CS 208 : Automata Theory and Logic	15th February 2024

Lecture - 17

Topic: NFA with Epsilon Transitions

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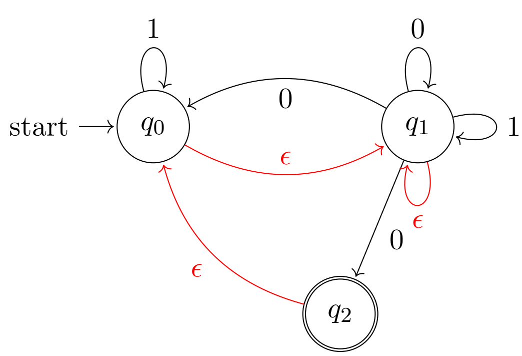
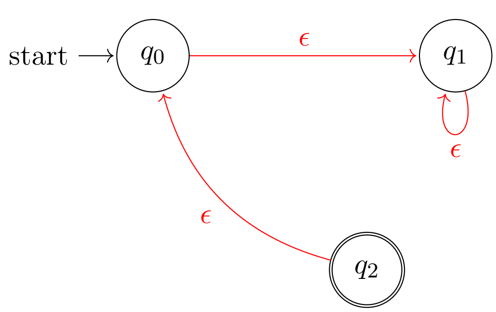
1 Epsilon Transitions

1.1 Epsilon Closure

The epsilon closure of a state q in a non-deterministic finite automaton (NFA) is the set of all states that are reachable from q by following epsilon transitions (transitions that don't consume any input). Formally, the epsilon closure of a state q, denoted by $\epsilon(q)$, can be recursively defined as follows:
BASIS: $q \in \epsilon(q)$
INDUCTION: If state $p \in \epsilon(q)$ and there is a transition from state p to state r labelled ϵ, then state $r \in \epsilon(q)$

For a given Non deterministic Finite Automaton, we remove all the edges except those, labelled ϵ.



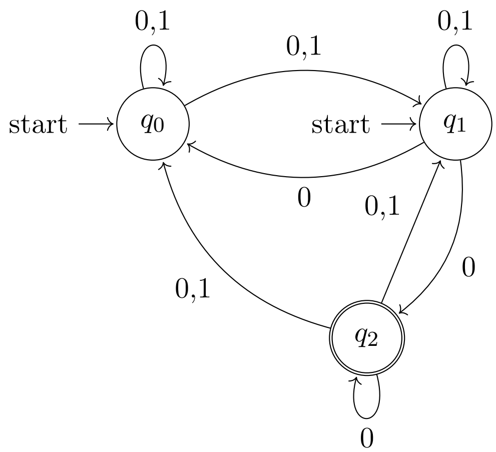


Now we can get the ϵ closures for each state by following the transitions.

$$\epsilon(q_0) \to \{q_0, q_1\}$$
$$\epsilon(q_1) \to \{q_1\}$$
$$\epsilon(q_2) \to \{q_0, q_1, q_2\}$$

1.2 Eliminating ϵ Transitions

We can convert the above epsilon NFA to NFA without epsilon edges. For each state q, for each state p $\in \epsilon(q)$, if there is an edge labelled a from state q to state r, we add an edge labelled a from state p to state r. After all such additions we can remove the edges labelled ϵ.



We need to define a new set of accepting sets F' for this NFA. If for some state p $\in F$ (initial set of accepting states), p $\in \epsilon(q)$, then q $\in F'$ Also, the set of start states is now the union of $\epsilon(q)$, q $\in Q_0$ (initial set of start states).

2 Conversion of Epsilon NFA to DFA

It can be shown that, any Non Deterministic Finite Automaton with ϵ edges can be equivalently represented as a Deterministic Finite Automaton. As discussed in previous lectures, any Non Deterministic Finite Automaton without ϵ transitions can be represented by a Deterministic Finite Automaton, we can say,

$$NFA \text{ with } \epsilon \equiv NFA \text{ without } \epsilon \equiv DFA$$

(Note: The section below was not discussed in detail in the class and is inspired from the book "Introduction to Automata Theory, Languages and Computation by John E. Hopcraft, Rajeev Motwani, Jefferey D. Ullman")

Given any ϵ-NFA E, we can find a DFA D that accepts the same language as E. The construction is quite similar to the case of NFA without ϵ transitions.
Let $E = (Q_E, \Sigma, \delta_E, q_0, F_E)$. Then the equivalent DFA $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$ is defined as follows:

1. Q_D is the set of subsets of Q_E. More precisely, it is the set of ϵ-closed subsets of Q_E, that is, sets $S \subseteq Q_E$ such that $S = \text{EClose}(S)$. The ϵ-closed sets of states S are those such that any ϵ-transition out of one of the states in S leads to a state that is also in S.

2. $q_D = \text{ECLOSE}(q_0)$; that is, we get the start state of D by closing the set consisting of only the start state of E.

3. F_D is those sets of states that contain at least one accepting state of E. That is, $F_D = \{S \mid S \text{ is in } Q_D \text{ and } S \cap F_E \neq \varnothing\}$.

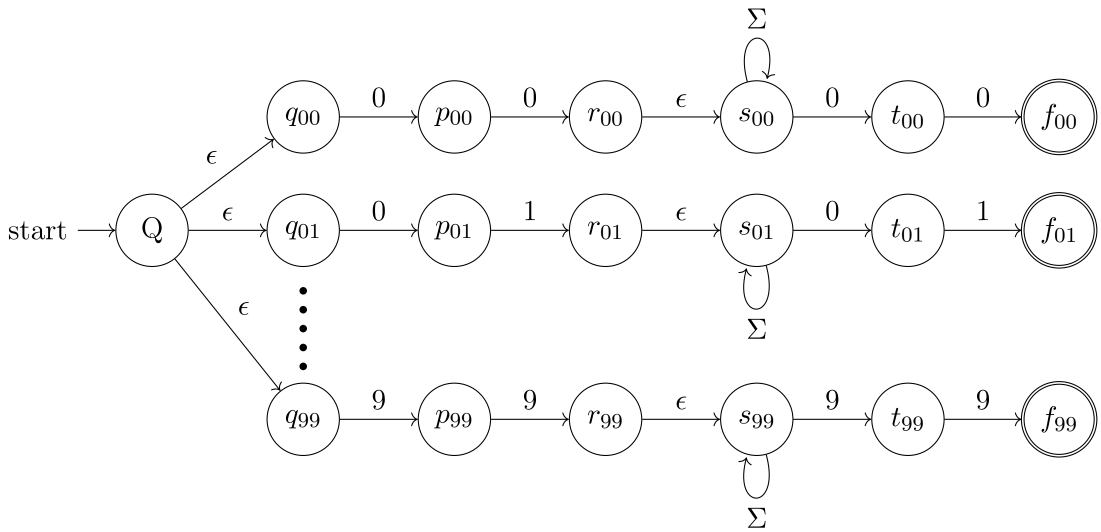
4. $\delta_D(S, a)$ is computed, for all a in Σ and sets S in Q_D by:

   (a) Let $S = \{p_1, p_2, \ldots, p_k\}$.
   (b) Compute $\bigcup_{i=1}^{k} \delta_E(p_i, a)$; let this set be $\{r_1, r_2, \ldots, r_m\}$.
   (c) Then $\delta_D(S, a) = \bigcup_{j=1}^{m} \text{ECLOSE}(r_j)$.

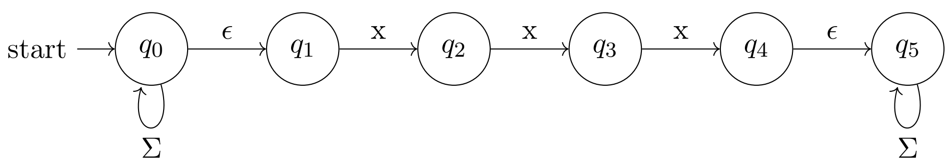
3 Representing languages using ϵ-NFA

- Example:

$$\Sigma = \{0, 1, 2, \ldots .9\}$$
$$L = \left\{\Phi^* \mid w \in \Sigma^*, w = u.v.u, u \in \Sigma^+, v \in \Sigma^*, |u| \leq 2\right\}$$



- Example: Design an $\epsilon - NFA$ that accepts all strings which contains "xxx", where alphabet of the language is the English Alphabet.



4 References

"Introduction to Automata Theory, Languages and Computation by John E. Hopcraft, Rajeev Motwani, Jefferey D. Ullman"