Jan 18, 2024

Scribes: 926, 923, 922, 925

$\vdash \varphi \vee \neg\varphi$

$\{\varphi_0, \varphi_1\} \vdash \varphi_2$

φ_0 --- premise
φ_1 --- premise
$\vdots$
φ_2 --- conclusion

set of formulas $\leftarrow \Sigma \vdash \varphi$
syntactic entailment

$p, q \vdash p \wedge q$.

$$\frac{\varphi_1 \quad \varphi_2}{\varphi_1 \wedge \varphi_2}$$

$\Sigma \models \varphi$
semantic entailment

$\Sigma \vdash \varphi$ implies $\checkmark$ $\Sigma \models \varphi$: Soundness of proof rules

$\Sigma \models \varphi$ implies ? $\Sigma \vdash \varphi$: Completeness of proof rules

p
q

$\Sigma = \{p \to q,\ \neg q\} \models \neg p$

p	q	$p \to q$	$\neg q$	$\neg p$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	0

$(p \to q) \wedge \neg q$
$\vdots$
$\neg p$

$(p \to q) \wedge \neg q \to \neg p$

$\neg p,\ \neg q \overset{?}{\vdash} p \to q$

$\neg p, \neg q \vdash \neg q$

$\neg p$
$\neg q$

p
$\perp$
q

$p \to q$

$\neg p$
$\neg q.$
$\vdots$
$p \to q$

$(p \to q) \wedge \neg q$
$\vdots$
$\neg p$

$(p \to q) \to \neg p$

$\neg p, \neg q \vdash (p \to q) \to \neg p$

$\neg p$
q

$p \to q$

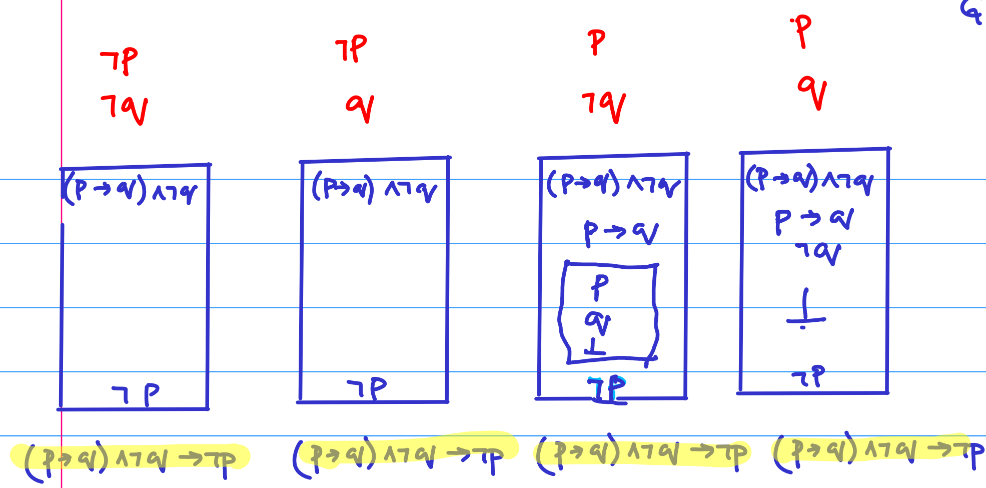
Given:

$\Sigma = \{p \to q, \neg q\} \models \neg p$

$\models (p \to q) \land \neg q \to \neg p$

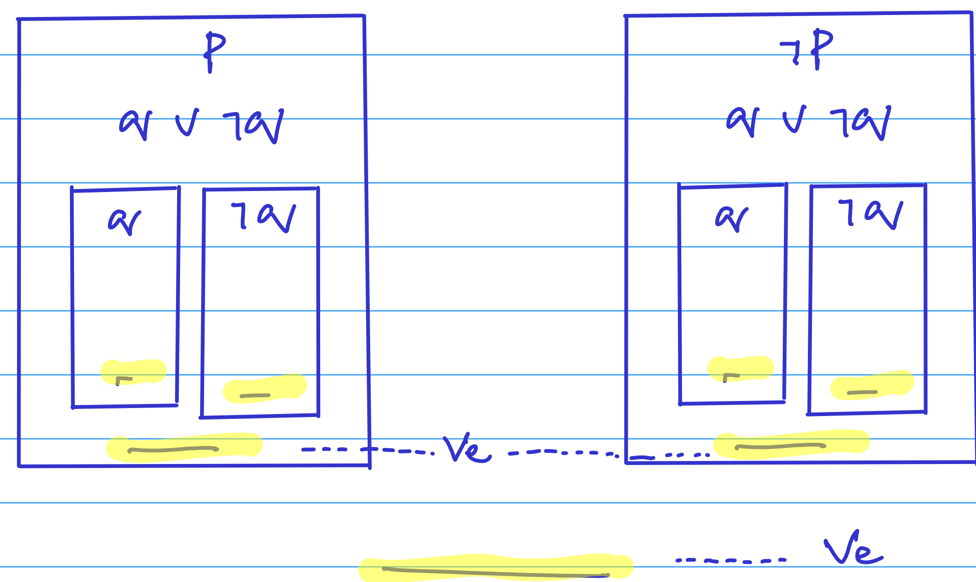
Show:

$p \to q,\ \neg q \vdash \neg p$



$\vdash \varphi \lor \neg \varphi$ ..... Recall proof of this.

$p \quad \lor \quad \neg p$



To show

$\varphi_1, \varphi_2, \varphi_3 \vdash \varphi_4$

it suffices to show

$\vdash (\varphi_1 \land \varphi_2 \land \varphi_3) \to \varphi_4$

and vice-versa

To show

$\varphi_1, \varphi_2, \varphi_3 \models \varphi_4$

it suffices to show

$\models (\varphi_1 \land \varphi_2 \land \varphi_3) \to \varphi_4$

and vice-versa

WHY?