Chapter 5
Analog Transmission

Jirasak Sittigorn
Data Communications
Department of Computer Engineering, KMITL

Analog Transmission

- DIGITAL-TO-ANALOG CONVERSION
  - Amplitude Shift Keying
  - Frequency Shift Keying
  - Phase Shift Keying
  - Quadrature Amplitude Modulation
- ANALOG-TO-ANALOG CONVERSION
  - Amplitude Modulation
  - Frequency Modulation
  - Phase Modulation

DIGITAL-TO-ANALOG CONVERSION

- ASPECTS OF DIGITAL-TO-ANALOG CONVERSION
- AMPLITUDE SHIFT KEYING
- FREQUENCY SHIFT KEYING
- PHASE SHIFT KEYING
- QUADRATURE AMPLITUDE MODULATION

Digital Transmission vs Analog Transmission

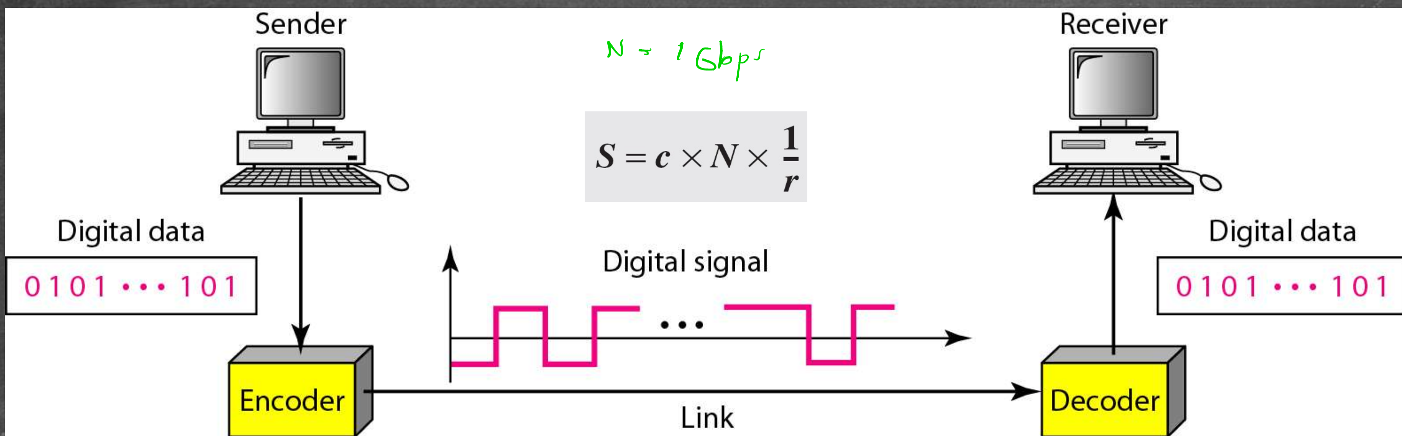
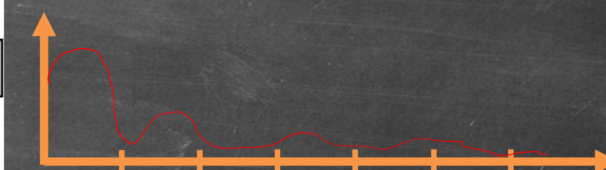


Figure 4.1 Line coding and decoding



802.11ac : Bw 20, 40, 80, 80+80 MHz / 5179-5835 MHz

AM : Bw 10kHz / 540-1600 kHz

FM : Bw 200kHz / 88.1-108.1 Mhz

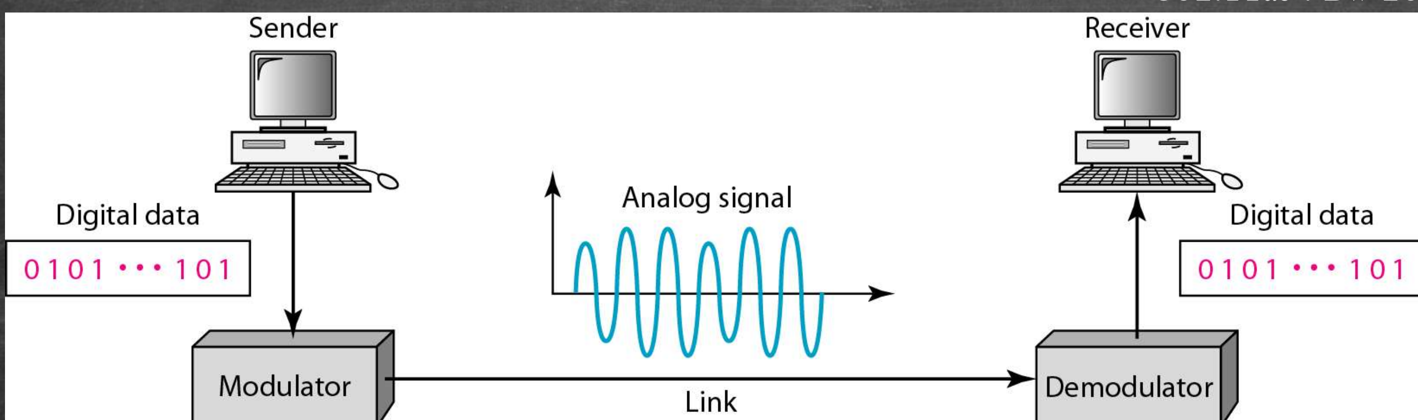


Figure 5.1 Digital-to-analog conversion

DIGITAL-TO-ANALOG CONVERSION

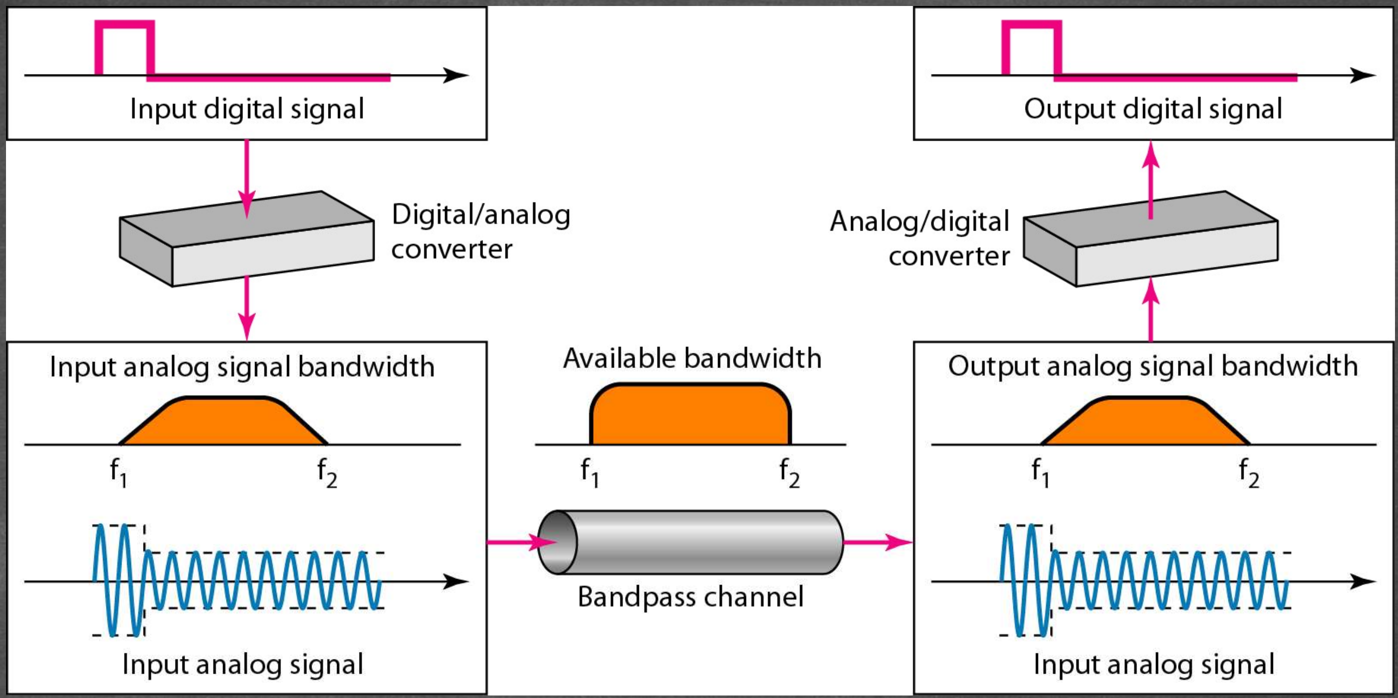


Figure 3.24 Modulation of a digital signal for transmission on a bandpass channel

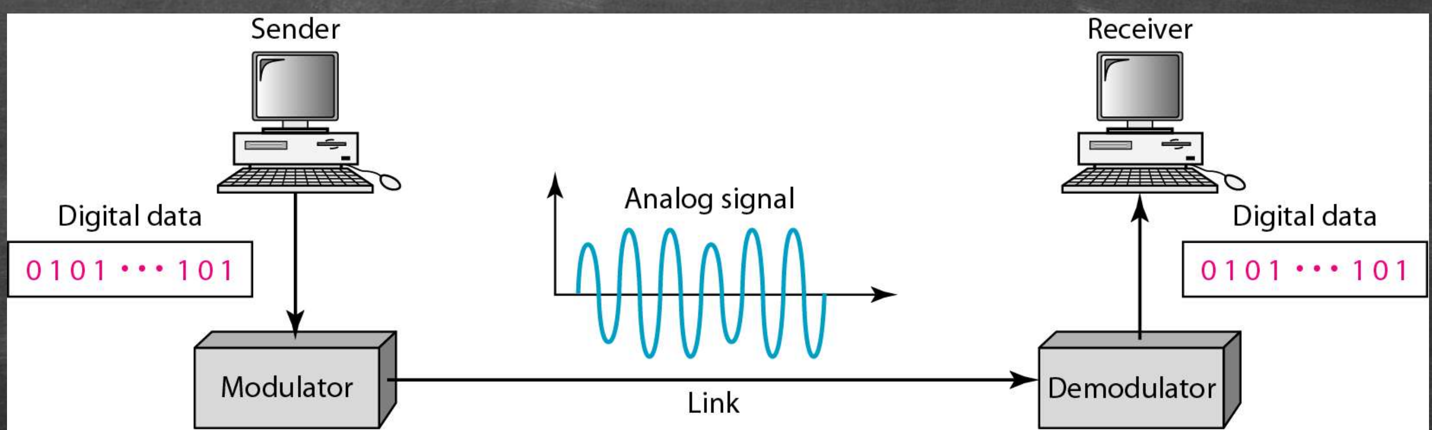


Figure 5.1 Digital-to-analog conversion

DIGITAL-TO-ANALOG CONVERSION

$$s(t) = A\sin(\omega t + \varphi) = A\sin(2\pi f t + \varphi) = A\sin(\frac{2\pi}{T}t + \varphi)$$

- A sine wave is defined by
  - Amplitude
  - Frequency
  - Phase
- Digital Modulation
  - digital data => an analog signal (Carrier signal)

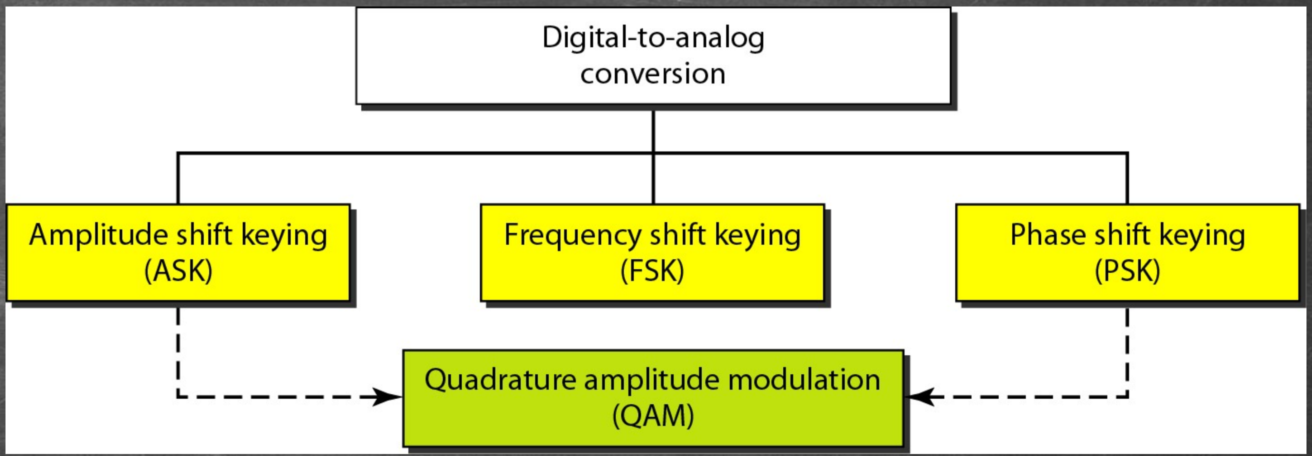
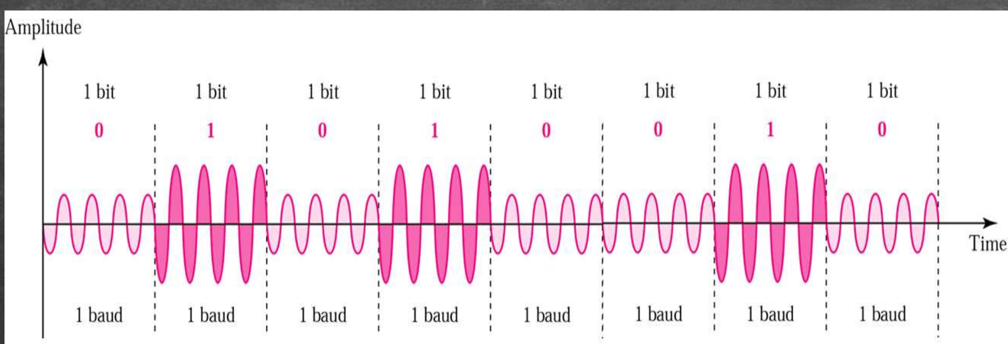


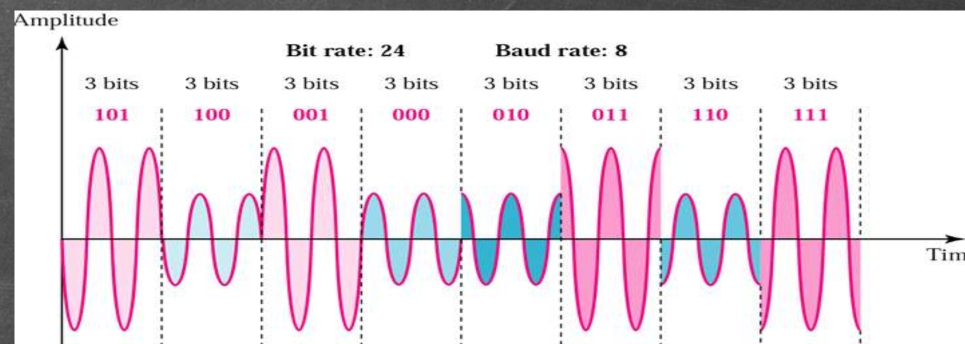
Figure 5.2 Types of digital-to-analog conversion

Aspects of Digital-to-Analog Conversion

- Bit rate (N) is the number of bits per second.
- Baud rate (S) is the number of signal elements per second.

$$S = N \times \frac{1}{r}$$





- In the analog transmission of digital data, the baud rate is less than or equal to the bit rate.

Example 5.1

- An analog signal carries 4 bits per signal element. If 1000 signal elements are sent per second, find the bit rate.
- Solution
  - In this case, r = 4, S = 1000, and N is unknown. We can find the value of N from

$$S = N \times \frac{1}{r} \qquad \text{or} \qquad N = S \times r = 1000 \times 4 = 4000 \text{ bps}$$

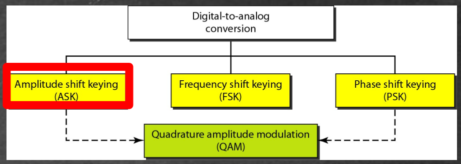
Example 5.2

- An analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?
- Solution
  - In this example, S = 1000, N = 8000, and r and L are unknown. We find first the value of r and then the value of L.

$$S = N \times \frac{1}{r} \rightarrow r = \frac{N}{S} = \frac{8000}{1000} = 8 \text{ bits/baud}$$

$$r = \log_2 L \rightarrow L = 2^r = 2^8 = 256$$

Amplitude Shift Keying



N = 5 bps S = 5 x $\frac{1}{1}$ = 5

- Binary ASK (BASK)
  - on-off keying (OOK)
    - "0" => A_1 = 0 V : 0 sin(2π15t) = 0
    - "1" => A_2 = A V : A sin(2π15t) = A sin(30πt)

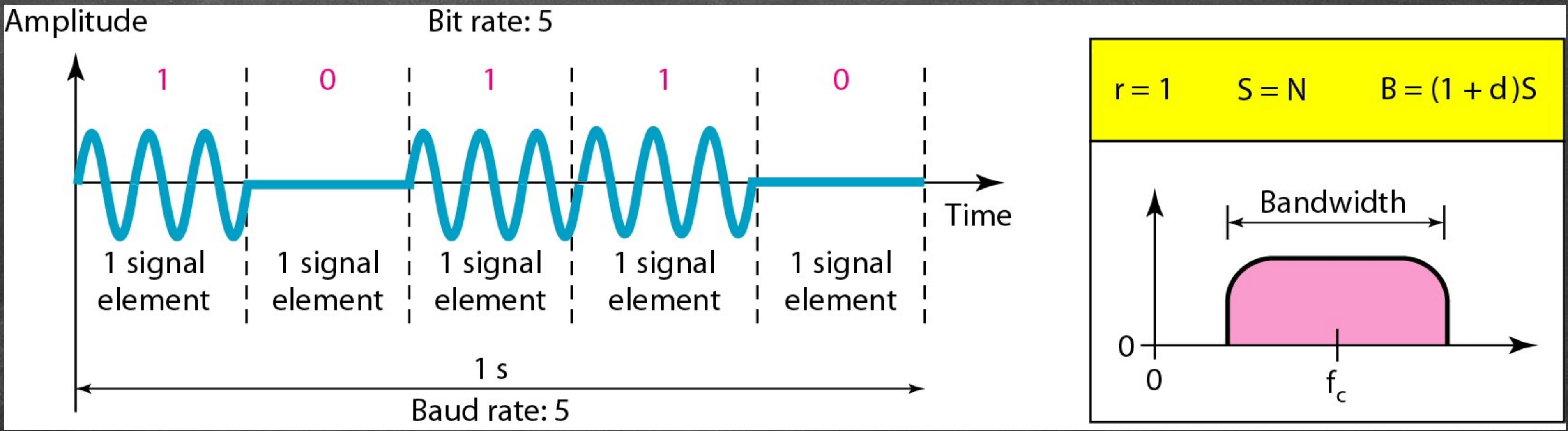
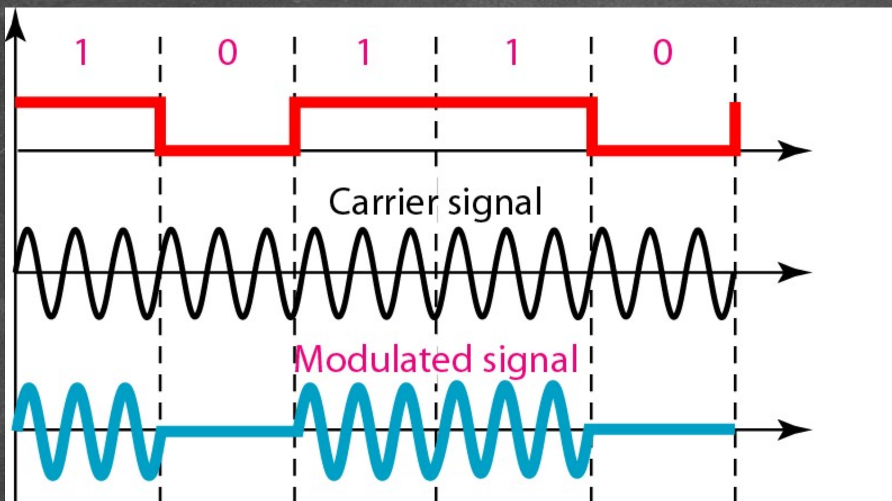


Figure 5.3 Binary amplitude shift keying

Implementation of binary ASK

- ASK Modulation



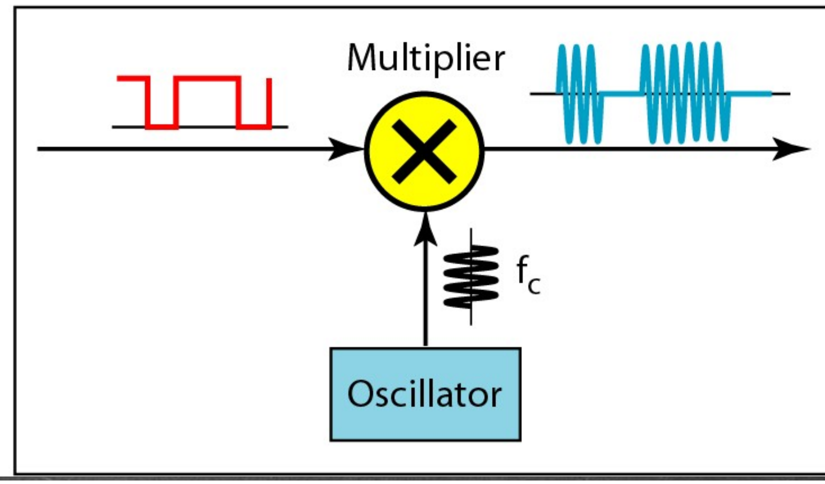
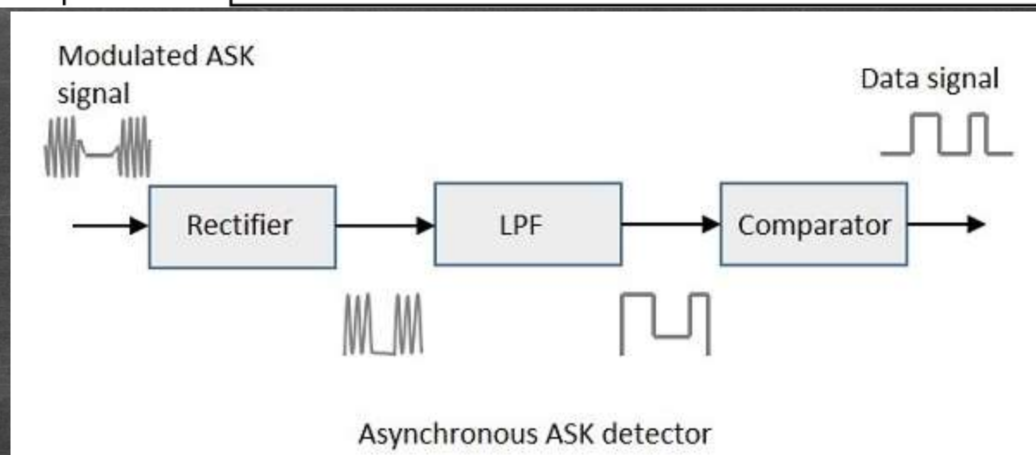
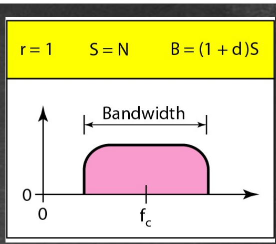


Figure 5.4 Implementation of binary ASK

- ASK Demodulation



https://www.tutorialspoint.com/digital_communication/digital_communication_amplitude_shift

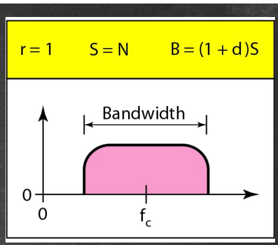


Example 5.3

- We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What are the carrier frequency and the bit rate if we modulated our data by using BASK with d = 1?
- Solution
  - The middle of the bandwidth is located at 250 kHz. This means that our carrier frequency can be at f_c = 250 kHz. We can use the formula for bandwidth to find the bit rate (with d = 1 and r = 1).

$$B = (1 + d) \times S = 2 \times N \times \frac{1}{r} = 2 \times N = 100 \text{ kHz} \quad \rightarrow \quad N = 50 \text{ kbps}$$

Example 5.4



- In data communications, we normally use full-duplex links with communication in both directions. We need to divide the bandwidth into two with two carrier frequencies, as shown in Figure 5.5. The figure shows the positions of two carrier frequencies and the bandwidths. The available bandwidth for each direction is now 50 kHz, which leaves us with a data rate of 25 kbps in each direction.

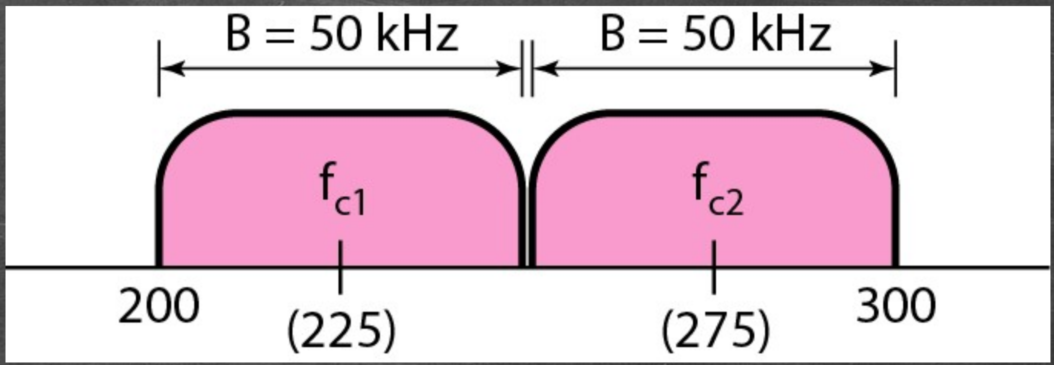


Figure 5.5 Bandwidth of full-duplex ASK used in Example 5.4

Amplitude Shift Keying

- Binary ASK (BASK)
- Multilevel ASK
  - 4ASK

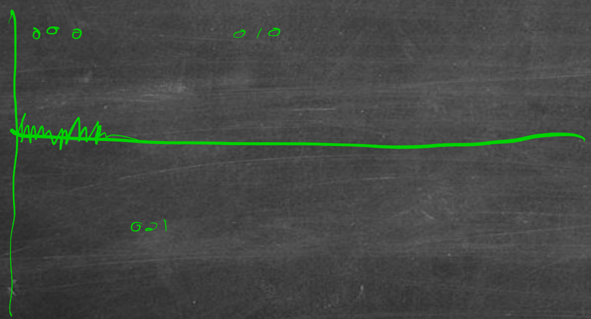
    $00 \rightarrow A_1 \quad \sin(2\pi f_0 t)$

    $01 \rightarrow A_2$

    $10 \rightarrow A_3$

    $11 \rightarrow A_4$

  - 8ASK



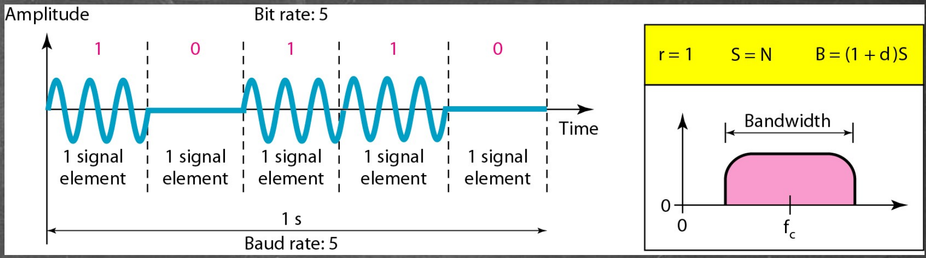
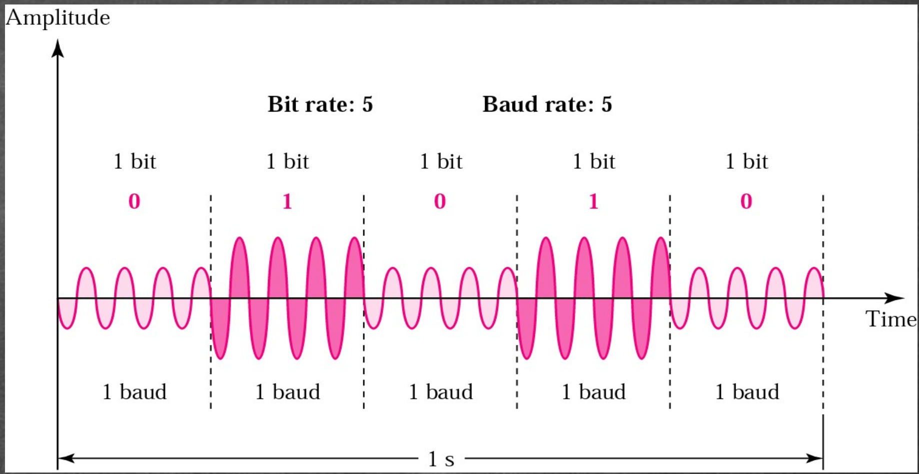
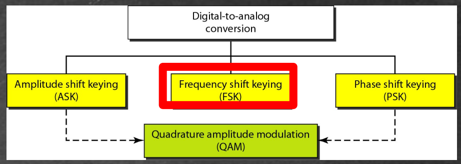


Figure 5.3 Binary amplitude shift keying



Frequency Shift Keying



- Binary FSK (BFSK)
  - "0" => f_1 = 10 Hz : $A \sin(2\pi 10t) = A \sin(20\pi t)$
  - "1" => f_2 = 20 Hz : $A \sin(2\pi 20t) = A \sin(40\pi t)$

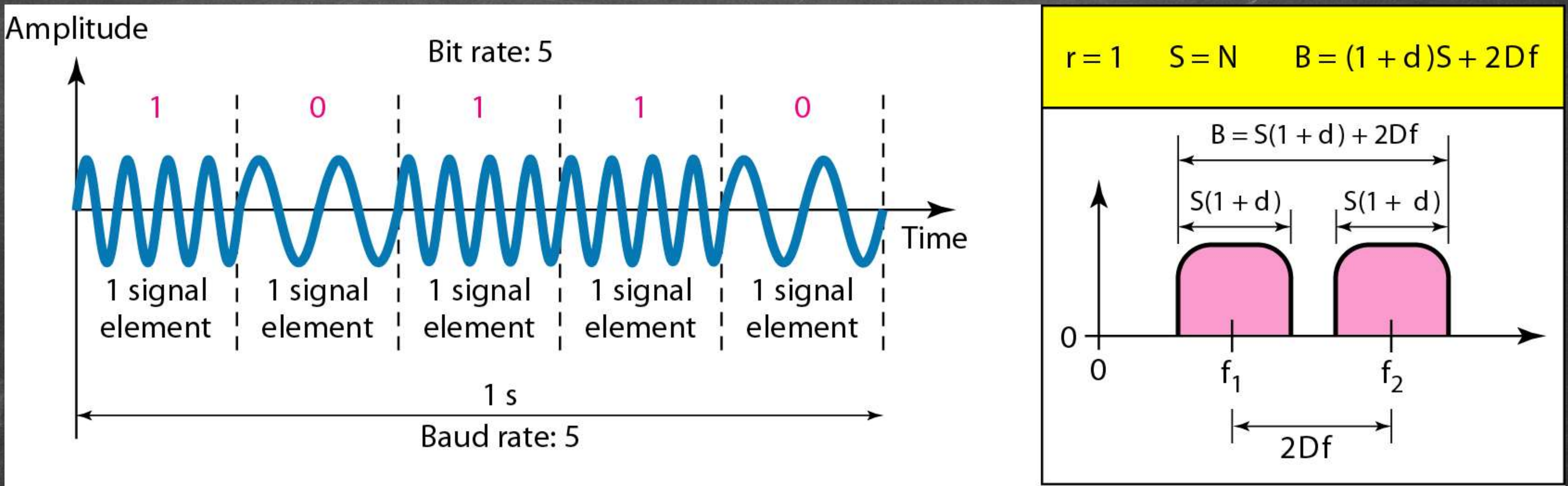
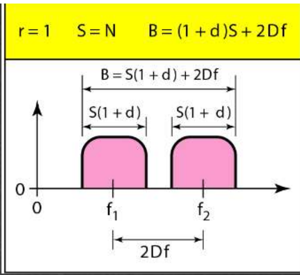


Figure 5.6 Binary frequency shift keying

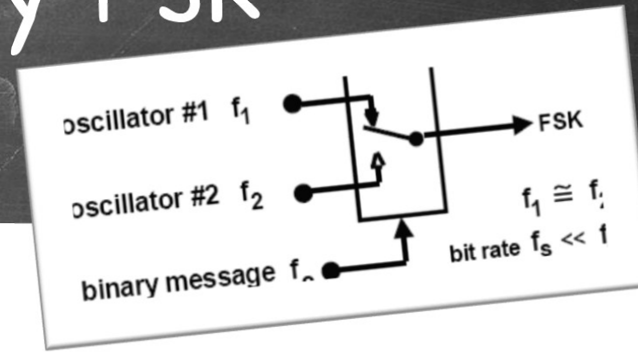


Example 5.5

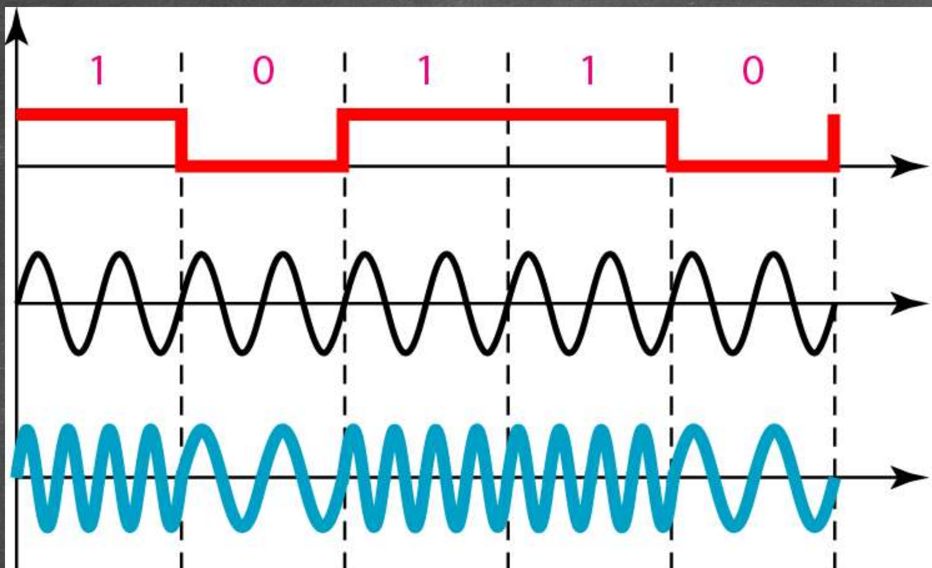
- We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What should be the carrier frequency and the bit rate if we modulated our data by using BFSK with d = 1?
- Solution
  - This problem is similar to Example 5.3, but we are modulating by using BFSK. The midpoint of the band is at 250 kHz. We choose $2\Delta f$ to be 50 kHz; this means

$$B = (1 + d) \times S + 2\Delta f = 100 \rightarrow 2S = 50 \text{ kHz} \quad S = 25 \text{ kbaud} \quad N = 25 \text{ kbps}$$

Implementation of binary FSK



- FSK Modulation



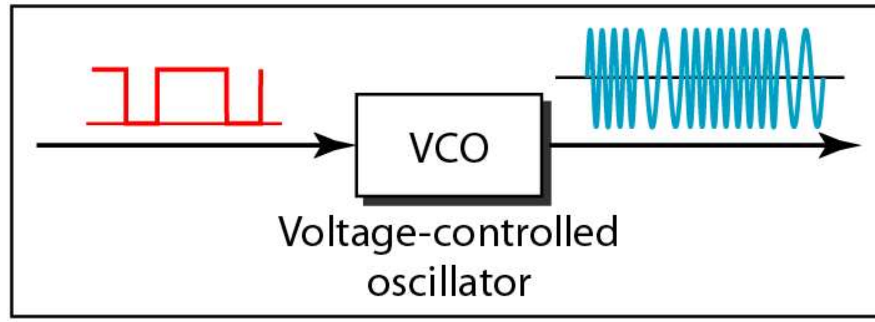
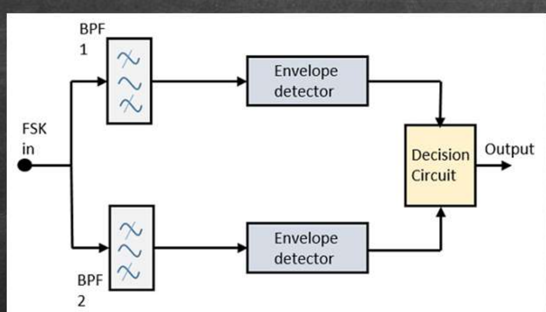
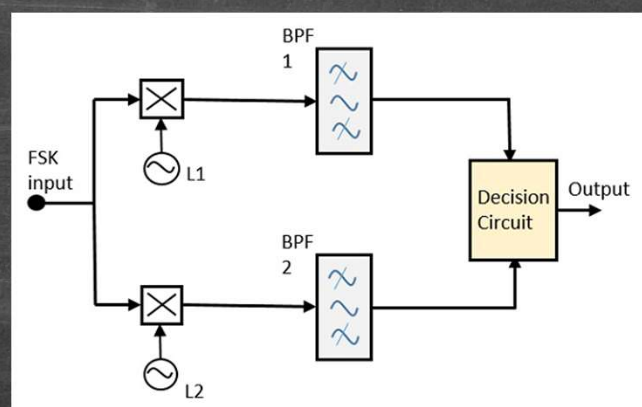


Figure 5.7 Bandwidth of MFSK used in Example 5.6

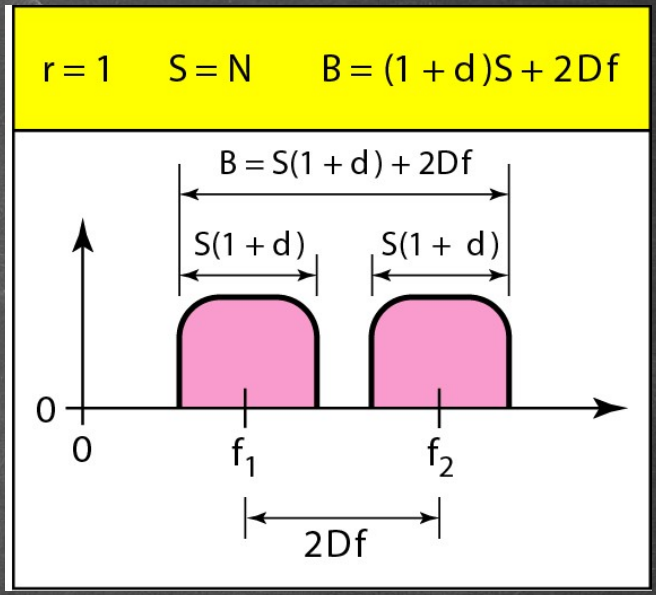
- FSK Demodulation

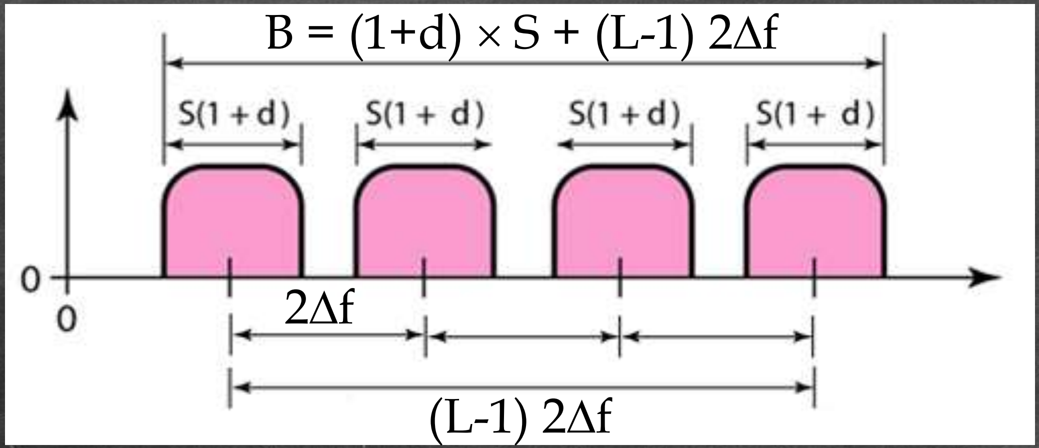




Frequency Shift Keying

- Binary FSK (BFSK)
  - $B = (1+d) \times S + 2\Delta f$
- Multilevel FSK
  - $B = (1+d) \times S + (L-1)\, 2\Delta f$

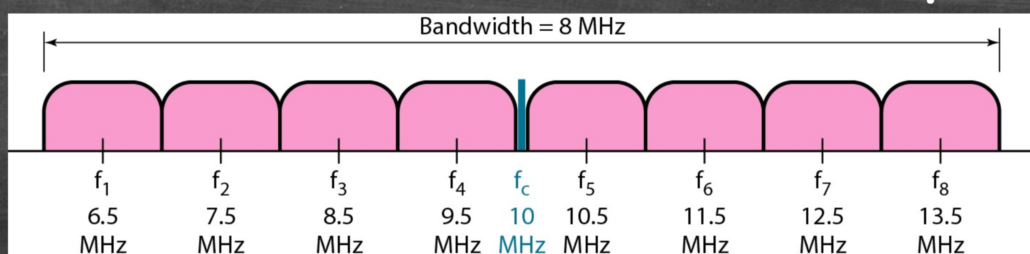




$L = 2^3$

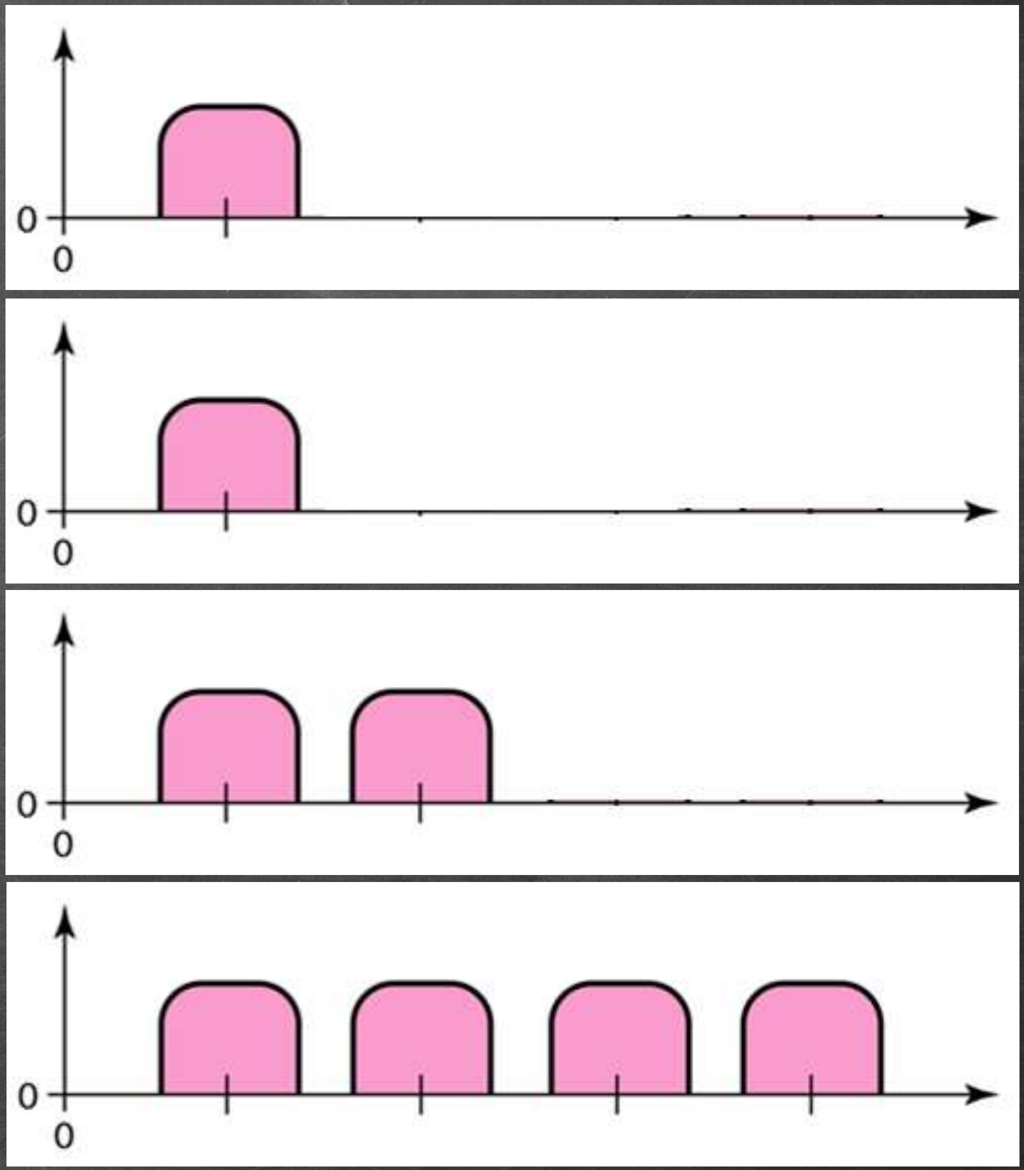
Example 5.6

- We need to send data 3 bits at a time at a bit rate of 3 Mbps. The carrier frequency is 10 MHz. Calculate the number of levels (different frequencies), the baud rate, and the bandwidth.
- Solution
  - We can have $L = 2^3 = 8$. The baud rate is $S = 3$ MHz/3 = 1000 Mbaud. This means that the carrier frequencies must be 1 MHz apart ($2\Delta f = 1$ MHz). The bandwidth is $B = 8 \times 1000 = 8000$. Figure 5.8 shows the allocation of frequencies and bandwidth.

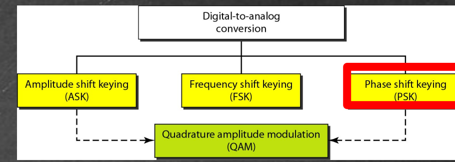


ASK VS FSK

- Binary ASK (BASK)
  - $B = (1+d) \times S$
- Multilevel ASK
  - $B = (1+d) \times S$
- Binary FSK (BFSK)
  - $B = (1+d) \times S + 2\Delta f$
- Multilevel FSK (4FSK)
  - $B = (1+d) \times S + (L-1)\, 2\Delta f$



Phase Shift Keying



- Binary PSK (BPSK)
  - "0" => $\varphi_1 = \pi/2$: $A\cos(30\pi t + \pi/2) = A\sin(30\pi t + \pi)$
  - "1" => $\varphi_2 = -\pi/2$: $A\cos(30\pi t - \pi/2) = A\sin(30\pi t)$

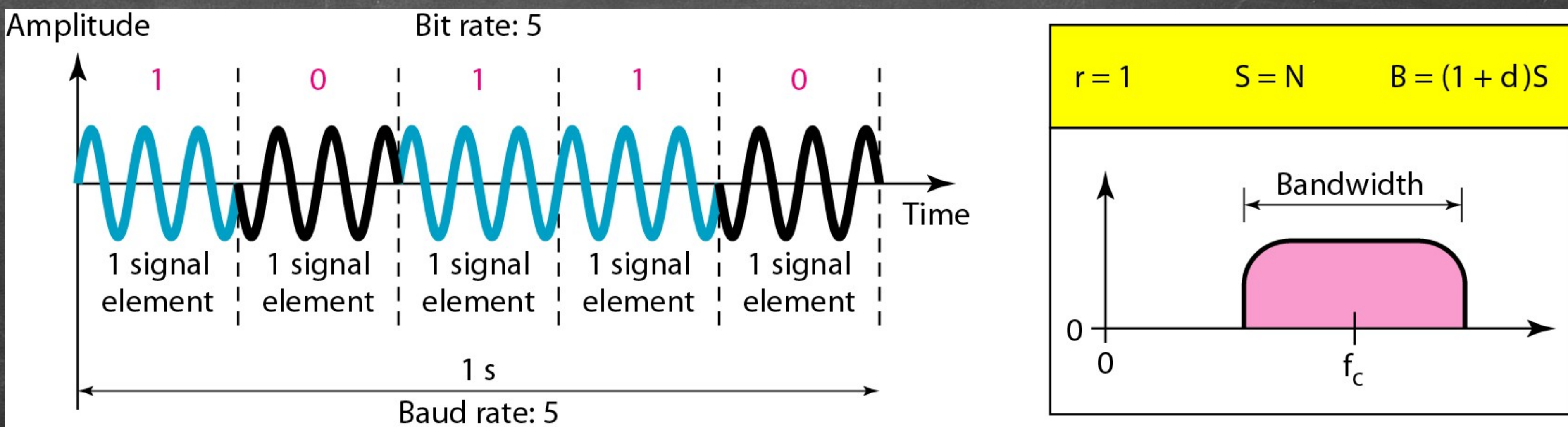


Figure 5.9 Binary phase shift keying

Implementation of binary PSK

$\cos\alpha \cos\beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin\alpha \sin\beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$

- PSK Modulation
  - Binary Data => polar NRZ ("1" -> +1 / "0" -> -1) : X
  - Multiply X with Carrier Signal : $X \cos(2\pi f_c t)$

"1" $\cos(2\pi f_c t - \pi/2) = \cos(2\pi f_c t - \pi/2)$

"0" $-\cos(2\pi f_c t - \pi/2) = \cos(2\pi f_c t + \pi/2)$

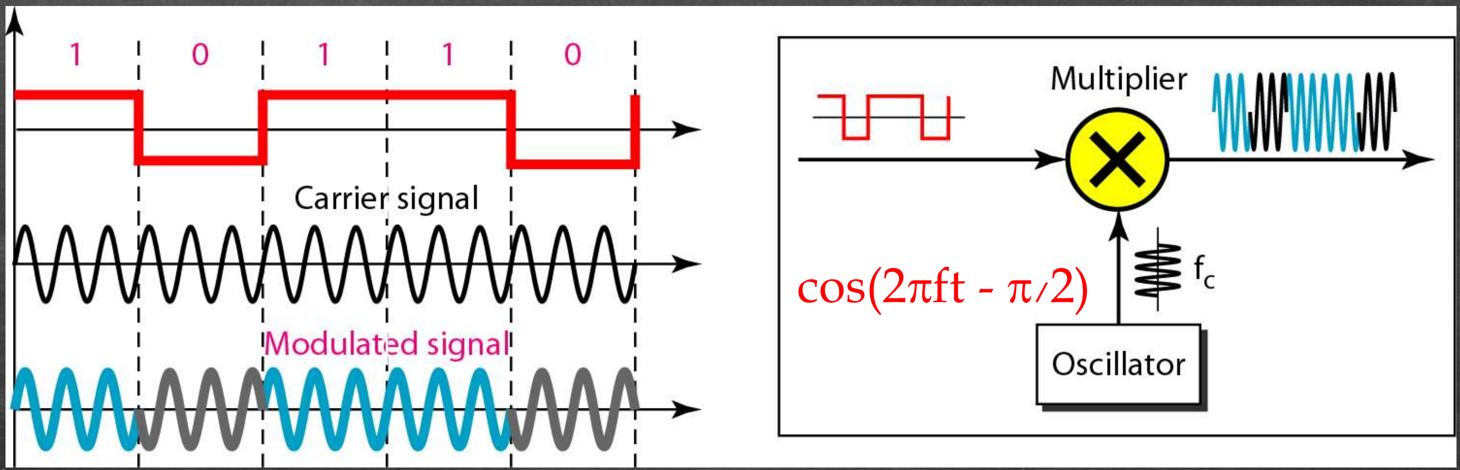
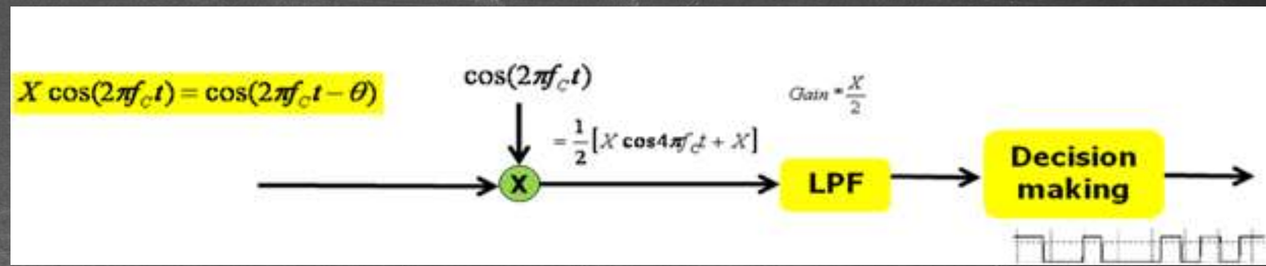


Figure 5.10 Implementation of BASK

Implementation of binary PSK

- PSK Modulation
  - $X\cos(2\pi f_c t)$



- PSK Demodulation
  - Multiply PSK with Carrier Signal : $X\cos(2\pi f_c t) \times \cos(2\pi f_c t)$

$= \frac{1}{2} X [\cos(2\pi f_c t - 2\pi f_c t) + \cos(2\pi f_c t + 2\pi f_c t)]$

$= \frac{1}{2} X [\cos(0) + \cos(4\pi f_c t)]$

$= \frac{1}{2} X [1 + \cos(2\pi 2 f_c t)]$

$= \frac{1}{2} \times [x + x\cos(2\pi 2 f_c t)]$

  - Low pass filter
  - Decision making

Quadrature Phase Shift Keying (QPSK)

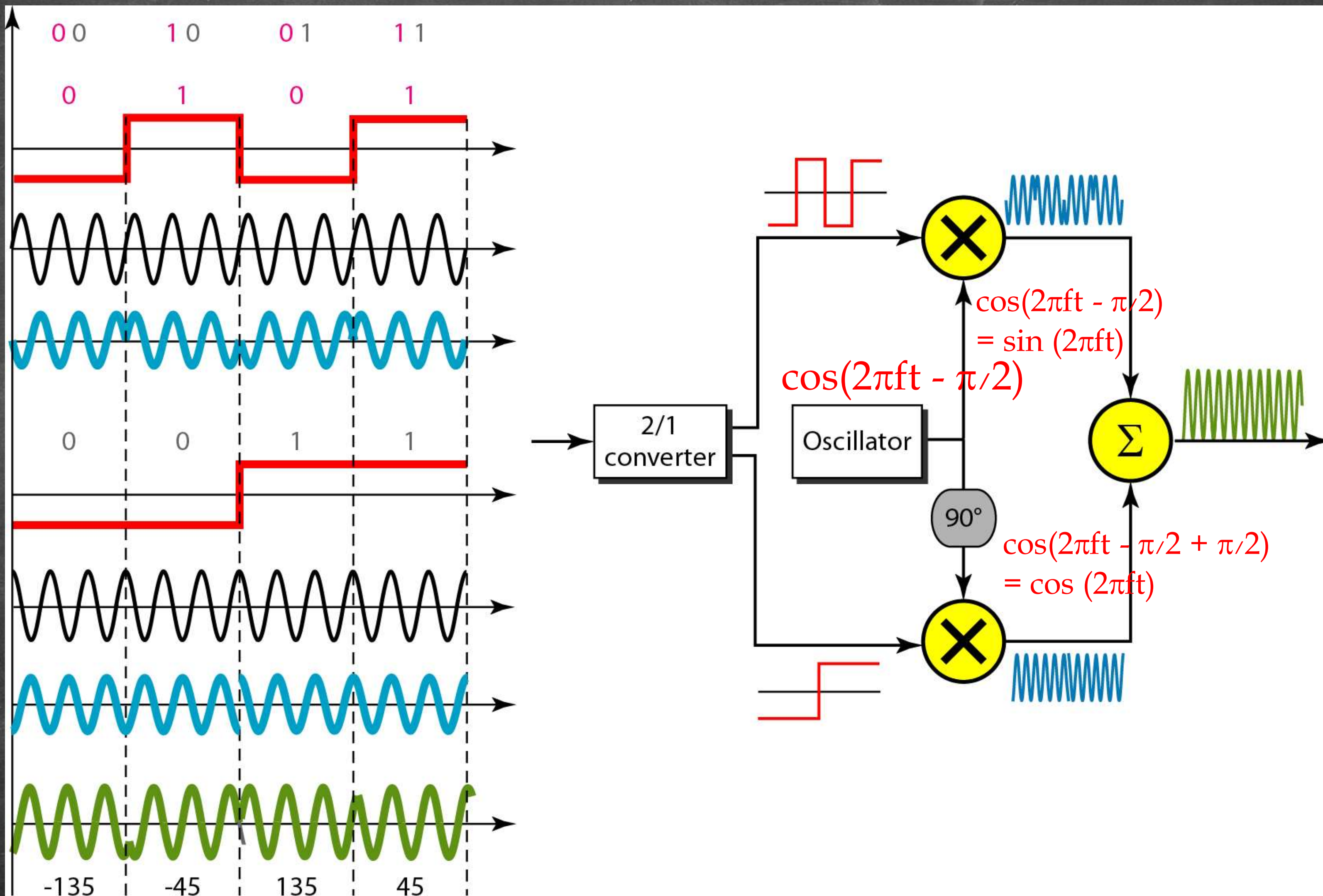


Figure 5.11 QPSK and its implementation

Example 5.7

- Find the bandwidth for a signal transmitting at 12 Mbps for QPSK. The value of d = 0.
- Solution
  - For QPSK, 2 bits is carried by one signal element. This means that r = 2.
  - So the signal rate (baud rate) is S = Nx(1/r) = 6 Mbaud. With a value of d = 0, we have B = S = 6 MHz.

Constellation diagram

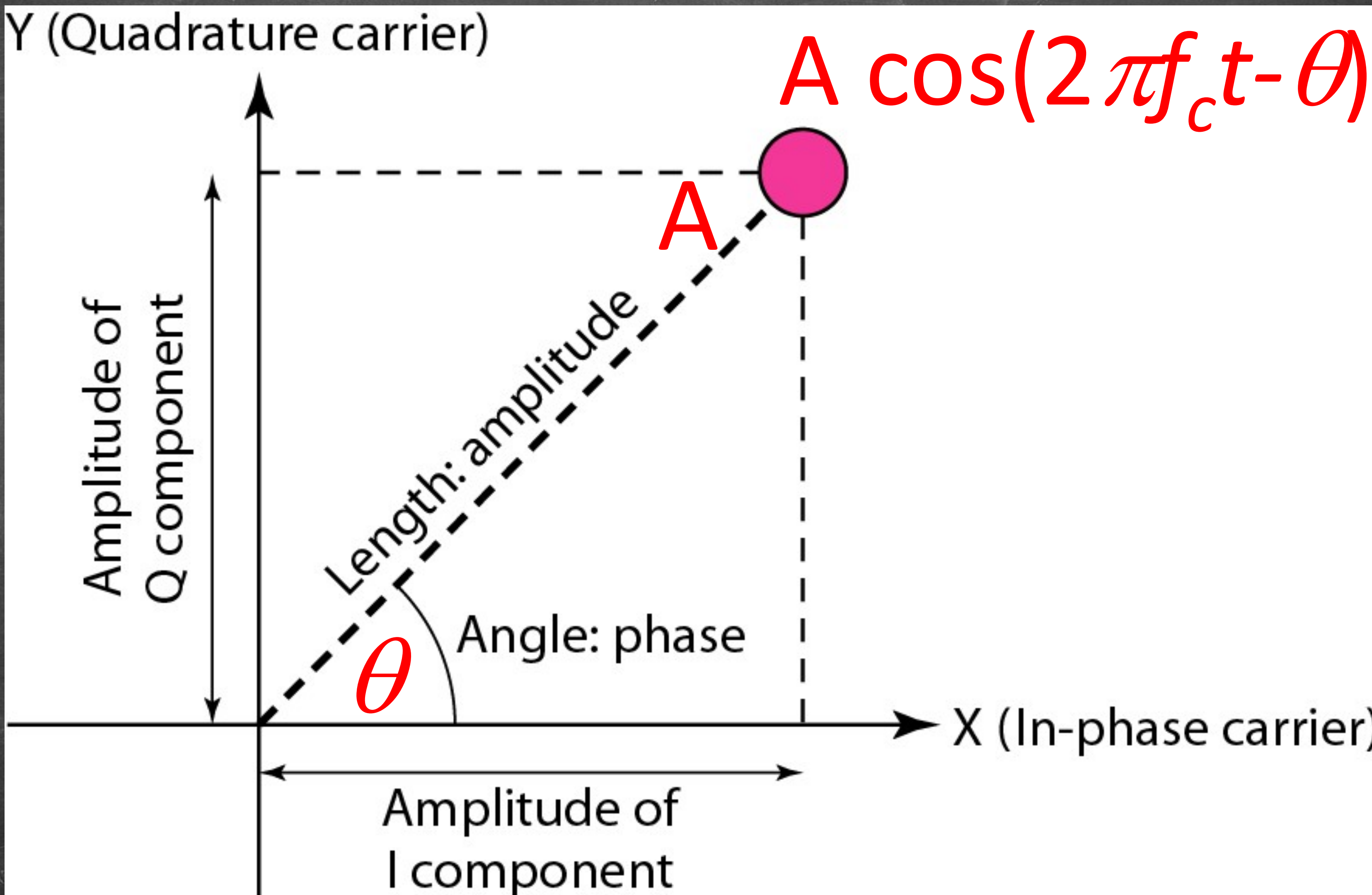
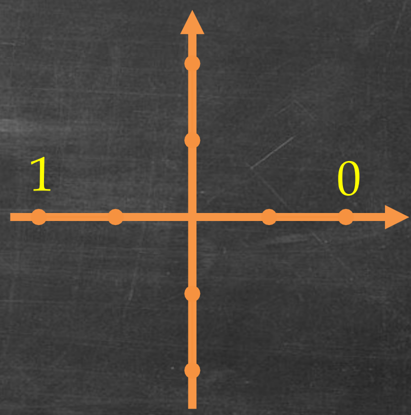


Figure 5.12 Concept of a constellation diagram

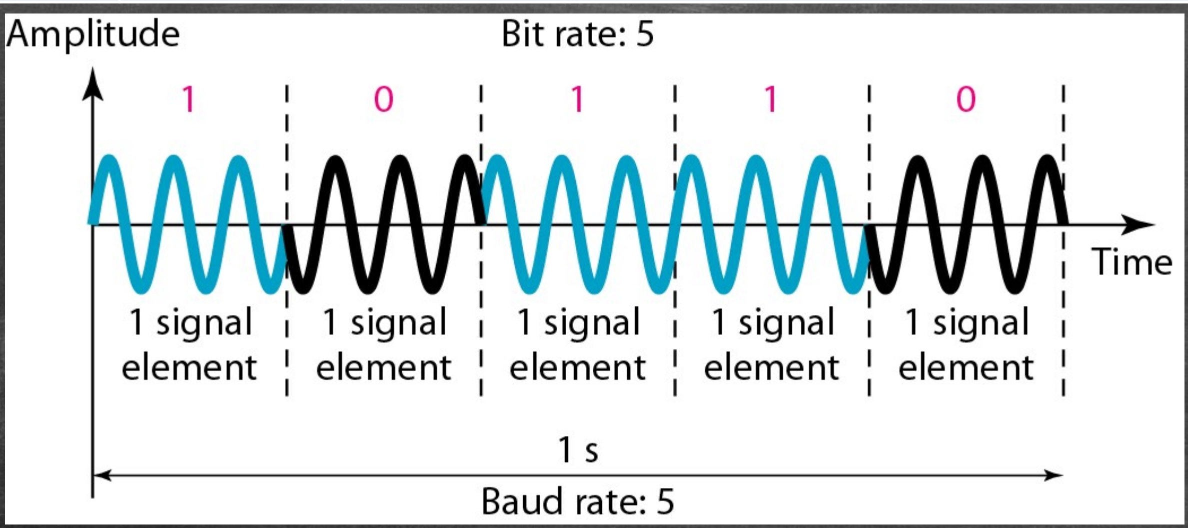
Constellation diagram of BPSK

Data [0, 1]	X [-1, 1]	BPSK $X\cos(2\pi f_c t)$	$\cos(2\pi f_c t-\theta)$	θ
"0"	-1	$-\cos(2\pi f_c t)$	$\cos(2\pi f_c t-\pi)$	$180°$
"1"	1	$\cos(2\pi f_c t)$	$\cos(2\pi f_c t-0)$	$0°$



Constellation diagram of BPSK

Data [0, 1]	X [-A, A]	BPSK $X\cos(2\pi f_c t)$	BPSK $A\cos(2\pi f_c t-\theta)$	θ
"0"	-1	$-\cos(2\pi f_c t-\pi/2)$ $=-\sin(2\pi f_c t)$	$A\cos(2\pi f_c t-(-\pi/2))$	$-90°$
"1"	1	$\cos(2\pi f_c t-\pi/2)$ $=\sin(2\pi f_c t)$	$A\cos(2\pi f_c t-\pi/2)$	$90°$



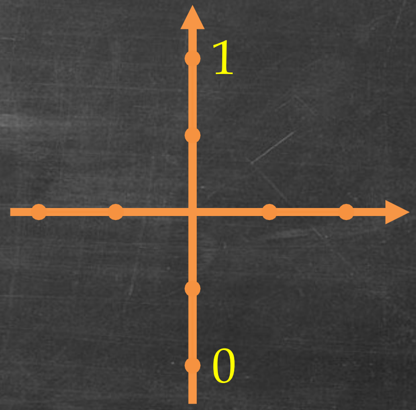


Figure 5.9 Binary phase shift keying

B. A. Forouzan, Data Communications and Networking, 4th edition, McGRAW-HILL

Constellation diagram of BPSK

Data [0, 1]	X [0 A]	BASK $X\cos(2\pi f_c t)$	$\cos(2\pi f_c t-\theta)$	θ
"0"	0	0	0	0°
"1"	A	$A\cos(2\pi f_c t-\pi/2)$	$A\cos(2\pi f_c t - \pi/2)$	90°

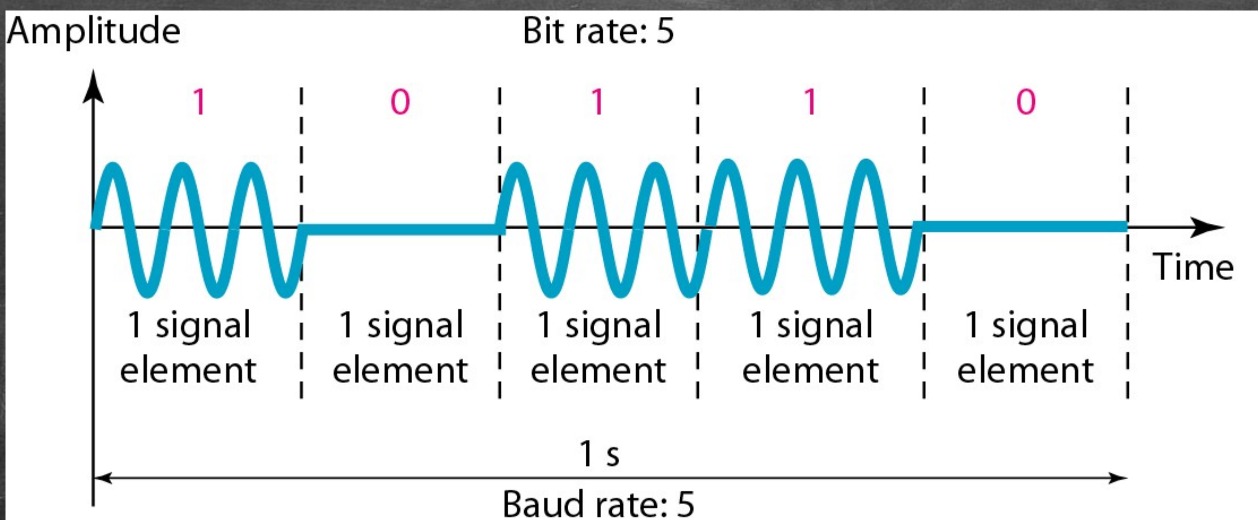
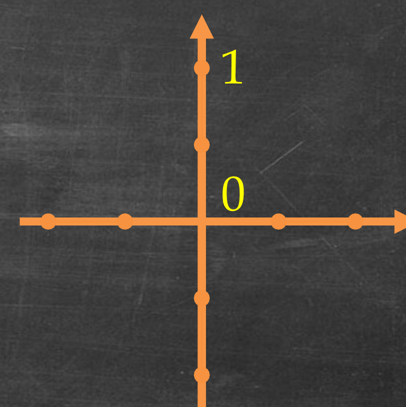


Figure 5.3 Binary amplitude shift keying
B. A. Forouzan, Data Communications and Networking, 4th edition, McGRAW-HILL



Quadrature Amplitude Modulation

- Quadrature amplitude modulation is a combination of ASK and PSK.
- Bandwidth for QAM
  - Same ASK and PSK

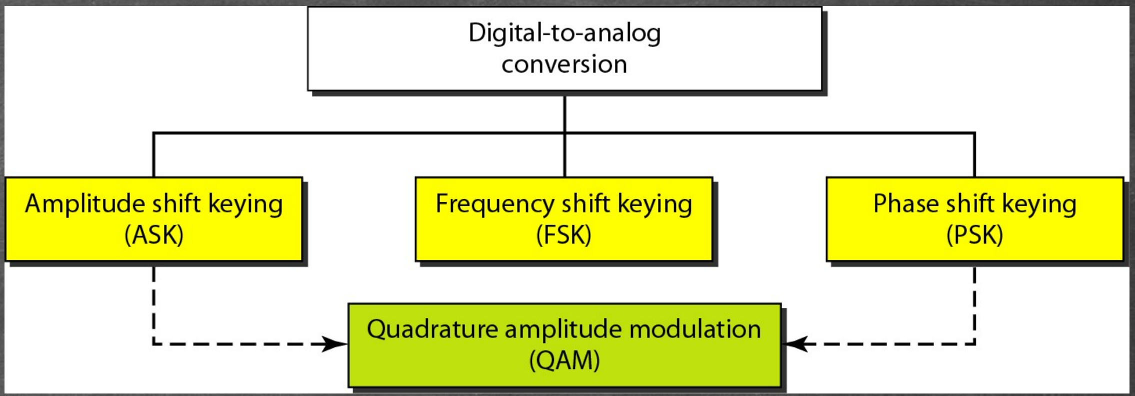


Figure 5.2 Types of digital-to-analog conversion

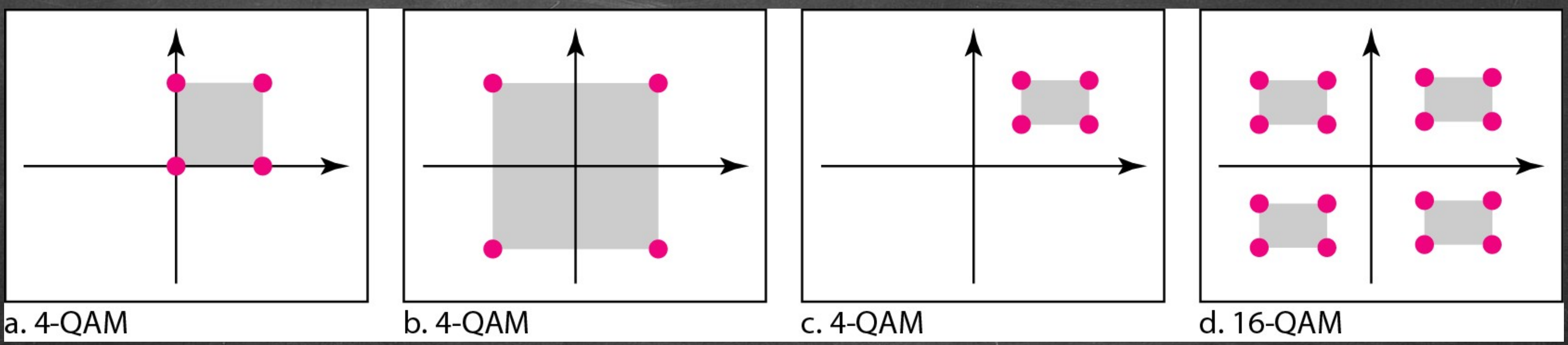


Figure 5.14 Constellation diagrams for some QAMs

4-QAM and 8-QAM constellations

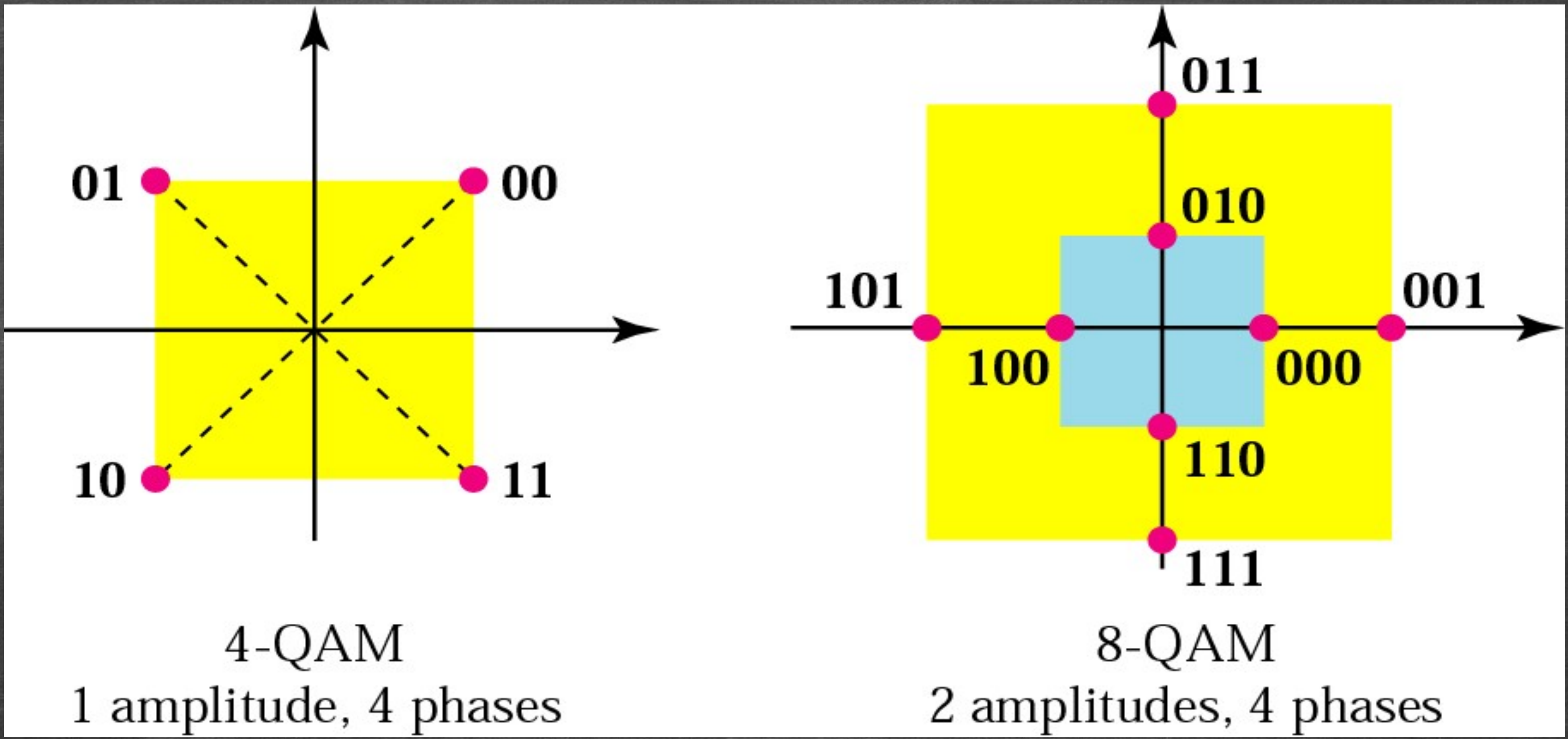
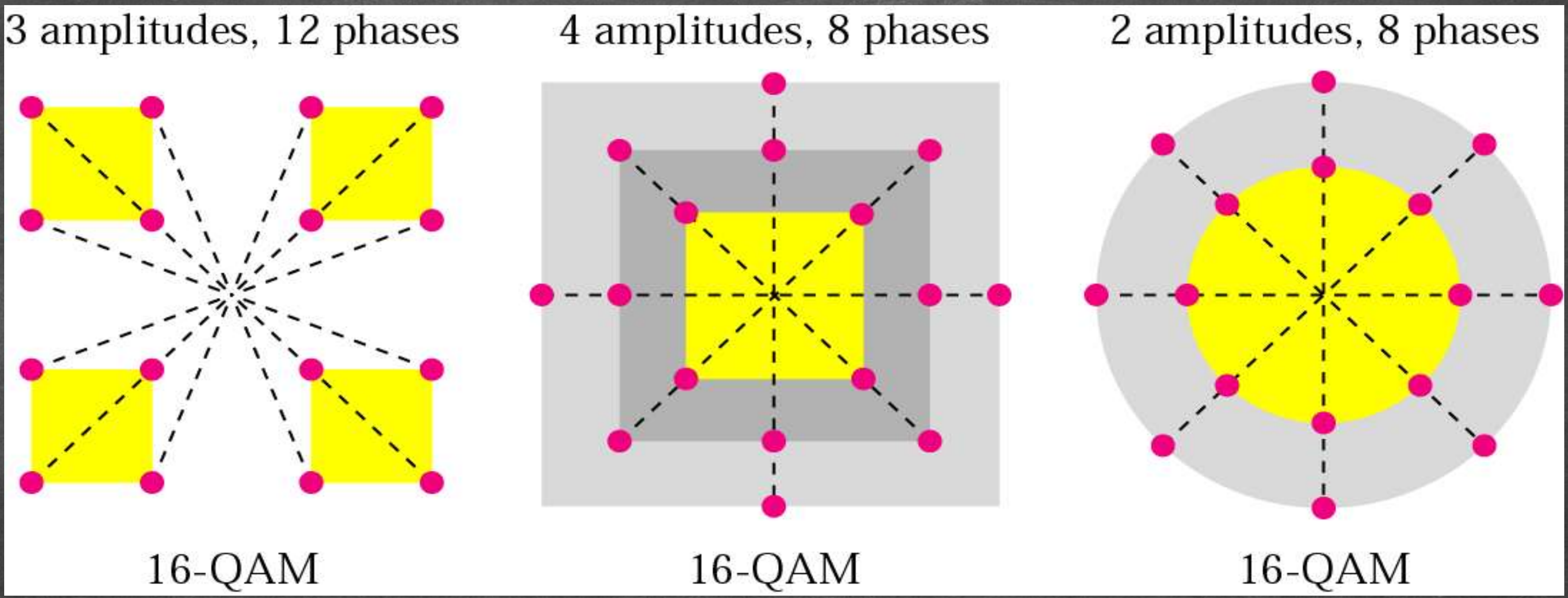


Figure 5.14 The 4-QAM and 8-QAM constellations

16-QAM constellations

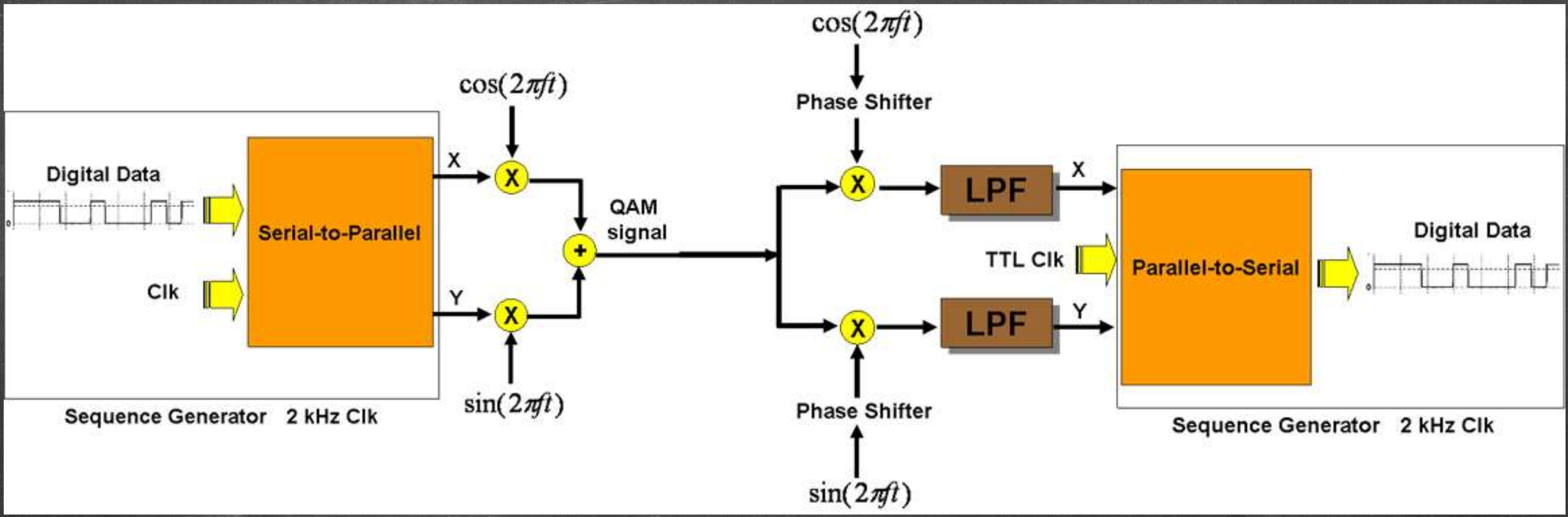


ITU-T recommendation

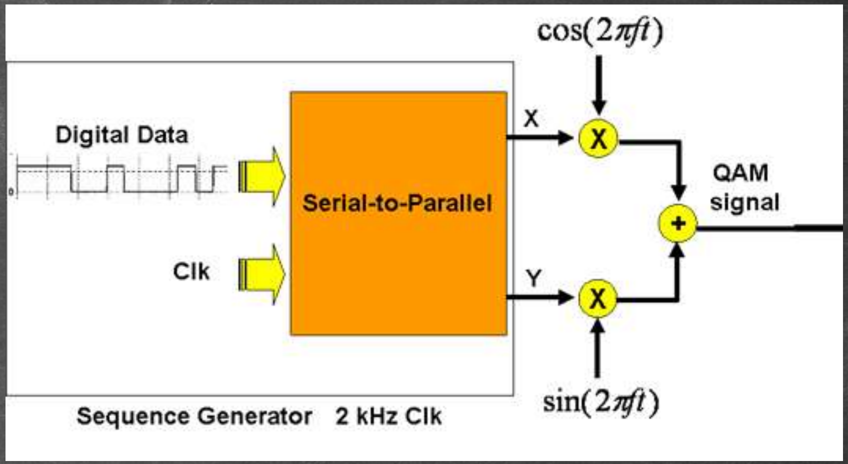
OSI recommendation

Figure 5.16 16-QAM constellations

Implementation of simple QAM

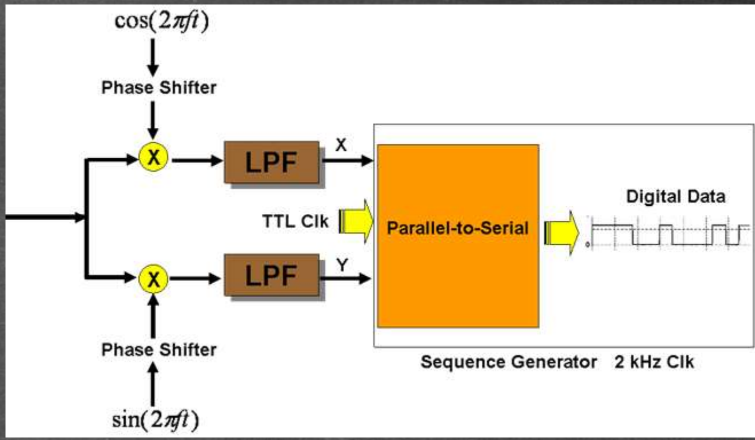


Implementation of simple QAM



- QAM Modulation
  - 2 Binary Data => polar NRZ
    - Odd bit : X ("1" -> +1 / "0" -> -1)
    - Even bit : Y ("1" -> +1 / "0" -> -1)
  - Multiply X with Carrier Signal : $X\,A1\cos(2\pi f_c t)$
  - Multiply Y with Carrier Signal : $Y\,A2\sin(2\pi f_c t)$
  - QAM : $X\,A1\cos(2\pi f_c t) + Y\,A2\sin(2\pi f_c t)$

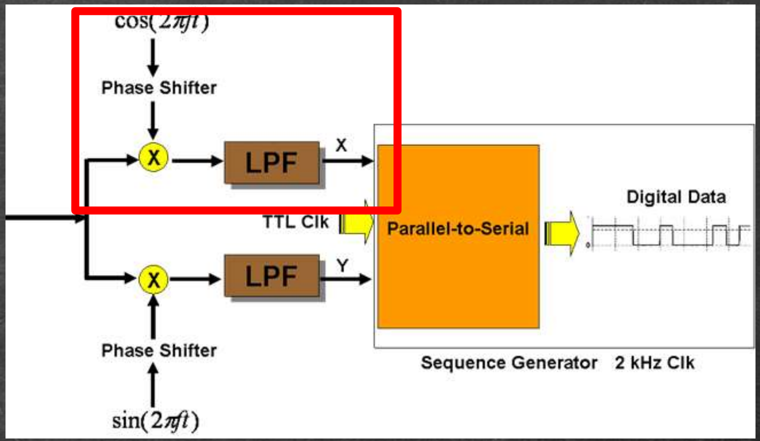
Implementation of simple QAM



- QAM Modulation
  - $X\ A1 \cos(2\pi f_c t) + Y\ A2 \sin(2\pi f_c t)$
- QAM Demodulation
  - Odd bit
    - $[X\ A1 \cos(2\pi f_c t) + Y\ A2 \sin(2\pi f_c t)] \times \cos(2\pi f_c t)$
    - Low pass filter => Odd bit
  - Even bit
    - $[X\ A1 \cos(2\pi f_c t) + Y\ A2 \sin(2\pi f_c t)] \times \sin(2\pi f_c t)$
    - Low pass filter => Even bit
  - Parallel to Serial

Implementation of simple QAM

- Odd bit (X)
  - $[X\ A1 \cos(2\pi f_c t) + Y\ A2 \sin(2\pi f_c t)] \times \underline{\cos(2\pi f_c t)}$
    $= X\ A1 \cos(2\pi f_c t)\ \underline{\cos(2\pi f_c t)} + Y\ A2 \sin(2\pi f_c t)\ \underline{\cos(2\pi f_c t)}$
    $= X\ A1 \{ ½ [\cos(0) + \cos(4\pi f_c t)] \} + Y\ A2 \{ ½ \sin(4\pi f_c t) \}$
    $= ½ [X\ A1 + X\ A1 \cos(4\pi f_c t) + Y\ A2 \sin(4\pi f_c t)]$
  - Low pass filter => ½ X A1



$\cos\alpha \cos\beta = ½(\cos(\alpha - \beta) + \cos(\alpha + \beta))$

$\sin\alpha \cos\beta = ½(\sin(\alpha + \beta) + \sin(\alpha - \beta))$

$\cos\alpha \sin\beta = ½(\sin(\alpha + \beta) - \sin(\alpha - \beta))$

$\sin\alpha \sin\beta = ½(\cos(\alpha - \beta) - \cos(\alpha + \beta))$

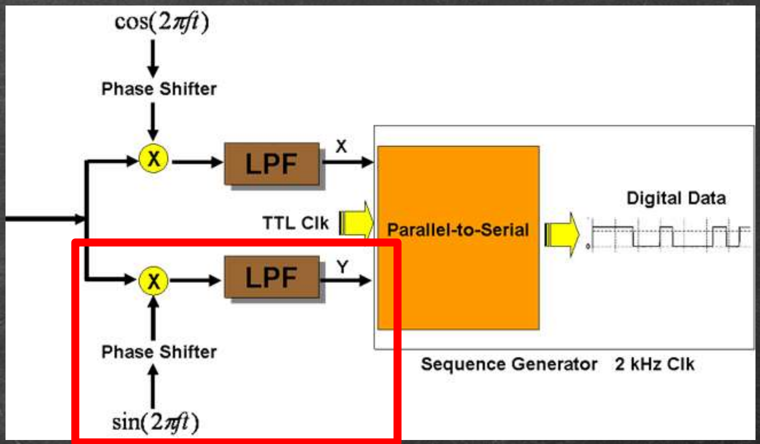
Implementation of simple QAM

- Even bit (Y)
  - $[X\ A1\ \cos(2\pi f_c t) + Y\ A2\ \sin(2\pi f_c t)] \times \underline{\sin(2\pi f_c t)}$

    $= X\ A1\ \cos(2\pi f_c t)\ \underline{\sin(2\pi f_c t)} + Y\ A2\ \sin(2\pi f_c t)\ \underline{\sin(2\pi f_c t)}$

    $= X\ A1\ \{\ ½\ \sin(4\pi f_c t)\ \} + Y\ A2\ \{\ ½\ [\ \cos(0) - \cos(4\pi f_c t)\ ]\ \}$

    $= ½\ [\ X\ A1\ \cos(4\pi f_c t) + Y\ A2 - Y\ A2\ \sin(4\pi f_c t)\ ]$
  - Low pass filter => ½ Y A2
  -



$\cos\alpha \cos\beta = ½(\cos(\alpha - \beta) + \cos(\alpha + \beta))$

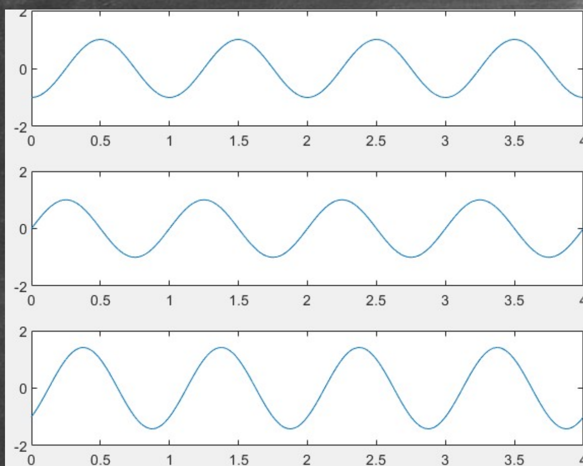
$\sin\alpha \cos\beta = ½(\sin(\alpha + \beta) + \sin(\alpha - \beta))$

$\cos\alpha \sin\beta = ½(\sin(\alpha + \beta) - \sin(\alpha - \beta))$

$\sin\alpha \sin\beta = ½(\cos(\alpha - \beta) - \cos(\alpha + \beta))$

Implementation of simple QAM

- Define : A1 = A2 = 1 => Same as QPSK

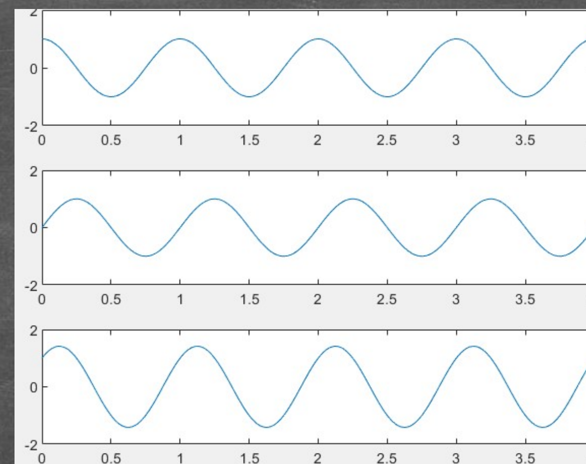


“0 1”

s1 = -cos(2*pi*t);

s2 = sin(2*pi*t);

s3 = s1+s2;

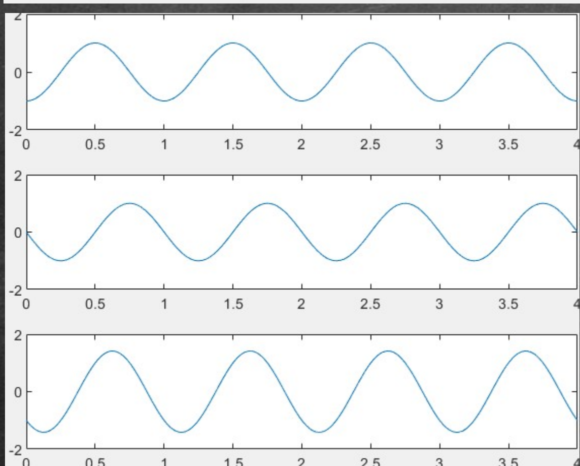


“1 1“

s1 = cos(2*pi*t);

s2 = sin(2*pi*t);

s3 = s1+s2;

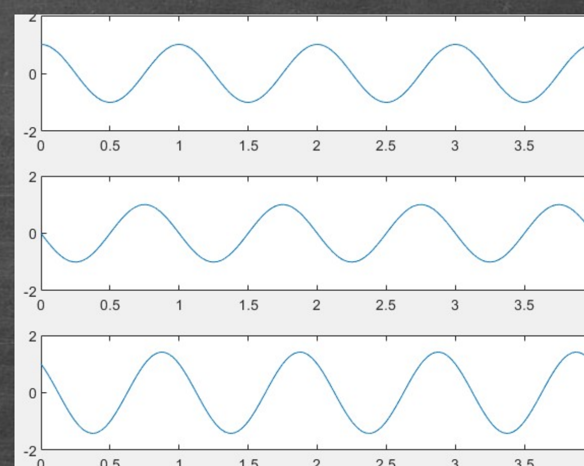


“0 0”

s1 = -cos(2*pi*t);

s2 = -sin(2*pi*t);

s3 = s1+s2;



“1 0”

s1 = cos(2*pi*t);

s2 = -sin(2*pi*t);

s3 = s1+s2;

Constellation diagram of simple QAM

Data [0, 1]	X, Y [-1, 1]	simple QAM $A\cos(2\pi f_c t - \theta)$	θ
"0 0"	−1, −1	$A\cos(2\pi f_c t + \frac{3}{4}\pi)$	−135°
"0 1"	−1, 1	$A\cos(2\pi f_c t - \frac{3}{4}\pi)$	135°
"1 0"	1, −1	$A\cos(2\pi f_c t + \frac{1}{4}\pi)$	−45°
"1 1"	1, 1	$A\cos(2\pi f_c t - \frac{1}{4}\pi)$	45°