What Is This Module About?

Have you encountered numbers in sequence? Were you not puzzled as to how these numbers are arranged?

Some numbers are arranged following special patterns. Some of the special patterns can be determined using your knowledge of sequences. In this module, you will learn about arithmetic sequences and series.

The following lessons in the module will help you understand better the concepts of arithmetic sequences and series:

Lesson 1 – *Patterns, Patterns in a Set*

Lesson 2 – *The n^th^ Term in an Arithmetic Sequence*

Lesson 3 – *The Sum of the First n Terms*

Wait!

Before studying this module, please be sure that you have completed the module on *Positive and Negative Integers*.

What Will You Learn From This Module?

After studying this module, you will be able to:

- state whether the given sequence is an arithmetic sequence or not;
- use the formula for finding the n^{th} term of an arithmetic sequence;
- compute the sum of the first n terms of an arithmetic sequence; and
- solve problems which involve arithmetic sequences.

Let's See What You Already Know

Try to answer the following exercises.

A. Determine if the given sequence is an arithmetic sequence. Write **Yes** if it is an arithmetic sequence and **No** if it is not.

1. 2, 6, 10, 14, . . .

2. –4, 8, –16, 32, –64, . . .

3. 2, 1, $^1/_2$, $^1/_4$, $^1/_8$, . . .

4. $20, 13, 6, -1, -8, \ldots$

5. $2, 2\,{}^{1}/_{2}, 3, 3\,{}^{1}/_{2}, \ldots$

B. Find the n^{th} term in the following arithmetic sequences.

1. 5^{th} term in the arithmetic sequence if the first term is 5 and the common difference is 4.

2. 20^{th} term in the sequence 2, 7, 12, 17, . . .

3. 4^{th} term when the 1^{st} term is 5 and the common difference is 4.

4. 7^{th} term in the sequence 3, 15, 27, . . .

C. Find the sum of the 1^{st} *n* terms in the following arithmetic sequences.

1. sum of the first 32 terms in the arithmetic sequence if the common difference is –3 and the 1^{st} term is 7.

2. sum of the first 6 terms of an arithmetic sequence if the 1^{st} term is 4 and the common difference is –3.

D. Solve the following problems.

1. Mark takes a job with a starting salary of ₱50.00 per hour. He was promised an increase of ₱5.00 per hour every three months for 5 years. What will be his wage per hour at the end of 5 years?

2. Sally had ₱3.00 on August 1 and was determined to add to this every day. She had ₱3.25 on August 2; ₱3.50 on August 3; and ₱3.75 on August 4 and so on until August 31. How much would she have at the end of August?

Well, how was it? Do you think you fared well? Compare your answers with those in the *Answer Key* on page 37 to find out.

If all your answers are correct, very good! This shows that you already know much about the topic in this module. You may still study the module to review what you already know. Who knows, you might learn a few more new things as well.

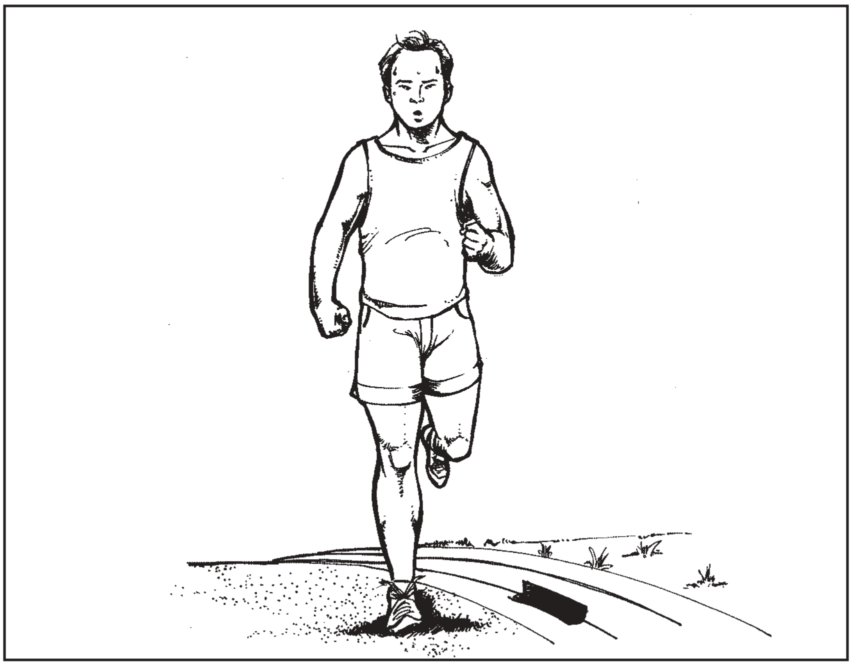
If you got a low score, don’t feel bad. This means that this module is for you. It will help you understand some important concepts that you can apply in your daily life. If you study this module carefully, you would learn the answers to all the items in the test and a lot more! Are you ready?

You may go now to the next page to begin Lesson 1.

LESSON 1

Arithmetic Sequence

This lesson will introduce you to sets of numbers following certain patterns called **arithmetic sequences.** After this lesson, you should be able to tell whether a certain set of numbers is an arithmetic sequence or not.



Suppose you jog every day to maintain good health. On the 1st day of the week you ran 150 meters; on the 2nd day, 155 meters; on the 3rd day, 160 meters; and on the 4th day, 165 meters. Now, pause for a while and look at the number of meters you covered each day. Do you see a pattern?

Let us present the values as in the table below.

Day	1	2	3	4
Meters covered	150	155	160	165

What did you observe with the given numbers? What is the difference between the number of meters covered on the second day and the number of meters covered on the first day? What about the number of meters covered on the fourth day compared to that on the third day?

Note that the number of meters covered each day increases by 5 meters. Notice that on Day 1, you covered 150 meters. On Day 2, you covered 155 meters. This is 5 meters more than 150, the number of meters you covered on the first day. On Day 3, you covered 160 meters which is 5 meters more than 155. On Day 4, you covered 165 meters which is also 5 meters more than 160.

Suppose you continue jogging for two more days, each time increasing the distance you will cover by 5 meters.

Can you determine the distance or the number of meters you will cover on the fifth day and on the sixth day? Write your answers in the spaces provided in the chart below.

Day	1	2	3	4	5	6
Number of meters covered	150	155	160	165		

I'm sure your answers are correct. But let's do it together. Once the numbers are arranged in a series, we follow these steps:

STEP 1 Determine the **common difference**.

To do this, we find the difference between any two consecutive numbers in the sequence.

150 155 160 165

155 – 150 = 5 160 – 155 = 5 165 – 160 = 5

5 is the common difference.

STEP 2 To get the next number in the series, add the common difference to the last number in the series.

In our problem, the common difference is 5. Following Step 2, we will add 5 to the last number in the series, which is 165. We will then get 170, which should be the number of meters you will cover on the 6^{th} day. To get the number of meters you should cover on the 7^{th} day, simply add the common difference (5) again to the last number. Hence, 170 + 5 = 175. This is the next number in the series.

It's very easy, isn't it?

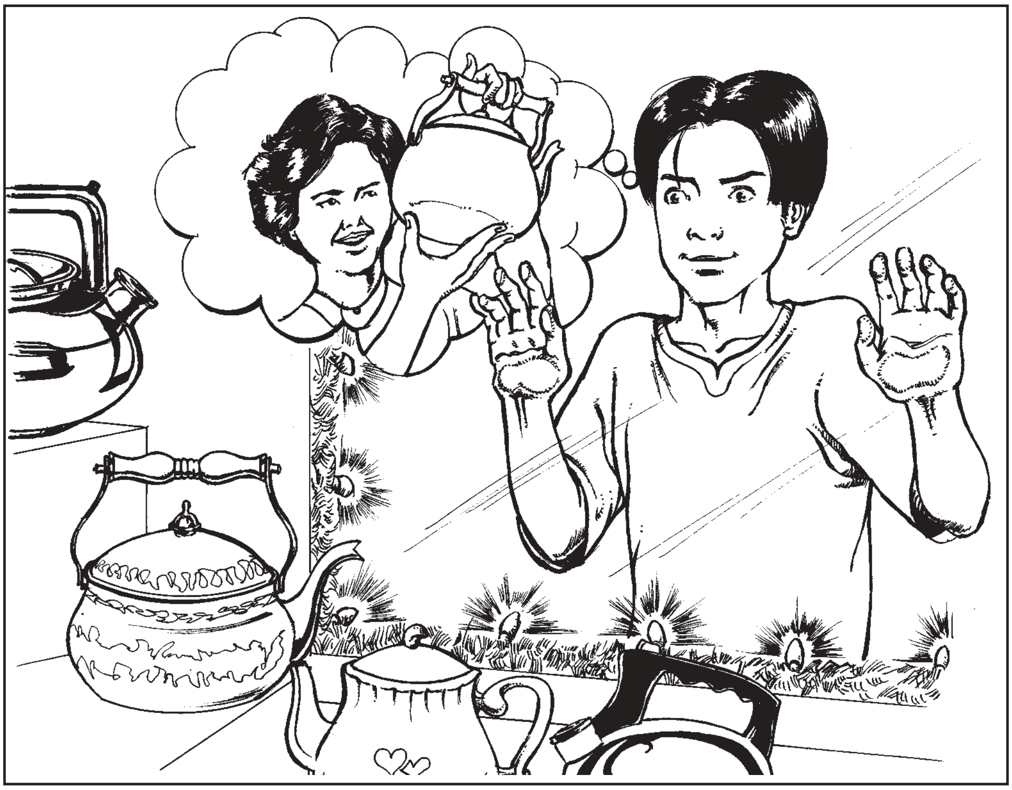
To repeat, the difference, which is 5 meters is called the **common difference** and is usually denoted by *d*. This is constant throughout the series. This means that it will always be 5 for this particular series of numbers.

Let's Remember

To find the common difference, simply get the difference between any two consecutive terms in the sequence.

For example, in the given problem, two consecutive terms may be 150 and 155. The difference is 5. To check, we take other consecutive terms—160 and 165. Note that the difference is also 5. We can therefore safely conclude that the common difference is 5.

Now, let us try another example.



Suppose you want to save a certain amount of money for the coming Christmas season because you want to buy a special gift for your mother. Initially, you had ₱10.00. On the next day, from your pocket money, you decided to set aside ₱5.00, and add this to your ₱10.00. Then, to increase your savings, you promised yourself that every day you will add ₱5.00 to your savings. How much money do you think you will have after 10 days?

To determine the amount of money you will have saved after ten days, let us first analyze the problem.

First, state the given facts.

₱10.00 — initial amount set aside as savings

₱5.00 — amount you add every day to your savings

10 — number of days you will save

Second, find out what is being asked for in the problem. In this case: How much money will you save after 10 days?

Third, solve the problem.

To solve the problem, present the information in tabular form as in:

Day	Amount Saved	
1	₱10.00	initial amount
2	15.00	10.00 + 5.00
3	20.00	15.00 + 5.00
4	25.00	20.00 + 5.00
5	30.00	25.00 + 5.00
6	35.00	30. 00 + 5.00
7	40.00	35.00 + 5.00
8	45.00	40.00 + 5.00
9	50.00	45.00 + 5.00
10	55.00	50.00 + 5.00

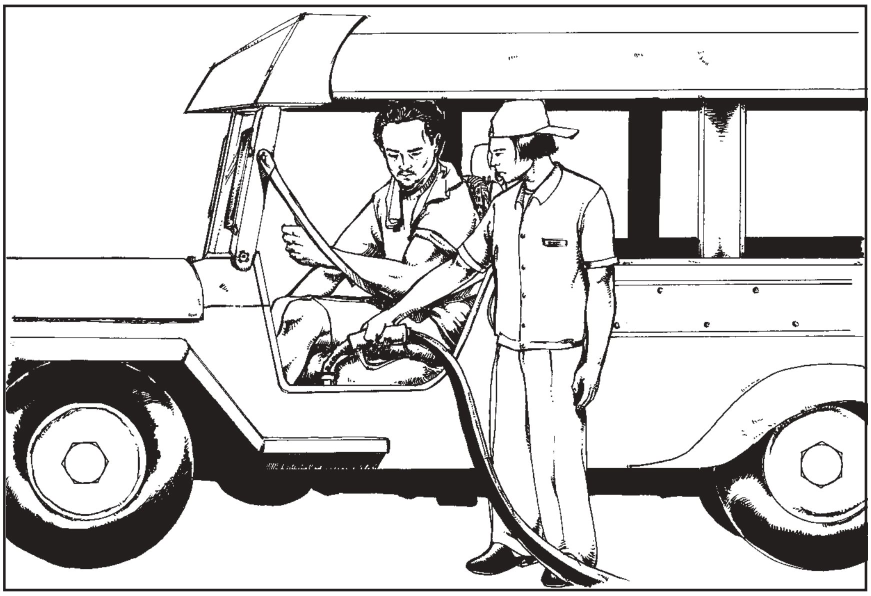
(₱5.00 is the amount of money you decided to add to your savings every day. This is the common difference between the amount saved on the second day and that on the first day)

From the table, you can see that after 10 days you will have saved ₱55.00.

Note, too, that you added ₱5.00 to your total savings every day.

Now, can you try working on the next problem by yourself?

 Let's Try This



Mario is a jeepney driver plying the route from Fairview to Quiapo. Every day he passes by the gasoline station to buy diesel gas for his jeepney. With the increasing prices of gasoline and diesel, he wants to monitor the number of liters of diesel he uses every day.

So, every day, he takes note of the number of liters of diesel his jeep uses up. On Monday, his jeep used up 7 liters of diesel gas. On Tuesday, it used up 7.5 liters of diesel gas. On Wednesday, he bought 8 liters of diesel gas. On Thursday, it used up 8.5 liters. How many liters of diesel gas do you think will he buy on Friday? What about on Saturday?

It will be helpful to you if you will first determine the following information:

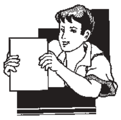
a. initial liters of diesel used = __________

b. number of liters being added every day or the common difference = __________

Having determined the common difference, fill in the blanks in the table below:

Days	Number of Liters of Diesel Gas Used
Monday	7
Tuesday	7.5
Wednesday	8
Thursday	8.5
Friday	
Saturday	

Is your answer for Friday 9 liters and 9.5 liters for Saturday? If you did, you're doing well!



Let's Learn

The three examples you worked on earlier are examples of an arithmetic sequence or arithmetic progression.

An **arithmetic sequence** or **arithmetic progression** is a set or series of numbers following a certain pattern depending on the common difference. The common difference is constant or a fixed number.

Any series of numbers that do not have a common difference is not an arithmetic sequence or progression.

Here are some more examples of arithmetic sequences.

a. 5, 10, 15, 20, 25, 30, 35

b. 220, 230, 240, 250, 260, 270, 280

c. 55, 65, 75, 85, 95, 105, 115

d. 100, 200, 300, 400, 500, 600, 700

Let's Review

Determine if the following series of numbers are arithmetic sequences or not. If the given is an arithmetic sequence, write **S** on the space provided. If not, write **N**.

______ 1. 2, 4, 6, 8, 10, 12, 14

______ 2. 5, 4, 7, 9, 11, 10, 8

______ 3. 25, 28, 31, 34, 37, 40, 43

______ 4. 14, 15, 17, 17, 19, 20, 21

______ 5. 124, 126, 128, 130, 132, 134, 136

______ 6. 10, 20, 30, 40, 50, 60, 70

______ 7. 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5

______ 8. P15.00, P17.00, P19.00, P21.00

_______ 9. 17, 18, 35, 34, 21, 22, 16

_______ 10. 4, 8, 12, 16, 20, 24, 28

Compare your answers with those found in the *Answer Key* on page 37. If you got 6 to 10 correct answers, you're doing great. You can proceed to the next lesson. If you got only 6 or less, read Lesson 1 again and after doing that, try to solve the exercises given above again.

LESSON 2

The n^{th} Term in an Arithmetic Sequence

In this lesson, you will learn how to determine the missing numbers in an arithmetic sequence without going through the process of constructing a table as you did earlier.

Consider the following set of numbers that represent the number of meters you cover while jogging each day: 150, 155, 160, 165, 170, 175.

This represents an arithmetic sequence. Each number in an arithmetic sequence is called a **term**. The 1st number is called the **1st term**, the 2nd, the **2nd term,** and so on.

The 1st term in the sequence above is 150. The 2nd term is 155. The 3rd term is 160 and so on.

If we use a letter, like the letter *a* to replace any term in the sequence, we need to indicate whether it is the first, the second, or the third term in the sequence. This is done by writing a number at the lower right of the letter. This number is called a **subscript**. In the example, a_1 is the first term in the sequence. We can therefore say that $a_1 = 150$.

The symbol for the 2nd term is a_2. In our example, $a_2 = 155$.

What about the symbols for the following terms? The third and fourth terms have been done for you.

3rd term: a_3

4th term: a_4

5th term: _____

6th term: _____

•

•

9th term: _____

•

•

•

n^{th} term: _____

If your answers are a_5, a_6 , a_9 and a_n, then you got the answers right.

Remember our previous example on the number of meters covered when jogging? We can now represent those numbers as terms in a series. The numbers can be represented as:

$$a_1 = 150 \quad \text{(first term)}$$
$$a_2 = 155 \quad (150 + 5)$$
$$a_3 = 160 \quad (155 + 5)$$
$$a_4 = 165$$
$$a_5 = 170$$
$$a_6 = 175$$

Observe how a term in the sequence above is obtained.

2nd term: $155 = 150 + 5$

This means that the 2nd term is obtained by adding 5 to the 1st term. Thus, we have:

$$a_2 = 150 + 5$$

Recall that in Lesson 1, we determined the common difference $d = 5$. Then by substituting 5 to d, we get:

$$a_2 = 150 + d$$

150 is a_1, the first term. Substituting 150 to a_1, we get the following number sentence for a_2:

$$a_2 = a_1 + d$$

$$a_2 = 150 + 5 = 155$$

The 3rd term, which is 160, is obtained by adding 155 + 5. But 155 = 150 + 5.

So, substituting 155 with (150 + 5), changes our number sentence to:

$$160 = (150 + 5) + 5$$

This means that the 3rd term is obtained by adding 5 two times to the 1st term.

Thus we have:

$$a_{\underline{3}} = 150 + \underline{2}\,(5) \text{ or } a_{\underline{3}} = a_1 + \underline{2}d$$

The fourth term is:

4th term: $165 = 160 + 5$

$165 = (155 + 5) + 5$

$165 = (150 + 5) + 5 + 5.$

This means that the 4th term is obtained by adding 5 three times to the 1st term. So we have:

$$a_{\underline{4}} = 150 + \underline{3}\,(5) \text{ or } a_{\underline{4}} = a_1 + \underline{3}d$$

The fifth term is:

5th term: $170 = 165 + 5$

$170 = (160 + 5) + 5$

$170 = (155 + 5) + 5 + 5$

$170 = (150 + 5) + 5 + 5 + 5$

This means that the 5th term is obtained by adding 5 four times to the 1st term.

$$a_{\underline{5}} = 150 + \underline{4}\,(5) \text{ or } a_{\underline{5}} = a_1 + \underline{4}d$$

So, how do you think can we obtain the 6th term in the same arithmetic sequence?

$$a_6 = 150 + \_\_\_\ (5) \text{ or } a_6 = a_1 + \_\_\_d$$

How many times do we add 5 to the 1st term? You're right. We will add 5 five times to the 1st term.

So the 6th term is: $a_{\underline{6}} = 150 + \underline{5}\,(5)$ or $a_{\underline{6}} = a_1 + \underline{5}d$

And what if you are to obtain the 7th term? How many times will you add 5 to the 1st term?

Give your answer by filling in the blanks below.

7th term: $a_{\underline{7}} = 150 + \_\_\_\ (5)$ or $a_{\underline{7}} = a_1 + \_\_\_\_d$

You're doing great! So, let's continue.

Now, what have you noticed about our number sentences for the previous terms?

Let us examine them closely.

$$\begin{aligned} a_2 &= a_1 + d \\ a_3 &= a_1 + 2d \\ a_4 &= a_1 + 3d \\ a_5 &= a_1 + 4d \\ a_6 &= a_1 + 5d \end{aligned}$$

where a_1 is the first term, and d is the common difference

What have you noticed about the number of times you multiplied d? Compare this with the order of the terms.

For example, $a_6 = a_1 + 5d$.

This is the sixth term, right? But you are multiplying d five times. And 5 is one less than 6 or 6 – 1 = 5.

Let's try another term: $a_7 = a_1 + 6d$

So, we can write: $a_7 = a_1 + (7-1)\,d$

For the rest of the terms, we can also write them as:

$$a_8 = a_1 + (8-1)\,d$$
$$a_9 = a_1 + (9-1)\,d$$
$$a_{10} = a_1 + (10-1)\,d$$

Now, suppose we want to find the n^{th} term (n designates any number in the term), our n^{th} term would be:

$$a_n = a_1 + (n-1)\,d$$

This now becomes our guide or rule in finding any term in an arithmetic sequence for as long as you have already determined d, or the common difference.

This is also known as the formula for finding the n^{th} term in an arithmetic sequence.

$$\boxed{a_n = a_1 + (n-1)\,d \text{ (this is equation 1)}}$$

Let us try using the formula.

Exercise

Find the following n^{th} terms in an arithmetic sequence with 25 as the first term and 3 as the common difference (d). Numbers 1 and 2 have been done for you.

1. 3rd term or $a_3 = 31$

$$\begin{aligned} a_3 &= 25 + (3-1)3 \\ &= 25 + 2 \times 3 \\ &= 25 + 6 \\ a_3 &= 31 \end{aligned}$$

2. fifth term or $a_5 = 37$

$$\begin{aligned} a_5 &= 25 + (5-1)\,3 \\ &= 25 + 4 \times 3 \\ &= 25 + 12 \\ a_5 &= 37 \end{aligned}$$

3. $a_7 =$ ______

4. $a_9 =$ ______

5. $a_{11} =$ ______

Compare your answers with those found in the *Answer Key* on page 37.

EXAMPLE 1 Consider the sequence 2, 6, 10, 14, ... What is the 12th term in the given sequence?

Follow the steps below to solve this problem.

STEP 1 Find the common difference.

$6 – 2 =$ ____

$10 – 6 =$ ____

$14 – 10 =$ ____

The common difference is: $d = 4$.

STEP 2 Determine the 1st term in the given arithmetic sequence.

The 1st term in the given arithmetic sequence is 2. This means that $a_1 = 2$.

STEP 3 Find the symbol for the unknown term in the sequence.

You are asked for the 12th term in the given arithmetic sequence. Thus, the symbol for the unknown term is a_{12}.

STEP 4 Write the equation or the number sentence for the unknown term in the sequence.

The equation for a_{12} is:

$$a_{12} = a_1 + (12 – 1)\, d$$

$$a_{12} = a_1 + 11d$$

STEP 5 Substitute the values in the equation and solve for the answer.

From Step 1, $d = 4$ and from Step 2, $a_1 = 2$. Thus we have

$a_{12} = a_1 + 11d$

$a_{12} = 2 + 11\,(4) = 46$

This means that the 12th term of the arithmetic sequence 2, 6, 10, 14, ... is 46.

Now, supposing the common difference is a negative number. What do we do? We follow the same steps. But we should recall how to add and multiply signed numbers.

Consider the following examples.

EXAMPLE 2 What is the 15th term of the sequence 5, 3, 1, –1, –3, –5, . . .?

Following the steps, we will have:

STEP 1 Find the common difference.

3 – 5 = ____

1 – 3 = ____

–3 – (–1) = ____

The common difference $d = -2$.

STEP 2 Determine the 1st term in the given arithmetic sequence.

The 1st term in the given arithmetic sequence is 5. This means that $a_1 = 5$.

STEP 3 Find the symbol for the unknown term in the sequence.

You are asked for the 15th term in the given arithmetic sequence. Thus, we solve for a_{15}.

STEP 4 Write the equation for the unknown term in the sequence.

The equation for a_{15} is:

$$a_{15} = a_1 + (15 - 1)\,d \qquad = \quad a_{15} = a_1 + 14d$$

STEP 5 Substitute the values in the equation and solve for the result.

From Step 1, $d = -2$ and from Step 2, $a_1 = 5$. Thus we have:

$$a_{15} = a_1 + 14d$$

$$a_{15} = 5 + 14\,(-2) = -23.$$

The 15th term of the sequence 5, 3, 1, –1, –3, –5, . . . is –23.

Now, try doing the next example by yourself. Just follow the steps.

EXAMPLE 3 What is the 17th term of the sequence 2, –1, –4, –7, . . . ?

STEP 1 Find the common difference.

$-1 - (2) =$ _____

$-4 - (-1) =$ _____

_____ – _____ = _____

The common difference $d =$ _____

STEP 2 Determine the 1st term in the given arithmetic sequence.

The 1st term in the given arithmetic sequence is _____. This means that $a_1 =$ _____.

STEP 3 Find the symbol for the unknown term in the sequence.

You are asked for the 17th term in the given arithmetic sequence. Thus, we solve for _____.

STEP 4 Write the equation for the unknown term in the sequence.

The equation for a_{17} is:

$a_{17} =$ _______________.

STEP 5 Substitute the values in the equation and solve for the result.

From Step 1, $d =$ ____ and from Step 2, $a_1 =$ ____. Thus we have

$a_{17} = a_1 +$ ____d

$a_{17} =$ ___________ = ____

The 17th term of the arithmetic sequence 2, –1, –4, –7, . . . is _____ .

Compare your answers with those given in the *Answer Key* on page 38.

Now, suppose you are tired of using the steps in solving for the n^{th} term, what other method can you use?

Remember the formula:

$$a_n = a_1 + (n - 1)d \text{ (let us call this formula equation 1)}$$

This is the formula you will use in finding any term in the sequence provided you know a_1 and d.

Solve the following problem using the formula.

PROBLEM 1 What is the 20th term of the sequence 10, 13, 16, 19, . . . ?

Let's solve this problem together.

SOLUTION First, we need to know the following:

first term = ______	:	the first term is 10
d = ______	:	$d = 13 – 10 = 3$ $16 – 13 = 3$ $19 – 16 = 3$

Second, we should know what is being asked for in the problem.

The unknown in the problem is the 20th term in the sequence. This is represented as a_{20}.

Knowing the first term or $a_1 = 10$ and $d = 3$, we can now substitute these values in the equation as follows:

$$a_n = a_1 + (n – 1)d$$

Substituting these values in the formula, we have:

$$a_{20} = a_1 + 19d$$

$$a_{20} = 10 + 19 (3) = 67$$

Thus, the 20th term of the arithmetic sequence 10, 13, 16, 19, . . . is 67.

Do you understand the solutions to the examples presented? If you do, you may continue reading this module. If you do not, read the discussion and try answering the problems again. I'm sure that after the second reading, the topic will become clearer to you.

Let us try some more problems.

PROBLEM 2 Simon takes a job with a starting salary of ₱30.00 per hour. He was promised an increase of ₱5.00 per hour every three months for 5 years. What will be his wage per hour at the end of 5 years?

Let us analyze the problem.

The following are the given facts in the problem:

starting wage = ₱30.00 per hour;

increase of ₱5.00 per hour every 3 months for 5 years. This means that after 3 months, his wage will be ₱35.00 per hour. After 6 months, his wage will be ₱40.00 per hour and so on.

Now, since there are 12 months in 1 year, 5 years = 60 months.

But the increase is done every 3 months. Thus, 60/3 = 20. This means that in 5 years, his wage will increase by ₱5.00 per hour 20 times.

Therefore, we have the sequence 30, 35, 40, 45, 50, ...

STEP 1 Find the common difference.

35 – 30 = ____

____ – ____ = ____

____ – ____ = ____

The common difference is: d = _____

STEP 2 Determine the 1st term in the given arithmetic sequence.

The 1st term in the given arithmetic sequence is ____. This means that a_1 = ____.

STEP 3 Find the symbol for the unknown term in the sequence.

You are asked for the 20th term in the given arithmetic sequence. Thus, we solve for ____.

STEP 4 Write the equation for the unknown term in the sequence.

The equation for a_{20} is:

a_{20} = ______________

STEP 5 Substitute the values in the equation and solve.

From Step 1, d = ____ and from Step 2, a_1 = ____. Thus we have:

$a_{20} = a_1$ + ____d

a_{20} = __________ = ____

Simon's wage per hour at the end of 5 years will be _____.

Compare your answers with those found in the *Answer Key* on page 38. If your answers are wrong, read the parts which are not clear to you again. If you got them right, congratulations. You did a very good job. Try solving the next problem.

PROBLEM 3 Analiza saved ₱5.00 on 1 September, ₱5.25 on 2 September, ₱5.50 on 3 September and so on until 30 September. How much would she have saved by the end of the month?

Again, let's analyze the given problem.

What are the values given in the problem? You're right. We have 5, 5.25, 5.5, 5.75, ... What do these numbers form? The numbers form an arithmetic sequence.

Instead of following the 5 steps, let's just use the formula.

You are asked for the amount that Analiza would be able to save by 30 September. This is equivalent to the 30^{th} term of the arithmetic sequence 5, 5.25, 5.5, 5.75, . . . with $a_1 = 5$ and $d = 5.25 - 5 = .25$

Using the formula, we have:

$$a_n = a_1 + (n-1)\,d$$

$$a_{30} = 5 + 29\,(.25) = 12.25$$

This means that Analiza will be able to save ₱12.25 by 30 September.

Let's Remember

- To solve for the n^{th} term in an arithmentic sequence, we use the formula:

  $a_n = a_1 + (n-1)d$ where a_n = n^{th} term

  a_1 = 1^{st} term

  n = number of terms

  d = common difference

- We follow the following steps in using the equation above:

  STEP 1 Find the common difference.

  STEP 2 Determine the 1^{st} term in the given arithmetic sequence.

  STEP 3 Find the symbol for the unknown term in the sequence.

  STEP 4 Write the equation for the unknown term in the sequence.

  STEP 5 Substitute the values in the equation and solve for the result.

Let's See What You Have Learned

A. Use the 5 steps in applying the formula for finding the n^{th} term in an arithmetic sequence to solve the following.

1. Find the 28th term in an arithmetic sequence if the 1st term is 20 and the common difference is –5.

2. What is the 17th term in the arithmetic sequence 7, 7.3, 7.6, 7.9, . . . ?

B. Use the formula $a_n = a_1 + (n–1)d$ in finding the n^{th} term in the following arithmetic sequences.

1. A stack of bricks has 61 bricks in the bottom layer, 58 bricks in the 2nd layer, 55 bricks in the 3rd layer, and 10 bricks in the last layer. How many bricks are there in the 11th layer?

2. The seats in a theater are arranged so that there are 70 seats in the 1st row, 72 seats in the 2nd row and so on for 30 rows altogether. How many seats are there in the last row?

3. Once a month, a man puts some money in a cookie jar. During the 1st month he has ₱10.50 and each month he adds ₱0.50 more into the jar. How much money was placed in the jar during the last month of the 4th year?

Compare your answers with those found in the *Answer Key* on pages 39 and 40. If you got 4 or 5 correct answers, you're doing great. Continue reading this module. If you got only 4 or below, read Lesson 2 again, then try solving the exercises given again.

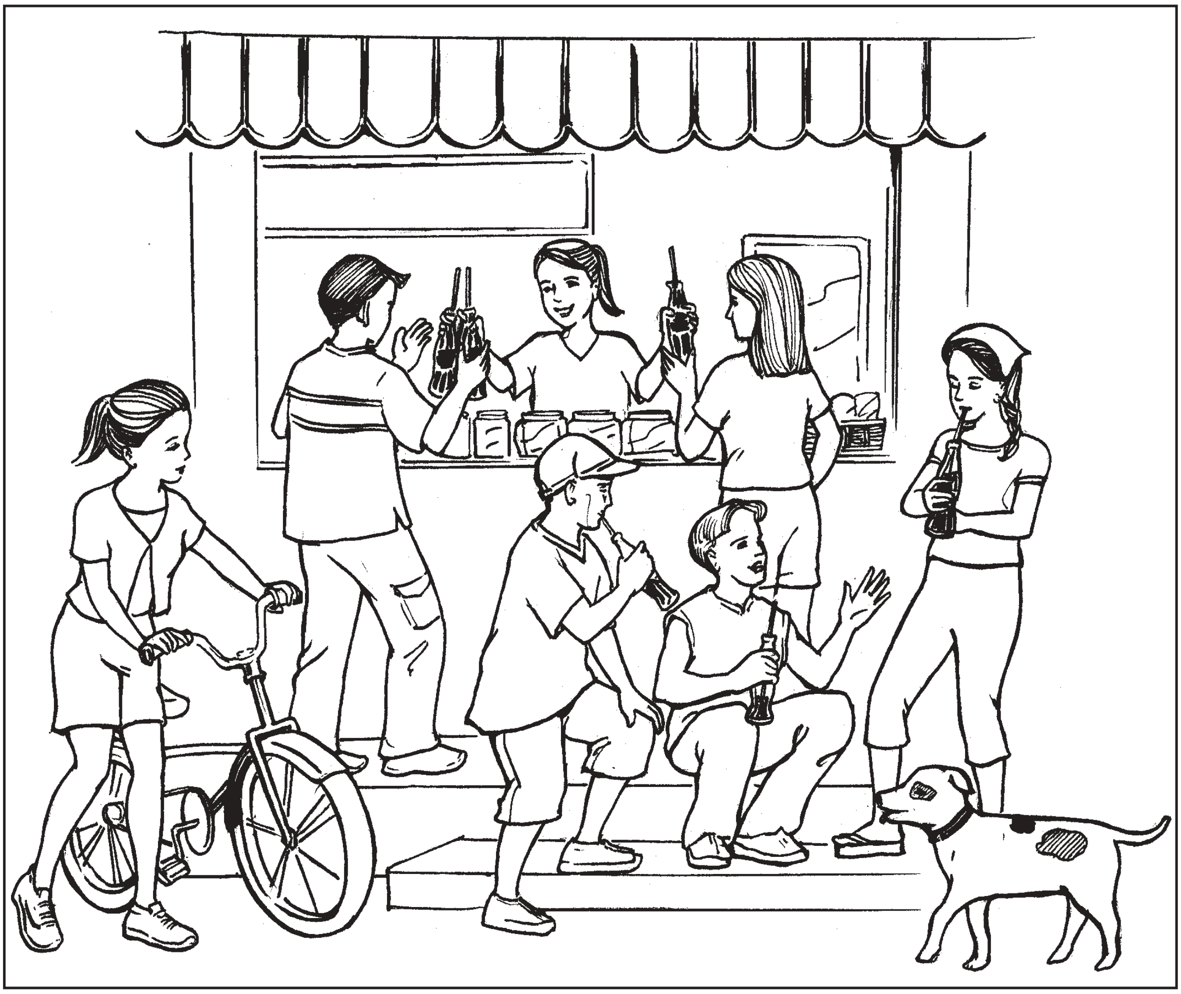
LESSON 3

The Sum of the First *n* Terms

In this lesson, you will learn how to find the sum of the first *n* terms in an arithmetic sequence. You will also learn how to apply that process in your daily life.

Let's Study and Analyze

Jaine-Anne noticed that when she changed the brand of soft drinks she was selling, her number of customers increased every day.



Shown in the table is a list of the number of bottles of soft drinks Jaine-Anne sold from Monday to Saturday.

Day	Number of Bottles
1	20
2	70
3	120
4	170
5	220
6	270

PROBLEM How many bottles of soft drinks will Jaine-Anne sell in 10 days?

Let's solve the problem together.

From the series of numbers, find out the following:

1. Is the series an arithmentic sequence?
2. Do the numbers in the series have a common difference? If yes, what is the common difference or *d?*
3. What is the first term or a_1?

Before answering Question 1, first determine if there is a common difference.

Let us compute for the common difference by finding the difference between two consecutive numbers.

70 – 20 = 50

120 – 70 = 50

170 – 120 = 50

So, there is a common difference or *d,* which is 50. Therefore, the numbers follow an arithmetic sequence.

Now, the first term is 20. This is a_1.

We can now solve for the number of soft drinks that Jaine-Anne can sell in 10 days.

Let us add:

20 + 70 = 90

90 is the sum of the 1st two terms.

20 + 70 + 120 = 210

210 is the sum of the 1st three terms.

20 + 70 + 120 + 170 = 380

380 is the sum of the 1st four terms.

20 + 70 + 120 + 170 + 220 = 600

600 is the sum of the 1st five terms.

20 + 70 + 120 + 170 + 220+ 270 = 870

870 is the sum of the 1st six terms.

Each of these sums is called an **arithmetic series**. An arithmetic series is the sum of the 1st *n* terms of an arithmetic sequence. In the example above, 90, 210, 380, 600, 870 and so on is an example of an arithmetic series.

Do you want to discover a formula for finding the sum of the 1st *n* terms of an arithmetic sequence? That's good! So, let's continue.

EXAMPLE 1

Consider the sequence in the example above.

20, 70, 120, 170, 220, 270, . . .

Earlier, we added the terms in the arithmetic sequence and we came up with the following sums:

90, 210, 380, 600, 870

Let us write S to replace the sum. Again, let's use a subscript, say 2.

S_2 means the sum of the 1st two terms in an arithmetic sequence.

How do you write the sum of the 1st three terms? S_3 is the sum of the 1st three terms in an arithmetic sequence.

How do you write the sum of the 1st four terms? _______

How do you write the sum of the 1st five terms? _______

How do you write the the sum of the 1st six terms? _______

How do you write the sum of the 1st ten terms? ______

How do you write then the sum of the 1st *n* terms? ______

If your answers are S_4, S_5, S_6, S_{10} and S_n, then you did a very good job.

STEP 2 Determine the 1st term and the common difference in the given sequence.

In the activity that you did, the 1st term and the common difference are given: $a_1 = 20$ and $d = 70 - 20 = 50$.

STEP 3 Find the formula for finding the unknown sum.

Now, note that in the activity that you did:

$S_2 = 90$; $S_3 = 210$; $S_4 = 380$

What is the sum of $a_1 = 20$ and $a_2 = 70$?

$20 + 70 = 90$

What is the quotient if you divide that sum by 2?

$(20 + 70) \div 2 = 90 \div 2 = 45$

How is your last answer related to S_2? We say that:

$S_2 = 90 = 45\ (2)$

What is the sum of $a_1 = 20$ and $a_3 = 120$?

$20 + 120 = 140$

What is the quotient if you divide that sum by 2?

$(20 + 120) \div 2 =$ ______

How is your last answer related to S_3? We say that,

$S_3 = 210 = 70\ (3)$

What is the sum of $a_1 = 20$ and $a_4 = 170$?

$20 + 170 =$ ______

What is the quotient if you divide that sum by 2?

$(20 + 170) \div 2 =$ ______

How is your last answer related to S_4? We say that:

$S_4 = 380 = 95\ (4)$

Let us again go back to our computations for S_n.

$$S_2 = \frac{a_1 + a_2}{2}(2)$$
$$S_3 = \frac{a_1 + a_3}{2}(3)$$
$$S_4 = \frac{a_1 + a_4}{2}(4)$$

What then is the formula for S_5, S_6 and S_7?

Are your answers the same as those written below? If yes, you've done a good job!

$$S_5 = \frac{a_1 + a_5}{2}(5)$$
$$S_6 = \frac{a_1 + a_6}{2}(6)$$
$$S_7 = \frac{a_1 + a_7}{2}(7)$$

So, now I'm sure you can write the formula for S_n.

$$S_n = \frac{\_\_ + a_n}{2}(n)$$

Here's the formula:

$$S_n = \frac{a_1 + a_n}{2}(n)$$

or

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n) \text{ (let us call this Equation 2)}}$$

Use this formula or equation if you have the following information given:

1. the value of the first term or a_1
2. the value of the n^th term or a_n

Can you think of any other formula for S_n?

Suppose a_n is not given. Let us see how we can do this.

In Lesson 2, you learned that $a_n = a_1 + (n - 1)d$. Using this in Equation 2, we have:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}[a_1 + a_1 + (n-1)d]$$

Thus, by adding the two a_1 we will have:

$$\boxed{S_n = \frac{n}{2}[2a_1 + (n-1)d] \text{ (let us call this Equation 3)}}$$

Equations 2 and 3 are used to compute for the sum of the 1st n terms in an arithmetic sequence.

When do we use Equation 2?

Equation 2 is used when the 1st term, a_1 and the n^{th} term, a_n are given.

When do we use Equation 3?

Equation 3 is used when the 1st term a_1 and the common difference, d are given.

If you cannot follow anymore, read the previous discussion again. If things are clear to you, that's good! Continue reading this module.

In Example 1, the 1st term and the common difference are given so we can use Equation 3. Thus, we have:

$$S_{10} = \frac{10}{2}[2a_1 + (10-1)d]$$

STEP 4 Substitute the given values in the formula and solve for the unknown.

From Step 2, we have $a_1 = 20$ and $d = 50$. From Step 3 we then have:

$$S_{10} = \frac{10}{2}[2a_1 + (10-1)d]$$

Substituting the values in the equation:

$$\begin{aligned} S_{10} &= \frac{10}{2}\left[2(20)+(10-1)50\right] \\ &= 5\left[40+9(50)\right] \\ S_{10} &= 2{,}450 \end{aligned}$$

This means that Jaine-Anne can sell up to 2,450 bottles of softdrinks in 10 days.

Let’s look at another example.

EXAMPLE 2 What is the sum of the 1st 32 terms in the arithmetic sequence if the common difference is 3 and the 1st term is 7?

To solve this, follow the steps below:

STEP 1 Determine what the unknown is and write its arithmetic notation.

In the given problem you are asked to determine the 1st 32 terms in the arithmetic sequence. Thus, you are to solve for S_{32}.

STEP 2 Find out what the given facts in the problem are.

The problem has the following given facts:

The 1st term or $a_1 = 7$.

The common difference or $d = 3$.

STEP 3 Determine which formula to use, given the first term, a_1 and the common difference, d.

Remember, there are two equations or formulas for finding the sum of the first n terms.

The first formula: $S_n = \frac{n}{2}(a_1 + a_n)$ or Equation 2 is used when we have the following information given:

a. the value of the 1st term or a_1

b. the value of the n^{th} term or a_n

The second formula, $S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$ or Equation 3 is used when we have the following information given:

a. the value of the 1st term or a_1

b. the value of the common difference or d

The example that we are working on has the following information given:

a. the first term or $a_1 = 7$

b. the common difference or $d = 3$

From the information given in the example, we choose the second formula or Equation 3 in solving the problem.

STEP4 Using Formula 2 or Equation 3 and substituting the given values, we have:

$$\begin{aligned} S_{32} &= \frac{32}{2}\left[2a_1 + (32-1)d\right] \\ &= \frac{32}{2}\left[2(7) + (32-1)3\right] \\ &= 16\left[14 + 3(31)\right] \\ S_{32} &= 1{,}712 \end{aligned}$$

Thus, the sum of the 1st 32 terms in the arithmetic sequence is 1,712.

Let's look at another example.

EXAMPLE 3 Find the sum of the 1st 21 terms in an arithmetic sequence if the 1st term is 7 and the last term is –53.

Let's analyze the problem.

STEP 1 Determine what the given facts are.

The 1st and last terms of the sequence are given.

$a_1 = 7$ and $a_{21} = -53$

STEP 2 Determine what is being asked for in the problem and write this in arithmetic notation.

In the given problem, you are asked for the 1st 21 terms in the arithmetic sequence. Thus, you are to solve for _____.

STEP 3 Decide which formula to use for finding the unknown sum given the 1st and last terms.

We are given the 1st and last terms in the sequence so, we use Equation 2.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{21} = \frac{21}{2}(a_1 + a_{21})$$

STEP 4 Substitute the given values in the formula and solve for the unknown.

In the problem, we are given $a_1 = 7$ and $a_{21} = -53$. From Step 3, we have:

$$S_{21} = \frac{21}{2}(a_1 + a_{21})$$

By substituting the values in the formula, we get,

$$\begin{aligned} S_{21} &= \frac{21}{2}[7 + (-53)] \\ &= 10.5(-46) \\ S_{21} &= -483 \end{aligned}$$

Thus, the sum of the 1st 21 terms in the arithmetic sequence is –483.

EXAMPLE 4 A wife earned ₱10,000.00 during her 1st year of working and receives ₱300.00 more every year. She saves all her earnings because she budgets her husband's salary wisely. How much money will she have at the end of 12 years?

Here is my analysis.

The wife saves ₱10,000 during the 1st year, ₱10,300 during the 2nd year, ₱10,600 during the 3rd year and so on until the 12th year. 10,000, 10,300, 10,600, . . . form an arithmetic sequence.

STEP 1 Determine what the given facts are.

The 1st term and the common difference of the sequence are given.

$a_1 = 10{,}000$ and $d = 10{,}300 - 10{,}000 = 300$

STEP 2 Determine what is being asked for in the problem and write this in arithmetic notation.

In the given problem, you are being asked for the sum of the 1st 12 terms in the arithmetic sequence. Thus, you are to solve for S_{12}.

STEP 3 Decide which formula to use for finding the unknown sum given the 1st term and the common difference.

We are given the 1st term and the common difference, so we will use Equation 3.

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2a_1 + (12-1)d\right]$$

STEP 4 Substitute the given values in the formula and solve for the unknown.

In the problem, we are given a_1 = 10,000 and d = 300.

From Step 3, we will have:

$$\begin{aligned} S_{12} &= \frac{12}{2}\left[2a_1 + (12-1)d\right] \\ &= \frac{12}{2}\left[2(₱10{,}000) + (12-1)₱300\right] \\ &= 6\left[₱20{,}000 + (11)₱300\right] \\ S_{12} &= ₱139{,}800 \end{aligned}$$

This means that the wife will have P139,800.00 at the end of 12 years.

Try solving the following by yourself.

EXERCISE Jose is saving for a pair of shoes. He sets aside ₱35.00 on the 1st week and increases his savings constantly every week. On the 8th week, he has saved ₱105.00 and discovers that he already has the exact amount for the pair of shoes. How much is the pair of shoes?

Analyze the problem.

The values given in the problem can be related to an arithmetic sequence because the savings are increased constantly. This means that there is a common difference.

STEP 1 Determine the given facts.

The 1st and the last terms of the sequence are given.

a_1 = ______ and a_8 = ______

STEP 2 Determine what is being asked for in the problem and write this in arithmetic notation.

In the given problem, you are being asked for the sum of the 1st eight terms in the arithmetic sequence. Thus, you are to solve for _____.

STEP 3 Decide which formula should be used in finding the unknown sum given the 1st and last terms.

We are given the 1st and last terms in the sequence so, we use Equation ____.

____

STEP 4 Substitute the given values in the formula and solve for the unknown.

In the problem, we are given a_1 = _____ and a_8 = _____.

From Step 3, we will have:

____ = __________________

By substituting the given values in the formula, we get:

____ = __________________ = _____

Thus, the pair of shoes cost _____.

Compare your answers with those given in the *Answer Key* on pages 40.

Let's Remember

♦ The formula $S_n = \frac{n}{2}(a_1 + a_n)$ (Equation 2)

or $S_n = \frac{n}{2}[2a_1 + (n-1)d]$ (Equation 3)

where: n = number of terms

a_1 = 1st term

a_n = n^{th} term

d = common difference

are the two equations used to find the sum of the 1st n terms in an arithmetic sequence.

- ♦ We follow the steps below in using the given equations.

STEP 1 Determine what the given facts in the problem are. If the 1^{st} and last terms are given, use Equation 2.

If the 1^{st} term and the common difference are given, use Equation 3.

STEP 2 Determine what is being asked for in the problem and write this in arithmetic notation.

STEP 3 Decide which formula to use in finding the unknown sum based on the given facts or information.

STEP 4 Substitute the given values in the formula and solve for the unknown.

Let's See What You Have Learned

A. Use the four steps you just learned in finding the sum of the 1^{st} *n* terms in an arithmetic sequence to solve the following:

1. Find the sum of the 1^{st} 26 terms in an arithmetic sequence whose 1^{st} term is 15 and common difference is $-{}^{1}/_{5}$.

2. What is the sum of the 1^{st} 11 terms in an arithmetic sequence whose 1^{st} term is –40 and the 11^{th} term is –73?

B. Use the formula (Equation 2 or Equation 3) for finding the sum of the 1^{st} *n* terms in an arithmetic sequence to solve the following problems.

1. A stack of bricks has 61 bricks in the bottom layer, 58 bricks in the 2^{nd} layer, 55 bricks in the 3^{rd} layer and 10 bricks in the last or 18^{th} layer. How many bricks are there in all?

2. The seats in a theater are arranged so that there are 70 seats in the 1^{st} row, 72 seats in the 2^{nd} row and so on for 30 rows altogether. How many seats in all are there in the theater?

5. Once a month, a man puts money into a cookie jar. During the 1^{st} month he has ₱10.50 and each month he adds an amount ₱0.50 more. How much money had he put in the jar at the end of 4 years?

Compare your answers with those given in the *Answer Key* on pages 40 and 41. If you got 4 or 5 correct answers, you're doing great. Continue reading this module. If you get a score below 4, read Lesson 3 again. Review the parts you did not understand very well.

Let's Sum Up

Let us summarize what you have learned in this module.

- An **arithmetic sequence** is a set of terms in which any 2 consecutive terms have a common difference. Each term in the sequence, except the 1st one, is obtained by adding the common difference to the previous term.

  The equation $a_n = a_1 + (n - 1)d$ where

  a_n = n^{th} term

  a_1 = 1st term

  n = number of terms

  d = common difference

  is used to compute for the n^{th} term in an arithmetic sequence.

  We follow the following steps in using the equation above:

  STEP 1 Find the common difference.

  STEP 2 Determine the 1st term in the given arithmetic sequence.

  STEP 3 Find the symbol for the unknown term in the sequence.

  STEP 4 Write the equation for the unknown term in the sequence.

  STEP 5 Substitute the values in the equation and solve for the unknown.

- To solve for the n^{th} term of an arithmetic sequence, we use the formula:

  $a_n = a_1 + (n - 1)d$ where

  a_n = n^{th} term

  a_1 = 1st term

  n = number of terms

  d = common difference

  We follow the following steps in using the given equation:

  STEP 1 Find the common difference.

  STEP 2 Determine the 1st term in the given arithmetic sequence.

  STEP 3 Find the symbol for the unknown term in the sequence.

STEP 4 Write the equation for the unknown term in the sequence.

STEP 5 Substitute the values in the equation and solve for the unknown.

- The formula:

$$S_n = \frac{n}{2}(a_1 + a_n) \text{ (Equation 2)}$$

or $$S_n = \frac{n}{2}[2a_1 + (n-1)d] \text{ (Equation 3)}$$

where n = number of terms
a_1 = 1^{st} term
a_n = n^{th} term
d = common difference

are the two equations used to find the sum of the 1^{st} n terms of an arithmetic sequence.

We follow the steps below in using the equations above:

STEP 1 Determine the given facts in the problem. If the 1^{st} and last terms are given, use Equation 2.

If the 1^{st} term and the common difference are given, use Equation 3.

STEP 2 Determine what is being asked for in the problem and write in arithmetic notation.

STEP 3 Decide which formula to use in finding the unknown sum based on the given facts or information.

STEP 4 Substitute the given values in the formula and solve for the unknown.

What Have You Learned?

Let us check if you understood the topics discussed in this module. Answer the exercises given below.

A. Determine if the given sequence is an arithmetic sequence or not. Write **Yes** if it is and **No** if it is not.

1. 3, –1, –5, –9, . . .
2. ½, 2, 8, 16, . . .
3. 5¼, 5½, 5¾, 6, . . .
4. $^3/_2$, $–^3/_4$, $^3/_8$, $–^3/_{16}$, . . .
5. 6, –18, 54, –162, . . .

B. Use the formula for finding the n^{th} term of the following arithmetic sequences.

1. 10^{th} term in the arithmetic sequence with –9 as the 1^{st} term and 4 as the common difference.
2. 12^{th} term in a sequence if the first term is 5 and the common difference is 5.
3. 6^{th} term in the sequence 3, 6, 9, ...
4. 5^{th} term in the sequence if the 1^{st} term is 64 and the common difference is 4.

C. Use the appropriate formula in finding the sum of the 1^{st} n terms of an arithmetic sequence given the following:

1. 4, 1, –2, . . .
2. 5, 9, 13, 17, . . .

D. Solve the following problems.

1. Lydia is preparing for a coming marathon. She plans to run for seven days. She decides to cover 2 km on the first day and constantly increase the number of kilometers to cover each day by ½ km. How many kilometers will she have to cover on the last day?
2. In a potato race, the first and last potatoes are 5 m and 15 m away, respectively, from the starting line and the rest are equally spaced 1 m away from each other. What is the total distance traveled by a runner who brings them one at a time to the finishing line?
3. For finishing a certain job, Pepe earns ₱10.00 on the first day, ₱20.00 on the second day, ₱30.00 on the third and so on. How much will Pepe earn in 5 days?

Compare your answers with those found in the *Answer Key* on pages 42 to 44. If your score falls between 14 – 15, congratulations. You did great. You really understood the topics discussed in this module. However, if you scored below 14, look at the descriptive ratings given below:

11 – 13	Very satisfactory. Just review the parts of the module you did not understand very well.
8 – 10	Satisfactory. Review the parts you did not understand and solve other exercises similar to those.
1 – 7	Study the whole module again.

Answer Key

A. Let's See What You Already Know *(pages 1-2)*

A. 1. Yes
2. No
3. No
4. Yes
5. Yes

B. 1. 21
2. 97
3. 17
4. 75

C. 1. –1,264
2. –21

D. 1. ₱145.00
2. ₱10.50

B. Lesson 1

Let's Review (pages 8-9)

1.	S	6.	S
2.	N	7.	S
3.	S	8.	S
4.	N	9.	N
5.	S	10.	S

C. Lesson 2

Exercise (page 13)

3. $a_7 = a_1 + 6\,(3)$

$= 25 + 6\,(3)$

$= 25 + 18$

$a_7 = 43$

4. $a_9 = a_1 + (9 - 1)d$

$= 25 + (8)\ 3$

$= 25 + 24$

$a_9 = 49$

5. $a_{11} = a_1 + (11 - 1)d$

$= 25 + (10)\ 3$

$= 25 + 30$

$a_{11} = 55$

Example 3 (pages 15–16)

Step 1: $-1 - (2) = -3$

$-4 - (-1) = -3$

$7 - (-4) = -3$

$d = -3$

Step 2: $a_1 = 2$

Step 3: a_{17}

Step 4: $a_{17} = a_1 + 16d$

Step 5: $d = -3$

$a_1 = 2$

$a_{17} = a_1 + 16d$

$= 2 + 16(-3)$

$= 2 + (-48)$

$a_{17} = -46$

Problem 2 (page 17-18)

Step 1: $35 - 30 = 5$

$40 - 35 = 5$

$45 - 40 = 5$

The common difference is 5.

Step 2: the first term or $a_1 = 30$

Step 3: a_{20}

Step 4: $a_{20} = a_1 + (20 - 1)d$

Step 5: $a_{20} = a_1 + 19d$

$= 30 + 19(5)$

$= 30 + 95$

$= 125$

₱125.00 per hour will be his salary at the end of 5 years.

Let's See What You Have Learned (page 20)

A. 1. Step 1: $d = -5$

Step 2: $a_1 = 20$

Step 3: a_{28}

Step 4: $a_{28} = a_1 + 27d$

Step 5: $a_{28} = 20 + 27(-5)$

$a_{28} = 115$

2. Step 1: $d = 0.3$

Step 2: $a_1 = 7$

Step 3: a_{17}

Step 4: $a_{17} = a_1 + 16d$

Step 5: $a_{17} = 7 + 16(0.3)$

$a_{17} = 11.8$

B. 1. $a_1 = 61$; $d = 58 - 61 = -3$; $n = 11$

$$\begin{aligned} a_{11} &= 61 + (11-1)(-3) \\ &= 61 + (-30) \\ a_{11} &= 31 \end{aligned}$$

2. $a_1 = 70$; $n = 30$; $d = 72 - 70 = 2$

$$\begin{aligned} a_{30} &= 70 + (30-1)(2) \\ &= 70 + (29)(2) \\ a_{30} &= 128 \end{aligned}$$

There are 128 seats in the last row.

5. $a_1 = 10.5$; $d = 0.5$; $n = 4(12) = 48$ because there are 12 months in a year

$$\begin{aligned} a_{48} &= 10.5 + (48-1)(0.5) \\ &= 10.5 + (47)(0.5) \\ &= 10.5 + 23.5 \\ a_{48} &= 34 \end{aligned}$$

He put ₱34.00 on the last month of the 4th year.

D. Lesson 3

Exercise (pages 30 – 31)

Step 1: a_1 = P35.00; a_8 = P105.00

Step 2: S_8

Step 3: Equation 2

$$S_n = \frac{n}{2}[a_1 + a_n]$$

Step 4:

$$\begin{aligned} S_8 &= \frac{8}{2}[a_1 + a_n] \\ &= 4[\text{P35 + P105}] \\ &= 4[\text{P140}] \\ &= ₱560 \end{aligned}$$

₱560.00 is the cost of a pair of shoes

Let's See What You Have Learned (page 32)

A. 1. $a_1 = 15;\ d = -\frac{1}{5}$

$$\begin{aligned} S_{26} &= \frac{26}{2}\left[2(15) + (26-1)\left(-\frac{1}{5}\right)\right] \\ &= 13\left[30 + 25\left(-\frac{1}{5}\right)\right] \\ &= 13\left[30 + \left(\frac{-25}{5}\right)\right] \\ &= 13(30 - 5) \\ &= 13(25) \\ S_{26} &= 325 \end{aligned}$$

2. $a_1 = -40;\ a_{11} = -73$

$$S_{11} = \frac{11}{2}[a_1 + a_{11}]$$
$$= 5.5[(-40)+(-73)]$$
$$= 5.5[-113]$$
$$= 621.5$$

B. 1. $a_1 = 61;\ a_2 = 58;\ a_3 = 55;\ a_{18} = 10$

$$S_{18} = \frac{18}{2}[a_1 + a_{18}]$$
$$= \frac{18}{2}[61+10]$$
$$= 9[71]$$
$$S_{18} = 639 \text{ bricks in all}$$

2. $a_1 = 70;\ d = 2;\ a_2 = 72$

$$S_{30} = \frac{30}{2}[2a_1 + (30-1)d]$$
$$= \frac{30}{2}[2(70)+(29)2]$$
$$= 15[140+58]$$
$$S_{30} = 2{,}970 \text{ seats in all}$$

5. $a_1 = 10.50;\ d = 0.50;\ n = 48$ months in 4 years

$$\mathrm{S}_{48} = \frac{48}{2}[2(10.50)+(48-1)(0.50)]$$
$$= 24[2(10.50)+(47)(0.50)]$$
$$= 24[21 + 23.50]$$
$$\mathrm{S}_{48} = ₱1{,}068$$

E. **What Have You Learned?** *(pages 35–36)*

A. 1. Yes
   2. No
   3. Yes
   4. No
   5. No

B. 1. Given: $a_1 = -9$; $d = 4$

$$\begin{aligned} a_{10} &= a_1 + 9(4) \\ &= -9 + 36 \\ a_{10} &= 27 \end{aligned}$$

2. Given: $a_1 = 5$; $d = 5$

$$\begin{aligned} a_{12} &= a_1 + 11d \\ &= 5 + 11(5) \\ &= 5 + 55 \\ a_{12} &= 60 \end{aligned}$$

3. Given: $a_1 = 3$; $d = 3$

$$\begin{aligned} a_6 &= a_1 + 5d \\ &= 3 + 5(3) \\ &= 3 + 15 \\ a_6 &= 18 \end{aligned}$$

4. Given: $a_1 = 64$; $d = 4$

$$\begin{aligned} a_5 &= a_1 + 4d \\ &= 64 + 4(4) \\ &= 64 + 16 \\ a_5 &= 80 \end{aligned}$$

C. 1. Given: $a_1 = 4$; $d = -3$; $n = 3$

$$\begin{aligned} S_3 &= \frac{3}{2}[2a_1 + (n-1)d] \\ &= \frac{3}{2}[2(4) + (2)(-3)] \\ &= \frac{3}{2}[8 + (-6)] \\ &= \frac{3}{2}(2) \\ &= \frac{6}{2} \\ S_3 &= 3 \end{aligned}$$

2. Given: $a_1 = 5$; $d = 4$; $n = 4$

$$\begin{aligned} S_4 &= \frac{4}{2}[2(5) + (4-1)4] \\ &= 2[10 + 3(4)] \\ &= 2[10 + 12] \\ &= 2(22) \\ S_4 &= 44 \end{aligned}$$

D. 1. Given: $a_1 = 2$ km; $d = 0.5$ km; $n = 7$ days

$$\begin{aligned} a_7 &= a_1 + (n-1)d \\ &= 2 + (6)(0.5) \\ &= 2 + 3 \\ a_7 &= 5 \text{ kms} \end{aligned}$$

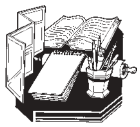
2. Given: $a_1 = 5$ m; $a_n = 15$ m; $d = 1$; $n = 11$

$n = 11$ because starting from 5 meters which is the first term in the series, there will be 10 more meters from 5 meters to 15 meters. So, the number of terms in the series is 11.

$$\begin{aligned} S_{11} &= \frac{11}{2}[2(5) + (10)1] \\ &= \frac{11}{2}[10 + 10] \\ &= 5.5(20) \\ S_{11} &= 110 \text{ meters in all} \end{aligned}$$

3. Given: a_1 = ₱10.00; a_2 = ₱20.00; a_3 = ₱30.00; S_5 = ?;
d = ₱ 10.00

$$
\begin{aligned}
S_5 &= \frac{n}{2}\left[2a_1 + (n-1)d\right] \\
&= \frac{5}{2}\left[2(10) + (5-1)10\right] \\
&= 2.5(20+40) \\
&= 2.5(60) \\
S_5 &= ₱150
\end{aligned}
$$



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