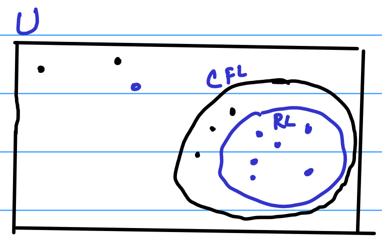
## Post's Correspondence Problem (PCP)

not rec. [ not r.e. : $L_d = \{ i \mid M_i \text{ doesn't halt on } w_i \}$
r.e. $\overline{L_d} = \{ \ldots M_i \text{ halts on } w_i \}$ ]

not rec. [ r.e. $L_u = \{ (i,j) \mid M_i \text{ halts on } w_j \}$
not r.e. $\overline{L_u}$ ]

$L = H(M)$




set of all possible r.e. languages on $\Sigma$.

Property of languages defines a subset of U

Property $P$ of languages is non-trivial
iff $\exists L_1$ s.t. $L_1 \notin P$ — $\exists M_1$ s.t. $H(M_1) \in P$
and $\exists L_2$ s.t. $L_2 \in P$ — $\exists M_2$ s.t. $H(M_2) \notin P$

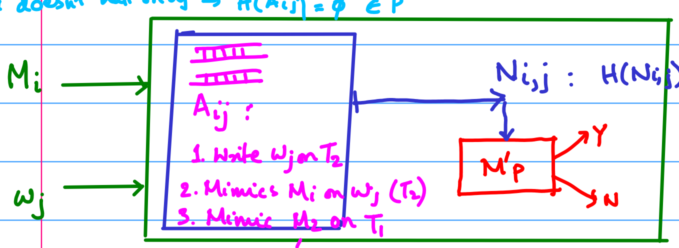
$\phi$ < $\in P$ ✓ / $\notin P$

Rice's Theorem [ Given a non-trivial prop $P$ of U $(P \subset U)$ $(P \neq \phi)$
is $P$ decidable?
Given a TM $M$, does $H(M) \in P$? ]

$M \rightarrow \boxed{M'_P}$ → Y iff $H(M) \in P$; → N iff $H(M) \notin P$

$\exists M'_P$?

$M_i$ halts on $w_j \Rightarrow H(A_{ij}) = H(M_2) \notin P$
$M_i$ doesn't halt on $w_j \Rightarrow H(A_{ij}) = \phi \in P$




$H(N_{ij}) \in P$ iff $M_i$ doesn't halt on $w_j$

$H(M_2) \notin P$

$T_1$ is tape on which $A_{ij}$ processes its input