Elementary Lower Bound for e

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Consider $f(x) = \frac{1}{x}$. Since f is concave for $x > 0$, tangents to f lie below it. The tangent at a point $(a, f(a))$ is given by

$$y = f(a) + f'(a)(x - a).$$

Therefore,

$$\log\left(\frac{3}{2}\right(\left) = \int_1^{3/2} \frac{1}{x}\, dx > \int_1^{3/2} \text{tangent}(x)\, dx$$

Picking $a = 1.25$, we get

$$\text{tangent}(x) = 0.8\ - 0.64\,(x - 1.25)$$

Integrating this,

$$\log(3/2) > 0.4 \iff e^{2/5} < 1.5 \iff e < (1.5)^{2.5} \approx 2.756.$$

$$\boxed{e < 2.756}$$