SIMPLE
LINEAR REGRESSION

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Introduction

This chapter considers relationship between two variables in two ways:

- by using regression analysis and
- by computing correlation coefficient.

- By using **regression model**, we can evaluate magnitude of change in one variable due to a certain change in another variable.

- **Correlation coefficient**, tells us how strongly two variables are related.

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Introduction

Regression Model

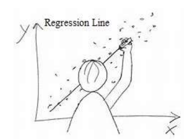
- For example, an economist can estimate the amount of change in food expenditure due to certain change in income of household by using regression model.

Estimate

food expenditure

Due to

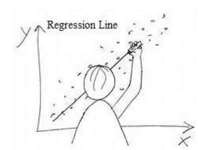
income of household



- Sociologist may want to estimate increase in crime rate due to particular increase in unemployment rate.

Estimate

Due to



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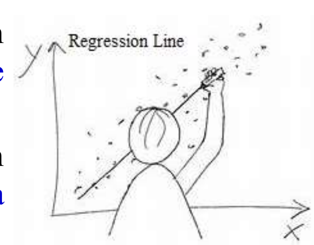
Introduction

Regression Model

- Besides answering these questions, regression model also helps predict value of one variable for a given value of another variable.
- For example, by using regression line, we can predict (approximate) food expenditure of a household with given income.



- **Correlation coefficient**, tells us how strongly two variables are related.
- For example, correlation coefficient tells us how strongly
  income and food expenditure or
  crime rate and unemployment rate are related.

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13.1 Simple Linear Regression

- Simple Regression
- Linear Regression

Definition

- **Regression Model** is mathematical equation that describes relationship between two or more variables.
- ***Simple Regression*** model includes only two variables: one independent and one dependent.
- Dependent variable is the one being explained, and independent variable is the one used to explain the variation in dependent variable.

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Linear Regression

- Relationship between two variables in regression analysis is expressed by mathematical equation called a **regression equation or model.**
- **Regression equation, when plotted, may assume** one of many possible shapes, including straight line.
- Regression equation that gives straight-line relationship between two variables is called **Linear Regression Model; otherwise, m**odel is called **Nonlinear Regression Model.**
- **In this chapter, only linear regression models** are studied.

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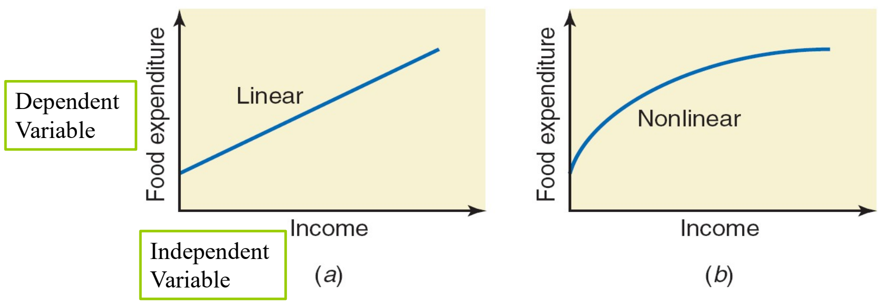
Linear Regression

Definition

- A (simple) regression model that gives a straight-line relationship between two variables is called a ***linear regression*** model.



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Linear Regression

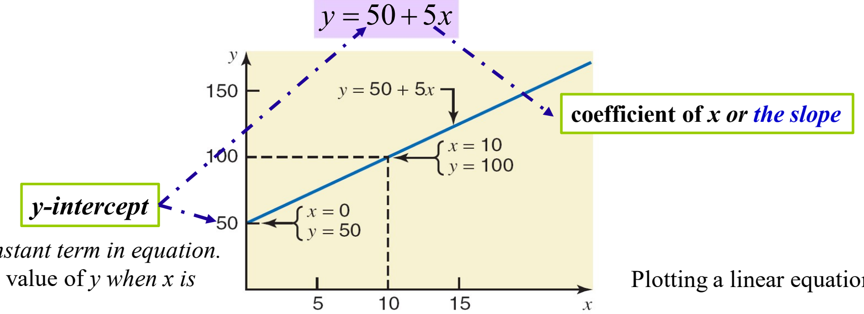
- Equation of a linear relationship between two variables *x and y is written as*

y-intercept **coefficient of *x or the slope***

$$y = a + bx$$

- Each set of values of ***a*** and ***b*** gives different straight line.
- For instance, when a = 50 and b = 5, this equation becomes

$$y = 50 + 5x$$



○ *constant term in equation.*
○*It is* value of *y when x is zero.*

Plotting a linear equation. 9

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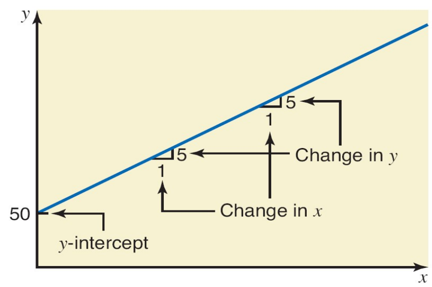
Figure 13.3 y-intercept and slope of a line.

$y = 50 + 5x$

y-intercept **coefficient of *x or the slope***



If $x = 10$, then $y = 50 + 5(10) = 100$.
If $x = 11$, then $y = 50 + 5(11) = 105$.

Hence, as x increases by 1 unit (from 10 to 11), y increases by 5 units (from 100 to 105). This is true for any value of x. Such changes in x and y are shown in Figure 13.3.

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SIMPLE LINEAR REGRESSION ANALYSIS

- Linear Regression Model

Constant term or y-intercept — Slope

$$\underset{\text{Dependent variable}}{y} = A + B\underset{\text{Independent variable}}{x}$$

Deterministic Model

(1)

- It gives **exact relationship between *x and y*.**
- This model simply states that *y is determined exactly by x, and for a given value of x there is* one and **only one** (unique) value of *y*.

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SIMPLE LINEAR REGRESSION ANALYSIS

- However, in many cases relationship between variables is not exact.

$$y = A + Bx \qquad (1)$$

- For instance, if y is food expenditure and x is income, then model (1) would state that food expenditure is determined by income only and that all households with the same income spend the same amount on food.

- As mentioned earlier, however, food expenditure is determined by **many variables**, only one of which is included in model (1).

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$$y = A + Bx \qquad (1)$$

SIMPLE LINEAR REGRESSION ANALYSIS

- In reality, different households with the same income spend different amounts of money on food because of differences in
  - sizes of household,
  - the assets they own, and
  - their preferences and tastes.

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SIMPLE LINEAR REGRESSION ANALYSIS

$$y = A + Bx \qquad (1)$$

- Hence, to take these variables into consideration and to make our model complete, we add **random error term** to right side of model (1).
- **It is denoted by ε *(Greek letter epsilon).***
- Complete regression model is written as

y-intercept | Slope | Independent variable

Dependent variable

$$y = A + Bx + \varepsilon$$

Random Error Term

Probabilistic Model or Statistical Relationship

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SIMPLE LINEAR REGRESSION ANALYSIS

- Regression line obtained for model (2) by using population data is called **Population Regression Line**.
- Values of A and B in population regression line are called true values of **y-intercept** and **slope**, respectively.

$$y = A + Bx + \varepsilon \qquad (2)$$

Population Parameters

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SIMPLE LINEAR REGRESSION ANALYSIS

- However, **population data** are **difficult to obtain**.
- As a result, we almost always use sample data to estimate model (2).
- Values of y-intercept and slope calculated from sample data on x and y are called estimated values of A and B and are denoted by a and b, respectively.

Estimated or Predicted Value of y $\hat{y} = a + bx$ **Estimated Regression Model** (3)

Definition

Estimates of *A* and *B* In the model $\hat{y} = a + bx$, a and b, which are calculated using sample data, are called the *estimates of A and B*, respectively.

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Table 13.1 Incomes (in hundreds of dollars) and Food Expenditures of Seven Households

- Suppose we take sample of seven households from small city and collect information on their incomes and food expenditures for the last month.

(in hundreds of dollars).

Income	Food Expenditure
55	14
83	24
38	13
61	16
33	9
49	15
67	17

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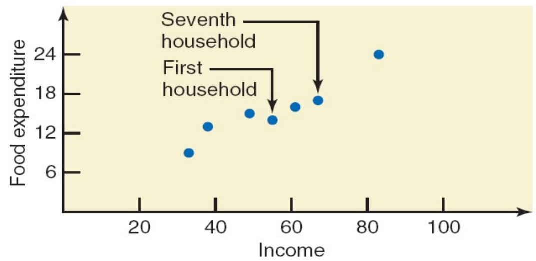
Scatter Diagram

- By plotting all seven pairs of values, we obtain **scatter diagram** or **scatterplot**.

Income	Food Expenditure
55	14
83	24
38	13
61	16
33	9
49	15
67	17



Definition

Scatter Diagram A plot of paired observations is called a *scatter diagram*.

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Figure 13.5 Scatter diagram and straight lines.

- Line obtained by using least squares method is called **Least Squares Regression line.**

$$\hat{y} = a + bx \quad \text{.........} (3)$$

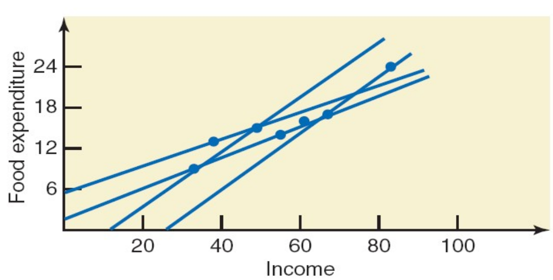

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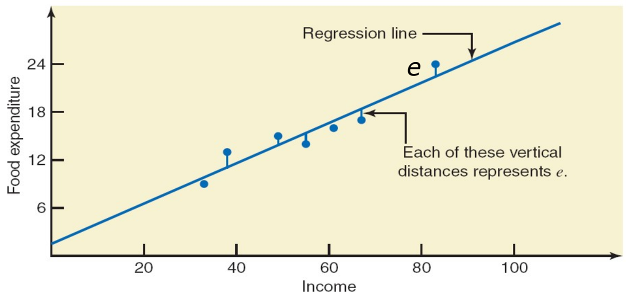
Figure 13.6 Regression Line and random errors.



Random error for sample regression model

e = Actual food expenditure − Predicted food expenditure = $y - \hat{y}$

estimator of ε

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Sum of Squares Error (SSE)

- ***Sum of Squares Error***, denoted SSE, is

$$SSE = \sum e^2 = \sum (y - \hat{y})^2$$

$\hat{y} = a + bx$

- Values of a and b that give minimum SSE are called ***Least Square Estimates*** of A and B, and regression line obtained with these estimates is called ***Least Squares Line***.

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Least Squares Line

- For the least squares regression line $\hat{y} = a + bx$,

$$b = \frac{SS_{xy}}{SS_{xx}} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

where

$$SS_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n} \quad \text{and} \quad SS_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$$

- SS stands for "Sum of Squares."
- Least squares regression line $\hat{y} = a + bx$ is also called **regression of y on x**.

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Least Squares Line

Step ① বের $\sum x$, $\sum y$, $\bar{x}$, $\bar{y}$

Step ② বের $\sum_{i=1}^{n} x_i y_i$, $\sum_{i=1}^{n} x_i^2$

Step ③ SS_{xy} এবং SS_{xx}

Step ④ $b = \frac{SS_{xy}}{SS_{xx}}$

$a = \bar{y} - b\bar{x}$

$$SS_{xy} = \sum_{i=1}^{n} xy - \frac{\left(\sum_{i=1}^{n} x\right)\left(\sum_{i=1}^{n} y\right)}{n}$$

$$SS_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x\right)^2}{n}$$

$\hat{y} = a + bx$,

Least squares regression line

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Example 13-1

- Find least squares regression line for data on incomes and food expenditure on seven households given in Table 13.1.
- Use income as independent variable and food expenditure as dependent variable.

Table 13.1 Incomes and Food Expenditures of Seven Households

Income	Food Expenditure
55	14
83	24
38	13
61	16
33	9
49	15
67	17

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Example 13-1

- We are to find values of a and b for regression model Table 13.2 shows calculations required for computation of a and b.
- We denote independent variable (income) by x and dependent variable (food expenditure) by y, both in hundreds of dollars.

Table 13.2

Income x	Food Expenditure y	xy	x^2
55	14	770	3025
83	24	1992	6889
38	13	494	1444
61	16	976	3721
33	9	297	1089
49	15	735	2401
67	17	1139	4489
$\Sigma x = 386$	$\Sigma y = 108$	$\Sigma xy = 6403$	$\Sigma x^2 = 23{,}058$

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Example 13-1: Solution

- Step 1. Compute $\sum x, \sum y, \bar{x}, \bar{y}$

Example 13-1: Solution

- Step 2. Compute $\sum xy, \sum x^2$

Example 13-1: Solution

- Step 3. Compute SS_{xy} *and* SS_{xx}

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Example 13-1: Solution

- Step 4. Compute *a* and *b*.

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Example 13-1: Solution

Estimated regression model $\hat{y} = a + bx$ is

Regression line $\hat{y} = 1.5050 + 0.2525\,x$ ⇨ least squares regression line

- It gives *regression of food expenditure on income.*

- Ex : suppose we randomly select household whose monthly income is $6100, so that $x = 61$ (recall that x denotes income in hundreds of dollars).

- **Predicted value** of food expenditure for this household is

$$\hat{y} = 1.5050 + (.2525)(61) = \$16.9075 \text{ hundred} = \$1690.75$$

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Figure 13.7 Error of prediction.

$$\hat{y} = 1.5050 + (.2525)(61) = \$16.9075 \text{ hundred} = \$1690.75$$

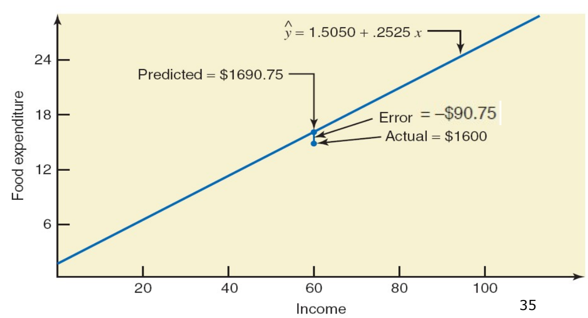
- In our data on seven households, there is one household whose income is $6100.
- **Actual** food expenditure for that household is $1600.
- Difference between actual and predicted values gives **error of prediction**.

$$e = y - \hat{y} = 16 - 16.9075 = -\$.9075 \text{ hundred} = -\$90.75$$

Table 13.1 Incomes and Food Expenditures of Seven Households

Income	Food Expenditure
55	14
83	24
38	13
61	16
33	9
49	15
67	17

y



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error of prediction for this household

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Interpretation of a and b

Table 13.1 Incomes and Food Expenditures of Seven Households

Income		Food Expenditure
55		14
83	maximum	24
38		13
61		16
33	minimum	9
49		15
67		17

$\hat{y} = a + bx$ ⇨ $\hat{y} = \mathbf{1.5050 + 0.2525}\, x$

Interpretation of *a*

- Consider a household with zero income.
- Using estimated regression line obtained in Example 13-1,
  - $\hat{y} = 1.5050 + 0.2525(0) = \$1.5050$ hundred.
- Thus, we can state that a household with no income is expected to spend $150.50 per month on food.
- Regression line is valid only for values of x between 33 and 83.

If we predict *y for value of x outside this* range, prediction usually will not hold true.

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Interpretation of a and b

Interpretation of *b*

- Value of b in regression model gives change in y (dependent variable) due to change of one unit in x (independent variable).
- We can state that, on average, a $100 (or $1) increase in income of a household will increase food expenditure by $25.25 (or $.2525).

When $x = 50$, $\hat{y} = 1.5050 + .2525(50) = 14.1300$

When $x = 51$, $\hat{y} = 1.5050 + .2525(51) = 14.3825$

- when x increased by one unit, from 50 to 51, increased by 14.3825 -14.1300 = 0.2525, which is the value of b.

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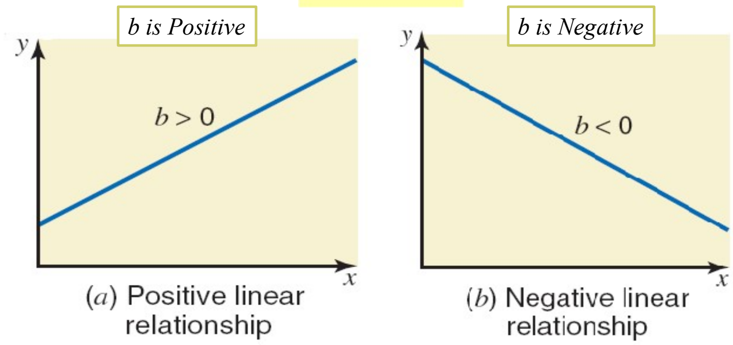
Figure 13.8 Positive and negative linear relationships between x and y.

$$\hat{y} = a + bx$$



when *b is positive, movements in x and y are in* the same direction

when *b is negative, movements in x and y are in opposit* direction

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Assumptions of Regression Model

- Like any other theory, linear regression analysis is also based on certain assumptions.
- Consider population regression model

$$y = A + Bx + \varepsilon$$

- Four assumptions are made about this model.

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Population Regression Model $\Rightarrow y = A + Bx + \varepsilon$

Assumptions of Regression Model

- Note that these assumptions are made about population regression model and not about the sample regression model.

- **Assumption 1:** Random error term ε has mean equal to zero for each x
- **Assumption 2:** Errors associated with different observations are independent
- **Assumption 3:** For any given x, distribution of errors is normal
- **Assumption 4:** Distribution of population errors for each x has the same (constant) standard deviation, which is denoted σ_ϵ

Assumptions of Regression Model

- Figure 13.11 illustrates the meanings of the first, third, and fourth assumptions for households with incomes of \$4000 and \$7500 per month.

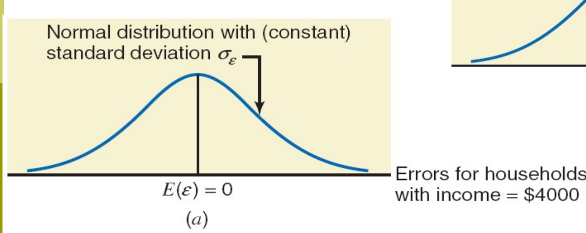

Figure 13.11 (a) Errors for households with income of \$4000 per month.

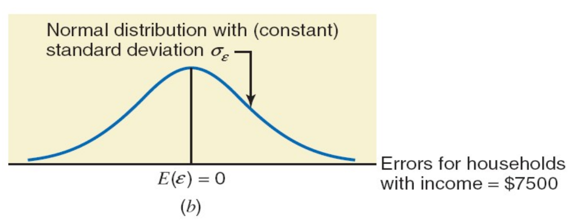

Figure 13.11 (b) Errors for households with income of \$ 7500 per month.

Assumptions of Regression Model

- Points on vertical line through $x = 40$ give food expenditures for various households in population, each of which has the same monthly income of $4000.
- The same is true about vertical line through $x = 75$ or any other vertical line for some other value of x.

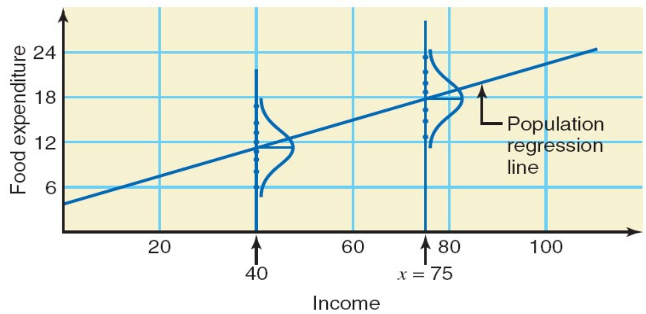

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Figure 13.12 Distribution of errors around population regression line.

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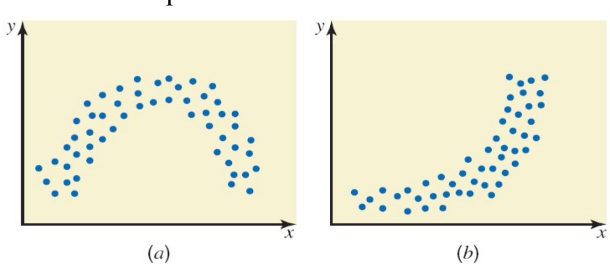
Note on the Use of Simple Linear Regression

- We should apply linear regression with caution.
- When we use simple linear regression, we assume that relationship between two variables is described by straight line.
- In real world, relationship between variables may not be linear.
- Hence, before we use simple linear regression, it is better to construct **scatter diagram** and look at plot of data points.
- We should estimate linear regression model only if scatter diagram indicates such relationship.



- Scatter diagrams give two examples for which relationship between x and y is not linear.
- Consequently, fitting linear regression in such cases would be wrong.

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Figure 13.13 Nonlinear relations between x and y.

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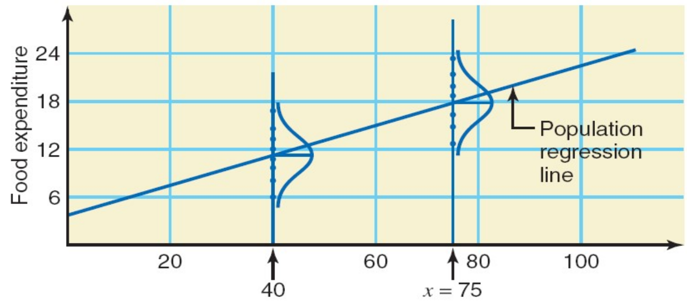
13.2 Standard Deviation of Errors and Coefficient of Determination

- **Standard deviation of errors tells us how widely** errors and values of *y are spread for a given x*.
- **Degrees of Freedom for Simple Linear Regression Model**
- *Degrees of freedom* for simple linear regression model are ***df* = *n* – 2**



- Standard deviation of errors σ_ϵ measures spread of such points around population regression line

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STANDARD DEVIATION OF ERRORS AND COEFFICIENT OF DETERMINATION

- ***Standard Deviation of Errors*** is calculated as

$$s_e = \sqrt{\frac{SS_{yy} - bSS_{xy}}{n-2}}$$

where

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} \qquad b = \frac{SS_{xy}}{SS_{xx}}$$

$$SS_{xy} = \sum xy - \frac{\left(\sum x\right)\left(\sum y\right)}{n} \qquad SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$$

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STANDARD DEVIATION OF ERRORS AND COEFFICIENT OF DETERMINATION

- ขั้นตอนการหา $S_e = \sqrt{\frac{SS_{yy} - bSS_{xy}}{n-2}}$

Example 13-2

- Compute standard deviation of errors s_e for data on monthly incomes and food expenditures of the seven households given in Table 13.1.

Table 13.1 Incomes and Food Expenditures of Seven Households

Income	Food Expenditure
55	14
83	24
38	13
61	16
33	9
49	15
67	17

Income x	Food Expenditure y	y^2
55	14	196
83	24	576
38	13	169
61	16	256
33	9	81
49	15	225
67	17	289
$\Sigma x = 386$	$\Sigma y = 108$	$\Sigma y^2 = 1792$

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Example 13-2: Solution

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Example 13-2: Solution

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Example 13-2: Solution

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Example 13-2: Solution

- To compute se, we need to know the values of SS_{yy}, SS_{xy}, and b.
- In Example 13–1, we computed SS_{xy} and b.
- These values are

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} = 6403 - \frac{(386)(108)}{7} = 447.5714$$

$$SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 23{,}058 - \frac{(386)^2}{7} = 1772.8571$$

$$b = \frac{SS_{xy}}{SS_{xx}} = \frac{447.5714}{1772.8571} = 0.2525$$

$$s_e = \sqrt{\frac{SS_{yy} - bSS_{xy}}{n-2}}$$

$$SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = 1792 - \frac{(108)^2}{7} = 125.7143$$

$$s_e = \sqrt{\frac{SS_{yy} - bSS_{xy}}{n-2}} \sqrt{\frac{125.7143 - .2525(447.5714)}{7-2}} = 1.5939$$

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COEFFICIENT OF DETERMINATION

- How good is regression model?
- In other words: How well does independent variable explain dependent variable in regression model?
- Coefficient of determination is one concept that answers this question.

- **Sum of Squares Total (SST)**
- ***Sum of squares total***, denoted by **SST**, is calculated as

$$SST = SS_{yy} = \Sigma(y - \bar{y})^2 \quad \Rightarrow \quad SST = \sum y^2 - \frac{\left(\sum y\right)^2}{n}$$

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COEFFICIENT OF DETERMINATION

- Value of SS_{yy}, which is 125.7143, was calculated in Example 13–2. Consequently, the value of SST is

$$SST = SS_{yy} = \Sigma(y - \bar{y})^2 \quad \Rightarrow \quad \boxed{SST = 125.7143}$$

$$\hat{y} = 1.5050 + .2525x$$

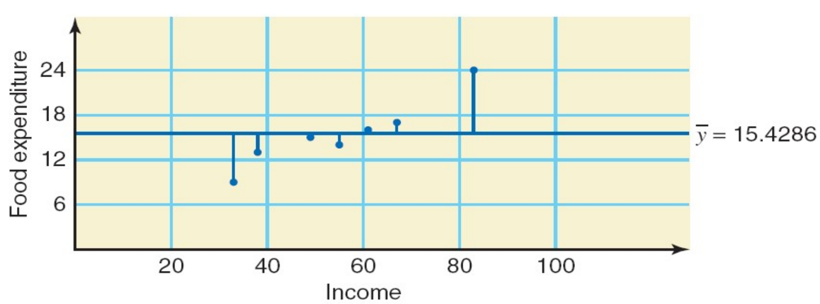

Figure 13.15 Total errors.

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COEFFICIENT OF DETERMINATION

Predicted food expenditures → $\hat{y} = 1.5050 + .2525x$

errors and error squares

x	y	$\hat{y} = 1.5050 + .2525x$	$e = y - \hat{y}$	$e^2 = (y - \hat{y})^2$
55	14	15.3925	−1.3925	1.9391
83	24	22.4625	1.5375	2.3639
38	13	11.1000	1.9000	3.6100
61	16	16.9075	−.9075	.8236
33	9	9.8375	−.8375	.7014
49	15	13.8775	1.1225	1.2600
67	17	18.4225	−1.4225	2.0235
				$\Sigma e^2 = \Sigma(y - \hat{y})^2 = 12.7215$

Sum of Square Error → $SSE = \Sigma(y - \hat{y})^2 = 12.7215$

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COEFFICIENT OF DETERMINATION

Sum of Error $SSE = \Sigma(y - \hat{y})^2 = 12.7215$

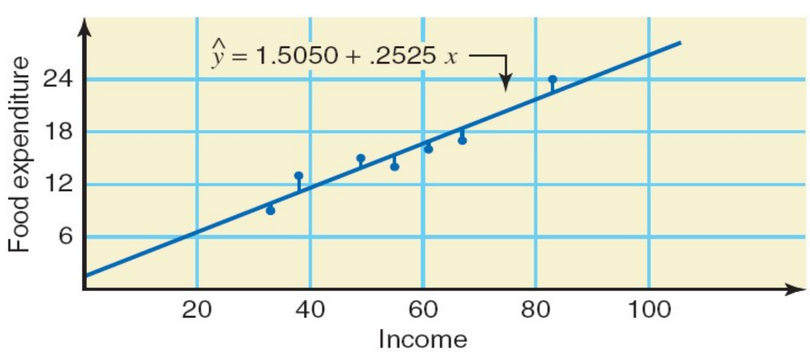

Figure 13.16 Errors of prediction when regression model is used.

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COEFFICIENT OF DETERMINATION

- Thus, from the foregoing calculations,

SST = 125.7143 and **SSE = 12.7215**

These values indicate that the sum of squared errors decreased from 125.7143 to 12.7215 when we used $\hat{y}$ in place of $\bar{y}$ to predict food expenditures. This reduction in squared errors is called the **regression sum of squares** and is denoted by **SSR**. Thus,

$$SSR = SST - SSE = 125.7143 - 12.7215 = 112.9928$$

The value of SSR can also be computed by using the formula

$$SSR = \Sigma(\hat{y} - \bar{y})^2$$

Regression Sum of Squares (SSR)

The ***regression sum of squares*** , denoted by **SSR**, is

$$SSR = SST - SSE$$

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COEFFICIENT OF DETERMINATION

- Ratio of SSR to SST gives coefficient of determination.
- Coefficient of determination calculated for population data is denoted by ρ^2 .
- Coefficient of determination calculated for sample data is denoted by r^2.
- Coefficient of determination gives the proportion of SST that is explained by the use of the regression model.

$$r^2 = \frac{SSR}{SST} \quad \text{or} \quad \frac{SST - SSE}{SST} \longrightarrow \text{range zero to one.}$$

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COEFFICIENT OF DETERMINATION

- **Coefficient of Determination**
- The ***coefficient of determination***, denoted by r^2, represents the proportion of SST that is explained by the use of the regression model.
- The computational formula for r^2 is

$$r^2 = \frac{bSS_{xy}}{SS_{yy}}$$

and $0 \le r^2 \le 1$

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Example 13-3

- For the data of Table 13.1 on monthly incomes and food expenditures of seven households, calculate coefficient of determination.
- From earlier calculations made in Examples 13-1 and 13-2,
- $\boldsymbol{b = 0.2525, \quad SS_{xx} = 447.5714, \quad SS_{yy} = 125.7143}$

$$r^2 = \frac{b\ SS_{xy}}{SS_{yy}} = \frac{(.2525)(447.5714)}{125.7143} = .90$$

- Thus, we can state that SST is reduced by approximately 90% (from 125.7143 to 12.7215) when we use $\hat{y}$ instead of $\bar{y}$ to predict the food expenditures of households.

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13.4 Linear Correlation

- Linear Correlation Coefficient
- Hypothesis Testing About the Linear Correlation Coefficient

- Another measure of relationship between two variables is correlation coefficient.
- This section describes simple linear correlation (linear correlation), which measures strength of linear association between two variables.
- In other words, linear correlation coefficient measures how closely points in scatter diagram are spread around regression line.

Value of Correlation Coefficient

The ***value of the correlation coefficient*** always lies in the range of –1 to 1; that is,

$$-1 \le \rho \le 1 \quad \text{and} \quad -1 \le r \le 1$$

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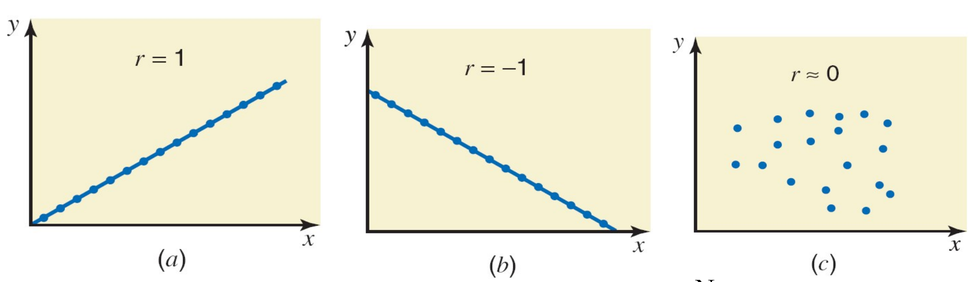
Figure 13.18 Linear correlation between two variables.



Perfect positive linear correlation, $r = 1$

Perfect negative linear correlation, $r = -1$

No linear correlation, , $r \approx 0$

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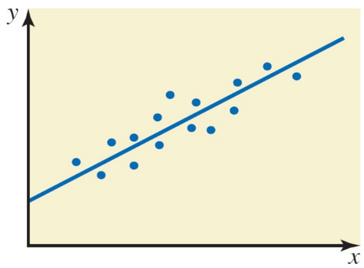
Figure 13.19 Linear correlation between variables.



(a) Strong positive linear correlation
(r is close to 1)

(b) Weak positive linear correlation
(r is positive but close to zero)

If correlation between two variables is positive and **close to 1**, we say that the variables have a ***strong positive linear correlation.***

If correlation between two variables is positive but **close to zero**, then the variables have a ***weak positive linear correlation***.

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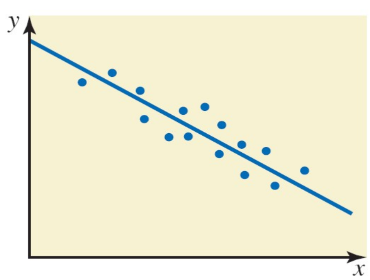
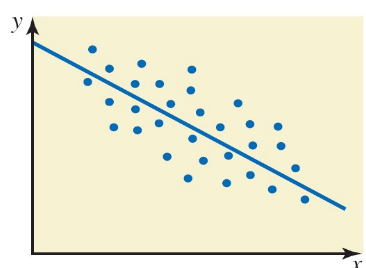
Figure 13.19 Linear correlation between variables.



(c) Strong negative linear correlation
(r is close to –1)



(d) Weak negative linear correlation
(r is negative and close to zero)

if correlation between two variables is **negative** and **close to -1**, then variables are said to have ***strong negative linear correlation.***

If correlation between two variables is **negative** but **close to zero**, there exists a ***weak negative linear correlation*** *between the variables.*

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Linear Correlation Coefficient

- **Simple linear correlation coefficient**, denoted by r, measures strength of linear relationship between two variables for **sample** and is calculated as

Sample

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

Population

$$\rho = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

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Example 13-6

- Calculate correlation coefficient for example on incomes and food expenditures of seven households.
- From earlier calculations made in Examples 13–1 and 13–2,

$$SS_{xy} = 447.5714, \quad SS_{xx} = 1772.8571, \quad \text{and} \quad SS_{yy} = 125.7143$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{447.5714}{\sqrt{(1{,}772.8571)(125.7143)}} = 0.95$$

- Linear correlation coefficient is 0.95.
- Correlation coefficient of 0.95 for incomes and food expenditures of seven households indicates that income and food expenditure are **very strongly and positively correlated.**

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Hypothesis Testing about Linear Correlation Coefficient

- how to perform test of hypothesis about population correlation coefficient ρ using sample correlation coefficient r.
- We can use ***t* distribution** to make this test.
- However, to use t distribution, both variables should be normally distributed.

Test Statistic for *r*

If both variables are normally distributed and null hypothesis is H_0: $\rho = 0$, then value of test statistic t is calculated as

$$t = r\sqrt{\frac{n-2}{1-r^2}}$$

$H_1 : \rho < 0$

$H_1 : \rho > 0$

$H_1 : \rho \neq 0$

Here $n - 2$ are the degrees of freedom.

Example 13-7

- Using the 1% level of significance and data from Example 13-1, test whether linear correlation coefficient between incomes and food expenditures is **positive**.
- Assume that populations of both variables are normally distributed.

Solution : From Examples 13–1 and 13–6,

$$n = 7 \quad \text{and} \quad r = 0.95$$

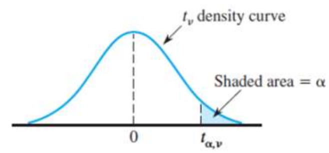
Example 13-7: Solution

Example 13-7: Solution

Example 13-7: Solution

Example 13-7: Solution

Table A.5 Critical Values for t Distributions



ν \ α	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041

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End of Simple Linear Regression

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