What Is This Module About?

This module is a continuation of the module on arithmetic sequences and series. Before you read this module, be sure that you have finished reading the module on "Arithmetic Sequence." It would help you understand this module more easily.

The following topics are discussed in this module:

Lesson 1—*Geometric Sequence*

Lesson 2—*The n^{th} Term of a Geometric Sequence*

Lesson 3—*The Sum of the First n Terms in a Geometric Sequence*

What Will You Learn From This Module?

After studying this module, you should be able to:

- ♦ state whether a given sequence is geometric or not;
- ♦ use the formula for finding the n^{th} term of a geometric sequence;
- ♦ compute the sum of the first n terms of a geometric sequence; and
- ♦ solve problems which involve geometric sequences.

Wait!

Before you start studying this module, be sure to have read the following modules: "Exponents and Radicals" and "Arithmetic Sequence." Reading these modules before this module will help you understand the topics in this module better.

Let's See What You Already Know

Before reading the module, answer the following exercises first to determine how much you already know about the topics.

A. Determine if the given sequence is geometric or not. Write **G** in the blank if it is and **N** if it is not.

_____ 1. 1, 3, 9, 27, 81, . . .

_____ 2. –4, 8, –16, 32, –64, . . .

______ 3. 1, 4, 16, 64, . . .

______ 4. 20, 13, 6, –1, –8, . . .

______ 5. –5, 0, 5, 10, 15, . . .

B. Find the n^{th} term in the following geometric sequences.

1. 5^{th} term if the 1^{st} term is 5 and the common ratio is 4

2. 6^{th} term of the sequence 1, 4, 16, 64 . . .

3. 4^{th} term of the sequence if the 1^{st} term is 5 and the common ratio is 2

4. 7^{th} term of the sequence 3, 15, 75, . . .

C. Find the sum of the 1^{st} n terms of the following geometric sequences.

1. The sum of the first 7 terms if $a_1 = ¼$, $r = –4$

2. The sum of the first 6 terms if the 1^{st} term is 4 and the common ratio is –3.

D. Solve this problem: The value of a P50,000 machine depreciates by 20% every year. What will be its value at the end of 5 years?

Well, how was it? Do you think you fared well? Compare your answers with those in the *Answer Key* on pages 28 to 30 to find out.

If all your answers are correct, very good! This shows that you already know much about the topics. You may still study the module to review what you already know. Who knows, you might learn a few more new things as well.

If you got a low score, don't feel bad. This means that this module is for you. It will help you understand important concepts that you can apply in your daily life. If you study this module carefully, you will learn the answers to all the items in the test and a lot more! Are you ready?

You may go now to the next page to begin Lesson 1.

Geometric Sequence

Can you still remember what an arithmetic sequence is? Well, an **arithmetic sequence** is a set of numbers wherein the difference between any two consecutive numbers or terms is constant (common difference).

In this lesson, you will learn about another type of number sequence—the geometric sequence. If you understood what an arithmetic sequence is, I'm sure it will also be easy for you to understand what a geometric sequence is.

Are you ready now?

Let's Try This

EXAMPLE 1

Sharon is raising chickens as a source of additional income. Among the chickens that she was raising, she chose to observe several of them as possible egg layers. She was very happy when they started laying their eggs. Sharon knew that day after day, these hens would be laying more and more eggs. She made a tally of the number of eggs her hens laid each day. The table below shows her tally.

Day	Number of Eggs
1	4
2	8
3	16
4	32
5	64

What can you say about the numbers? Is there anything in particular that you have noticed?

Let's look at the numbers closely.

4 8 16 32 64

From the numbers, we can say that the second number is twice the first number. This means that $4 \times 2 = 8$.

Again, look at the third number. Isn't it $8 \times 2 = 16$?

What about 32? Isn't this equal to 2×16?

So, we can make the following table:

$$
\begin{array}{rcl}
4 & = & \text{first term} \\
8 & = & 2 \times 4 \\
16 & = & 2 \times 8 \\
32 & = & 2 \times 16 \\
64 & = & 2 \times 32
\end{array}
$$

Based on the table above, can you determine the number of eggs Sharon's hen will lay on the sixth and seventh days?

Let's do it together. Of course, it will be better if we present the numbers in a table again.

Day	Number of Eggs	
1	4	the first term
2	8	2 × 4
3	16	2 × 8
4	32	2 × 16
5	64	2 × 32
6	___	2 × ____
7	___	2 × ____

Are your answers 128 and 256 for the 6th and 7th days, respectively? If yes, then you're doing great.

Let's try another example.

EXAMPLE 2 Examine the following sequence of numbers: –2, –6, –18, –54 . . .

What can you say about it?

Looking at the numbers, we can say that there is a commonality among them. Let's put the numbers in a table as in:

–2	=	first term
–6	=	3 × (– 2)
–18	=	3 × (– 6)
–54	=	3 × (–18)

Based on the two examples, what can we say about the sequences of numbers we worked on?

We can say that they are sequenced wherein each term after the first can be obtained by multiplying the preceding term by a constant called the **common ratio.**

In the first example, the constant is *2*. This is the number that we used as multiplier to get the succeeding terms in the sequences. In the second example, the constant is *3*. This is the number that we used as multiplier to get the succeeding terms.

A sequence in which each term after the first can be obtained by multiplying the preceding term by a fixed constant (common ratio) is called a **geometric sequence.**

Remember

A **geometric sequence** is a sequence in which each term after the first can be obtained by multiplying the preceding term by a fixed constant called the *common ratio* denoted by *r*.

Can you recall anything familiar given this definition of a geometric sequence? I bet you can! Isn't it similar to the definition of an arithmetic sequence?

Recall that an **arithmetic sequence** is a sequence of numbers wherein the difference between any two consecutive terms is constant (common difference denoted by *d*). You obtain *d* by getting the difference between any two consecutive numbers in the sequence.

In a geometric sequence, you obtain *r* by dividing any term by the preceding term. Recall our example about Sharon's hen. We had the following sequence of numbers: 4, 8, 16, 32, 64 . . .

To obtain *r*, the common ratio, divide any term by its preceding term. Let's say, we divide 8 by 4. The quotient is 2. (We call the answer to a division process a **quotient**.) Again, we divide 16 by 8. The answer is also 2. If we divide 64 by 32, the answer is again 2. This means that our common ratio is therefore 2. So, we can denote 2 as *r*.

Let's recall Example 2 in which we had the following sequence of numbers: –2, –6, –18, –54 . . .

Can you determine what the common ratio of this geometric sequence is?

Let's do it together.

$$(-6) \div (-2) = 3$$
$$(-18) \div (-6) = 3$$
$$(-54) \div (-18) = 3$$

When dividing numbers with like signs, the sign of the quotient is always positive.

So, 3 is the common ratio or $r = 3$.

Let’s Try This

Find the common ratio of the following geometric sequences.

A. 3, 9, 27, 81

B. 1, 4, 16, 64

Did you answer *3* as the common ratio for A, and *4* as the common ratio for B? If you did, you’re doing well!

Let’s See What You Have Learned

Identify if the following sequences of numbers are arithmetic sequences or geometric sequences. Write **A** in the blanks if they are arithmetic sequences and **G** if they are geometric sequences.

A. ____ 1. 2, 4, 6, 8, 10, . . .

____ 2. 5, 15, 45, 135, . . .

____ 3. 4, 16, 64, 256, . . .

____ 4. 2, 1, .5, .25, .125, . . .

____ 5. 3, 6, 9, 12, 15, . . .

B. Using the same sets of numbers in Exercise A, indicate the common differences (d) of the arithmetic sequences and the common ratios (r) of the geometric sequences. Indicate whether your answers are common differences or ratios.

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

Compare your answers with those found in the *Answer Key* on page 30. If you get 8 to 10 correct answers, you’re doing great. Continue reading this module. If you get a score below 8, go back to Lesson 1 then try to solve the exercises again.

Let’s Remember

- A **geometric sequence** is a sequence of numbers wherein each term after the first is obtained by multiplying the preceding term by a fixed constant called the **common ratio** denoted by r.

- The *common ratio* or r is obtained by dividing any term in a geometric sequence by the preceding term.

LESSON 2

The n^{th} Term of a Geometric Sequence

You already know what a geometric sequence is and how to obtain its succeeding terms. You also learned how to determine the common ratio or r.

In this lesson, you will learn how to determine the n^{th} term in a geometric sequence.

Recall that in an arithmetic sequence, we call the first term a_1. The same is also true for a geometric sequence. We call the first term a_1. The second term is denoted as a_2, the third term a_3 and so on until the n^{th} term a_n.

EXAMPLE 1

A couple had two children. Each of the children got married and gave birth to two children each. Following this pattern, how many children will there be in the 6^{th} generation? (Consider the first couple as the first generation.)

Let us analyze the given example following the steps below.

STEP 1 Determine the first term in the sequence.

If we are to designate the couple as the first term in the sequence, we denote it as a_1.

$$a_1 = 2$$

STEP 2 Determine the symbol for the unknown term in the sequence.

You are asked for the number of children in the 6^{th} generation or the 6^{th} term in the geometric sequence. Thus, we will solve for a_6.

$$a_6 = ?$$

STEP 3 Find the common ratio.

Remember that we obtain the next term in a geometric sequence by multiplying the preceding term by the common ratio which is constant. In the example, the common ratio is 2 because each couple in a generation has two children each. We designate $r = 2$.

$$r = 2$$

STEP 4 Work out the sequence starting from the first term, multiplying this by 2 to find the next number in the sequence and so on.

See the table below.

1^{st} generation (a_1) = 2

2^{nd} generation (a_2) = $2 \times 2 = 4$

3^{rd} generation (a_3) = $2 \times 2 \times 2 = 8$

4^{th} generation (a_4) = $2 \times 2 \times 2 \times 2 = 16$

5^{th} generation (a_5) = $2 \times 2 \times 2 \times 2 \times 2 = 32$

6^{th} generation (a_6) = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

So, the 6^{th} term in the sequence or the number of children in the 6^{th} generation is 64.

Suppose we cannot use the steps in solving the problem? How else can we solve for the unknown?

We can use a formula (Equation 1) for finding the n^{th} term in a geometric sequence. Let us see how this can be done.

Let us go back to Example 1 in this lesson.

Study the table below.

(a_1) = 2 = first term

(a_2) = $2 \times 2 = 4$

(a_3) = $2 \times 2 \times 2 = 8$

(a_4) = $2 \times 2 \times 2 \times 2 = 16$

(a_5) = $2 \times 2 \times 2 \times 2 \times 2 = 32$

(a_6) = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

We already know that to find the succeeding terms in the sequence, we multiply each term by a common ratio, *r*.

In the example, the common ratio is 2.

To get a_2, we multiply a_1 by $r = 2$. This can be denoted as:

$a_2 = a_1 \times r$

To get a_3, we multiply a_1 by r two times or twice. This can be denoted as:

$$a_3 = a_1 \times r \times r$$

At this point, this module will introduce to you the term *exponent*. An **exponent** is a number written on the upper right of a number to indicate the number of times you will multiply the number by itself.

In the term a_3, r is multiplied twice ($r \times r$), which can also be written as r^2.

So, $a_3 = a_1 \times r^2$

Now, let's proceed to the 4th term. We have the following number sentence for the 4th term.

$$a_4 = a_1 \times r \times r \times r$$

or

$$a_4 = a_1 \times r^3$$

Now try writing the formula for solving the 5th and 6th terms on your own.

Let's Try This

1. $a_5 = a_1 \times r \times r \times r \times r$ or $a_5 = a_1 \times$ _____
2. $a_6 = a_1 \times r \times r \times r \times r \times r$ or $a_6 = a_1 \times$ _____

Are your answers the same as the following?

1. $a_5 = a_1 \times r^4$
2. $a_6 = a_1 \times r^5$

If they are, very good. You're doing well!

Now, let us look at the number sentences we came up with.

$$a_1 = \text{first term}$$
$$a_2 = a_1 \times r^1$$
$$a_3 = a_1 \times r^2$$
$$a_4 = a_1 \times r^3$$
$$a_5 = a_1 \times r^4$$
$$a_6 = a_1 \times r^5$$

What did you notice about the numbers designating the sequence of the terms and exponents in their corresponding equations?

Note that: $a_2 = a_1 \times r^1$ (the exponent is 2 – 1 = 1)

$a_3 = a_1 \times r^2$ (the exponent is 3 – 1 = 2)

So, if we are to find the n^{th} term in a geometric sequence, our number sentence would be:

$$a_n = a_1 \times r^{n-1}$$

or $a_n = a_1 r^{n-1}$ (let us denote this as Equation 1)

We remove the multiplication sign, ×, since it is understood that the numbers are to be multiplied.

We can now use Equation 1 to find any term in a geometric sequence provided the following information are given:

- the first term denoted by a_1; and
- the common ratio, r.

Now, let us try using the formula or Equation 1.

EXAMPLE 2 Find the 6th term in the sequence –2, –6, –18, –54, . . .

Let us analyze the given example by doing the following steps.

STEP 1 Find the common ratio r.

Remember: The common ratio or r is obtained by dividing any term by its preceding term.

So, divide:

$$(-6) \div (-2) = 3$$

$$(-18) \div (-6) = 3$$

$$(-54) \div (-18) = 3$$

The common ratio or $r = 3$.

STEP 2 Determine the value of the 1st term.

The value of the 1st term, $a_1 = -2$.

STEP 3 Determine the symbol for the unknown term in the sequence.

You are asked for the 6th term in the given geometric sequence. Thus, you are to solve for a_6.

STEP 4 Find the exponent of the common ratio.

The exponent of the common ratio $n - 1 = 6 - 1 = 5$.

STEP 5 Write an equation for the unknown term in the geometric sequence.

From Equation 1, $a_n = a_1 r^{n-1}$ we have

$$a_6 = a_1 r^5$$

STEP 6 Substitute the values obtained in Steps 1 and 2 and solve for the unknown.

Using the equation in Step 5, $a_6 = a_1 r^5$, we will get:

$$\begin{aligned} a_6 &= (-2)(3)^{6-1} \\ &= (-2)(3)^5 \\ &= (-2)(243) \\ &= -486 \end{aligned}$$

Thus, the 6th term in the given geometric sequence is –486.

EXAMPLE 3 Mang Pepe deposited ₱10,000 in a bank. He planned to save that amount for his son's future. The bank gives a 7% interest per annum. How much will his savings be at the end of 5 years?

Analyze the given problem as in:

Mang Pepe's initial deposit is ₱10,000. The rate of interest per year is 7%. 7% can also be written as .07 in decimal form. This is the form we will use in the calculation.

The interest for the 1st year is ₱10,000 × .07 = ₱700. Thus, his money in the bank at the end of one year will be:

₱10,000 + ₱700 = ₱10,700

The interest for the 2nd year is ₱10,700 × .07 = ₱749. This means that Mang Pepe's money in the bank at the end of two years will be:

₱10,700 + ₱749 = ₱11,449

The interest for the 3rd year is ₱11,449 × .07 = ₱801.43. This means that Mang Pepe's money at the end of three years will be:

₱11,449.00 + ₱801.43 = ₱12,250.43

Let us look at the results of the computations by putting them in a table as in:

Year 1	=	₱10,700
Year 2	=	₱11,449
Year 3	=	₱12,250.43

Now, let us determine if there is a common ratio. To do this, we divide the second term by the first term as in:

₱11,449 ÷ ₱10,700 = 1.07

Divide the third term by the second term as in:

₱12,250.43 ÷ ₱11,449 = 1.07

This means that the values form a geometric sequence with common ratio or r = 1.07 and a_1 = ₱10,700 as the first term.

We can then use Equation 1, $a_n = a_1r^{n-1}$ to find any term in the sequence.

In the given problem, you are asked to find Mang Pepe's total savings at the end of five years. This is equivalent to the 5th term or a_5. So, n = 5. Using the values, we have:

$$
\begin{aligned}
a_5 &= a_1r^{n-1} \\
&= (₱10{,}700)(1.07)^{5-1} \\
&= (₱10{,}700)(1.07)^{4} \\
&= (₱10{,}700)(1.310796) \\
a_5 &= ₱14{,}025.52
\end{aligned}
$$

Mang Pepe's savings at the end of five years will therefore be ₱14,025.52

Let's Remember

- The formula $a_n = a_1r^{n-1}$

  where: n = number of terms;

  r = common ratio; and

  a_1= 1st term

  is used to find the n^{th} term in a geometric sequence.

- We follow the following steps in solving the n^{th} term in a geometric sequence.

  STEP 1 Find the common ratio (r).

  STEP 2 Determine the value of the 1st term (a_1).

STEP 3 Determine the symbol for the unknown term in the sequence.

STEP 4 Find the exponent of the common ratio (n–1).

STEP 5 Write an equation for the unknown term in the geometric sequence.

STEP 6 Substitute the values obtained in Steps 1 and 2 and solve for the unknown.

We will now see how well you understood Lesson 2.

Let's See What You Have Learned

A. Using the six steps in solving for the n^{th} term in a geometric sequence, solve for the following:

1. the 6th term in the sequence ${}^{1}/_{2}, -{}^{1}/_{4}, {}^{1}/_{8}, \ldots$

2. the population in a certain barrio in the year 2000 if the population in 1995 was 10,000 and the population increases annually by 2% due to migration.

B. Use Equation 1 to find the n^{th} term in the following geometric sequences.

1. A house worth ₱200,000 devaluates by 5% every year. How much will it be worth at the end of four years?

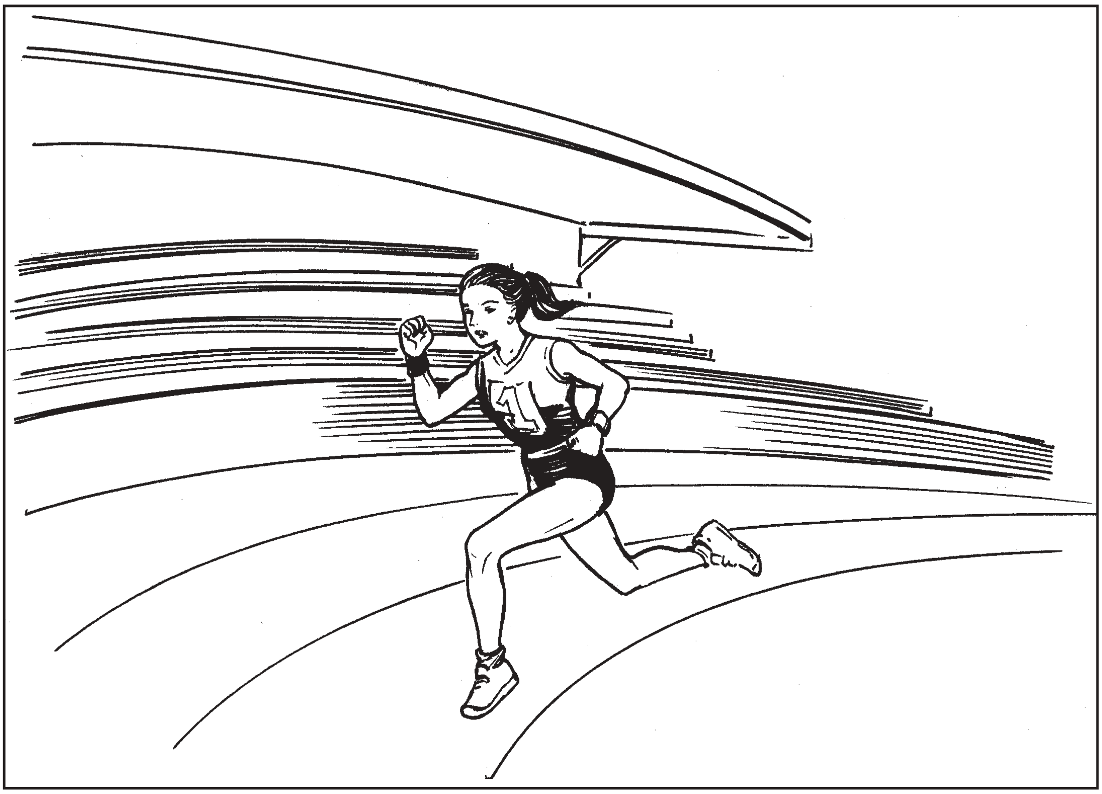
2. Suppose you are holding a piece of paper which is .02 inch thick. Each time you fold the paper in half, its thickness is doubled. What will be its thickness if you fold it ten times?

3. Suppose someone offered you a job for seven days under the following conditions: you will be paid ₱100 on the 1st day, ₱200 on the 2nd, ₱400 on the 3rd and so on. This means your salary doubles each day. What will be your salary on the 7th day?

Compare your answers with those in the *Answer Key* on pages 31 to 33. If you got 4 or 5 correct answers, you did great. Continue reading this module. If you got a score below 4, read Lesson 2 again and try to solve more exercises similar to those given in the lesson.

The Sum of the First *n* Terms in a Geometric Sequence

This lesson will discuss how to find the sum of the first *n* terms of a geometric sequence. It will also discuss how you can apply this skill in your daily life.



Speedy is the star runner of the Philippine team that will compete in the Olympics this year. A sports analyst studied her track records while she was preparing for competition. The analyst used the following formula to compute the total distance Speedy covered:

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$

or

$$S_n = \frac{a_1 r^n - a_1}{r - 1}, \text{ where } r \neq 1$$

where a_1 = distance covered on the 1st day

r = increase in distance covered per day

For this year's competition, Speedy covered 4,000 m on the 1st day of preparation. She decided to constantly increase the distance she will cover each day by 10%. Let us help the sports analyst compute for the total distance that Speedy should cover at the end of the 6th day.

EXAMPLE 1 Consider the number of meters covered by Speedy each day.

1st day: 4,000

2nd day: 4,000 + (.10 × 4,000) = 4,400

3rd day: 4,400 + (.10 × 4,400) = 4,840

The numbers 4,000, 4,400, 4,840, . . . form a geometric sequence.

Find the common ratio, *r*.

The common ratio (*r*) in the given problem is: 4,400/4,000 = 1.1.

STEP 2 Identify the 1st term (a_1) in the geometric sequence.

$a_1 = 4{,}000$

STEP 3 Write the equation for the sum of the 1st *n* terms in the geometric sequence.

1st 2 days: $S_2 = \dfrac{a\left(r^2 - 1\right)}{r - 1}$

1st 3 days: $S_3 = \dfrac{a\left(r^3 - 1\right)}{r - 1}$

1st 4 days: $S_4 = \dfrac{a\left(r^4 - 1\right)}{r - 1}$

STEP 4 Substitute the values in Steps 1 and 2 to determine the unknown.

1st day: 4,000 m, using the values in Step 1, r = 1.1 and in Step 2, a_1 = 4,000. By substituting these values in the equations above, we will get:

1st 2 days:

$$S_2 = \frac{4000\left(1.1^2 - 1\right)}{1.1 - 1} = \frac{4000(1.21 - 1)}{.1} = \frac{4000(.21)}{.1} = \frac{840}{.1}$$
$$S_2 = 8{,}400 \text{ m}$$

1st 3 days:

$$S_3 = \frac{4000\left(1.1^3 - 1\right)}{1.1 - 1} = \frac{4000(1.331 - 1)}{.1} = \frac{4000(.331)}{.1} = \frac{1324}{.1}$$
$$S_3 = 13{,}240 \text{ m}$$

1st 4 days:

$$S_4 = \frac{4000\left(1.1^4 - 1\right)}{1.1 - 1} = \frac{4000(1.4641 - 1)}{.1} = \frac{4000(.4641)}{.1} = \frac{185}{.1}$$
$$S_4 = 18{,}564 \text{ m}$$

Let us list these values in a table. Compute for the missing values.

Day	***S_n***
1st	4,000 m
2nd	8,400 m
3rd	13,240 m
4th	18,564 m
5th	
6th	

Are your answers 24,420 m and 30,840 m, respectively? If yes, that's very good. Continue reading this module. If not, review your computations so you will get them right.

Now, let's go back to the formula the analyst used.

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1} \quad \text{Call this Equation 2}$$

Equation 2 is the formula for finding the sum of the 1st n terms in a geometric sequence.

Let's have more examples.

EXAMPLE 2 What is the sum of the 1st 8 terms of a geometric sequence whose 1st term is 5 and common ratio is 2?

Here is the solution.

STEP 1 Find the common ratio, r.

The common ratio (r) is given in the problem.

$$r = 2$$

STEP 2 Find the 1st term (a_1) in the geometric sequence.

$$a_1 = 5$$

STEP 3 Write the equation for the sum of the 1st n terms in the geometric sequence. We are asked to solve for the sum of the 1st 8 terms so we will use the following equation:

$$S_8 = \frac{a\left(r^8 - 1\right)}{r - 1}$$

STEP 4 Substitute the values obtained in Steps 1 and 2 to determine the unknown.

We will then have:

$$S_8 = \frac{5\left(2^8 - 1\right)}{2 - 1} = \frac{5(256 - 1)}{1} = \frac{5(255)}{1} = \frac{1275}{1} = 1{,}275$$

This means that the 8th term in the given sequence is 1,275.

EXAMPLE 3 Suppose someone offered you a job for 7 days under the following conditions: you will be paid ₱100 on the 1st day, ₱200 on the 2nd, ₱400 on the 3rd and so on. This means, for each succeeding day, your salary will be doubled. How much will your salary be after 7 days?

Here is the solution.

STEP 1 Find the common ratio, *r*.

The common ratio (*r*) is given in the problem.

$$r = \frac{200}{100} = 2$$

STEP 2 Find the 1st term in the geometric sequence.

The given in the problem 100, 200, 400, form a geometric sequence with the 1st term $a_1 = 100$.

STEP 3 Write the equation for the sum of the 1st *n* terms in the geometric sequence.

We are asked to solve for the sum of the 1st 7 terms. Thus, we will have:

$$S_7 = \frac{a\left(r^7 - 1\right)}{r - 1}$$

STEP 4 Substitute the values obtained in Steps 1 and 2 to determine the unknown sum of the geometric sequence.

We will then have:

$$\begin{aligned} S_7 &= \frac{₱100\left(2^7 - 1\right)}{2 - 1} \\ &= \frac{₱100(128 - 1)}{1} \\ &= \frac{₱100(127)}{1} \\ &= \frac{₱12{,}700}{1} \\ &= ₱12{,}700 \end{aligned}$$

So, your salary after 7 days will be ₱12,700.

What if we just use Equation 2 instead of following the 4 steps? Let us look at the next example.

EXAMPLE 4 Mrs. de Claro opened a savings account with an initial deposit of ₱1,000. She plans to increase her deposit by 15% every month within a period of 8 months. How much will her total savings be at the end of 8 months?

Let us use the table presented below.

Month	Initial Deposit	Increase of Monthly Deposit at the Rate of 15%	Monthly Deposit
1st	1,000	—	1,000
2nd	—	1,000 × .15 = 150	1,000 + 150 = 1,150
3rd	—	1,150 × .15 = 172.50	1,150 + 172 = 1,322.50

Note the following:

- ♦ To compute the increase in the monthly deposit of Mrs. de Claro at the rate of 15% per month, change this to its decimal form first by dividing 15/100 = .15 and drop the percent sign. Multiply .15 by the initial deposit and add the product to ₱1,000 to get the amount Mrs. de Claro deposited for the second month and so on.
- ♦ It can be noted that the monthly deposit of 1,000, 1,150 and 1,322.50 form a geometric sequence with $r = 1.15$ and $a_1 = 1{,}000$.
- ♦ You are asked to find the total amount Mrs. de Claro deposited within a period of 8 months. This is the same as finding the sum of the 1st 8 terms of the given geometric sequence. Thus, we will solve for S_8 using the formula:

$$S_8 = \frac{a\left(r^8 - 1\right)}{r - 1}$$

♦ By substituting the values, we will get:

$$S_8 = \frac{₱1{,}000\left(1.15^8 - 1\right)}{1.15 - 1} = \frac{₱1{,}000\left(3.059 - 1\right)}{.15}$$
$$= \frac{₱1{,}000\left(2.059\right)}{.15} = \frac{₱2{,}059}{.15} = ₱13{,}726.67$$

This means that Mrs. de Claro will have a total of ₱13,726.67 in her savings accounts.

Let's Remember

♦ The formula for finding the sum of the 1st n terms of a geometric sequence is:

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

where: n = number of terms

a_1 = 1st term

r = common ratio

♦ The following steps should be followed in using the above formula:

STEP 1 Find the common ratio, r.

STEP 2 Find the 1st term in the geometric sequence.

STEP 3 Write the equation for finding the sum of the 1st n terms of the geometric sequence.

STEP 4 Substitute the values obtained in Steps 1 and 2 to solve for the unknown using the equation.

Let’s See What You Have Learned

A. Using the 4 steps in finding the sum of the 1st *n* terms of a geometric sequence, solve the following problems.

1. What is the sum of the 1st 6 terms of the geometric sequence 2, .2, .02, . . . ?

2. A ball is dropped from a height of 50 ft and rebounds 80% of the distance it traveled. What is its total rebound distance travelled after rebounding the 5th time?

B. Solve the following problems using Equation 2.

1. A housemaid has an initial monthly salary of ₱1,000. Her employer plans to give her a 20% salary raise every year. How much will her total salary be after three years?

2. A man earned a commission of ₱200 on the 1st week, ₱600 on the 2nd week, ₱1800 on the 3rd week and so on until the 10th week. What is his total commission after 7 weeks?

3. Manny gave Minda 3 red roses on their 1st wedding anniversary, 6 on their 2nd, 12 on their 3rd and so on. How many roses will Minda receive on their 6th wedding anniversary?

Compare your solutions with those found in the *Answer Key* on pages 34 to 36. If you got a score of:

4–5 Very good! You learned a lot from this lesson.

3 Good! Review the parts which you did not understand very well.

0–2 Read the lesson again and solve more exercises related to the topic discussed in the lesson.

Let’s Sum Up

In this module, you learned the following:

♦ A **geometric sequence** is a sequence of number wherein each term after the first can be obtained by multiplying the preceding term by a fixed constant called the *common ratio* denoted by *r*.

- The common ratio, r, is obtained by dividing any term in a geometric sequence by its preceding term.

- In finding the n^{th} term in a geometric sequence, we use the formula and follow the steps below:

$a_n = a_1 r^{n-1}$ where: n = number of terms

r = common ratio

$a_1 = 1^{st}$ term

STEP 1 Find the common ratio, r.

STEP 2 Determine the value of the 1^{st} term.

STEP 3 Find the symbol for the unknown term in the sequence.

STEP 4 Find the exponent of the common ratio.

STEP 5 Write an equation for solving the unknown term.

STEP 6 Substitute the values obtained in Steps 1 and 2 to the equation and solve for the unknown.

- To find the sum of the 1^{st} n terms of a geometric sequence, we use the formula and follow the steps below:

$$S_n = \frac{a_1(r^n - 1)}{r - 1}$$ where: n = number of terms

a_1 = 1st term

r = common ratio

STEP 1 Find the common ratio, r.

STEP 2 Find the 1^{st} term in the geometric sequence.

STEP 3 Write the equation for finding the sum of the 1^{st} n terms of the geometric sequence.

STEP 4 Substitute the values obtained in Steps 1 and 2 to solve for the unknown.

What Have You Learned?

Let us check if you have understood the topic discussed in this module. Answer the exercises below.

A. Determine if the given set of numbers is an arithmetic sequence or a geometric sequence. Write **A** in the blank if the sequence is arithmetic and **G** if it is geometric.

_____ 1. 3, –1, –5, –9, . . .

_____ 2. ½, 2, 8, 32, . . .

_____ 3. 5¼, 5½, 5¾, 6, . . .

_____ 4. 3/2, –3/4, 3/8, –3/16, . . .

_____ 5. 6, –18, 54, –162, . . .

B. Using the formula for finding the n^{th} term in a geometric sequence, solve for the following:

1. 6^{th} term in the sequence 3, 6, 12, . . .

2. 5^{th} term of the geometric sequence whose 1^{st} term is 64 and common ratio is $-{}^{1}/_{2}$

C. Using the formula for finding the sum of the 1^{st} n terms of a geometric sequence, solve for the following:

1. sum of the first 7 terms of the geometric sequence ${}^{1}/_{18}$, $-{}^{1}/_{6}$, ${}^{1}/_{2}$, . . .

2. sum of the first 8 terms of the geometric sequence 400, 500, 625, . . .

D. Solve the following problems.

1. Rico placed ₱20,000 in a time deposit with an interest of 8% computed quarterly. If his time deposit will mature in one year, what will be the value of his deposit upon maturity?

2. The Nonformal Accreditation and Equivalency System served 10,236 learners on its first year of implementation. The DECS-Bureau of Nonformal Education is targeting a 14% increase of learners to be served in 5 years. What will be the total number of learners served after 5 years?

3. Ricardo bought a brand-new car worth ₱350,000. Its value depreciates 15% every year. What will be its value at the end of the 7^{th} year?

Compare your answers with those found in the *Answer Key* on pages 36 to 39. If you got 7 to 10 correct answers, congratulations. You did great. You really understood the topics discussed in this module. However, if you got a score of:

6–7	Just review the parts of the module you did not understand very well.
4–5	Review the parts you did not understand and solve other exercises similar to those in this module.
1–3	Study the whole module again.

Answer Key

A. How Much Do you Know About This Topic? *(pages 1–2)*

A. 1. G

2. G

3. G

4. N

5. N

B. 1. $a_5 = 1{,}280$

Solution: $a_1 = 5$; $r = 4$

$$\begin{aligned} a_5 &= a_1 r^{5-1} \\ &= 5(4^4) \\ &= 5(4 \times 4 \times 4 \times 4) \\ &= 5(256) \\ &= 1{,}280 \end{aligned}$$

2. $a_6 = 1{,}024$

Solution: $a_1 = 1$; $r = 4$

$r = 4 \div 1 = 4$; $16 \div 4 = 4$

$r = 4$

$$\begin{aligned} a_6 &= a_1(r^{6-1}) \\ &= 1(4^5) \\ &= 1(4 \times 4 \times 4 \times 4 \times 4) \\ &= 1(1{,}024) \\ &= 1{,}024 \end{aligned}$$

3. $a_4 = 40$

4. $a_7 = 46{,}875$

C. 1. $S_7 = 819.25$

Solution: $a_1 = \frac{1}{4}$; $r = -4$; $n = 7$

$$S_n = \frac{a_1\left(r^n - 1\right)}{r - 1}$$

$$S_7 = \frac{\frac{1}{4}\left[(-4)^7 - 1\right]}{\left[(-4) - 1\right]}$$

$$= \frac{\frac{1}{4}\left[(-16{,}384) - 1\right]}{\left[-5\right]}$$

$$= \frac{\frac{1}{4}\left[-16{,}385\right]}{-5}$$

$$= \frac{-4{,}096.25}{-5}$$

$$= 819.25$$

2. $S_6 = -728$

Solution: $a_1 = 4$; $r = -3$

$$S_6 = \frac{a_1\left(r^6 - 1\right)}{r - 1}$$

$$= \frac{4\left[(-3)^6 - 1\right]}{(-3) - 1}$$

$$= \frac{4\left[729 - 1\right]}{-4}$$

$$= \frac{4(728)}{-4}$$

$$= \frac{2{,}912}{-4}$$

$$= -728$$

D. Solution: Value of machine = ₱50,000

Depreciation = 20% every year

Year	Depreciation	Value at the End of Year
1	₱50,000 × .20 = ₱10,000	₱50,000 – ₱10,000 = ₱40,000
2	₱40,000 × .20 = ₱ 8,000	₱40,000 – ₱ 8,000 = ₱32,000
3	₱32,000 × .20 = ₱ 6,400	₱40,000 – ₱ 6,400 = ₱25,600

$a_1 = 40,000$

$r = .8$

$$\begin{aligned} a_5 &= a_1 r^{(5-1)} \\ &= 40,000(.8^4) \\ &= 40,000(.4096) \\ &= P16,384 \end{aligned}$$

B. Lesson 1

Let's See What You Have Learned (page 7)

A. 1. A

2. G

3. G

4. G

5. A

B. 1. $d = 2$

2. $r = 3$

3. $r = 4$

4. $r = .5$ or $^1/_2$

5. $d = 3$

C. Lesson 2

Let's See What You Have Learned (page 16)

A. 1. Step 1:

$$\left.\begin{array}{l} -\frac{1}{4} \div \frac{1}{2} = -\frac{1}{4} \times \frac{2}{1} = -\frac{2}{4} \text{ or } -\frac{1}{2} \\ \frac{1}{8} \div -\frac{1}{4} = \frac{1}{8} \times -\frac{4}{1} = -\frac{4}{8} \text{ or } -\frac{1}{2} \end{array}\right\} \text{common ratio or r}$$

or you may compute this by using the equivalent decimal form.

$$^1/_2 = .5; \ -\frac{1}{4} = -.25; \ \frac{1}{8} = .125$$

$$\left.\begin{array}{r} -.25 \div .5 = -.5 \\ .125 \div -.25 = -.5 \end{array}\right\} \text{common ratio or } r$$

Step 2: $a_1 = {}^1/_2$ or .5

Step 3: a_6

Step 4: $-\frac{1}{2}^{(6-1)} = -\frac{1}{2}^{5}$ or $-.5^{(6-1)} = -.5^5$

Step 5: $a_6 = a_1 r^{(6-1)}$

$$a_6 = \frac{1}{2}\left(-\frac{1^{6-1}}{2}\right) \text{ or } .5\left(-.5^5\right)$$

Step 6: $= \frac{1}{2}\left(-\frac{1}{32}\right)$ or $.5(-.03125)$

$$a_6 = -\frac{1}{64} \text{ or } -.015625$$

The 6th term is $-\frac{1}{64}$ in fraction form or –.015625 in decimal form.

2. Given: 10,000 – population in 1995
2% – rate of increase per year

- ♦ In computing percentages, find their equivalents in fraction form and simplify them, like 2% = 2/100 = .02.
- ♦ In order to get the common ratio or r, compute first the population for the first 3 years at the rate of 2% increase per year.
- ♦ Population at the end of 1996: 10,200

$$10{,}000 \times .02 = 200$$
$$10{,}000 + 200 = 10{,}200$$

- ♦ Population at the end of 1997: 10,404

$$10{,}200 \times .02 = 204$$
$$10{,}200 + 204 = 10{,}404$$

- ♦ Population at the end of 1998: 10,612.08

$$10{,}404 \times .02 = 208.08$$
$$10{,}404 + 208.08 = 10{,}612.08$$

Step 1:

$$\left.\begin{array}{l} 10{,}404 \div 10{,}200 = 1.02 \\ 10{,}612 \div 10{,}404 = 1.02 \end{array}\right\} \text{common ratio or r}$$

Step 2: $a_1 = 10{,}200$

Step 3: $a_5 = ?$

Step 4: $1.02^{(5-1)}$ or 1.02^4

Step 5: $a_n = a_1 r^{(n-1)}$

Step 6: $a_n = 10{,}200(1.02^4)$
$= 10{,}200(1.08)$
$= 11{,}016$

The population of the barrio in year 2000 will be 11,016.

B. 1. $n = 4$; a_1 = ₱200,000

Decrease in the value of the house in the 1st year

= ₱200,000(.05)

= ₱10,000

Thus, a_1 = ₱200,000 – ₱10,000 = ₱190,000

Decrease in the value of the house in the 2nd year

= ₱190,000(.05)

= ₱9,500

So, a_2 = ₱190,000 – ₱9,500 = ₱180,500

$$r = \frac{₱180{,}500}{₱190{,}000} = .95$$

$$\begin{aligned} a_4 &= ₱190{,}000(.95)^{4-1} \\ &= ₱190{,}000(.95)^3 \\ &= ₱190{,}000(.857375) \\ &= ₱162{,}901.25 \end{aligned}$$

The house will be worth ₱162,901.25 at the end of 4 years.

2. $a_1 = .02(2) = .04$; $r = 2$; $n = 10$

$a_{10} = .04(2)^{10-1} = .04(2)^9 = .04(512) = 20.48$ inches

It will be 20.48 in. thick.

3. a_1 = ₱100; $n = 7$; $r = 2$

a_7 = ₱100$(2)^{7-1}$ = ₱100$(2)^6$ = ₱100(64) = ₱6,400.00

Your salary will be ₱6,400 on the 7th day.

D. Lesson 3

Let's See What You Have Learned (page 24)

A. 1. $S_6 = 2.2222$

Step 1: $r = \frac{.2}{2} = .1$

Step 2: $a_1 = 2$

Step 3: $r = \frac{.2}{2} = .1$

Step 4: $S_6 = \frac{2\left(.1^6 - 1\right)}{.1 - 1}$

$$= \frac{2(0.000001 - 1)}{-0.9}$$

$$= \frac{2(-.999999)}{-.09}$$

$$= \frac{1.999998}{-0.9}$$

$$= 2.22222$$

2. $S_5 = 134.464$ ft.

Step 1: $r = .8$

Step 2: $a_1 = 50(.8) = 40$

Step 3: $S_5 = \frac{40\left(r^5 - 1\right)}{r - 1}$

Step 4: $S_5 = \frac{40\left(.8^5 - 1\right)}{.8 - 1}$

$$= \frac{40(0.32768 - 1)}{-0.2}$$

$$= \frac{40(-0.67232)}{-0.2}$$

$$= \frac{-26.8928}{-0.2}$$

$$= 134.464 \text{ ft.}$$

B. 1. ₱1,000 × 12 mos. = ₱12,000 total salary for Year 1

a_1 = 12,000; r = 1.2; n = 3

$$S_3 = \frac{12{,}000(1.2^3 - 1)}{1.2 - 1}$$

$$= \frac{12{,}000(1.728 - 1)}{1.2 - 1}$$

$$= \frac{12{,}000(.728)}{.2}$$

$$= ₱43{,}680$$

Her total salary at the end of three years will be ₱43,680.

2. a_1 = ₱200; $r = \frac{600}{200} = 3$; n = 7

$$S_7 = \frac{₱200(3^7 - 1)}{3 - 1}$$

$$= \frac{₱200(2{,}187 - 1)}{2}$$

$$= \frac{₱200(2{,}186)}{2}$$

$$= \frac{₱437{,}200}{2}$$

$$= ₱218{,}600$$

His total commission after 7 weeks will be ₱218,600.

5. a_1 = 3; $r = \frac{6}{3} = 2$; n = 6

$$S_6 = \frac{3(2^6 - 1)}{2 - 1}$$

$$= \frac{3(64 - 1)}{1}$$

$$= \frac{3(63)}{1}$$

$$= \frac{189}{1}$$

$$= 189$$

Minda will receive 189 red roses from Manny on their 6th wedding anniversary.

E. What Have You Learned? *(pages 26–27)*

A. 1. A

2. G

3. A

4. G

5. G

B. 1. $a_1 = 3$; $n = 6$; $r = 2$

$$\begin{aligned} a_6 &= 3(2^{6-1}) \\ &= 3(2^5) \\ &= 3(32) \\ &= 96 \end{aligned}$$

2. $a_1 = 64$; $r = -\frac{1}{2}$ $n = 5$

$$\begin{aligned} a_5 &= 64\left[\left(-\frac{1}{2}\right)^4\right] \\ &= \frac{64}{16} \\ &= 4 \end{aligned}$$

C. 1. $a_1 = \frac{1}{18}$; $r = -3$; $n = 7$

$$S_7 = \frac{\frac{1}{18}\left[\left(-3^7\right)-1\right]}{(-3)-1}$$

$$= \frac{\frac{1}{18}\left[(-2{,}187)-1\right]}{-4}$$

$$= \frac{\frac{1}{18}(-2{,}188)}{-4}$$

$$= \frac{\frac{-2{,}188}{18}}{-4}$$

$$= \frac{-121.55555}{-4}$$

$$= 30.38889$$

2. $r = \frac{500}{400} = 1.25; \frac{625}{500} = 1.25$

$$r = 1.25$$
$$a_1 = 400$$
$$n = 8$$

$$S_8 = \frac{a_1\left(r^8 - 1\right)}{r-1}$$

$$= \frac{400\left(1.25^8 - 1\right)}{1.25-1}$$

$$= \frac{400(5.960464478-1)}{0.25}$$

$$= \frac{400(4.960464478)}{0.25}$$

$$= \frac{1984.185791}{0.25}$$

$$= 7936.743164$$

D. 1.

Quarter	Interest Earned (8%)	Total Value of Time Deposit
1	₱20,000 × .08 = ₱1,600	₱20,000 + ₱1,600 = ₱21,600
2	₱21,600 × .08 = ₱1,728	₱21,600 + ₱1,728 = ₱23,328
3	₱23,328 × .08 = ₱1,866	₱23,328 + ₱1,866.24 = ₱25,194.24

₱23,328 ÷ ₱21,600 = 1.08
₱25,194.24 ÷ ₱23,328 = 1.08

$r = 1.08$; $a_1 = ₱21,600$; $n = 4$

$$a_n = a_1 r^{(n-1)}$$
$$a_4 = ₱21,600(1.08^3)$$
$$= ₱21,600(1.259)$$
$$= ₱27,194.40$$

2.

Year	Targetted increase of learners (14%)	Total number of learners to be served
1		10,236
2	10,236 × .14 = 1,433.04	10,236 + 1,433.04 = 1,669.04
3	11,669.04 × .14 = 1,633.6656	11,669.04 + 1,633.6656 = 13,302.705

$r = 1.14$; $a_1 = 10,236$; $n = 5$

$$S_5 = \frac{a_1(r^5 - 1)}{r - 1}$$
$$= \frac{10,236(1.14^5 - 1)}{1.14 - 1}$$
$$= \frac{10,236(1.925 - 1)}{.14}$$
$$= 67,630 \text{ learners}$$

3.

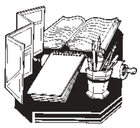
Year	Depreciation at 15% Per Year	Value at End of Year
1	₱350,000 × .15 = ₱52,500	₱350,000 – ₱52,500 = ₱297,500
2	₱297,500 × .15 = ₱44,625	₱297,500 – ₱44,625 = ₱252,875
3	₱252,875 × .15 = ₱37,931.25	₱252,875 – ₱37,931.25 = ₱214,943.75

₱252,875 ÷ ₱297,500 = .85

₱214,943.75 ÷ ₱252,875 = .85

$r = .85$; $a_1 = ₱297{,}500$; $n = 7$

$$
\begin{aligned}
a_n &= a_1 r^{(n-1)} \\
a_7 &= ₱297{,}500\left(.85^6\right) \\
&= ₱297{,}500(.3771494) \\
&= ₱112{,}201.94
\end{aligned}
$$



References

Sia, Lucy O., et al. *21st Century Mathematics, Second Year.* Quezon City: Phoenix Publishing House, Inc. Reprinted 2000.

Capitulo, F. M. *Algebra, a Simplified Approach.* Manila: National Bookstore, 1989.