Role of Mathematics in Image Processing

Manisha Vashisht, Madhulika Bhatia
Manav Racha International Institute of Research and Studies
Faridabad, India
manisha.vashisht@gmail.com, madhulikacse.fet@mriu.edu.in

Abstract**—**The various Image Processing technologies have revolutionized the way in which a particular image is captured, stored, or retrieved from a certain source. Most commonly used Image Processing tasks include; Image Restoration, Image Segmentation, Image Enhancement, De-Blurring, De-noising, etc. Such Images are well utilized in various ways by several of the Imaging techniques, like; Angiography, MRI (Magnetic Resonance Imaging), Arterial Spin Labeling (ASL), Computer Assisted Tomography (CT), Deep Brain Stimulation (DBS), Electroencephalography (EEG), etc. But, it is the, Mathematics that has played a crucial role in various Image Processing tasks mentioned above. Despite the innovation and rapid advances in the various Imaging Technology, one thing that has remained important throughout, is, the use of Mathematics.

It has been observed that image processing has got strong connection with Mathematics. Many of the image processing methods rely on the basic Mathematical Techniques of Histogram Equalization, Probability and Statistics, Discrete Cosine Transforms, Fourier Transforms, Differential Equations, Integration, Matrix and Algebra. For computational purposes, Matlab is one of the most commonly used tool by the researchers in the area of Image Processing. Other Tools which are commonly used include SciLab, GNU Octave, SageMath, etc.

This paper critically reviews the role of Mathematics in various Image Processing Applications. This paper would also present the basic Mathematical tools and techniques, used in Image processing, along with the literature review of the same.

Keywords—Image processing, mathematics, Imaging Techniques, Probability, Statistics, Fourier Transforms, Differential Equations, Matlab, GNU Octave, SCILAB, SageMath

I. Introduction

The increasing communication based technology poses a great challenge for developing better, faster and highly cost effective image transmission and storage systems. Also, the constantly enhancing complexity of software products and global IT industry competition has been pushing product organizations to implement advanced techniques to improve image quality and performance.

It is being assumed that mathematical techniques discussed in this paper could be easily understood by the researchers and be suitably adapted for solving many of the image processing related challenges. Our emphasis here is on the fundamentals of the various mathematical techniques.

One of the great wizard in the field of mathematics, Carl Friedrich Gauss cited mathematics as "the Queen of the Sciences". He further referred concept of number theory to be the queen of the Mathematics. The subject area of Mathematics has been known to cover subdivisions that link to other core fields, just like, logic, set theory (as foundations), the empirical mathematics of the several sciences (like the applied mathematics), and the very current, the study of uncertainty.

A. *Role of Mathematics in Computer Sciences*

The concept of Mathematics has emerged as a critical basis to almost all disciplines of Engineering and Technology. In almost all engineering courses, mathematics as a subject is taught in the very first year of academic's curriculum to create the foundation of applying mathematical concepts in discrete engineering areas.

It has been widely accepted fact that for understanding and solving computer sciences problems, Mathematics concepts is a must requirement. But, all these very essential foundations of mathematics' are often taught separately and the most relevant connections to computing technology, which is actually required to motivate or inspire the mathematics and its users, are usually not made [1]. This paper provides a motivation and a general structure of the mathematical tools and techniques for the advanced versions of Computer Sciences.

B. *Role of Mathematics in Image Processing*

During the previous decade, there has been increased integration of the fields of Image Processing (a subject of Computer Sciences) and those of Mathematics. Mathematics is quite inherent and deeply connected to various core Image Processing tasks: just like, De-noising, De-blurring, Enhancement, Segmentation and Edge Detection etc. The study of such Image Processing tasks provides a unique opportunity of incorporating Mathematical tools and techniques to address several of the Image Processing applications in various scientific fields of study.

Multiple Mathematical Techniques are used, i.e. for Image Filtering in the Spatial domain (using first- and second order partial derivatives, the gradient, Laplacian, and their discrete approximations by finite differences, averaging filters, order statistics filters, convolution), and in the frequency domain (Fourier transform, low-pass and high-pass filters), zero-crossings of the Laplacian, etc. [2].

978-1-7281-0211-5/19/$31.00 2019 ©**IEEE**

II. RELATED WORK

In order to carry out effective Image Processing, it is highly essential that the Mathematical Techniques and Algorithms are identified and applied accurately. The overall effectiveness of Image Processing process depends upon how well the mathematical approach is aligned for particular problem solving.

With the increasing interests in technology, mathematical tools and techniques has become a crucial aspect in the image processing. In view of the fact, that each image processing problem has its own set of advantages and disadvantages, it is hard to decide superficially, which mathematical technique should be used for a particular type of problem scenario, especially, as every image processing approach tends to be unique. For the reasons mentioned earlier, image processing is a very active research area.

Also, a short review is given on the rationale for using mathematical tools and techniques for solving image processing problems. In order to design appropriate image processing problem solution, we need to know the concepts and theories of Mathematics. [3].

A. Mathematical Techniques

1) Histogram equalization

The histogram equalization technique is a well-known statistical tool widely used for improving the contrast of images. The input image is $f\ (x,\ y)$ (as a low contrast image, dark image, or light image). The output is a high contrasted image $g\ (x,\ y)$. [2]

Assume that the gray-level range consists of L gray-levels. In the discrete case, let $rk = f\ (x,\ y)$ be a gray level of f, $k = 0,\ 1,\ 2,\ ...L - 1$. Let $sk = g\ (x,\ y)$ be the desired gray level of output image g. The transformation T: $[0,\ L - 1]\ 7\to\ [0,\ L - 1]$ such that $sk = T(rk)$, for all $k = 0,\ 1,\ 2,\ ...,\ L - 1$. It is defined that $h(rk) = nk$ where rk is the k^{th} gray level, and nk is the number of pixels in the image f taking the value rk. The discrete function $rk\ 7\to\ h(rk)$ is visualized for all $k = 0,\ 1,\ 2,\ ...,\ L - 1$. This provides the histogram. The normalized histogram is defined as: let $p(rk) = nk\ n$, where n is the total number of pixels in the image (thus $n = MN$). The visualization of the discrete mapping $rk\ 7\to\ p(rk)$ for all $k = 0,\ 1,\ 2,\ ...,\ L - 1$ gives the normalized histogram. It is noted that $0 \leqslant p(rk) \leqslant 1$ and $L\ X{-1}\ k{=}0\ p(rk) = 1$

2) Spatial Linear Filters

A Spatial Filter comprises of a distinct vicinity of a pixel and a pre-defined process that is performed more on the image pixels those are contained by the neighborhood. The output of the operation provides the value of the output image $g\ (x,\ y)$ [2].

Input Image	Output Image	Integer Coordinates
f (x, y)	g (x, y)	(x, y) with $0 \leq x \leq M - 1$ *and* $0 \leq y \leq N - 1$.

Both images f and g are of size $M \cdot N$. A neighborhood centered is defined at the point $(x,\ y)$ by $S\ (x,\ y) = n\ (x + s,\ y + t),\ -a \leqslant s \leqslant a,\ -b \leqslant t \leqslant b\ o$, where $a,\ b \geqslant 0$ are integers. The size of the patch $S\ (x,\ y)$ is $(2a + 1)\ (2b + 1)$, is denoted by $m = 2a + 1$ and $n = 2b + 1$ (odd integers), thus the size of the patch becomes $m\ \cdot\ n$, and $a = m{-}1\ 2,\ b = n{-}1\ 2$.
For example, the restriction $f|S\ (x,\ y)$ for a 3×3 neighborhood $S\ (x,\ y)$ is represented by:

$$f|S\,(x,y) = f\,(x-1,y-1)\,f\,(x-1,y)\,f\,(x-1,y+1)\,f\,(x,y-1)\,f\,(x,y)\,f\,(x,y+1)\,f\,(x+1,y-1)\,f\,(x+1,y)\,f\,(x+1,y+1).$$

A window mask is defined as $w = w\ (s,\ t)$, for all $-a \leqslant s \leqslant a,\ -b \leqslant t \leqslant b$, of size $m \times n$. For the 3×3 case, this window is:

$$w = w\,(-1,-1)\,w\,(-1,0)\,w\,(-1,+1)\,w(0,-1)\,w(0,0)\,w(0,+1)\,w(+1,-1)\,w(+1,0)\,w(+1,+1).$$

The output image g is defined by $g\ (x,\ y) = Xa\ s{=}{-}a\ X\ b\ t{=}{-}b\ w\ (s,\ t)\ f\ (x + s,\ y + t$. The image f can be extended by zero in a band outside of its original domain, or by periodicity, or by reflection (mirroring). In all these cases, all values $f\ (x + s,\ y + t)$ will be defined. For a 3 × 3 window mask and neighborhood, the filter becomes:

$$g\,(x,y) = w\,(-1,-1)f\,(x-1,y-1) + w\,(-1,0)f\,(x-1,y) + w\,(-1,1)f\,(x-1,y+1) + w\,(0,-1)f\,(x,y-1) + w\,(0,0)f\,(x,y) + w\,(0,1)f\,(x,y+1) + w\,(1,-1)f\,(x+1,y-1) + w\,(1,0)f\,(x+1,y) + w\,(1,1)f\,(x+1,y+1).$$

There are two classes of spatial linear filters:

(i) smoothing linear spatial filters

(ii) sharpening linear spatial filters

Smoothing linear spatial filters

Smoothing is known to cause local averaging (or blurring), which is similar with spatial summation or spatial integration. During the smoothing process, small details and noise gets lost, but sharp edges becomes blurry. Sharpening and smoothing operations results in opposite effects. Using smoothing, a blurry image f is made sharper, where edge of the image f would get enhanced [2].

Two examples of smoothing spatial masks w of size 3×3 are considered. The first one gives the average filter or the box filter as below:

$$w = 1 9 1 1 1 1 1 1 1 1 1$$

The second one can be seen as a discrete version of a 2-dimension Gaussian function as below:

$$e - x 2 + y 2 2\sigma 2 : w = 1 16 1 2 1 2 4 2 1 2 1$$

(The coefficients of w decrease as there is a movement away from the center). It may be noted that for both these filter masks w, the sum of the coefficients

$$P1 s = -1 P1 t = -1 w (s, t) = 1.$$

Thus, the input image f and the output image g have the same range of intensity values. It is evident from these two examples that it is convenient to directly normalize the filter by considering the general weighted average filter of size

$$m \times n : g (x, y) = Pa s = -a Pb t = -b w (s, t) f (x + s, y + t) Pa s = -a Pb t = -b w (s, t)$$

The above equation is computed for all (x, y), where

$$x = 0, 1, 2, ..., M - 1$$

$$y = 0, 1, 2, ..., N - 1.$$

It may be noted that the denominator is a constant, thus it has to be computed only once for all (x, y).

Sharpening linear spatial filters

Sharpening linear filters for image enhancement uses the Laplacian technique. It is assumed that image function f has second order partial derivatives. The Laplacian of f in the continuous case is defined by:

$$4f (x, y) = \partial 2 f \partial x2 (x, y) + \partial 2 f \partial y2 (x, y)$$

The result shows that the mapping f 7→ 4f is linear and rotationally invariant. Using the numerical analysis for one dimension, the second order derivative of f, f 00 at the point x, can be approximated by finite differences of values of f at x:

$$x + h, x - h : f 00(x) \approx f (x + h) - 2f(x) + f (x - h) h 2, (3)$$

where h > 0 is a small discretization parameter. [2]

3) Discrete Cosine Transform (DCT)

With the advent of the web and industry wide digital transformation, the JPEG (Joint Photographic Experts Group) image format has emerged as a most commonly used format for sharing images with lossy compression over the web.

Discrete Cosine Transform (DCT) is one of the powerful techniques in domain of signal processing [4]. DCT performs execution using mathematical operation technique known as Fast Fourier Transform (FFT). FFT takes one signal as an input and then transforms the representation from one type to another. In this transformation process, a signal from the spatial domain gets transformed to the frequency domain. During this process, information redundancy is kept minimum because of kernel functions (cosines) comprise an orthogonal basis. This technique was first developed by J.W. Cooley and J.W. Tukey in 1965. FFT found wide acceptance since it helped in reducing the computation time and increased the ease of performing the digital Fourier analysis much more feasible.

Over the years, the DCT compression algorithm has been extensively studied by various researchers. Along with the wide usage of the 2-D DCT algorithms for image compression, it is also used as feature extraction method in pattern recognition applications in image water-marking and data hiding and in various image processing applications [4 – 10]. The major advantage of DCT transform is that it is easier to implement and at the same time provides high energy compactness, which results in DCT coefficients completely describing the signal used in the process. It has been observed that once execution of DCT is completed and resultant coefficients are captured, the output statistical distribution has been found to be advantageous for designing the quantizer and removing noise for enhancing the image quality [11-12].

There have been wide areas of application of two dimension DCT algorithm in the industrial world. This increased usage has further inspired research scholars to design and develop enhanced algorithms and solutions to improve the required time to compute the image coefficients. During literature review, it has been observed that scholars have proposed multiple algorithms that use hardware based solutions for improving the overall DCT compute time and distributions of the wavelet and DCT coefficients of video frame [13-16].

4) Laplacian Distribution

The Laplacian distribution is one of the most commonly used approach used for performing image analysis. Since Laplacian approach uses invariant Gaussian kernels, there are known issues with regard to inability to represent the edges of images suitably. Hence, this approach is not recommended for edge centric operations like preservation of image edges, image, smoothing and tone mapping.

In order to address these challenges, researchers have come up with multiple alternative solutions, such as anisotropic diffusion, neighborhood filtering, and specialized wavelet bases. These alternative approaches have provided the results but this has also increased the complexity, computational cost and the need for performing the post-processing operations for getting desired outcomes [17].

It has been observed during the conducted literature review that Laplacian approach has been widely used for image analysis using different scales for multiple applications such as compression texture synthesis, and harmonization [18-20].

It has been observed by research scholars that for applications such as edge preservation and tone mapping, where images edges are critical, Lapalacian is not considered to be the best choice for Image processing. This is due to the Gaussian kernels on which the laplacian distribution approach is built. The Gaussian Kernels are known to be anisotropic by nature. [21]. These known issues have resulted in usage of effective schemes such as anisotropic diffusion, neighborhood filters, edge-preserving optimization, and edge-aware wavelets [22-24]. Among the sophisticated techniques, the bilateral filter uses an optimization based approach using spatially varying kernel, to build functions for each new image [25 -26].

This approach though is considered better but has got its own set of drawbacks. There is an increased level of difficulty to configure the parameters of anisotropic diffusion. This is due to the increased complexity of the algorithm resulting from the iterative process causing the neighborhood filters to over-sharpen the image edges, [27]. Post processing image operation can result in reduction of these known issues where bilateral filtered edges can be smoothed using additional computation and configuring the parameters [28-30].

B. Mathematical Tools

1) Matlab

Matlab is package from Mathworks. This is one of the most popular tools used for building models using AI technologies such as ANN, Fuzzy and hybrid algorithms such as ANFIS. Matlab provides Image Processing Toolbox as a part of its package. This toolbox makes available a wide range of Mathematical algorithms and features which are widely used by research scholar and scientist in field of Image Processing.

The toolbox provides capability to perform Image Processing operations, including:

- Image Segmentation,
- Image Enhancement,
- Noise Reduction,
- Three dimensional Image processing [31-32].

Images may be considered as matrices whose elements are the pixel values of the image. The matrix capabilities of Matlab allow investigating images and their properties. The tool handles 24-bit RGB images in much the same way as greyscale and does not distinguish between greyscale and binary images. Further, conversion of images from one image type to another can be easily performed using Matlab [33].

2) Scilab Image Processing (SIP) toolbox

SIP is another open source tool used widely in area of Image Processing. This tool supports rapid prototyping of imaging solutions and has capability to perform multiple operations such as providing quick diagnosis by processing MRI images, Image Segmentation, Image filtering, Edge detection, etc. [34].

SIP provides unified bindings to several image processing libraries such as ImageMagick, OpenCV, animal, and Leptonica. Being an open source solution, it has got responsive developer and user community. Also being cost effective as compared many proprietary software packages, many people in universities and some industries have been using SIP, especially researchers and Image Processing students [35].

3) GNU Octave

GNU Octave is mainly used for performing numerical computing. The tool uses a high level programming language that uses CLI (Command Line Interface), for providing solution to multiple numerical, linear and non-linear problems. The programming language provides support for Image Processing operations with pre-packaged image plotting and visualization tools. GNU Octave belongs to open-source and supports multiple operating systems including, Windows, Linux and Macintosh, [36-37]. It has been used for Image Processing applications in areas of analysis of laser deposition experiments, anomaly detection in an image dataset and to calculate the structural similarity index (SSIM), the values of which are showing how various degrees of distortion/damage affect the quality of the image, where the lower values usually translate to lower image quality [38-39].

4) SageMath

SageMath is built from several open-source packages, and it provides a range of capabilities for Image Processing using mathematics. It can also produce a 2-D as well as a 3-D graphics/images, and even animated plots.

SageMath has been used by researchers to: develop Image Processing application for determining the letter represented in an ornamental letter image; in order to improve the perceptual quality of the visualization of the panorama; and also to apply the recovery technique on images for solving the set of linear Boolean equations [40-42].

III. RESULTS AND CONCLUSION

In view of the above study from detailed literature survey, it was found that mathematics continues to play a pivotal role in future advancements in the field of Image Processing. It was observed that most of the times to a normal user, it may not be visible looking at the image editing process but actually it is various mathematical algorithms that helps to get the final output.

This paper covered the role of mathematics in various Image Processing Applications. The most common applications of Image Processing, using Mathematics are: image sharpening and restoration, medical diagnostics, remote sensing, transmission and encoding, medical fields, pattern recognition, color processing etc. The paper also covered how Image Processing has evolved with contributions from a number of pioneers in the field of mathematics, how it has become a major challenging research topic and why it continues to be so even today. This paper gives the summary of some of the Image Processing mathematical tools and techniques. It is believed that this paper would help the research scholars and scientists working in different fields related to Image Processing.

REFERENCES

[1] P. B. Henderson, “The Role of Mathematics in Computer Science and Software Engineering Education,” Advances in Computers, pp. 349–395, 2005.

[2] Vese, Luminita, and T. Wittman. "An introduction to mathematical image processing." Under graduate summer school (2010).

[3] A. K. Jain, “Image data compression: A review,” Proceedings of the IEEE, vol. 69, no. 3, pp. 349–389, Mar. 1981.

[4] C. Sanderson and K. K. Paliwal, “Features for robust face-based identity verification,” Signal Processing, vol. 83, no. 5, pp. 931–940, May 2003

[5] D. V. Jadhav and R. S. Holambe, “Radon and discrete cosine transforms based feature extraction and dimensionality reduction approach for face recognition,” Signal Processing, vol. 88, no. 10, pp. 2604–2609, Oct. 2008.

[6] F. Liu and Y. Liu, “A Watermarking Algorithm for Digital Image Based on DCT and SVD,” 2008 Congress on Image and Signal Processing, 2008.

[7] Phu Ngoc Le, E. Ambikairajah, and E. Choi, “An improved soft threshold method for DCT speech enhancement,” 2008 Second International Conference on Communications and Electronics, Jun. 2008

[8] E.-L. Tan, W.-S. Gan, and M.-T. Wong, “Fast Arbitrary Resizing of Images in DCT Domain,” Multimedia and Expo, 2007 IEEE International Conference on, Jul. 2007

[9] R. Krupiński and J. Purczyński, “Modeling the distribution of DCT coefficients for JPEG reconstruction,” Signal Processing: Image Communication, vol. 22, no. 5, pp. 439–447, Jun. 2007.

[10] W. K. Pratt, Digital Image Processing. New York: Wiley, 1978

[11] G. Lakhani, “Adjustments for JPEG de-quantization coefficients,” in 1998 Proceedings Data Compression Conference, J. A. Storer and M. Cohn, Eds. Los Alamitos, CA: IEEE Comput. Soc. Press, Mar. 1998, p. 557.

[12] R. Brown and A. Boden, “A posteriori restoration of block transformcompressed data,” in 1995 Proceedings Data Compression Conference, J. A. Storer and M. Cohn, Eds. Los Alamitos, CA: IEEE Comput. Soc. Press, Mar. 1995, p. 426

[13] J. A. Nikara, J. H. Takala, and J. T. Astola, “Discrete cosine and sine transforms—regular algorithms and pipeline architectures,” Signal Processing, vol. 86, no. 2, pp. 230–249, Feb. 2006.

[14] G. Plonka and M. Tasche, “Fast and numerically stable algorithms for discrete cosine transforms,” Linear Algebra and its Applications, vol. 394, pp. 309–345, Jan. 2005.

[15] S. G. Mallat, “A theory for multiresolution signal decomposition: The wavelet representation,” IEEE Trans. Pattern Anal. Machine Intell., vol. 11, pp. 674–693, July 1989.

[16] F. Bellifemine, A. Capellino, A. Chimienti, R. Picco, and R. Ponti, “Statistical analysis of the 2D-DCT coefficients of the differential signal for images,” Signal Process. Image Commun., vol. 4, pp. 477–488, Nov. 1992.

[17] S. Paris, S. W. Hasinoff, and J. Kautz, “Local Laplacian filters,” ACM SIGGRAPH 2011 papers on - SIGGRAPH '11, 2011.

[18] Burt, P. J., and Adelson, e. H. 1983. The Laplacian pyramid asacompactimagecode. IEEETransactionsonCommunication 31, 4, 532–540

[19] Heeger, D. J., and Bergen, J. R. 1995. Pyramid-based texture analysis/synthesis. In Proceedings of the ACM SIGGRAPH conference.

[20] Sunkavalli, K., Johnson, M. K., Matusik, W., Pfister, H. 2010. Multi-scale image harmonization. ACM Transactions on Graphics (Proc. SIGGRAPH) 29, 3.

[21] Y. Li, L. Sharan, and E. H. Adelson, “Compressing and companding high dynamic range images with subband architectures,” ACM SIGGRAPH 2005 Papers on - SIGGRAPH ’05, 2005.

[22] Farbman, Z., Fattal, R., Lischinski, D., Szeliski, R. 2008. Edge-preserving decompositions for multi-scale tone and detailmanipulation. ACMTransactionsonGraphics(Proc.SIGGRAPH) 27, 3

[23] Perona, P., and Malik, J. 1990. Scale-spaceandedgedetection using anisotropic diffusion. IEEE Transactions Pattern Analysis Machine Intelligence 12, 7, 629–639.

[24] G. Aubert and P. Kornprobst, “Image Processing: Mathematics,” Encyclopedia of Mathematical Physics, pp. 1–10, 2006.

[25] C. Tomasi and R. Manduchi, “Bilateral filtering for gray and color images,” Sixth International Conference on Computer Vision (IEEE Cat. No.98CH36271).

[26] Z. Farbman, R. Fattal, D. Lischinski, and R. Szeliski, “Edge-preserving decompositions for multi-scale tone and detail manipulation,” ACM SIGGRAPH 2008 papers on - SIGGRAPH ’08, 2008.

[27] A. Buades, B. Coll, and J.-M. Morel, “The staircasing effect in neighborhood filters and its solution,” IEEE Transactions on Image Processing, vol. 15, no. 6, pp. 1499–1505, Jun. 2006.

[28] F. Durand and J. Dorsey, “Fast bilateral filtering for the display of high-dynamic-range images,” ACM Transactions on Graphics, vol. 21, no. 3, Jul. 2002.

[29] S. Bae, S. Paris, and F. Durand, “Two-scale tone management for photographic look,” ACM SIGGRAPH 2006 Papers on - SIGGRAPH ’06, 2006.

[30] M. Kass and J. Solomon, “Smoothed local histogram filters,” ACM SIGGRAPH 2010 papers on - SIGGRAPH ’10, 2010.

[31] In.mathworks.com. (2018). MATLAB Product Description- MATLAB & Simulink- MathWorks India. [online] Available at: https://in.mathworks.com/help/matlab/learn_matlab/product-description.html

[32] Madhulika, A. Bansal, D. Yadav, and . Madhurima, “Survey and Comparative Study on Statistical Tools for Medical Images,” Advanced Science Letters, vol. 21, no. 1, pp. 74–77, Jan. 2015.

[33] Mc Andrew,A.,2004. Introduction to Digital Image Processing with Matlab. Thompson Course Technology,Melbourne.

[34] Gabriel Peyré. The Numerical Tours of Signal Processing - Advanced Computational Signal and Image Processing. IEEE Computing in Science and Engineering, 2011, 13 (4), pp.94-97.

[35] Fabbri, Ricardo, Odemir Martinez Bruno, and Luciano da Fontoura Costa. "Scilab and SIP for image processing." arXiv preprint arXiv:1203.4009 (2012).

[36] Eaton, John Wesley, David Bateman, and Søren Hauberg. Gnu octave. London: Network thoery, 1997.

[37] GNU Octave, https://www.gnu.org/software/octave/

[38] Sparks, Todd, Heng Pan, and Frank W. Liou. "Development of image processing tools for analysis of laser deposition experiments." Proceedings of the Fifteenth Annual Solid Freeform Fabrication Symposium. 2004.

[39] Flynn, Jeremy R., et al. "Image quality assessment using the ssim and the just noticeable difference paradigm." International Conference on Engineering Psychology and Cognitive Ergonomics. Springer, Berlin, Heidelberg, 2013.

[40] Landré, Jérôme, Frédéric Morain-Nicolier, and Su Ruan. "Ornamental letters image classification using local dissimilarity maps." Document Analysis and Recognition, 2009. ICDAR'09. 10th International Conference on. IEEE, 2009.

[41] Elsheh, Esam, and A. Ben Hamza. "Comments on matrix-based secret sharing scheme for images." Iberoamerican Congress on Pattern Recognition. Springer, Berlin, Heidelberg, 2010.

[42] Penaranda, Luis, Luiz Velho, and Leonardo Sacht. "Real-time correction of panoramic images using hyperbolic moebius transformations." Journal of Real-Time Image Processing (2015): 1-14.