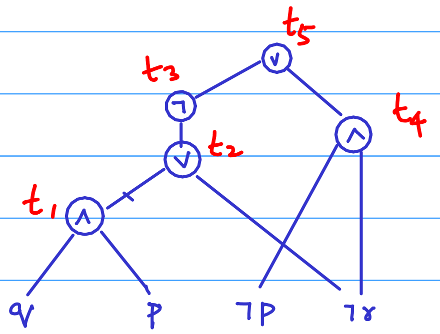
937, 938, 939, 940, 941, 936

$\varphi(p, q, r)$

$\varphi'(p, q, r, t_1, t_2, t_3, t_4, t_5)$: Equisatisfiable.

CNF



$$(t_1 \leftrightarrow p \wedge q) \wedge$$
$$(t_2 \leftrightarrow t_1 \vee \neg r) \wedge$$
$$(t_3 \leftrightarrow \neg t_2) \wedge$$
$$(t_4 \leftrightarrow \neg p \wedge \neg r) \wedge$$
$$(t_5 \leftrightarrow t_3 \vee t_4) \wedge t_5$$

$$\Leftrightarrow (t_1 \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow t_1)$$
$$(\neg t_1 \vee (p \wedge q)) \wedge (\neg(p \wedge q) \vee t_1) \Leftrightarrow \underbrace{(\neg t_1 \vee p) \wedge (\neg t_1 \vee q) \wedge (\neg p \vee \neg q \vee t_1)}_{\text{CNF}}$$

$$(t_2 \rightarrow (t_1 \vee \neg r)) \wedge ((t_1 \vee \neg r) \rightarrow t_2)$$
$$\Leftrightarrow (\neg t_2 \vee t_1 \vee \neg r) \wedge (\neg(t_1 \vee \neg r) \vee t_2) \Leftrightarrow \overbrace{(\neg t_2 \vee t_1 \vee \neg r) \wedge (\neg t_1 \vee t_2) \wedge (r \vee t_2)}$$

$$(t_3 \rightarrow \neg t_2) \wedge (\neg t_2 \rightarrow t_3) \Leftrightarrow \underbrace{(\neg t_3 \vee \neg t_2) \wedge (t_2 \vee t_3)}_{\text{CNF}}$$

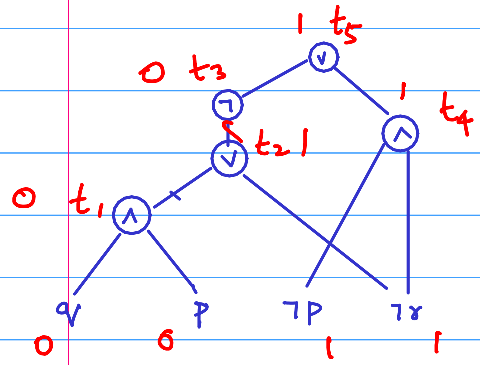
$\varphi(p, q, r) \rightarrow \boxed{\text{Tseitin Encoding}} \rightarrow \varphi'(p, q, r, t_1, \ldots, t_n)$

Equisatisfiable

not necessarily semantically equivalent

Size of φ' linear in size of φ

Tseitin Encoding



$\checkmark\ (t_1 \leftrightarrow p \wedge q) \wedge$

$\checkmark\ (t_2 \leftrightarrow t_1 \vee \neg r) \wedge$

$\checkmark\ (t_3 \leftrightarrow \neg t_2) \wedge$

$\checkmark\ (t_4 \leftrightarrow \neg p \wedge \neg r) \wedge$

$\checkmark\ (t_5 \leftrightarrow t_3 \vee t_4) \wedge t_5$

$p = q = r = 0$

Unique mapping

$p = q = r = 0$

$t_1 = 0,\ t_2 = 1,\ t_3 = 0,\ t_4 = 1$

$t_5 = 1$

Bijection between sat. assignments of φ and φ'

(Stronger than equisatisfiability)

Towards checking satisfiability of CNF

$(\neg \cancel{x_5} \vee x_3) \wedge (x_5) \wedge (\cancel{\neg x_5} \vee x_4)$

$(\neg x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_4 \vee x_5 \vee \neg x_2) \wedge (\neg x_1 \vee \neg \cancel{x_5})$ ✓

=

Horn clause ¦ At most one unnegated var. per clause

$$
\left.\begin{array}{l}
x_1 \wedge x_2 \to x_3 \\
x_4 \wedge x_3 \to x_5 \\
x_1 \wedge x_5 \to \perp \\
x_5 \to x_3 \\
\top \to x_5 \\
x_5 \to x_1
\end{array}\right\}
$$

Horn formula.

$x_1 = x_5 = x_3 = 1$ ✓

$\to \quad x_2 = x_4 = 0$

n vars, k clauses

Logic Programming $\boxed{\text{Prolog}}$

No Alarm
No B
No F
No E
No P

Alarm :

Burglary	Night
Fire	Day.
Earthquake	N/D.
Prank	N

$Nb \wedge Nf \wedge Ne \wedge NP \rightarrow Na$

$b=0$
$n=0$
$e=0$
$p=0$
$f=0$

$b \wedge n \rightarrow a$
$f \wedge d \rightarrow a$
$e \wedge d \rightarrow a$
$e \wedge n \rightarrow a$
$p \wedge n \rightarrow a.$
$d \wedge n \rightarrow \bot$
$p \wedge d \rightarrow \bot$
$b \wedge f \rightarrow \bot$
$T \rightarrow a$
$T \rightarrow d$

$b \leftrightarrow \neg Nb$

$b \rightarrow \neg Nb$
$\wedge$
$\neg Nb \rightarrow b$

$b=n=e=$
$p=f=0$

$Na=Nb=Ne$
$=Nf=0$

$a \wedge Na \rightarrow \bot$
$b \wedge Nb \rightarrow \bot$
$e \wedge Ne \rightarrow \bot$
$f \wedge Nf \rightarrow \bot$
$p \wedge Np \rightarrow \bot$

$(\neg b \vee \neg Nb)$
$\wedge$
$(Nb \vee b)$

947, 948, 949, 951

Davis Putnam Logemann Loveland (DPLL)

$C_1 \wedge C_2 \wedge - - - \wedge C_K$

$SAT(\varphi, PA)$ returns (status, assignment)

status: sat / unsat

assignment: $x_1=0, x_2=1, x_7=0, \ldots$

0. If $\varphi = \top$ return (sat, PA)
   If $\varphi = \bot$ return (unsat, PA)

1. If C_i is a unit clause (clause with single literal ℓ)

   return $SAT(\varphi[\ell=1], PA \cup \{\ell=1\})$

   Unit-propagation

   $\varphi[\ell=1]$: simplify φ after setting $\ell=1$

2. If a literal ℓ doesn't appear negated in any clause

   pure literal elimination

   return $SAT(\varphi[\ell=1], PA \cup \{\ell=1\})$

3. $x :=$ choose_variable(φ) — decision
   $v :=$ choose_value( ) .... $\in \{0,1\}$

   if $SAT(\varphi[x=v], PA \cup \{x=v\})$.status = sat
       return (sat, $PA \cup \{x=v\}$)

   else if $SAT(\varphi[x=1-v], PA \cup \{x=1-v\})$.status = sat
       return (sat, $PA \cup \{x=1-v\}$)

   else return (unsat, PA) — backtrack

Is there a Horn formula in which

(a) every clause has ≥ 2 literals

(b) every lit appear negated & un-negated?

$(a \vee \neg b)(\neg a \vee b)$

$$(\neg \cancel{x_1} \vee \neg \cancel{x_2} \vee \overset{T}{x_3})$$

$x_i \wedge x_j \wedge x_k \to x_l$

$$\underset{T}{\underline{x_1}} \wedge \underset{T}{\underline{x_2}} \to x_3$$

$x_i \wedge x_j \to T$

$\to (x_i \wedge x_j \wedge x_k \to \perp)$

$(\neg x_i \vee \neg x_j \vee \neg x_k)$

$x_i \to x_j$

$x_i \to \perp \quad (\neg x_i)$

DPLL in action

~~$C_1: (\neg P_1 \vee P_2)$~~

~~$C_2: (\neg P_1 \vee P_3 \vee \cancel{P_5})$~~

~~$C_3: (\neg P_2 \vee P_4)$~~

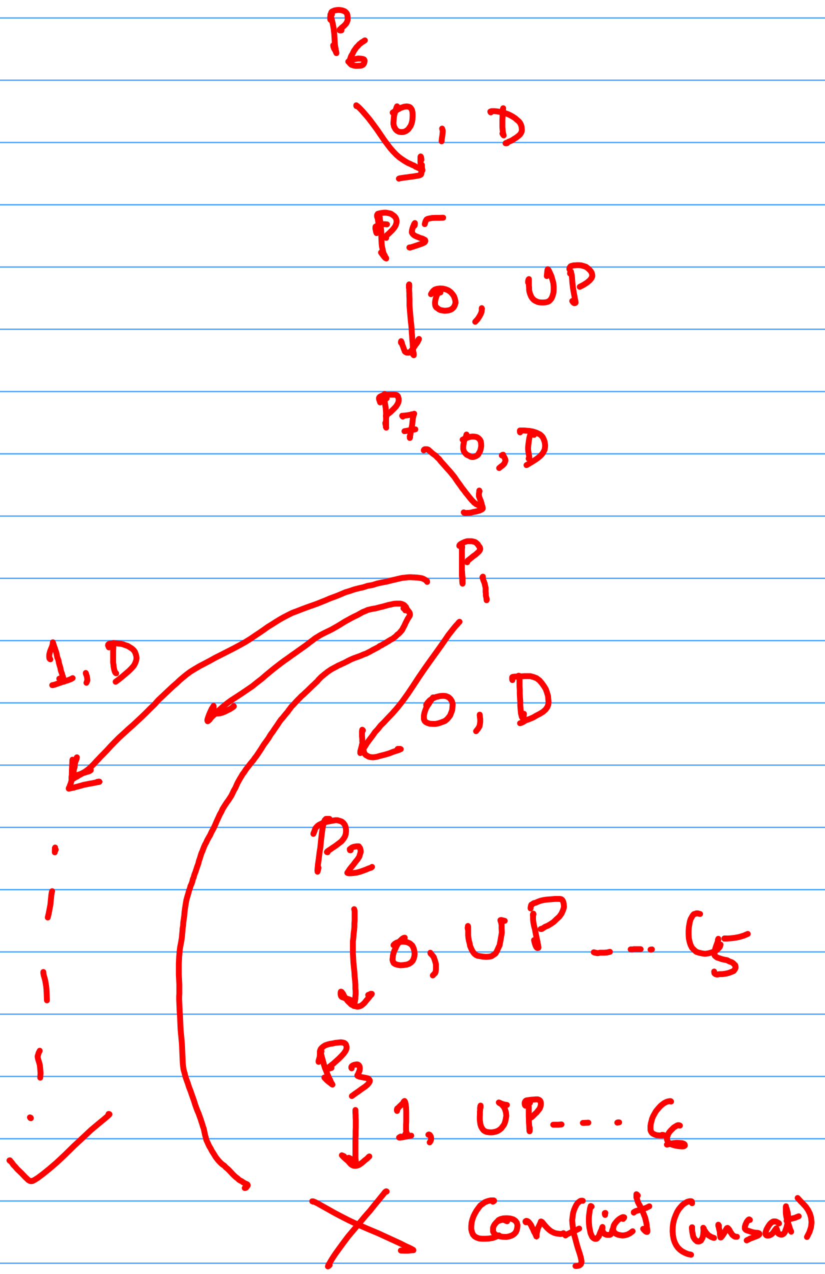
$C_4: (\neg \cancel{P_3} \vee P_4)$

~~$C_5: (\cancel{P_1} \vee \cancel{P_5} \vee \neg P_2)$~~

~~$C_6: (\cancel{P_2} \vee P_3)$~~

$C_7: (\cancel{P_2} \vee \cancel{\neg P_3} \vee \cancel{P_7})$

~~$C_8: (\cancel{P_6} \vee \neg P_5)$~~



$$\overset{\checkmark}{a_1} \wedge \overset{\checkmark}{a_2} \wedge \cdots \wedge \overset{\checkmark}{a_n}$$

$$\wedge (\neg a_1 \vee \neg a_2 \vee \cdots \vee \neg a_n)$$