3 Discrete Random Variables and Probability Distributions

Introduction

- whether experiment yields qualitative or quantitative outcomes, methods of statistical analysis require that we focus on certain numerical aspects of data such as
  - Sample proportion x/n
  - Mean $\overline{X}$
  - Standard deviation S
- The concept of random variable allows us to pass from

Experimental Outcomes → Numerical Outcomes

- There are two fundamentally different types of Random Variable
  - Discrete Random Variables (ตัวแปรสุ่มแบบไม่ต่อเนื่อง)
  - Continuous Random Variables (ตัวแปรสุ่มแบบต่อเนื่อง)

3.1 Random Variables

Random Variables

- In any experiment, there are numerous characteristics that can be observed or measured, but in most cases experimenter will focus on some specific aspect or aspects of a sample
- For example : in study of commuting patterns in a metropolitan area
  - Each individual in sample might be asked about
    - commuting distance
    - number of people commuting in same vehicle
  - but not about
    - IQ
    - Income
    - Family size, and other such characteristics

- Alternatively, researcher may test a sample of components and record only the number that have failed within 1000 hours, rather than record the individual failure times

4

Random Variables

- In general, each outcome of experiment can be associated with a number by specifying a rule of association e.g.,
  - Number among sample of ten components that fail to last 1000 hours or
  - Total weight of baggage for a sample of 25 airline passengers
- Such a rule of association is called a **Random** **Variable**

because observed value depends on which of possible experimental outcomes results

because different numerical values are possible

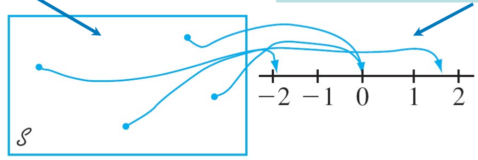


Figure 3.1 A random variable

Random Variables

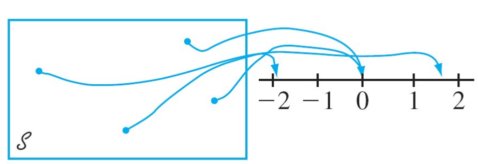
Definition

For given sample space $\mathcal{S}$ of some experiment,

Random Variable (rv) is any rule that associates a number with each outcome in $\mathcal{S}$.

In mathematical language,

Random Variable is a function whose domain is sample space and whose range is set of real numbers.



Random Variables

○ Random variables are customarily denoted by uppercase letters, such as *X* and *Y*, near the end of our alphabet.

○ In contrast to our previous use of a lowercase letter, such as *x*, to denote a variable, we will now use lowercase letters to represent some particular value of corresponding random variable.

○The notation *X*(*s*) = *x means that x is value associated with outcome s by random variable X.*

Random variable *X* → $X(s) = x$ ← *value associated with outcome s*

outcome s

7

Example 1

When a student calls a university help desk for technical support, he/she will either immediately be able to speak to someone (*S*, for success) or will be placed on hold (*F*, for failure).

With $\mathcal{S}$ = {*S*, *F*}, define an rv *X* by

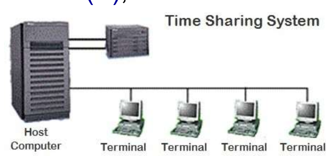
$X(S) = 1 \qquad X(F) = 0$

The rv *X* indicates whether (1) or not (0) student can immediately speak to someone.

8

Example 3.1

- When student attempts to log on to a computer time-sharing system, either
  all ports are busy (F) in which case student will fail to obtain access, or else
  there is at least one port free (S), in which case student will be successful in accessing the system



With $\mathscr{S}$ = {S, F}, define an rv X by $X(S) = 1$ $X(F) = 0$

rv X indicates whether (1) or not (0) student can log on

rv X in ex. 3.1 was specified by explicitly listing each element of $\mathscr{S}$ and associated number

Such listing is tedious if $\mathscr{S}$ contains more than a few outcomes, but it can frequently be avoided

9

Example 3.2

Consider the experiment in which a telephone number in a certain area code is dialed using a random number dialer (such devices are used extensively by polling organizations), and define an rv Y by

$$Y = \begin{cases} 1 \text{ if the selected number is unlisted} \\ 0 \text{ if the selected number is listed in the directory} \end{cases}$$

<u>For example</u>

If 5282966 appears in the telephone directory, then Y(5282966) = 0, whereas Y(7727350) = 1 tells us that number 7727350 is unlisted

A word description of this sort is more economical than a complete listing, so we will use such a description whenever possible

In ex.3.1 and 3.2, the only possible values of this random variable were 0 and 1

Such a random variable arises frequently enough to be given a special name, after the individual who first studied it.

10

Random Variables

Definition

Any random variable whose only possible values are

0 and 1

is called

Bernoulli Random Variable

Random variable X → $X(s) = x$ ← *value associated with outcome s*

outcome s

11

We will sometimes want to define and study several different random variables from the same sample space

Example 3.3 described experiment in which the number of pumps in use at each of two six-pump gas stations was determined. Define rv's *X, Y*, and *U* by

X = Total number of pumps in use at two stations

Y = Difference between the number of pumps in use at station 1 and the number in use at station 2

U = Maximum of numbers of pumps in use at two stations

outcome

If this experiment is performed and s = (2,3) results, then

$X(s) = x$ ⇨ $X((2,3)) = 2+3 = 5$ ⇨ The observed value of X was x = 5

$Y(s) = y$ ⇨ $Y((2,3)) = 2-3 = -1$ ⇨ The observed value of Y was y = -1

$U(s) = u$ ⇨ $U((2,3)) = \max(2,3) = 3$ ⇨ The observed value of U was u = 3

2

Example 3.4

In Example 2.4, we considered the experiment in which batteries were examined until a good one (S) was obtained

The sample space was $\mathcal{S} = \{S, FS, FFS, FFFS, \ldots\}$

Define an random variable (rv) X by

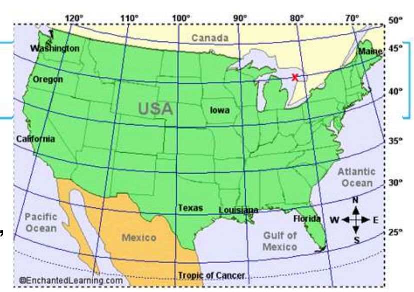
X = Number of batteries examined before the experiment terminates

Then

$$X(S) = 1$$
$$X(FS) = 2$$
$$X(FFS) = 3$$
$$\vdots$$
$$X(FFFFFFS) = 7$$
$$\vdots$$

13

Example 3.5



Suppose that in some random fashion, a location (latitude and longitude) in the continental United States in selected,

Define an random variable (rv) Y by

Y = The height above sea level at the selected location

For example, if selected location were (39^0 $50^/$ N, 98^0 $35^/$ W), then we might have

$$Y((39^0 50' N, 98^0 35' W)) = 1{,}748.26\ ft$$

The largest possible value of Y is 14,494 (Mt. Whitney)

The smallest possible value of Y is -282 (Death Valley)

The set of all possible values of Y is the set of all numbers in the interval between -282 and 14,494

$$\{y : y \text{ is a number}, -282 \le y \le 14{,}494\}$$

and there are an infinite number of numbers in this interval

14

Two Types of Random Variables

Two Types of Random Variables

- **Discrete** Random Variable (ตัวแปรสุ่มแบบไม่ต่อเนื่อง)

- **Continuous** Random Variable (ตัวแปรสุ่มแบบต่อเนื่อง)

Definition

Discrete random variable is an random variable whose possible values either constitute a finite set or else can be listed in an infinite sequence in which there is a first element, a second element, and so on ("countably" infinite).

Two Types of Random Variables

Definition

Random variable is **continuous** if *both* of the following apply:

1. Its set of possible values consists either of
all numbers in a single interval on the number line
(possibly infinite in extent, e.g., from $-\infty$ to ∞) or
all numbers in a disjoint union of such intervals
(e.g., $[0, 10] \cup [20, 30]$).

2. No possible value of the variable has positive probability,
that is, $P(X = c) = 0$ for any possible value c.

17

Example 3.6

All random variables in Examples 3.1 –3.4 are discrete.

As another example, suppose we select married couples at random and do a blood test on each person until we find a husband and wife who both have the same Rh factor.

With
X = the number of blood tests to be performed, possible values of X are $D = \{2, 4, 6, 8, \ldots\}$.

Since the possible values have been listed in sequence, X is a discrete rv.

19

3.2 Probability Distributions for Discrete Random Variables

การแจกแจงความน่าจะเป็นสำหรับตัวแปรสุ่มแบบไม่ต่อเนื่อง

Probability Distributions for Discrete Random Variables

Probabilities assigned to various outcomes in $\mathcal{S}$ in turn determine probabilities associated with values of any particular random variable *X*.

Probability Distribution of X says how the total probability of 1 is distributed among (allocated to) the various possible *X* values.

Probability Distributions for Discrete Random Variables

Suppose, for example, that a business has just purchased four laser printers, and

let X be number among these that require service during warranty period.

Possible X values are then 0, 1, 2, 3, and 4.

Probability distribution will tell us

- how probability of 1 is subdivided among these five possible values
- how much probability is associated with the X value 0,
- how much is apportioned to the X value 1, and so on.

We will use following notation for probabilities in distribution:

$p(0)$ = Probability of X value 0 = $P(X = 0)$

$p(1)$ = Probability of X value 1 = $P(X = 1)$ and so on.

In general, $p(x)$ will denote probability assigned to the value x.

$P(X = x) = p(x)$

22

Example 3.7

The Cal Poly Department of Statistics has a lab with six computers reserved for statistics majors.

Let X denote number of these computers that are in use at a particular time of day.

Suppose that probability distribution of X is as given in following table;

Random variable

possible X values

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

$P(X = x)$

probability of each such value

23

Example 3.7

cont'd

We can now use elementary probability properties to calculate other probabilities of interest.

มากที่สุด

For example, probability that **at most** 2 computers are in use is

	x	0	1	2	3	4	5	6
$P(X = x) \longrightarrow$	$p(x)$	.05	.10	.15	.25	.20	.15	.10

Example 3.7

cont'd

อย่างน้อย

Since the event *at least 3 computers are in use* is complementary to *at most 2 computers are in use,*

which can, of course, also be obtained by adding together probabilities for the values, 3, 4, 5, and 6.

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

or

Example 3.7

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

cont'd

(รวมทั้ง)

Probability that between 2 and 5 computers *inclusive* are in use is

whereas the probability that the number of computers in use is *strictly between* 2 and 5 is

28

Probability Distributions for Discrete Random Variables

Definition การแจกแจงความน่าจะเป็น หรือ ฟังก์ชันมวลของความน่าจะเป็น

Probability Distribution or **Probability Mass Function** (pmf) of discrete random variable is defined for every number x by

outcome | Random variable X | *value associated with outcome s*

$$p(x) = P(X = x) = P(all\ s \in \mathcal{S}: X(s) = x)$$

In words, for every possible value x of random variable, **pmf** specifies **probability of observing that value** when the experiment is performed.

The conditions $p(x) \geq 0$ and $\sum_{all\ possible\ x} p(x) = 1$ are required of any pmf.

30

Example 3.8

- Six lots of components are ready to be shipped by a certain supplier.
- Number of defective components in each lot is as follows:

Lot	1	2	3	4	5	6
Number of defectives	0	2	0	1	2	0

- One of these lots is to be randomly selected for shipment to particular customer
- Let X be number of defectives in selected lot
- Three possible X values are 0, 1, and 2

$$X(1,3,6) = 0)$$
$$X(4) = 1)$$
$$X(2,5) = 2)$$

$$p(x) = P(X = x) = P(all\ s \in \{1,2,3,4,5,6\} : X(s) = x)$$

Cont.

Example 3.8

$$p(x) = P(X = x) = P(all\ s \in \{1,2,3,4,5,6\} : X(s) = x)$$

- Of the six equally likely simple events,

Three result in X=0

One in X=1

Lot	1	2	3	4	5	6
Number of defectives	0	2	0	1	2	0

Two in X=2

- Then

$$p(0) = P(X = 0) = P(lot\ 1\ or\ 3\ or\ 6\ is\ sent) = \frac{3}{6} = 0.500$$
Probability of 0.5 is distributed to X value 0

$$p(1) = P(X = 1) = P(lot\ 4\ is\ sent) = \frac{1}{6} = 0.167$$
Probability of 0.167 is placed on X value 1

$$p(2) = P(X = 2) = P(lot\ 2\ or\ 5\ is\ sent) = \frac{2}{6} = 0.333$$
Probability of 0.333 is associated with X value 2

- Value of X along with their probabilities collectively specify the pmf
- If this experiment were repeated over and over again, in the long run
  - X = 0 occur one-half of the time,
  - X = 1 one-sixth of the time, and
  - X = 2 one-third of the time

Example 3.9

$p(x) = P(X = x) = P(all\ s \in \{desktop, laptop\} : X(s) = x)$

- Consider whether the next person buying a computer at a university book store buys a laptop or a desktop model

  Let

$$X = \begin{cases} 1 & if\ the\ customer\ purchases\ a\ laptop\ computer \\ 0 & if\ the\ customer\ purchases\ a\ desktop\ computer \end{cases}$$

  - If 20% of all purchasers during that week select a laptop, the pmf for X is

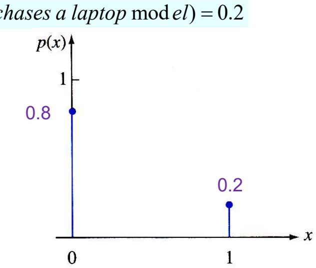
$$p(0) = P(X = 0) = P(next\ customer\ purchases\ a\ desktop\ \text{model}) = 0.8$$

$$p(1) = P(X = 1) = P(next\ customer\ purchases\ a\ laptop\ \text{model}) = 0.2$$

$$p(x) = P(X = x) = 0\ for\ x \neq 0\ or\ 1$$

- An equivalent description is

$$p(x) = \begin{cases} 0.8 & if\ x = 0 \\ 0.2 & if\ x = 1 \\ 0 & if\ x \neq 0\ or\ 1 \end{cases}$$



Line graph for pmf of Example 3.9

35

Example 3.10

$p(y) = P(Y = y) = P(all\ s \in \{a, b, c, d, e\} : Y(s) = y)$

- Consider a group of five potential blood donors – a, b, c, d, and e – of whom only a and b have type O+ blood.
- Five blood samples, one from each individual, will be typed in random order until and O+ individual is identified
- Let rv Y = Number of typings necessary to identify an O+ individual.
- Then the pmf of Y is

$$p(1) = P(Y = 1) = P(a\ or\ b\ typed\ first) = \frac{2}{5} = 0.4$$

$$p(2) = P(Y = 2) = P(c, d, or\ e\ first, and\ then\ a\ or\ b)$$

$$= P(c, d, or\ e\ first) \cdot P(a\ or\ b\ next \mid c, d, or\ e\ first) = \frac{3}{5} \cdot \frac{2}{4} = 0.3$$

$$p(3) = P(Y = 3) = P(c, d, or\ e\ first\ and\ \text{second}, and\ then\ a\ or\ b)$$

$$= \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right) = 0.2$$

$$p(4) = P(Y = 4) = P(c, d, and\ e\ all\ done\ first) = \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) = 0.1$$

$$p(y) = 0 \quad if\ y \neq 1, 2, 3, 4$$

36

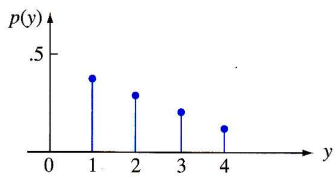
Cont.

Example 3.10

- In tabular form, the pmf is

y	1	2	3	4
p(y)	0.4	0.3	0.2	0.1

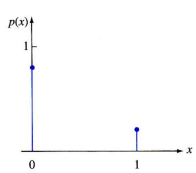
- where any y value not listed receives zero probability



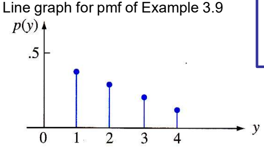
Line graph for pmf in Example 3.10

Probability Histogram

- Another useful pictorial representation of a pmf, called a **Probability Histogram**, is similar to histograms

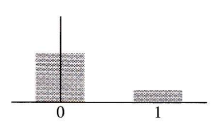


Line graph for pmf of Example 3.9

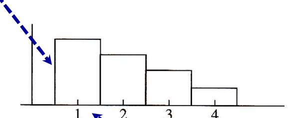


Line graph for pmf in Example 3.10

Height of each rectangle is proportional to p(y), and base is the same for all rectangles



Probability histogram for Example 3.9



Probability histogram for Example 3.10

above each y with p(y) > 0, construct a rectangle centered at y.

- When possible values are equally spaced, base is frequently chosen as distance between successive y values

Parameter of a Probability Distribution

Parameter of a Probability Distribution

- In Example 3.9, the pmf of Bernoulli rv X was
  - $p(0) = 0.8$ and
  - $p(1) = 0.2$ because 20% of all purchasers selected a desktop computer.
- At another store, it may be the case that
  - $p(0) = 0.9$ and
  - $p(1) = 0.1$.
- More generally, pmf of any Bernoulli rv can be expressed in the form
  - $p\ (1) = \alpha$ and
  - $p\ (0) = 1 - \alpha$, where $0 < \alpha < 1$.
- Because pmf depends on the particular value of α we often write

$p\ (x;\ \alpha)$ rather than just **$p\ (x)$**:

$$p(x; \alpha) = \begin{cases} 1 - \alpha & \text{if } x = 0 \\ \alpha & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

Parameter of a Probability Distribution

$$p(x;\alpha) = \begin{cases} 1-\alpha & \text{if } x = 0 \\ \alpha & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

- Quantity α in Expression (3.1) is a parameter.
- Each different number α between 0 and 1 determines different member of the family of distributions
- Two such number are

$$p(x;0.6) = \begin{cases} 0.4 & if\ x=0 \\ 0.6 & if\ x=1 \\ 0 & otherwise \end{cases}$$ and $$p(x;0.5) = \begin{cases} 0.5 & if\ x=0 \\ 0.5 & if\ x=1 \\ 0 & otherwise \end{cases}$$

- Each probability distribution for a Bernoulli random variable has the form of Expression (3.1), so it is called the **Bernoulli Family of Distributions**

Example 3.12

Starting at a fixed time, we observe the gender of each newborn child at a certain hospital until a boy (*B*) is born.

Let *p* = *P* (*B*),

assume that successive births are independent, and define random variable *X* by *x* = number of births observed.

$$p(1) = P(X=1) = P(B) = p$$

$$p(2) = P(X=2) = P(GB) = P(G)\cdot P(B) = (1-p)\cdot p$$

$$p(3) = P(X=3) = P(GGB) = P(G)\cdot P(G)\cdot P(B) = (1-p)^2\cdot p$$

$$\vdots$$

Continuing in this way, a general formula emerges:S

Quantity p represents number between 0 and 1 and is **Parameter of Probability Distribution**

$$p(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, 3, \dots \\ 0 & otherwise \end{cases} \qquad (3.2)$$

Expression (3.2) describes the family of ***Geometric Distributions***.

Cumulative Distribution Function

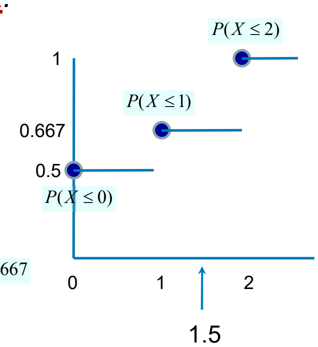
ฟังก์ชันการแจกแจงสะสม

46

Cumulative Distribution Function

- For some fixed value *x*, we often wish to compute probability that observed value of *X* will be **at most *x***.
- For example, pmf in Example 3.8 was

$$p(x)=\begin{cases}0.500 & x=0\\ 0.167 & x=1\\ 0.333 & x=2\\ 0 & otherwise\end{cases}$$



- Probability that X is at most 1 is then

$$P(X \le 1) = P(X=0)+P(X=1)=p(0)+p(1)=0.500+0.167=0.667$$

- In this example, $P(X \le 1.5)\ iff\ X \le 1$, so

$$P(X \le 1.5) = P(X \le 1) = 0.667$$

- Similarly,

$$P(X \le 0) = P(X=0) = 0.5$$
$$P(X \le 0.75) = P(X=0) = 0.5$$

47

Cumulative Distribution Function

Definition

Cumulative Distribution Function (cdf) *F*(*x*) of a discrete rv variable *X* with pmf *p*(*x*) is defined for every number *x* by

$$F(x) = P(X \leq x) = \sum_{y:y \leq x} p(y) \qquad \textbf{(3.3)}$$

For any number x, F(x) is the probability that the observed value of X will be **at most x**

49

Example 3.13

A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory.

The accompanying table gives distribution of *Y* = the amount of memory in a purchased drive:

ขนาดของหน่วยความจำของ Flash Drive ที่ขาย

y	1	2	4	8	16
$p(y)$	.05	.10	.35	.40	.10

$$F(y) = P(Y \leq y)$$

Cumulative Distribution Function (cdf)

50

Example 3.13

cont'd

Let's first determine $F(y)$ for each of the five possible values of Y:

y	1	2	4	8	16
$p(y)$	.05	.10	.35	.40	.10

$F(y) = P(Y \le y)$

$$\begin{aligned} F(1) &= P(Y \le 1) \\ &= P(Y = 1) \\ &= p(1) \\ &= 0.05 \end{aligned}$$

$$\begin{aligned} F(2) &= P(Y \le 2) \\ &= P(Y = 1 \text{ or } 2) \\ &= p(1) + p(2) = 0.05+0.10 \\ &= 0.15 \end{aligned}$$

Example 3.13

cont'd

y	1	2	4	8	16
$p(y)$	.05	.10	.35	.40	.10

$F(y) = P(Y \le y)$

$$\begin{aligned} F(4) &= P(Y \le 4) \\ &= P(Y = 1 \text{ or } 2 \text{ or } 4) \\ &= p(1) + p(2) + p(4) \\ &= 0.05 + 0.10 + 0.35 \\ &= 0.50 \end{aligned}$$

$$\begin{aligned} F(8) &= P(Y \le 8) \\ &= p(1) + p(2) + p(4) + p(8) \\ &= 0.05 + 0.10 + 0.35 + 0.40 + 0.10 \\ &= 0.90 \end{aligned}$$

$$\begin{aligned} F(16) &= P(Y \le 16) \\ &= \mathrm{p}(1) + \mathrm{p}(2) + \mathrm{p}(4) + \mathrm{p}(8) + \mathrm{p}(16) \\ &= 0.05 + 0.10 + 0.35 + 0.40 + 0.10 \\ &= 1 \end{aligned}$$

Example 3.13

cont'd

Now for any other number y,

$F(y)$ will equal the value of F at the closest possible value of Y to the left of y.

For example,

y	1	2	4	8	16
$p(y)$	.05	.10	.35	.40	.10

$$F(y) = P(Y \le y)$$

$$\begin{aligned} F(2.7) &= P(Y \le 2.7) \\ &= P(Y \le 2) \\ &= F(2) \\ &= 0.15 \end{aligned}$$

$$\begin{aligned} F(7.999) &= P(Y \le 7.999) \\ &= P(Y \le 4) \\ &= F(4) \\ &= 0.50 \end{aligned}$$

Example 3.14 In Example 3.12, any positive integer was a possible X value, and the pmf was

$$p(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

For any positive integer x,

$$F(x) = \sum_{y \le x} p(y) = \sum_{y=1}^{x} (1-p)^{y-1} p = p \sum_{y=0}^{x-1} (1-p)^y \qquad \textbf{(3.4)}$$

$$F(x) = p \cdot \frac{1-(1-p)^x}{1-(1-p)} = 1 - (1-p)^x \quad x \text{ a positive integer}$$

Cont.

Example 3.14

Since F is constant in between positive integers,

$$F(x) = \begin{cases} 0 & x < 1 \\ 1 - (1-p)^{[x]} & x \geq 1 \end{cases} \qquad (3.5)$$

where $[x]$ is the largest integer $\leq x$ (e.g., $[2.7] = 2$).

if $p = 0.51$ as in the birth example, then probability of having to examine at most five births to see the first boy is F(5)

$F(5) = 1-(1-0.51)^5 = 1-0.49^5 = 1-0.0282 = 0.9718$

$F(10) = 1-(1-0.51)^{10} = 1-0.49^{10} = 1-0.000798 = 0.999202$ $\quad F(10) \approx 1.0000.$

59

The Cumulative Distribution Function

For X a discrete rv, the graph of $F(x)$ will have a jump at every possible value of X and will be flat between possible values. Such a graph is called a **step function.**

Proposition

For any two numbers a and b with $a \leq b$,

$$P(a \leq X \leq b) = F(b) - F(a-)$$

where "a–" represents the largest possible X value that is strictly less than a.

62

Cumulative Distribution Function

In particular, if the only possible values are integers and if a and b are integers, then

$$P(a \leq X \leq b) = P(X = a \text{ or } a + 1 \text{ or} \ldots \text{ or } b)$$

$$= F(b) - F(a - 1)$$

Taking $a = b$ yields $P(X = a) = F(a) - F(a - 1)$ in this case.

63

Cumulative Distribution Function

The reason for subtracting $F(a–)$ rather than $F(a)$ is that we want to include $P(X = a)$

$$F(b) - F(a); \text{ gives } P(a < X \leq b).$$

This proposition will be used extensively when computing binomial and Poisson probabilities in Sections 3.4 and 3.6.

64

Example 15

- Let X = the number of days of sick leave taken by a randomly selected employee of a large company during a particular year.
- If the maximum number of allowable sick days per year is 14, possible values of X are 0, 1, . . . , 14.

With $F(0) = 0.58$,
$F(1) = 0.72$,
$F(2) = 0.76$,
$F(3) = 0.81$,
$F(4) = 0.88$,
$F(5) = 0.94$,

$$P(2 \le X \le 5) = P(X = 2, 3, 4, \text{ or } 5)$$
$$= F(5) - F(1)$$
$$= 0.94 - 0.72$$
$$= 0.22$$

and

$$P(X = 3) = F(3) - F(2)$$
$$= 0.81 - 0.76$$
$$= 0.05$$

65

66