1-Namunaviy hisob topshiriqlari

Oldin biror qator elementlarining bittasidan boshqasini nolga aylantirib, determinantni tartibini pasaytirish usuli b-n hisoblang:

1.21
$$\begin{vmatrix} 2 & -1 & 0 & 3 \\ 5 & 1 & 4 & -7 \\ 5 & -1 & 0 & 2 \\ 1 & -8 & 5 & 3 \end{vmatrix} = -\begin{vmatrix} 5 & 1 & 4 & -7 \\ 3 & -1 & 0 & 3 \\ 5 & -1 & 0 & 2 \\ 1 & -8 & 5 & 3 \end{vmatrix} = -\begin{vmatrix} 5 & 1 & 4 & -7 \\ 8 & 0 & 4 & -4 \\ 10 & 0 & 4 & -5 \\ 41 & 0 & 37 & -53 \end{vmatrix}$$

$$= -(-1)^3\begin{vmatrix} 8 & 4 & -4 \\ 10 & 4 & -5 \\ 41 & 37 & -53 \end{vmatrix} = +260$$

2-Namunaviy hisob topshiriqlari

Berilgan determinantni 2 usul b-n hisoblang:

a) i-satr elementlari bo'yicha yoyib

b) j-ustun elementlari bo'yicha yoyib

1.21
$$\begin{vmatrix} 2 & 6 & -10 & 3 \\ -1 & 3 & 2 & 7 \\ 5 & 1 & 0 & 3 \\ -2 & 0 & 3 & 6 \end{vmatrix} \qquad i=1 \quad j=2$$

a)
$$\begin{vmatrix} 2 & 6 & -10 & 3 \\ -1 & 3 & 2 & 7 \\ 5 & 1 & 0 & 3 \\ -2 & 0 & 3 & 6 \end{vmatrix} = 2\cdot(-1)^{1+1}\begin{vmatrix} 3 & 2 & 7 \\ 1 & 0 & 3 \\ 0 & 3 & 6 \end{vmatrix} + 6\cdot(-1)^{1+2}\begin{vmatrix} -1 & 2 & 7 \\ 5 & 0 & 3 \\ -2 & 3 & 6 \end{vmatrix} +$$

$$+(-10)\cdot(-1)^{1+3}\begin{vmatrix}-1 & 3 & 7\\ 5 & 1 & 3\\ -2 & 0 & 6\end{vmatrix}+3\cdot(-1)^{1+4}\begin{vmatrix}-1 & 3 & 2\\ 5 & 1 & 0\\ -2 & 0 & 3\end{vmatrix}=$$

$$=2\cdot(-18)-6\cdot 42-10\cdot(-100)-3\cdot(-44)=\text{[illegible]}\ 844$$

б)

$$\begin{vmatrix}2 & 6 & -10 & 3\\ -1 & 3 & 2 & 7\\ 5 & 1 & 0 & 3\\ -2 & 0 & 3 & 6\end{vmatrix}=6\cdot(-1)^{1+2}\begin{vmatrix}-1 & 2 & 7\\ 5 & 0 & 3\\ -2 & 3 & 6\end{vmatrix}+3\cdot(-1)^{2+2}\begin{vmatrix}2 & -10 & 3\\ 5 & 0 & 3\\ -2 & 3 & 6\end{vmatrix}+$$

$$+1\cdot(-1)^{3+2}\begin{vmatrix}2 & -10 & 3\\ -1 & 2 & 7\\ -2 & 3 & 6\end{vmatrix}+0=-6\cdot 42+3\cdot 387-1\cdot 65=844$$

3-Namunaviy hisob topshiriqlar.

Matritsiya berilgan A^{-1} teskari matritsani toping $A\cdot A^{-1}=E=A^{-1}A$ ekanini tekshiring

3.21

$$A=\begin{pmatrix}2 & 1 & 5\\ 1 & 3 & 1\\ 1 & 4 & 8\end{pmatrix}$$

$$\det A=\begin{vmatrix}2 & 1 & 5\\ 1 & 3 & 1\\ 1 & 4 & 8\end{vmatrix}=\begin{vmatrix}2 & 1 & 5 & 2 & 1\\ 1 & 3 & 1 & 1 & 3\\ 1 & 4 & 8 & 1 & 4\end{vmatrix}=(48+1+20)-(15+8+8)=$$

$$=49+20-31=38$$

$A_{11} = +2\begin{vmatrix} 3 & 1 \\ 4 & 8 \end{vmatrix} = 2 \cdot 20 = 40 \quad A_{12} = -\begin{vmatrix} 1 & 1 \\ 1 & 8 \end{vmatrix} = -7 \quad A_{13} = 5\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 5$

$A_{21} = -\begin{vmatrix} 1 & 5 \\ 4 & 8 \end{vmatrix} = 12 \quad A_{22} = 3\begin{vmatrix} 2 & 5 \\ 1 & 8 \end{vmatrix} = 33 \quad A_{23} = -\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} = -7$

$A_{31} = \begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix} = -14 \quad A_{32} = -4\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 12 \quad A_{33} = 8\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 40$

$A^{-1} = \frac{1}{38}\begin{pmatrix} 40 & 12 & -14 \\ -7 & 33 & 12 \\ 5 & -7 & 40 \end{pmatrix}^{*}$

$AA^{-1} = \frac{1}{38} \cdot \begin{pmatrix} 2 & 1 & 5 \\ 1 & 3 & 1 \\ 1 & 4 & 8 \end{pmatrix} \cdot \begin{pmatrix} 40 & 12 & -14 \\ -7 & 33 & 12 \\ 5 & -7 & 40 \end{pmatrix} =$

$= \frac{1}{38}\begin{pmatrix} 80-7+25 & 24+33-35 & -28+12+200 \\ 40-21+5 & 12+99-7 & -14+36+40 \\ 40-28+40 & 12+132-56 & -14+48+328 \end{pmatrix} =$

$= \frac{1}{38} \cdot \begin{pmatrix} 98 & 22 & 184 \\ 24 & 104 & 62 \\ 52 & 88 & 266 \end{pmatrix}$ * Итак $A^{-1} = \frac{1}{38}\begin{pmatrix} 40 & 12 & -14 \\ -7 & 33 & 12 \\ 5 & -7 & 40 \end{pmatrix}$

4-Namunaviy hisob topshiriqlari

A va B matritsalar berilgan AB va BA ko'paytmalarni topilg.

2.21

$$A=\begin{pmatrix}1 & 3 & -1\\ 2 & 0 & 1\\ -1 & 2 & 1\end{pmatrix} \qquad B=\begin{pmatrix}5 & -1 & 3\\ 1 & -2 & 0\\ 0 & 7 & -1\end{pmatrix}$$

$$AB=\begin{pmatrix}5+3+0 & -1-6-7 & 3+0+1\\ 10+0+0 & -2+0+7 & 6+0-1\\ -5+2+0 & 1-4+7 & -3+0-1\end{pmatrix}=\begin{pmatrix}8 & -14 & 4\\ 10 & 5 & 5\\ -3 & 4 & -4\end{pmatrix}$$

$$BA=\begin{pmatrix}5-2-3 & 15+0+6 & -5-1+3\\ 1-4+0 & 3+0+0 & -1-2+0\\ 0+14+1 & 0+0-2 & 0+7-1\end{pmatrix}=\begin{pmatrix}0 & 21 & -3\\ -3 & 3 & -3\\ 15 & -2 & 6\end{pmatrix}$$

6-Mavzuga doir topshiriqlar

Chiziqli tenglamalar sistemasini Kramer usulida yechishga doir misollar yechish

2.21

$$\begin{cases} x_1 + 4x_2 - x_3 = -13 \\ 2x_1 + x_2 - x_3 = 0 \\ 4x_1 + 2x_2 + 5x_3 = 3 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 2 & 1 & -1 \\ 4 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 4 & -1 & 1 & 4 \\ 2 & 1 & -1 & 2 & 1 \\ 4 & 2 & 5 & 4 & 2 \end{vmatrix} = (5 - 16 - 4) - (-4 - 2 + 40) =$$

$$= -15 - 34 = -49$$

$$\Delta x_1 = \begin{vmatrix} -13 & 4 & -1 \\ 0 & 1 & -1 \\ 3 & 2 & 5 \end{vmatrix} = -100$$

$$\Delta x_2 = \begin{vmatrix} 1 & -13 & -1 \\ 2 & 0 & -1 \\ 4 & 3 & 5 \end{vmatrix} = 179$$

$$\Delta x_3 = \begin{vmatrix} 1 & 4 & -13 \\ 2 & 1 & 0 \\ 4 & 2 & 3 \end{vmatrix} = -21$$

$x_1 = \frac{\Delta x_1}{\Delta} = \frac{100}{49}$ $x_2 = \frac{\Delta}{\Delta} = \frac{179}{49}$ $x_3 = \frac{21}{49}$ Javob: $\left(\frac{100}{49}; -\frac{179}{49}; \frac{21}{49}\right)$

7-Namunaviy kasb topshirig'lari.

Biz jinsli tenglamalar sistemasini yechamiz.

5.21

$$\begin{cases} x_1 - 3x_2 - 2x_3 = 0 \\ 3x_1 + x_2 + 4x_3 = 0 \\ 5x_1 - x_2 + x_3 = 0 \end{cases}$$

Ravshanki $x_1 = x_2 = x_3 = 0$ trivial yechim bosqa yechim bor yoqligini tekshiramiz.

$$\Delta = \begin{vmatrix} 1 & -3 & -2 \\ 3 & 1 & 4 \\ 5 & -1 & 1 \end{vmatrix} = (1 - 60 + 6) - (-10 - 4 - 9) = -30.$$

$\Delta \neq 0$ demak trivial bolmagan ildiz yo'q

Javob: $x_1 = x_2 = x_3 = 0$

9-Namunaviy kitob topshiriqlari.

$\alpha\bar{a}+\beta\bar{b}+\delta\bar{d}$ $\bar{a},\bar{b},\bar{c},\bar{d}$ vektorlar berilgan

a) $\bar{d}$ vektorning $\bar{a},\bar{b},\bar{c}$ vektorlar orqali yoyilmasini, b) $\alpha\bar{a}+\beta\bar{b}$ vektorning $\lambda\bar{c}+\delta\bar{d}$ vektor yo'nalishidagi proyeksiyasini toping.

$\bar{a}=(4;-5;-3)$ $\bar{c}=(3,-1,2)$ $\alpha=2$ $\beta=4$

$\bar{b}=(-2;3;1)$ $\bar{d}=(26;-23;-1)$ $\lambda=-3$ $\delta=5$.

a) $\bar{d}=m\bar{a}+n\bar{b}+k\bar{c}$

$$\begin{cases} 4m-2n+3k=26 \\ -5m+3n-k=-23 \mid (2) \\ -3m+n+2k=-1 \mid (1) \end{cases}$$

$-13m+7n=-27 \Rightarrow m=\frac{7n+27}{13}$

$-3\cdot\frac{7n+27}{13}+n+2k=-1 \Rightarrow \frac{-21n-81+13n}{13}+2k=-1$

$2k=-1+\frac{8n+81}{13} \Rightarrow k=\frac{8n+68}{26}$

$4\cdot\frac{7n+27}{13}-2n+3\cdot\frac{8n+68}{26}=26$

$2(28n+108)-52n+24n+204=26\cdot 26$

$28n=256$ $n=\frac{64}{7}$ $m=7$ $k=\frac{38}{7}$

$\bar{d}=7\bar{a}+\frac{64}{7}\bar{b}+\frac{38}{7}\bar{c}$

в) $\vec{p} = \alpha\bar{a} + \beta\bar{b} = 2\bar{a} + 4\bar{b} = (0; 2; -2)$

$\vec{g} = \gamma\bar{c} + \delta\bar{d} = -3\bar{c} + 5\bar{d} = (121; -112; -11)$

$$\Pr_{\bar{g}}\bar{p} = |\bar{p}|\cos\alpha = |\bar{p}|\frac{\bar{p}\,\bar{g}}{|\bar{p}|\,|\bar{g}|} = \frac{\bar{p}\cdot\bar{g}}{|\bar{g}|} = \frac{0 - 224 + 22}{\sqrt{27306}} =$$

$$= -\frac{202}{\sqrt{27306}} = -\frac{202}{3\sqrt{3034}}$$

Javob: а) $d = 7\bar{a} + \frac{64}{7}\bar{b} + \frac{38}{7}\bar{c}$

в) $-\frac{202}{3\sqrt{3034}}$

10-namunaviy fizik topshiriqlar

a) ABCD piramida berilgan qirralari orasidagi burchak kosinusi - ?

b) piramidaning berilgan yoqi yuzi.

A(-7, 2, 3) B(0, -2, 6) C(-1, 3, 7) D(-8, -4, -8)

a) ABC b) CBD

a) $\overline{AB} = (7, -4, 3)$ $\overline{AD} = (-1, -6, -8)$

$$\cos(\overline{AB} \wedge \overline{AD}) = \frac{\overline{AB}\,\overline{AD}}{|AB|\,|AD|} = \frac{-7 \cdot 1 + 4 \cdot 6 - 24}{\sqrt{49+16+9}\sqrt{1+36+64}} =$$

$$= \frac{28 + 24 - 24}{\sqrt{74} \cdot \sqrt{116}} = \frac{28}{2\sqrt{37 \cdot 29}} = \frac{14}{\sqrt{2146}}$$

b) $S_{CBD} = \frac{1}{2}|\overline{CB}| \cdot |\overline{CD}| \sin\alpha$ $\alpha = \overline{CB} \wedge \overline{CD}$

$$\cos\alpha = \frac{\overline{CB}\,\overline{CD}}{|\overline{CB}|\,|\overline{CD}|} = \frac{(1, -5, -1) \cdot (-2, -7, -12)}{\sqrt{1^2 + (-5)^2 + (-1)^2} \cdot \sqrt{(-2)^2 + (-7)^2 + (-12)^2}} =$$

$$= \frac{-2 + 35 + 12}{\sqrt{1+25+1} \cdot \sqrt{4+49+144}} = \frac{45}{\sqrt{27}\sqrt{197}} = \frac{15}{\sqrt{1973}} = \frac{15}{\sqrt{591}}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \frac{225}{591}} = \sqrt{\frac{366}{591}}$$

$$|\overline{CB}|\,|\overline{CD}| = \sqrt{1^2 + (-5)^2 + (-1)^2} \cdot \sqrt{(-2)^2 + (-7)^2 + (-12)^2} = 3\sqrt{591}$$

$$S = \frac{1}{2} \cdot 3\sqrt{591} \cdot \sqrt{\frac{366}{591}} = \frac{3}{2}\sqrt{366}$$

11-Namunaviy bisob topshiriqlari

$\bar{a}, \bar{b}, \bar{c}$ vektorlar bazis hosil qilishini tekshiring $\bar{d}$ (du) vektorni shu bazisdagi yoyilmasini toping

$\bar{a}=(1,0,4)$ $\bar{b}=(-1,1,3)$ $\bar{c}=(1,-2,0)$ $\bar{d}=(0,-7,29)$

$\lambda_1\bar{a}+\lambda_2\bar{b}+\lambda_3\bar{c}=0 \Rightarrow (\lambda_1,0,4\lambda_1)+(-\lambda_2,\lambda_2,3\lambda_2)+(\lambda_3,-2\lambda_3,0)=0$

$$\begin{cases}\lambda_1-\lambda_2+\lambda_3=0\\ 0+\lambda_2-2\lambda_3=0 \Rightarrow \lambda_1=\lambda_2=\lambda_3=0\\ 4\lambda_1+3\lambda_2+0=0\end{cases}$$

zlimok $\bar{a},\bar{b},\bar{c}$ lar bazis hosil qiladi

$\bar{d}=m\bar{a}+n\bar{b}+k\bar{c}$

$(0,-7,29)=(m,0,4m)+(-n,n,3n)+(k,-2k,0)$

$$\begin{cases}m-n+k=0\\ n-2k=-7 \Rightarrow n=2k-7\\ 4m+3n=29\end{cases}$$

$m-(2k-7)+k=0 \Rightarrow m=k-7$

$4\cdot(k-7)+3\cdot(2k-7)=29 \Rightarrow 4k-28+6k-21=29$

$10k=78 \quad k=7,8 \Rightarrow n=2\cdot 7,8-7=8,6$

$m=7,8-7=0,8$

$\bar{d}=0,8\bar{a}+8,6\bar{b}+7,8\bar{c}$

12-Misol. Kosinusni toping.

a) $\cos(\bar{a}\wedge\bar{b})=?$ b) $Pr_{\bar{b}}(\alpha\bar{a}+\beta\bar{b})=?$

$A=(1,1,4)$ $B=(-2,5,1)$ $C=(-1,3,3)$

$\bar{a}=\overline{AB}-\overline{AC}$ $\bar{b}=2\overline{BC}+3\overline{AB}$ $\alpha=3$ $\beta=-4$

$\overline{AB}=(-3,4,-3)$ $\overline{AC}=(-2,2,-1)$ $\overline{BC}=(1,-2,2)$

$\bar{a}=\overline{AB}-\overline{AC}=(-3,4,-3)-(-2,2,-1)=(-1,2,-2)$

$\bar{b}=2\overline{BC}+3\overline{AB}=(2,-4,4)+(-9,12,-9)=(-7,8,-5)$

a) $\cos(\bar{a}\wedge\bar{b})=\frac{\bar{a}\bar{b}}{|\bar{a}||\bar{b}|}=\frac{7+16+10}{\sqrt{1+4+4}\cdot\sqrt{49+64+25}}=\frac{23}{3\sqrt{138}}$

b) $\bar{c}=\alpha\bar{a}+\beta\bar{b}=3(-1,2,-2)+(-4)\cdot(-7,8,-5)=$

$=(-3,6,-6)+(28,-32,20)=(25,-26,14)$

$\cos(\bar{a}\wedge\bar{c})=\frac{-25-52-28}{\sqrt{1+4+4}\sqrt{625+676+196}}=\frac{-105}{3\sqrt{1497}}=-\frac{35}{\sqrt{1497}}$

$Pr_{\bar{b}}\bar{c}=|\bar{c}|\cos(\bar{a}\wedge\bar{c})=\sqrt{1497}\cdot\frac{-35}{\sqrt{1497}}=-35$

Javob: a) $\frac{23}{3\sqrt{138}}$ b) -35

13-Misol. Kosinusni toping

Agar $\bar{a},\bar{b},\alpha,\beta$ lar ma'lum bo'lsa $\bar{c}_1=\alpha_1\bar{a}+\beta_1\bar{b}$, $\bar{c}_2=\alpha_2\bar{a}+\beta_2\bar{b}$ vektorlar kollinear bo'lish bo'lmasligini tekshiring

3.21 $\bar{a}=(0,-2,6)$ $\bar{b}=(\ldots)$ $\ldots\beta_1=-6$ $\alpha_2=1$ $\beta_2=-2$

$\bar{c}_1 = \alpha_1\bar{a} + \beta_1\bar{b} = (0,-6,18) + (-12,-24,6) = (-12,-30,24)$

$\bar{c}_2 = \alpha_2\bar{a} + \beta_2\bar{b} = (0,-2,6) + (-4,-8,2) = (-4,-10,8)$

$-\frac{12}{-4} = \frac{-30}{-10} = \frac{24}{8} \Rightarrow 3 = 3 = 3$

Javob: Kollenear vektorlar bola oladi.

14-Namuna. mustaqil ish.

$\bar{a}, \bar{b}$ va $\bar{c}$ vektorlar komplanar bo'lish-bo'lmasligini tekshiring.

4.21 $\vec{a} = (17,-6,2)$, $\vec{b} = (1,0,1)$, $\vec{c} = (6,-2,1)$

komplanarlik sharti $\bar{a}\bar{b}\bar{c} = 0$

$$\begin{vmatrix} 17 & -6 & 2 \\ 1 & 0 & 1 \\ 6 & -2 & 1 \end{vmatrix} = (0-36-4) - (0-6-34) = 0$$

Javob: $\bar{a}, \bar{b}$ va $\bar{c}$ vektorlar komplanar bo'ladi.

15-Namuna. Bizob tapshiriqlari:

piramidaning uchlari A,B,C,D berilgan

a) berilgan yog yuzini, b) piramidaning l qirrasi va berilgan ikkita uchidan o'tuvchi yuzini v) piramidaning balandligi hisoblang. A(-2,4,2), B(6,2,5), C(-3,0,3) D(0,2,1) ABC g) $\angle CD$, AVB

Yechish: $\bar{AB}=(16,-2,-3)$ $\bar{AC}=(6,-4,-5)$

$S_{ABC}=\frac{1}{2}|\bar{AB}\cdot\bar{AC}|$

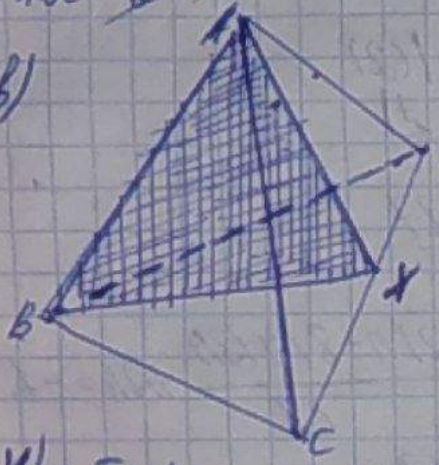
$$\bar{AB}\cdot\bar{AC}=\begin{vmatrix} i & j & k \\ 16 & -2 & -3 \\ 6 & -4 & -5 \end{vmatrix}=(i\cdot 10+j\cdot(-18)+k\cdot(-60))-(-12k-60j+12i)=$$

$$=10i-18j-60k+12k+60j-12i=-2i+42j-48k$$

$$|\bar{AB}\,\bar{AC}|=\sqrt{4+1764+2304}=\sqrt{4072}=2\sqrt{1018}$$

$$S_{ABC}=\frac{1}{2}\,2\sqrt{1018}=\sqrt{1018}$$

b)



Kesim yuzi kesim e tekisi qanday nisbatda kesib o'tishiga bog'liq yoni X ning joylashishiga qarab kesim yuzi o'zgaradi.

v) $\bar{AB}=(16,-2,-3)$ $\bar{AC}=(6,-4,-5)$ $\bar{AD}=(9,-2,-7)$

$$V_{piramida}=\pm\frac{1}{6}\bar{AB}\cdot\bar{AC}\,\bar{AD}=\pm\frac{1}{6}\begin{vmatrix} 16 & -2 & -3 \\ 6 & -4 & -5 \\ 9 & -2 & -7 \end{vmatrix}=\frac{1}{6}\cdot 204=34$$

Javob: a) $\sqrt{1018}$; b) cheksiz ko'pmoqda; v) 34