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# Hand-Eye Calibration

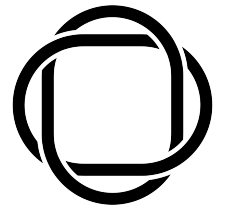
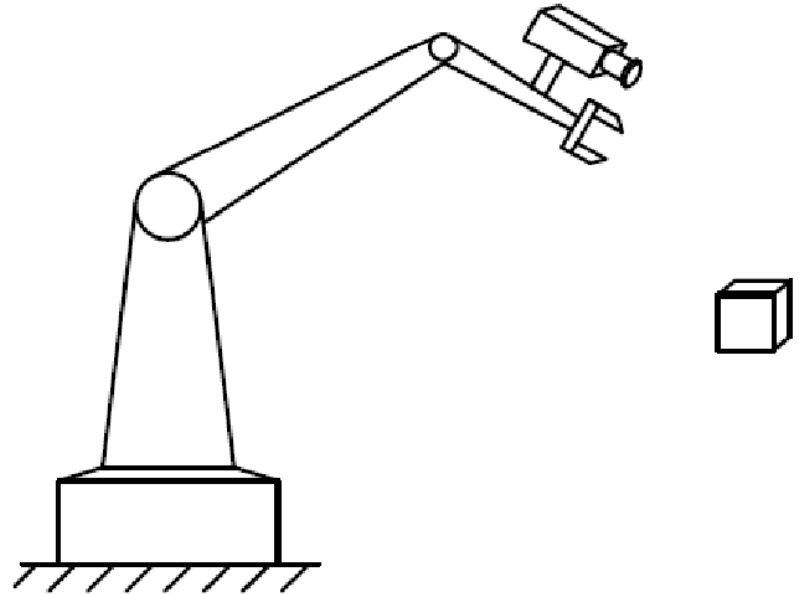
Friday 16 Mar 2018, 12:00PM

# Outlines

- Introduction

- Algorithm

- Experiment and evaluation



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# Introduction

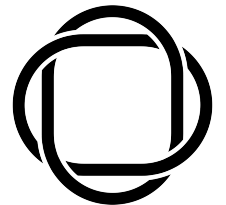
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## Hand-eye calibration

With Camera installed, robot may perform some specific tasks with visual access to environmental information:

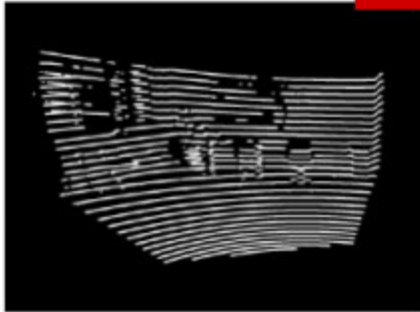
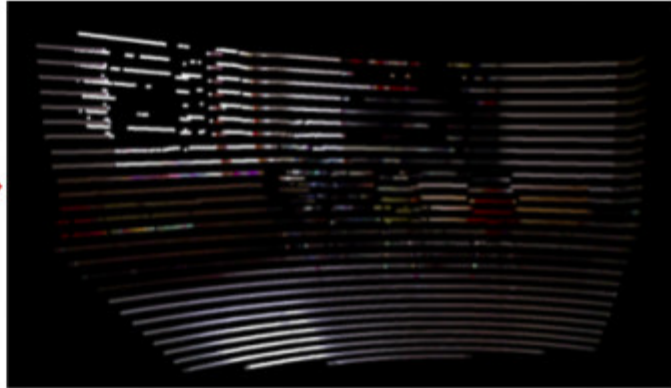
Measurement, tracking,  
Navigation, etc.



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# Introduction

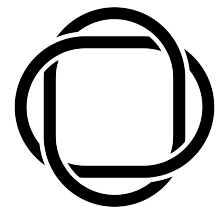
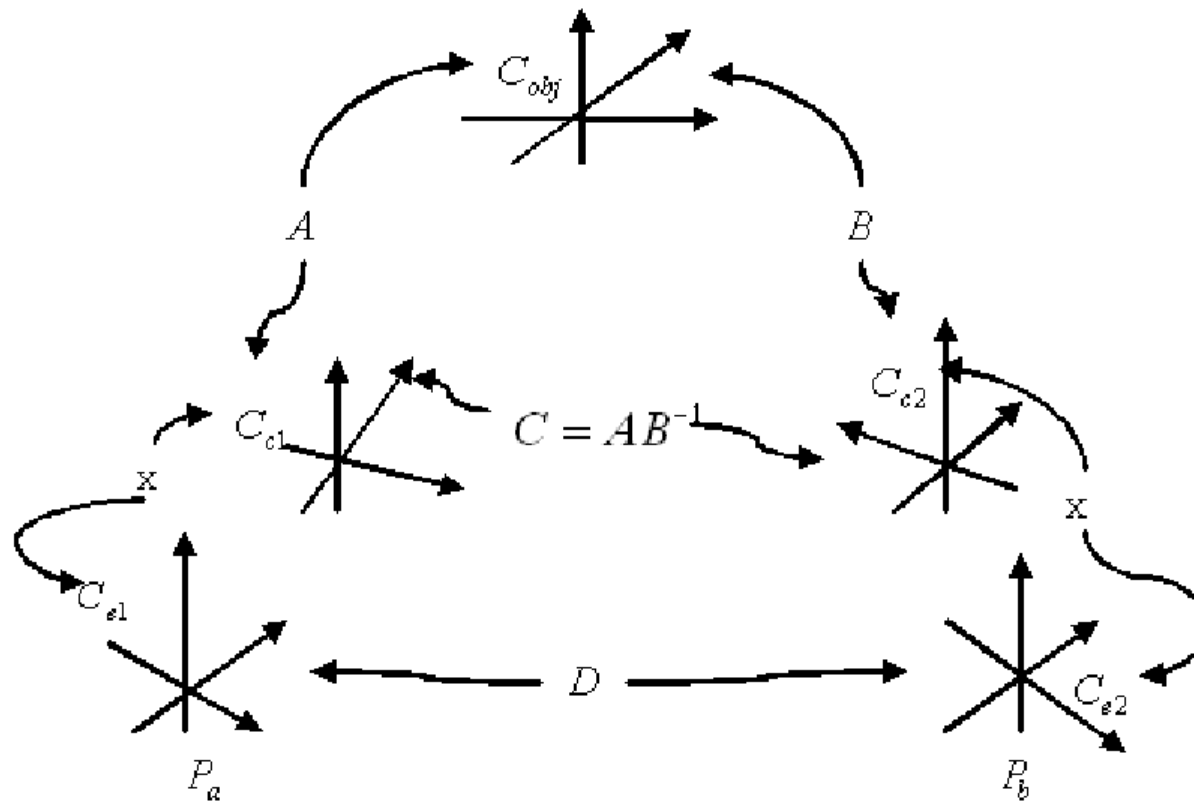
**Hand-eye calibration may help in sensor fusion**



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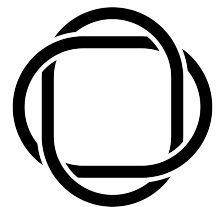
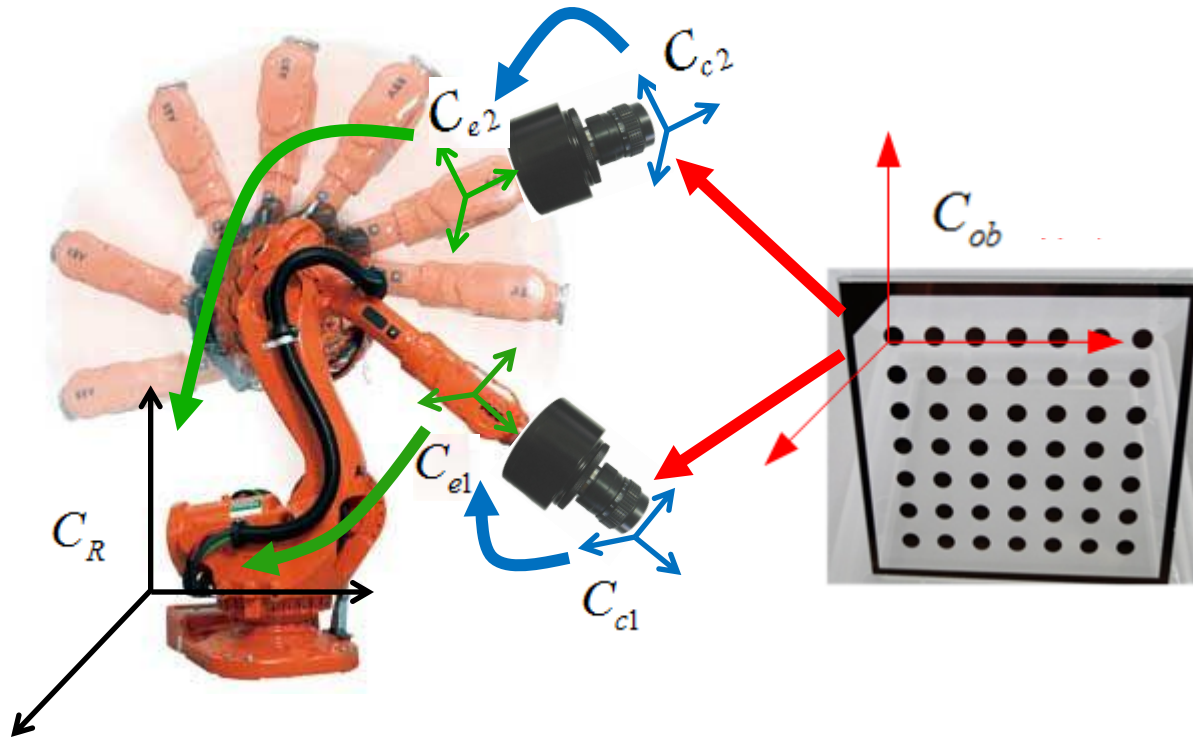
# Introduction

**Unifying coordinate system could be a problem**



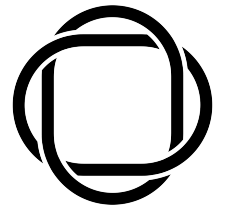
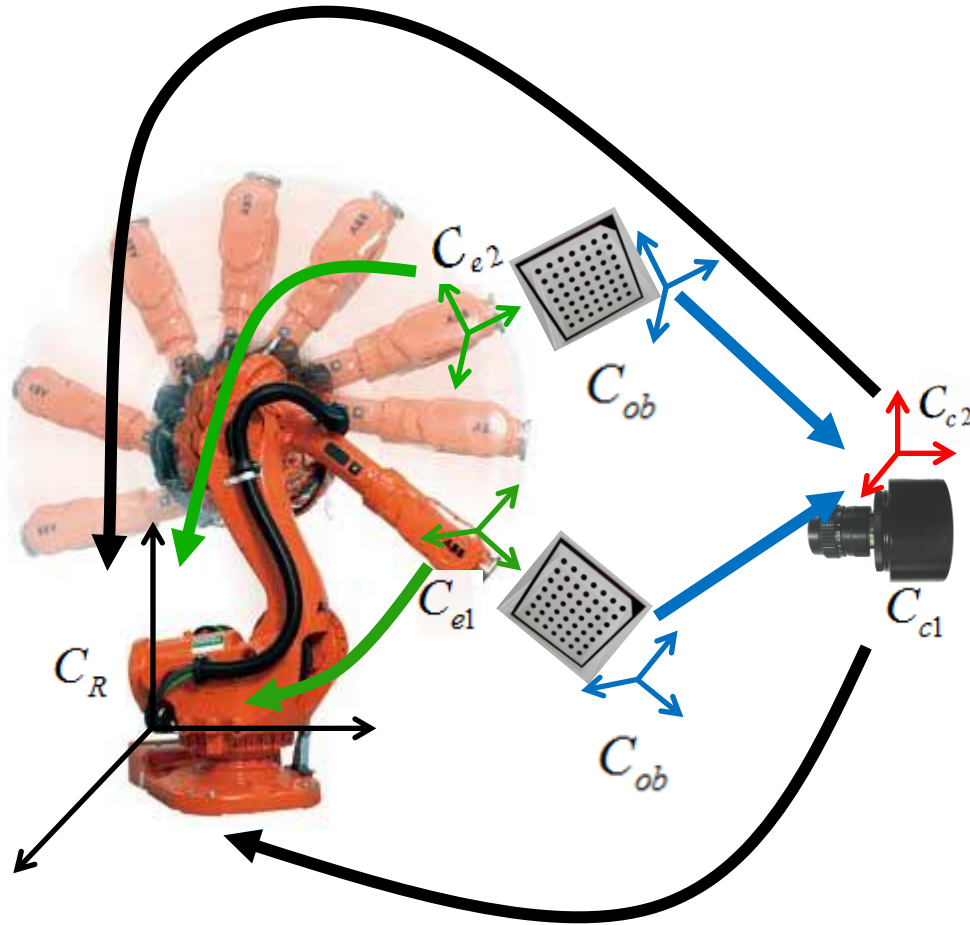
# Introduction

**Eye-in-hand:** the camera is either grasped by the gripper, or just fastened to it.



# Introduction

**Eye-to-hand: the camera is fixed in a position.**

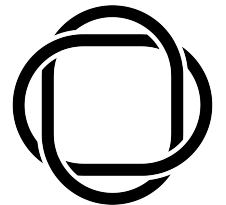


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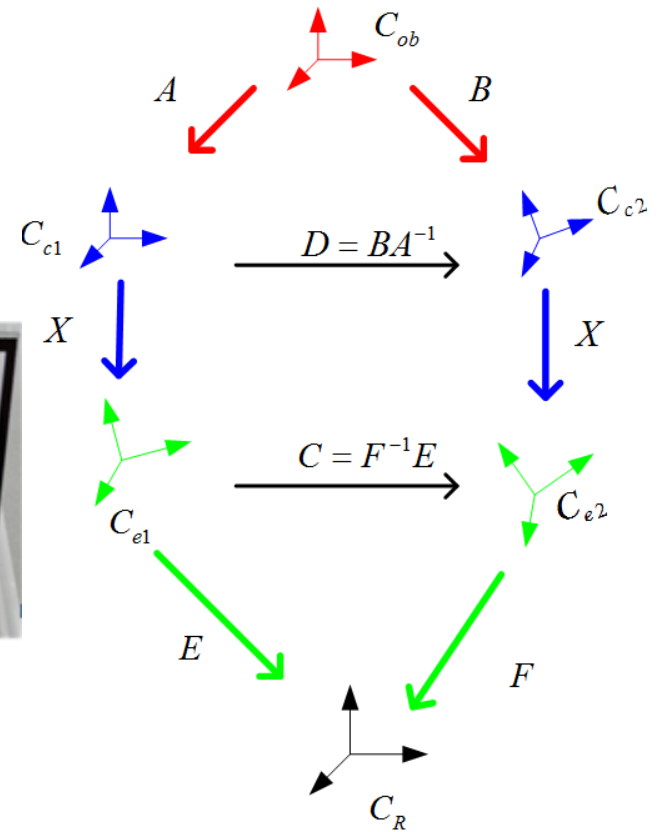
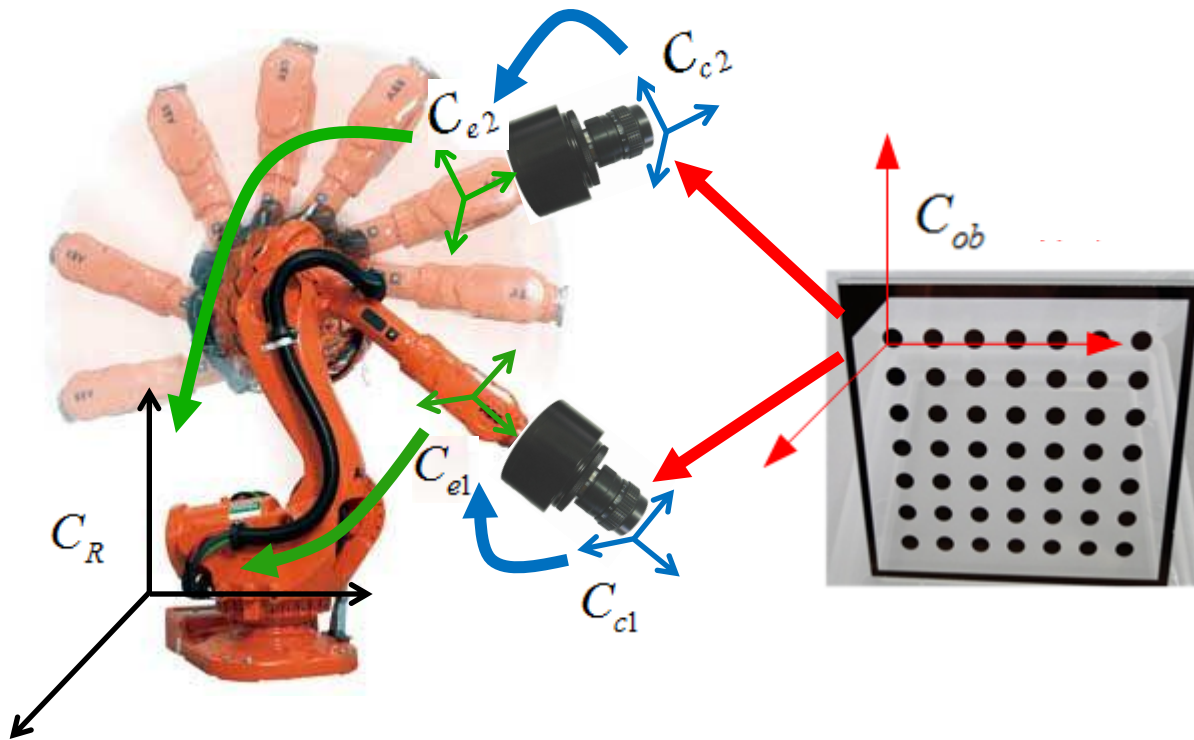


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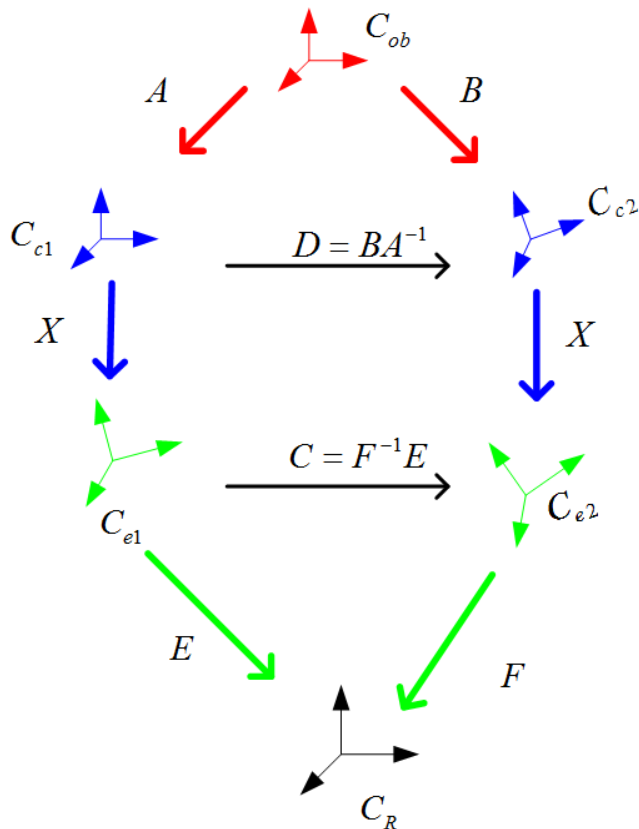


# Algorithm

## Eye-in-hand: sensor fusion



# Algorithm

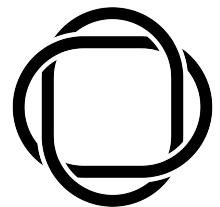


$$CX = XD$$

$$\begin{bmatrix} R_c & t_c \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} R_d & t_d \\ 0^T & 1 \end{bmatrix}$$

$$R_c R = R R_d$$

$$R_c t + t_c = R t_d + t$$



# Algorithm

Let  $R_c$  be expressed as an angle of rotation  $\omega_c$  around an arbitrary axis  $k_c$ , that is,  $R_c = \text{Rot}(k_c, \omega_c)$ . Similarly,  $R_d = \text{Rot}(k_d, \omega_d)$ .

$$\begin{cases} R_c = RR_dR^{-1} \\ (R_c - I)t = Rt_d - t_c \end{cases}$$

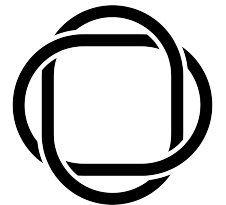
According to above equations, we can get:

$$k_c = Rk_d$$

When  $R_c - I = 0$ ,

$$t_c = Rt_d$$

$$[k_{c_1}, \dots, k_{c_N}, t_{c_1}, \dots, t_{c_M}] = R[k_{d_1}, \dots, k_{d_N}, t_{d_1}, \dots, t_{d_M}]$$



# Algorithm

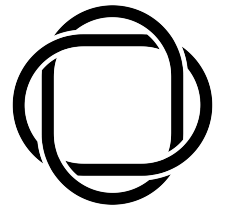
$$[k_{c_1}, \dots, k_{c_N}, t_{c_1}, \dots, t_{c_M}] = R[k_{d_1}, \dots, k_{d_N}, t_{d_1}, \dots, t_{d_M}]$$

$$M_1 = R \cdot M_2$$

Theoretically,  $\text{rank}(M_1) = \text{rank}(M_2)$ , but in fact,  $M_1$  is more accuracy than  $M_2$ , and now we focus on  $\text{rank}(M_1)$ .

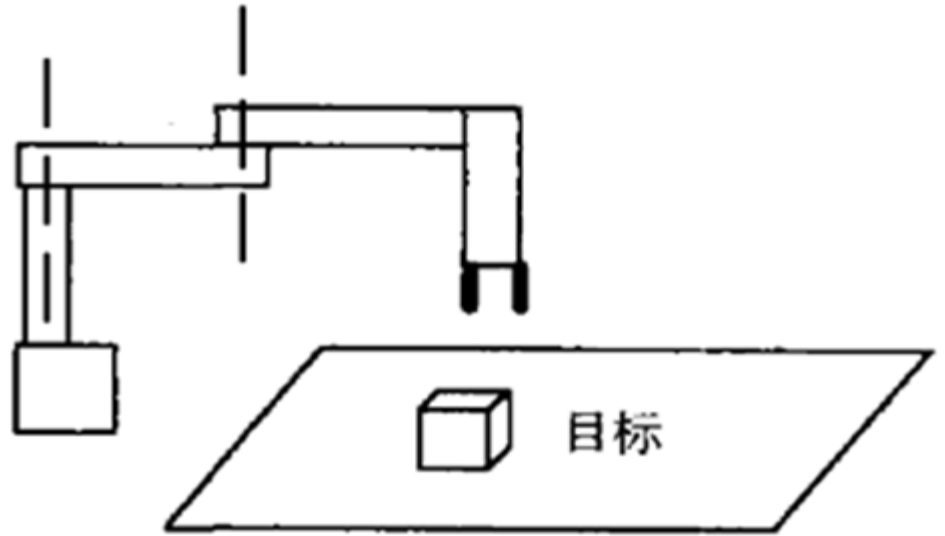
When  $\text{rank}(M_1) \neq 3$ :

$$\begin{cases} M_1^{**} = [n_{c_1}, n_{c_2}, n_c] \\ M_2^{**} = [n_{d_1}, n_{d_2}, n_d] \\ M_1^{**} = R \cdot M_2^{**} \end{cases} \quad n_c = n_{c_1} \times n_{c_2}, \quad n_d = n_{d_1} \times n_{d_2}$$



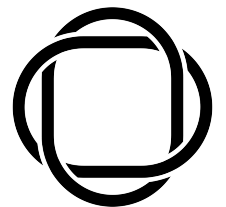
# Algorithm

$$\begin{bmatrix} R_{c_{12}} & -I \\ R_{c_{23}} & -I \\ \dots & \\ R_{c_{N-1N}} & -I \end{bmatrix} t = \begin{bmatrix} Rt_{d_{12}} - t_{c_{12}} \\ Rt_{d_{23}} - t_{c_{23}} \\ \dots \\ Rt_{d_{N-1N}} - t_{c_{N-1N}} \end{bmatrix}$$
$$M_3 \cdot t = M_4$$



When  $\text{rank}(M_3) = 2$  (4-DOF robot):

$$R_c = R \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} R^{-1}$$



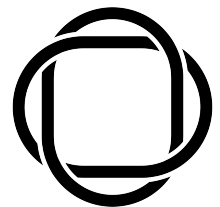
# Algorithm

$$R_c - I = R \begin{bmatrix} \cos\theta - 1 & -\sin\theta & 0 \\ \sin\theta & \cos\theta - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} R^{-1}$$

$$USV^T = R \begin{bmatrix} \cos\theta - 1 & -\sin\theta & 0 \\ \sin\theta & \cos\theta - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} R^{-1} \quad SV^T t = U^{-1}(Rt_d - t_c)$$

$$U^{-1}(Rt_d - t_c) = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \quad S = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

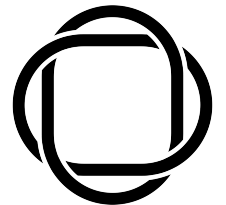
$$\begin{bmatrix} \lambda & 0 \\ 0 & \beta \end{bmatrix} V_{12}^T \cdot \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$



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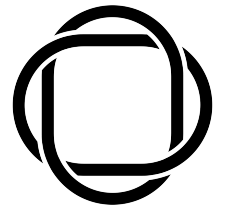
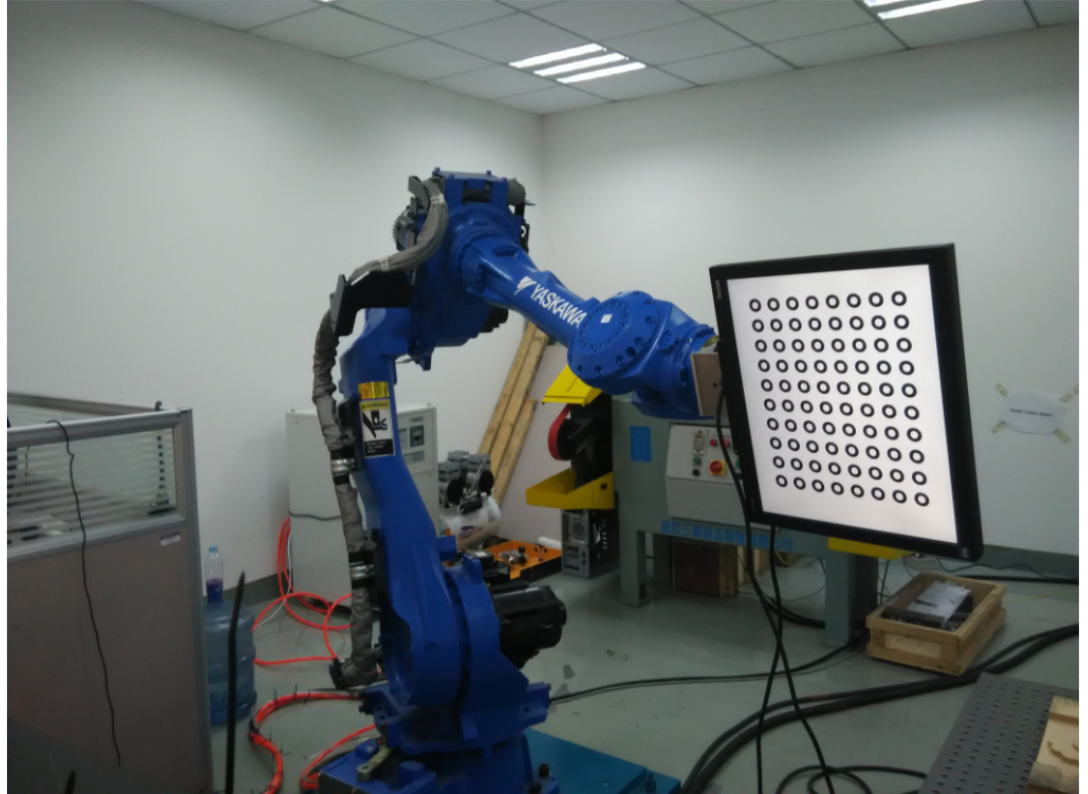


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# Experiment and evaluation

Calculate the transformation matrix of the calibration plate relative to the camera using PnP algorithm;

Read the position and orientation of the end effector from the robot controller.



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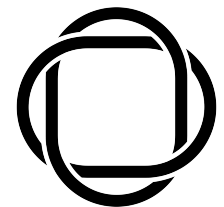


# Experiment and evaluation

$$err_i = ||(C_i X)^{-1} \cdot D_i X - I||$$

Table 1 Relative error of the hand-eye transformation matrix  $X$  in first experiment

	$   (C_i X)^{-1} \cdot D_i X - I   $
$err_1$	0.759559
$err_2$	0.940860
$err_3$	0.974952
均值	0.891790
标准差	0.115777

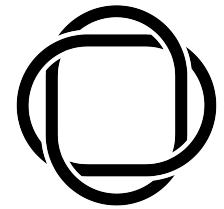


# Experiment and evaluation



Table 2 Relative error of the hand-eye transformation matrix  $X$  in second experiment<sup>o</sup>

<sup>o</sup>	$   (C_i X)^{-T} \cdot D_i X - I   $ <sup>o</sup>
$err_1$ <sup>o</sup>	0.349424 <sup>o</sup>
$err_2$ <sup>o</sup>	1.788770 <sup>o</sup>
$err_3$ <sup>o</sup>	1.507480 <sup>o</sup>
$err_4$ <sup>o</sup>	0.213286 <sup>o</sup>
$err_5$ <sup>o</sup>	1.252840 <sup>o</sup>
$err_6$ <sup>o</sup>	1.113794 <sup>o</sup>
均值 <sup>o</sup>	1.037598 <sup>o</sup>
标准差 <sup>o</sup>	0.630913 <sup>o</sup>



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# iMorpheus.ai Weekly Journal Club

Next Friday, 23/03/2018 12:00PM GMT+8

An Introduction to Blockchain

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