952, 953, 954, 955, 956,

Davis Putnam Logemann Loveland (DPLL)

$C_1 \wedge C_2 \wedge - - - \wedge C_k$

$\text{SAT}(\varphi, PA)$ returns (status, assignment)

status: sat / unsat; assignment: $x_1 = 0,\ x_2 = 1,\ x_7 = 0, \ldots$

0. If $\varphi = \top$ return (sat, PA)
   If $\varphi = \bot$ return (unsat, PA)

1. If C_i is a unit clause (clause with single literal l)

   return $\text{SAT}(\underbrace{\varphi[l=1]}_{\text{simplify } \varphi \text{ after setting } l=1}, PA \cup \{l=1\})$

   $\boxed{\text{Unit-propagation}}$

2. If a literal l doesn't appear negated in any clause

   $\boxed{\text{pure literal elimination}}$

   return $\text{SAT}(\varphi[l=1], PA \cup \{l=1\})$

3. $x := \text{choose\_variable}(\varphi)$ $\boxed{\text{decision}}$

   $v := \text{choose\_value}()$ .... $\in \{0,1\}$

   if $\text{SAT}(\varphi[x=v], PA \cup \{x=v\}).\text{status} = \text{sat}$

   return $(\text{sat}, PA \cup \{x=v\})$

   else if $\text{SAT}(\varphi[x=1-v], PA \cup \{x=1-v\}).\text{status} = \text{sat}$

   return $(\text{sat}, PA \cup \{x=1-v\})$

   else return (unsat, PA) $\boxed{\text{backtrack}}$

Illustrating how a Horn formula may force DPLL to backtrack
if we start with 1 as the first choice of value for a decision variable

		$p_1 = 1$	$p_1 = 0$
$p_1 \to p_2$	$(\neg p_1 \vee p_2)$	p_2	1
$p_2 \to p_1$	$(\neg p_2 \vee p_1)$	1	$\neg p_2$
$p_1 \wedge p_2 \to \bot$	$(\neg p_1 \vee \neg p_2)$	$\neg p_2$	1

Resolution.

$$\frac{(l_1 \vee l_2 \vee l_3 \vee x) \quad (l_4 \vee l_5 \vee l_6 \vee l_7 \vee \neg x)}{}$$

Resolvent $\rightarrow$ $(l_1 \vee l_2 \vee l_3 \vee l_4 \vee l_5 \vee l_6 \vee l_7)$

$(x_1 \vee x_2)$ $(\neg x_1 \vee x_2)$ $(x_1 \vee \neg x_2)$ $(\neg x_1 \vee \neg x_2)$

(x_2) $(x_2 \vee \neg x_2)$ $(\neg x_2)$

$(\ )$ --- empty clause

$\Rightarrow$ orignl. formula unsat.

n prop vars. 3^n clauses

$(x_1 \vee \bar{x_2})$ $(\neg x_1 \vee \neg \bar{x_2})$

(T)

Completeness of res. for unsatisfiability.

$(x_1) \wedge (\neg x_1)$ $(\ )$

UNSAT formula. — — — — — — — — — — . CNF on n vars

$(\ )$ $(\ \ x_n)$ ... $(\ \ \neg x_n)$

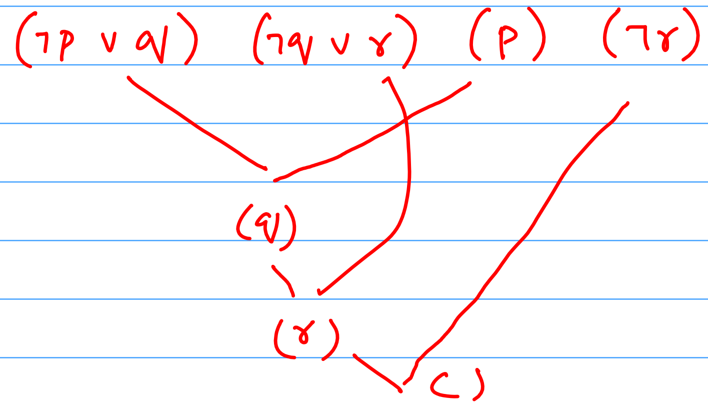
$(\ \ x_n)$

$(\ \ \neg x_n)$

$(\ )$ $(\neg x_n)$

$(\ )$

$p \to q \qquad q \to r$



$$L(\underbrace{\varphi(x_1, \ldots x_n)}) \subseteq \{0,1\}^n =$$

$$\{0,1\} \times \{0,1\} \times \cdots \{0,1\}$$

$$L(x_1 \oplus x_2 \oplus \cdots \oplus x_n) \qquad 0\,0\,1\,1\,0\,1\,0$$

$$=$$