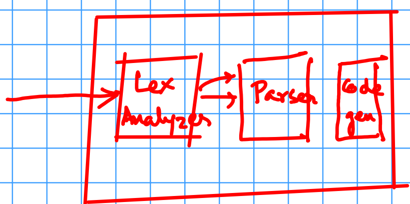
$|w| = k$ Write $w = w' \cdot a$, $a \in \Sigma$, $w' \in \Sigma^*$, $|w'| = k-1$

By hyp

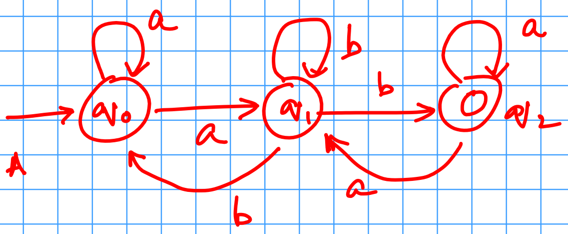
NFA $q_0 \xrightarrow{w'} q \xrightarrow{a} \hat{q}$

iff

DFA $q_0 \xrightarrow{w'} q \xrightarrow{a} \hat{q}$



n: Size of NFA

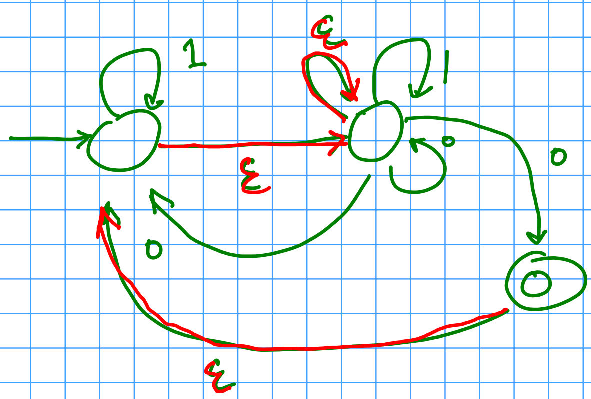


: NFA

ababab $\quad$ k = length of string.

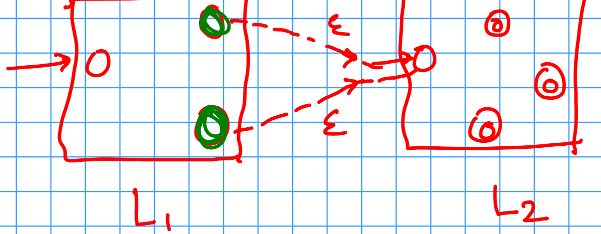
$\{q_0\} \xrightarrow{a} \{q_0, q_1\} \xrightarrow{b} \{q_0, q_1, q_2\} \xrightarrow{a} \{q_0, q_1, q_2\} \xrightarrow{b} \{q_0, q_1, q_2\} \xrightarrow{a} \{q_0, q_1, q_2\} \xrightarrow{b} \{q_0, q_1, q_2\}$

$n \cdot k$.



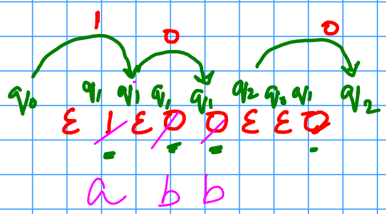
$10 \notin L$ without ε-edges

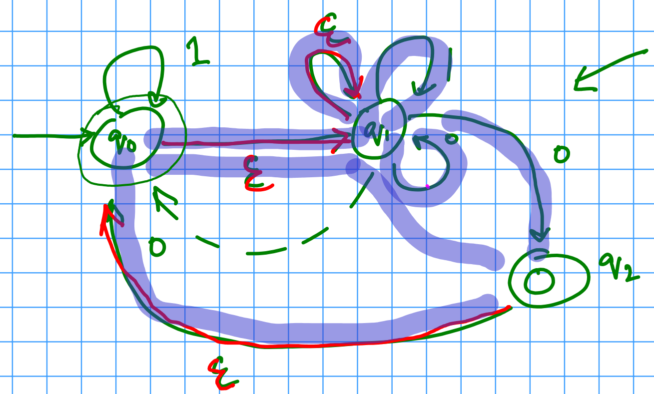
$10 \in L$ with "



$L_1 . L_2$

$w_1 . w_2$



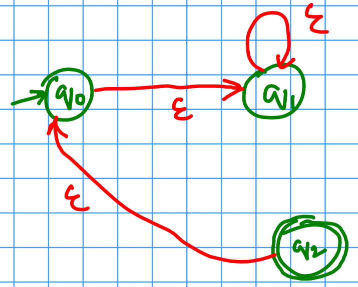


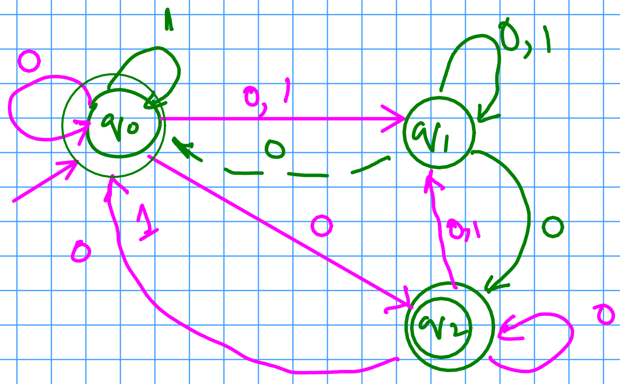
ε- closure.

$q_0 \rightsquigarrow \{q_0, q_1\} = \epsilon\text{-closure}(q_0)$

$q_1 \rightsquigarrow \{q_1\} = \epsilon\text{-cl}(q_1)$

$q_2 \rightsquigarrow \{q_0, q_1, q_2\} = \epsilon\text{-cl}(q_2)$





$q_1 \quad q_2 \quad q_0$

$0 \quad \varepsilon$

$$\boxed{\text{NFA with } \varepsilon \equiv \text{NFA without } \varepsilon \equiv \text{DFA}}$$

$\Sigma = \{0, 1, 2, \ldots 9\}$

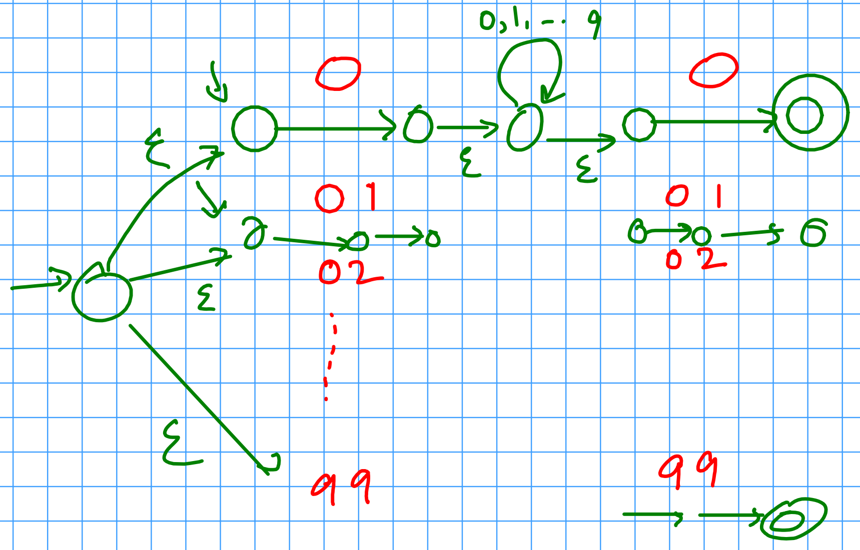
$L = \{ w \in \Sigma^* \mid w = u.v.w,\ u = w \in \Sigma^+,\ v \in \Sigma^*,\ |u| \le 2 \}$

0000

1234

$u = 0,\ v = 00,\ w = 0$

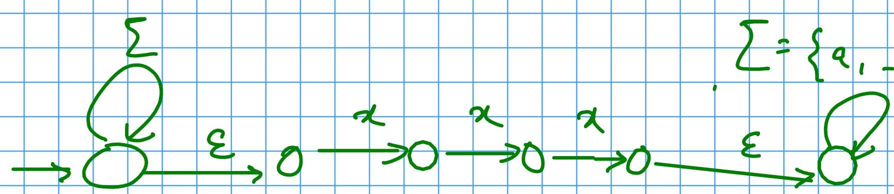
$u = 00,\ v = \epsilon,\ w = 00$



xxx

$xx\underline{x}\ \underline{x}$

$\Sigma = \{a, \ldots z\}$



Regular Expressions