What Is This Module About?

Polynomials have many uses. Scientists use polynomials to represent things or situations such as projectile motion (paths of things that are thrown or launched), paths of comets and planets and surface areas of planets and moons. Polynomials are also used in computing for interest and population growth.

Polynomials are indeed very interesting. You shall learn more about them in this module. This module is made up of three lessons:

Lesson 1—*Identifying and Evaluating Polynomials*

Lesson 2—*Performing Addition and Subtraction on Polynomials*

Lesson 3—*Performing Multiplication and Division on Polynomials*

What Will You Learn From This Module?

After studying this module, you should be able to:

- ♦ define algebraic expressions and other related terms;
- ♦ evaluate algebraic expressions; and
- ♦ perform operations on polynomials.

Wait!

Before you start studying this module, make sure that you have already read the following modules:

- ♦ *Sets, Sets and Sets*
- ♦ *Real Numbers*
- ♦ *Relations and Functions*
- ♦ *More on Functions*
- ♦ *Exponents and Radicals*

If you have read the modules listed above, you are now ready to proceed. Answer the questions in Let's See What You Already Know. This way you will know what you already know about the topics covered in this module.

Let's See What You Already Know

Before you start studying this module, take this simple test first to find out how much you already know about the topics to be discussed.

A. Define the following terms.

1. Polynomial

2. Monomial

3. Binomial

4. Trinomial

5. Multinomial

B. Write the degree of each of the following expressions.

1. $x^5 - 25x^3 - 24$
2. $x^6 + 22$
3. $x^3 - 64$

C. Solve the following word problem.

A farmer has a poultry farm whose area is expressed by $8x^2 + 97x + 12$ m^2. What is the land area of the poultry farm if $x = 2$ m?

D. Add/Subtract the following polynomials.

1. $(2x^2 + 3x) - (5x^2 + x)$
2. $(4x^2 + 5x^3 - x) + (4x^3 + 2x - x)$
3. $(x^4 + 3x^2 + x - 5) - (3x^4 + 2x^3 - x^2 + 3)$

E. Multiply the following expressions.

1. $(x + 1)(x - 1)$
2. $(x + 2)(2x + 1)$
3. $(x + 1)(x + 1)$

F. Divide the following using algorithm.

1. $17x^5 \div x^2$

2. $(16y^5 - 8y^4) \div 4y^3$

3. $(4x^2 - 7x - 2) \div (x - 2)$

G. Solve the following expression using synthetic division.

$(x^4 - 134x^2 - 100x + 240) \div (x + 2)$

Well, how was it? Do you think you fared well? Compare your answers with those in the *Answer Key* on pages 46 and 47 to find out.

If all your answers are correct, very good! You may still study the module to review what you already know. Who knows, you might learn a few more new things as well.

If you got a low score, don't feel bad. This only goes to show that this module is for you. It will help you understand some important concepts that you can apply in your daily life. If you study this module carefully, you will learn the answers to all the items in the test and a lot more! Are you ready?

You may now go to the next page to begin Lesson 1.

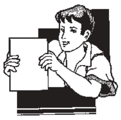
LESSON 1

Identifying and Evaluating Polynomials

Polynomials have several uses. For example, they are used to compute for areas and interest. If you have an account with a bank and its interest is expressed as $8x^4 + 5x^3 - x^2 + 56$, would you know how much interest you should get from the bank? Polynomials are also used in expressing the speed and acceleration of objects in motion. If the acceleration of an object is $x^2 - 16$, would you be able to compute for its acceleration? To solve these and similar problems, you need to understand what polynomials are first and know how to perform operations on them.

After studying this lesson, you should be able to:

- define algebraic expressions and other related terms;
- find the degree of a polynomial; and
- evaluate an algebraic expression.



Let's Learn

Algebraic Expressions and Other Related Terms

The expression x^2, $x + 6$ and $2x - 10$ are examples of **open phrases.** In the open phrase $2x - 10$, x is a variable and 10 is a constant. Another term for open phrase is **algebraic expression.** It is an expression which consists of numbers, variables and grouping and operation symbols.

The parts of an algebraic expression separated by plus or minus signs are called **terms.** A term may be an addend or a subtrahend. It is made up of numerical and literal coefficients. For example, in the expression $5x$, 5 is the numerical coefficient while x is the literal coefficient. The **numerical coefficient,** often simply called **coefficient,** is the number in a term. The **literal coefficient,** on the other hand, is the variable in a term, represented by a letter and its exponent. In the expression $4y^7$, the numerical coefficient is 4 while the literal coefficient is y^7.

The algebraic expression $8x^3 + x^2 - 3x + 4$ contains four terms.

The first term $8x^3$ has 8 as its numerical coefficient and x^3 as its literal coefficient. The second term x^2 has x^2 as its literal coefficient. If a term has no numerical coefficient before it, the numerical coefficient is understood to be 1 or –1. This means that the second term's numerical coefficient is 1. The third term may be renamed as $+ (-3x)$. Its numerical coefficient is –3 and its literal coefficient is x. The fourth term, 4, is called a **constant term.** Any term that has no variable or no literal coefficient is called a **constant.**

Let’s Try This

Below are algebraic expressions. Indicate the number of terms in each of them. Then identify the numerical and literal coefficients in each term. Write your answers in the designated columns. The first number has been done as an example for you.

		No. of Terms	Terms	Numerical Coefficients	Literal Coefficients
1.	$4x^2 + 2y^3$	2	$4x^2$	4	x^2
			$2y^3$	2	y^3
2.	$11x^3 + 4xy^2 + 8y^3$	______	______	______	______
			______	______	______
			______	______	______
3.	$a^2 + b^2 + c^2$	______	______	______	______
			______	______	______
			______	______	______
4.	$3x + 11$	______	______	______	______
			______	______	______
5.	$7x^5y + 6x^2 + 9$	______	______	______	______
			______	______	______
			______	______	______

Compare your answers with mine below.

	No. of Terms	Terms	Numerical Coefficients	Literal Coefficients
2.	3	$11x^3$	11	x^3
		$4xy^2$	4	xy^2
		$8y^3$	8	y^3
3.	3	a^2	1	a^2
		b^2	1	b^2
		c^2	1	c^2
4.	2	$3x$	3	x
		11	11	
5.	3	$7x^5y$	7	x^5y
		$6x^2$	6	x^2
		9	9	

If you got all the correct answers, then you can go to the next section. If not, review this part first and try to answer the exercises again before doing so.

Let's Learn

A **polynomial** is an algebraic expression involving only addition and multiplication of numbers and variables. If a term in an algebraic expression has a variable in the denominator, then the algebraic expression is *not* considered a polynomial. For example, $8x^3$ is considered a polynomial but $\frac{8}{x^3}$ is not a polynomial. $x^4 - 3$ is a polynomial but $\frac{4}{x^4} - 3$ is not a polynomial.

$3x^9 - 13$ is a polynomial but $\frac{3}{2x^9} - 13$ is not.

Let's Try This

Identify which of the following are polynomials. Put a 4 in the box if the expression is a polynomial and an 8 if it is not.

1. ☐ $4x^2 + 2xy$
2. ☐ $\frac{4x}{y^2} + \frac{6xy}{2x}$
3. ☐ $\frac{3x}{5} + 1$
4. ☐ $9xy + 3x^7$
5. ☐ $\frac{6x}{5y^9} + \frac{10}{x^3}$

Compare your answers with mine below.

If you checked numbers 1, 3 and 4 and crossed out numbers 2 and 5, then you got all the correct answers! Numbers 2 and 5 are not polynomials because they have variables in the denominator.

Is the subject now clear to you? This time, you will learn about the different kinds of polynomials.

Let's Learn

What are the different kinds of polynomials?

A polynomial is called a **monomial** if it contains only one term. Examples of monomials are $5x$, $29x^2$, $36xy$ and 25. Can you think of other examples of monomials?

A polynomial is called a **binomial** if it contains two terms. Examples of binomials are $35x + 30$, $4x^2 - 5y$ and $x^2y + 5xy$.

A polynomial is called a **trinomial** if it contains three terms. Examples of trinomials are $35x + 15y - 30$, $4x^2 - 5y + 10$ and $x^3 + 2xy - 2x^2$.

Finally, a polynomial is called a **multinomial** if it contains four or more terms. Examples of multinomials are $35x^2 - 15x + 15y - 30$, $4x^2 + 15x - 5y + 10$ and $x^3 + y^3 - 2x^2 + 25$. Can you think of other examples of trinomials and multinomials?

Sometimes, a binomial may be turned into a monomial. For example, $14x + 14y$ is a binomial because it has two terms. However, it can be turned into a monomial by taking out 14. If we take out 14, the term becomes $14(x + y)$, which is a monomial. $14(x + y)$ contains only one term because it is not separated by a plus or a minus sign.

When a monomial consists of only a constant term or a constant, the expression is called a **constant polynomial.** Examples of constant polynomials are 2, 306 and 1.

Let's Try This

Write in the blank what kind of polynomial each given expression belongs to.

_______________ 1. $x^2 + 29y$

_______________ 2. $57x^2y^7$

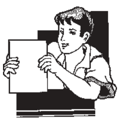
_______________ 3. 1200

_______________ 4. $x^3 + 21xy + 3y^7 + 31y^8$

_______________ 5. $x^6y + 7x^2y^5 + 12xy^8$

Compare your answers with mine below.

1. binomial
2. monomial
3. constant polynomial
4. multinomial
5. trinomial



Let's Learn

You have learned a lot about polynomials so far. Now, let us continue by studying the **degrees of polynomials.** It's true, polynomials are classified according to the number of terms they contain. But they may also be classified according to the highest exponent of any of the variables in it.

In the polynomial $3x^3 - 25x$, 3 is the highest exponent, so it is the degree of the polynomial. $3x^3 - 25x$ is said to be a binomial of the third degree. In the polynomial $x^2 + 6x + 9$, 2 is the highest exponent, so the expression $x^2 + 6x + 9$ is said to be a trinomial of the second degree. $6y^5 + y^3 + 4y^6 + y^2$ is a multinomial of the sixth degree since 6 is the highest exponent in it.

The **degree of a constant nonzero polynomial** is zero. Remember that the constant polynomial 5 can also be written as $5y^0$ or $5x^0$ since any variable raised to the 0 degree is equal to 1. The degree of 5 is zero, just like the numbers 23, 6 and 1046.

A **zero polynomial** has no definite degree.

Let’s Try This

Write the degree of the following polynomials.

_____ 1. $3y^2 + 2$

_____ 2. $3y^5 + 7y^4 + 9y^3 + 12$

_____ 3. 294

_____ 4. $8x^4 + 5x^5 + 11x^2$

_____ 5. $11x + 10$

Compare your answers with mine below.

1. 2
2. 5
3. 0
4. 5
5. 1

Did you get all the correct answers? If you did, then you can move on to the next part of the lesson. If you did not, then you have to review this part of the lesson and answer the exercises again.

Let’s Review

A. Identify the numerical and literal coefficients of the following terms. Write your answers in the spaces provided.

1.	$11xy^2$	numerical coefficient	____________________
		literal coefficient	____________________
2.	$a^2b^2c^2$	numerical coefficient	____________________
		literal coefficient	____________________
3.	$36z^2$	numerical coefficient	____________________
		literal coefficient	____________________

B. Put a 4 in the box if the expression is a polynomial and an 8 if it is not.

☐ 1. $26y^2 + 4x^2$

☐ 2. $\dfrac{3y}{4x^2} + \dfrac{6y}{2x}$

☐ 3. $15xy + 5xy^2$

C. Identify what kind of polynomial the following expressions belong to. Write your answers in the spaces provided.

1. ________________________ $a^2b^2c^2$

2. ________________________ 1500

3. ________________________ $a^2 + b^2 + c^2$

4. ________________________ $x^6y + x^5y^2 + x^4y^3 + x^3y^2 + xy$

5. ________________________ $25z^2 + 9$

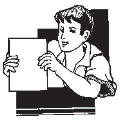
D. Identify the degree of the following polynomials. Write your answers in the spaces provided.

1. ________________________ $27x^3 + y^2$

2. ________________________ 150

3. ________________________ $11y^6 + x^7$

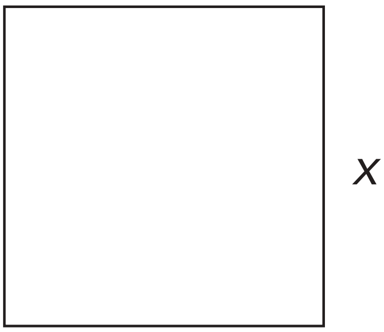
Compare your answers with those in the *Answer Key* on page 47. If you got all answers right, very good. Continue reading the module. If you made some mistakes, review the items you missed then answer the exercises again.



Let's Learn

Evaluating an Algebraic Expression

Look at the illustration below.



If the value of x is 500 meters, what is the area of the square? What is its perimeter? The area of a square is equal to the square of one of its sides. The area x^2 then becomes $(500)^2$ sq. m or 250000 sq. m. The perimeter of any polygon is equal to the sum of its sides. For a square, the perimeter is equal to $x + x + x + x$, which is also equal to $4x$. The perimeter becomes 4×500 m or 2000 m.

- The process of determining the value of a polynomial by replacing the variable/s with given numerical values is called **evaluating the polynomial.**

To further understand how to evaluate a polynomial, two sample problems are provided below with their step-by-step solutions. Study each problem carefully.

EXAMPLE 1 A farmer has a piece of land with an area expressed as $8x^3 + 25x^2 + 86$ sq. m. What is the area of the farmland, if $x = 2$ m?

SOLUTION

STEP 1 Write the given algebraic expression.

Given: $8x^3 + 25x^2 + 86$

STEP 2 Know what is being asked for in the problem.

Unknown: What is the area of the farmland if $x = 2$ m?

STEP 3 Solve the problem.

a. Substitute 2 for x.

$8(2)^3 + 25(2)^2 + 86$

$= 8(8) + 25(4) + 86$

$= 64 + 100 + 86$

b. Add the terms of the expression.

$64 + 100 + 86$

$= 250$

STEP 4 Review your answer.

STEP 5 Make a conclusion.

Final answer: The area of the farmland is 250 sq. m if $x = 2$ m.

EXAMPLE 2 Rene went to the bank to get the interest on his savings account. The interest on his savings account is expressed as $8x^4 + 5x^3 - x^2 + 56$ pesos. How much will Rene receive if $x = 2$ pesos?

SOLUTION

STEP 1 Write the given algebraic expression.

Given: $8x^4 + 5x^3 - x^2 + 56$

STEP 2 Know what is being asked for in the problem.

Unknown: How much interest will Rene receive if $x = 2$ pesos?

STEP 3 Solve the problem.

a. Substitute 2 for x.

$8(2)^4 + 5(2)^3 - 2^2 + 56$

$= 8(16) + 5(8) - 4 + 56$

$= 128 + 40 - 4 + 56$

b. Add the terms of the expression.

128 + 40 – 4 + 56

= 220

STEP 4 Review your answer.

STEP 5 Make a conclusion.

Final answer: Rene will receive ₱220 if x = 2 pesos.

Let's Try This

Answer the following problems.

1. Ana went to the grocery and bought items which cost $3x^4 + 2x^3 + 5x + 18$ pesos. How much do the items cost if x = 3 pesos?

2. The distance from Jerry's house to his school is $5y^3 + 4y^2 + 121$ m. How many meters does Jerry have to travel from his home to his school if x = 2 m?

Compare your solutions with mine below.

1. $3x^4 + 2x^3 + 5x + 18$

   $= 3(3)^4 + 2(3)^3 + 5(3) + 18$

   $= 3(81) + 2(27) + 15 + 18$

   $= 243 + 54 + 15 + 18$

   $= 330$

   The items cost ₱132 if x = 3 pesos.

2. $5x^3 + 4x^2 + 121$

   $= 5(2)^3 + 4(2)^2 + 121$

   $= 5(8) + 4(4) + 121$

   $= 40 + 16 + 121$

   $= 177$

   Jerry has to travel 177 m from his home to his school if x = 2 m.

Did you get all the correct answers? If you did, you may then proceed to the next part of the lesson. If you did not, then review this part and solve the problems again before moving on the next part of the lesson.

Let's See What You Have Learned

A. Underline the numerical coefficient once and the literal coefficient twice in each of the following expressions.

1. $3x^2 + 9y^3$
2. $12x^2 + 5x + 1$
3. $y^3 - 3$

B. Put a 4 in the box if the given expression is a polynomial and an 8 if it is not.

1. ☐ $2x^2 - \dfrac{3}{x^3}$
2. ☐ $\dfrac{4y^7}{9} + x^4 - 10$
3. ☐ $9x^2 + \dfrac{2y^5}{3x^4}$

C. Classify the following polynomials according to their number of terms and degree. The first one has been done as an example for you.

1. monomial of the third degree $2x^3$
2. ______________________ $4x^5 + 2x^4 + 3x^3 + x^2 + 7x + 1$
3. ______________________ 1203
4. ______________________ $3y^7 + 4y^8$
5. ______________________ $6y^8 + y^9 + 2$

D. Solve the following problem.

Mang Isko harvested $4x^3 + 9x^2 - 5x - 10$ bushels of corn yesterday. If $x = 2$, how many bushels did he harvest?

Compare your answers with those in the *Answer Key* on pages 47 and 48. If you got all the correct answers, very good! If you did not, read the lesson again. Then you can proceed to Lesson 2.

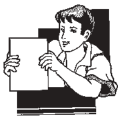
Let's Remember

- An **algebraic expression** is an expression made of numbers, variables and grouping and operation symbols.
- The parts of an algebraic expression separated by a plus or a minus sign are called **terms.**
- A term is made up of a **numerical coefficient** or numerical value and a **literal coefficient** or variable represented by a letter.
- A **polynomial** is an algebraic expression involving only addition and multiplication of numbers and variables. When a term in the expression has a variable in the denominator or when the exponent is a fraction the expression is not considered a polynomial.
- The **degree of a polynomial** is determined by the highest exponent of any of its variables. The degree of a constant nonzero polynomial is one while a zero polynomial does not have a definite degree.
- A polynomial consisting of only one term is called a **monomial.** One with two terms is called a **binomial.** One with three terms is called a **trinomial.** One with four or more terms is called a **multinomial.** A **constant polynomial** does not have a variable.
- To evaluate a polynomial, we replace the variable/s with the given numerical value/s.

Performing Addition and Subtraction on Polynomials

Cora went to the market and bought x pieces of live bangos worth ₱24 each. She also bought y pieces of tilapia worth ₱18 each. When she came home, she found out that her mother bought the same number of bangos and tilapia worth ₱12 each. What is the total cost of all the bangos and tilapia? How will you represent this amount as a polynomial? To solve problems like this, you need to know more about polynomials. This lesson will teach you more about this interesting topic.

After studying this lesson, you should be able to add and subtract polynomials using the concept of similar terms.



Let's Learn

Similar Terms

In $20y$, 20 is the numerical coefficient and y is the literal coefficient. In $-30y$, -30 is the numerical coefficient and y is the literal coefficient. Notice that the two terms, $20y$ and $-30y$, have the same literal coefficient y. Terms having the same literal coefficient are said to be **similar.** $10y^2$ and $12y$ are not similar terms because the literal coefficient of the first term is y^2, while that of the second term is y.

♦ Terms are **similar** if their literal coefficients or their variables and exponents are the same.

♦ Terms are **dissimilar** if they have different literal coefficients.

Let us look at some more examples below.

♦ $14x^3y^2$ and $-17x^3y^2$ are similar terms.

  $14x^3y^2$ and $-14x^2y^3$ are dissimilar terms.

♦ $10x^3y$ and $20yx^3$ are similar terms because $20yx^3$ can be rearranged to $20x^3y$.

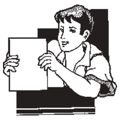
Note: A monomial is a polynomial containing only one term. Thus, similar terms may be called **similar monomials.** Monomials will only be similar if their literal coefficients are the same.

Let's Try This

Put a 4 in the box if the given terms are similar and an 8 if the terms are not.

1. ☐ $3x^2$ and $7x^2$
2. ☐ $-x^2y^3$ and $10x^3y^2$
3. ☐ $19x^5y^4$ and $12y^4x^5$
4. ☐ $1x$ and $11y$
5. ☐ $-7y^6$ and $7y^6$

If you checked numbers 1, 3 and 5 and crossed out numbers 2 and 4, then you got all the correct answers. Numbers 1, 3 and 5 contain similar terms because they have the same literal coefficients.



Let's Learn

Adding Polynomials With Similar Terms

If a polynomial contains two or more similar terms, the polynomial can be made simpler in form.

To further understand how to add polynomials, a sample problem is provided below.

EXAMPLE 1 Ana went to the market and bought x pieces of bangos worth ₱24 each. She also bought y pieces of tilapia worth ₱18 each. When she came home, she found out that her Nanay bought the same number of bangos and tilapia worth ₱12 each. What is the total cost of all the bangos and tilapia that Ana and her Nanay bought? Write the polynomial that will represent this amount.

SOLUTION

STEP 1 Determine what is being asked for in the problem.

Unknown: What is the total cost of all the bangos and tilapia that Ana and her Nanay bought? Write the polynomial that will represent this amount.

STEP 2 Write the polynomial.

- Each bangos is worth ₱24. To find the cost of x pieces, we have to multiply 24 by x as in:

  $24 \cdot x = 24x$

- Each tilapia is worth ₱18. To find the cost of y pieces, we have to multiply 18 by y as in:

  $18 \cdot y = 18y$

- Ana's mother also bought the same number of bangos and tilapia but they cost only ₱12 each. To find the cost, we multiply 12 by the number of bangos and tilapia then add them together.

  $12 \cdot x + 12 \cdot y = 12x + 12y$

- To find the total cost, we simply add all the terms as in:

  $24x + 18y + 12x + 12y$

STEP 3 See if the polynomial can be simplified.

a. Group similar terms together.

   $(24x + 12x) + (18y + 12y)$

b. Take out the variable and solve for the numerical coefficient by performing the necessary operation.

   $(24x + 12x) + (18y + 12y)$

   $= (24 + 12)x + (18 + 12)y$

   $= 36x + 30y$

STEP 4 Make a conclusion.

The polynomial that will represent the total cost of all the bangos and tilapia is $36x + 30y$.

Let's Try This

Simplify the following terms.

1. $2x + 4x$
2. $3x^3 + 5x^3$
3. $5xy + 7xy + 3y^2 + y^2$
4. $8y^8 + 9y^8 + 4y^8$
5. $6x^2y + 10x^2y + 12x^2y + 2xy^2 + 3xy^2$

Compare your answers with mine below.

1. $2x + 4x = (2 + 4)x$

   $= 6x$

2. $3x^3 + 5x^3 = (3 + 5)x^3$

   $= 8x^3$

3. $5xy + 7xy + 3y^2 + y^2 = (5 + 7)xy + (3 + 1)y^2$

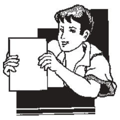
   $= 12xy + 4y^2$

4. $8y^8 + 9y^8 + 4y^8 = (8 + 9 + 4)y^8$

$= 21y^8$

5. $6x^2y + 10x^2y + 12x^2y + 2xy^2 + 3xy^2 = (6 + 10 + 12)x^2y + (2 + 3)xy^2$

$= 28x^2y + 5xy^2$



Let's Learn

Rearranging and Renaming Addends

Before adding polynomials, you should arrange the terms first from the one with the highest exponent to the one with the lowest. The terms should be arranged in descending order. If we want to add $(x^4 + x – 8x^2 + 5x^3 – 6)$ to $(12 – x^3 + 3x + 2x^2)$, for example, we should rearrange the terms:

- $x^4 + x – 8x^2 + 5x^3 – 6$ into $x^4 + 5x^3 – 8x^2 + x – 6$
- $12 – x^3 + 3x + 2x^2$ into $–x^3 + 2x^2 + 3x + 12$

If you were asked to add $(5x – 2x^2 + x^4 – 11)$ to $(10 – x^3)$, how would you go about it?

- If we rearrange $5x – 2x^2 + x^4 – 11$ in proper order, the polynomial would be $x^4 – 2x^2 + 5x – 11$. Notice that it does not have a term of the third degree. In cases like this, we can rename the polynomial by adding $0x^3$, which is just equal to zero. This would then be equal to:

  $x^4 + \mathbf{0x^3} – 2x^2 + 5x – 11$

- If we rearrange $10 – x^3$, the result would be $–x^3 + 10$. By including $0x^2$ and $0x$, we can rename the polynomial as:

  $–x^3 + \mathbf{0x^2} + \mathbf{0x} +10$

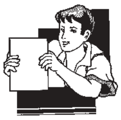
Let's Try This

Rearrange and rename the following polynomials by filling in the missing terms.

1. $x^5 + 3x^6 + 7x^3 + x^2 – 3x^4$
2. $y^3 + 1$
3. $a^3 – a^5 –1$
4. $x^5 – 10$
5. $z + 3z^3 – 4$

Compare your answers with mine below.

1. $3x^6 + x^5 - 3x^4 + 7x^3 + x^2 + 0x + 0$
2. $y^3 + 0y^2 + 0y + 1$
3. $-a^5 + 0a^4 + a^3 + 0a^2 + 0a - 1$
4. $x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 10$
5. $3z^3 + 0z^2 + z - 4$



Let's Learn

Adding Trinomials and Multinomials

You have already learned how to arrange polynomials. Now, let us learn how to add polynomials. If you remember how to add whole numbers well, you would easily learn how to add polynomials. Let us take the following as an example.

Adding 6463 to 3124 looks very simple. But if we write it in **expanded form,** we will get:

	6463	=	6(1000)	+	4(100)	+	6(10)	+	3(1)
+	3124	=	3(1000)	+	1(100)	+	2(10)	+	4(1)
	9587	=	(6 + 3)(1000)	+	(4 + 1)(100)	+	(6 + 2)(10)	+	(3 + 4)(1)
		=	9(1000) + 5(100) + 8(10) + 7(1)						
		=	9000 + 500 + 80 + 7						
		=	9587						

If we write it in **exponential form,** we will get:

Remember that: $1000 = 10^3$

$100 = 10^2$

$10 = 10^1$

$1 = 10^0$

	6463	=	$6(10^3)$	+	$4(10^2)$	+	$6(10^1)$	+	$3(10^0)$
+	3124	=	$3(10^3)$	+	$1(10^2)$	+	$2(10^1)$	+	$4(10^0)$
	9587	=	$(6 + 3)(10^3)$	+	$(4 + 1)(10^2)$	+	$(6 + 2)(10^1)$	+	$(3 + 4)(10^0)$
		=	$9(10^3) + 5(10^2) + 8(10^1) + 7(10^0)$						
		=	9(1000) + 5(100) + 8(10) + 7(1)						
		=	9587						

- Notice that in adding whole numbers, we also need to arrange numbers from the one with the highest exponent to the one with the lowest.

From the equation given above, we can substitute x for 10 if we let $x = 10$.

If we write it in terms of x, we will get:

$$\begin{array}{rrcccccccc} & 6463 & = & 6(x^3) & + & 4(x^2) & + & 6(x^1) & + & 3(x^0) \\ + & 3124 & = & 3(x^3) & + & 1(x^2) & + & 2(x^1) & + & 4(x^0) \\ \hline & 9587 & = & (6+3)(x^3) & + & (4+1)(x^2) & + & (6+2)(x^1) & + & (3+4)(x^0) \\ & & = & \multicolumn{7}{l}{9x^3 + 5x^2 + 8x^1 + 7x^0} \end{array}$$

Since x^0 is just equal to 1, we can perform substitution to the equation and get:

$$= 9x^3 + 5x^2 + 8x + 7(1)$$

$$= 9x^3 + 5x^2 + 8x + 7$$

It is now clear that adding polynomials is as easy as adding whole numbers. The only difference is that in adding polynomials, numbers are represented by variables such as x.

To further understand how to add polynomials, an example is given below. Study it carefully.

EXAMPLE 1 Alex was asked to prepare for a family reunion. He bought ingredients which cost $14x^2 + 3x^3 - 15 + 10x$. He also paid three waiters for their services. This payment is expressed by $15x^3 - 7x^2 - 15$. Find the polynomial that will represent the total cost.

SOLUTION

STEP 1 Determine what is being asked for in the problem.

Find the polynomial that will represent the total cost.

STEP 2 Write down the given facts.

a. cost of ingredients

$14x^2 + 30x^3 - 15 + 10x$

b. payment for the three waiters

$15x^3 - 7x^2 - 15$

STEP 3 Solve for the total cost.

a. Arrange the equations in descending order.

$14x^2 + 30x^3 - 15 + 10x = 30x^3 + 14x^2 + 10x - 15$

- ♦ The second equation does not have a term of the first degree so we have to rename it by adding $0x$.

$15x^3 - 7x^2 - 15 = 15x^3 - 7x^2 + 0x - 15$

b. Add the two expressions by adding similar terms.

$$\begin{array}{rrcrcrcr} & 30x^3 & + & 14x^2 & + & 10x & - & 15 \\ + & 15x^3 & - & 7x^2 & + & 0x & - & 15 \\ \hline \multicolumn{8}{l}{(30+15)x^3 + (14+(-7))x^2 + (10+0)x + (-15+(-15))} \end{array}$$

- Since **a + (–b)** is equal to **a – b,** the equation becomes:

  $= (30 + 15)x^3 + (14 - 7)x^2 + (10 + 0)x + (-15 - 15)$

  $= 45x^3 + 7x^2 + 10x - 30$

STEP 4 Make a conclusion.

The total cost Alex spent for the gathering is expressed as $45x^3 + 7x^2 + 10x - 30$.

Let's Review

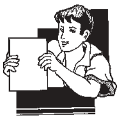
A. Add the following polynomials.

1. $(6x^2 + 5x^3 + 2x) + (x^2 - 1)$
2. $(x^5 + 3) + (3x^5 + 3x^3 - 6x^4 - 2x^2 - 13 + x)$
3. $(11y^3 - 19) + (4y^4 + 1)$
4. $(-10y^2 + 9y^4 + 12y^3 + 36y - 7) + (9y^3 + 8y^2 - y + 1 + 12y^4)$
5. $(12z^5 - 13z^6) + (9z^4 + z^2 - z + 1)$

B. Solve the following problem.

Atoy bought $9x^2 + 16x^4 - 5x^3 + 10$ pieces of red art paper and $2x^4 - 10x^2 - 17 + 13x$ pieces of green art paper. How many pieces of art paper does Atoy have in all?

Compare your answers with those in the *Answer Key* on pages 48 and 49. Did you get all the correct answers? If you did, then you may go to the next part of the lesson. If you did not, you may have to review the sections you didn't understand very well first.



Let's Learn

Subtracting Similar Terms

Getting the difference between polynomials is as easy as adding them. Since subtraction is the inverse operation of addition, subtracting a term from another is just like adding its opposite to that term.

- For example, $20x - 5x$ is equal to $20x + (-5x)$, $-5x$ is the opposite of $5x$.

  Then we can simply take out x to add the terms as in:

  $20x + (-5x)$

  $= (20 + (-5))x$

  $= (20 - 5)x$

  $= 15x$

As in addition, we can only subtract similar terms or terms having the same literal coefficients. Look at the two examples below.

- $25x^2 + 40y^2 - 30x^2 - 30y^2$

  a. First, group together similar terms.

     $(25x^2 - 30x^2) + (40y^2 - 30y^2)$

  b. Take out the variables.

     $(25 - 30)x^2 + (40 - 30)y^2$

  c. Subtract.

     $-5x^2 + 10y^2$

- $5x^2y + 3xy^2 - 4x^2y - 4xy^2$

  a. First, group together similar terms.

     $(5x^2y - 4x^2y) + (3xy^2 - 4xy^2)$

  b. Take out the variables.

     $(5 - 4)x^2y + (3 - 4)xy^2$

  c. Subtract.

     $x^2y - xy^2$

Let’s Try This

Subtract the following terms.

1. $5y^3 - 2y^3$
2. $4x^2 + 3y^5 - 2x^2 - y^5$
3. $-10xy + 19xy$
4. $4x^2 - 3y^2 + 15xy - 3x$
5. $11x^2y^3 - 10x^3y^2 + x^2y^3 + 2x^3y^2$

Compare your answers with mine below.

1. $$5y^3 - 2y^3 = (5 - 2)y^3 \\ = 3y^3$$

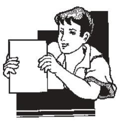
2. $$4x^2 + 3y^5 - 2x^2 - y^5 = (4x^2 - 2x^2) + (3y^5 - y^5) \\ = (4 - 2)x^2 + (3 - 1)y^5 \\ = 2x^2 + 2y^5$$

3. $$-10xy + 19xy = (-10 + 19)xy \\ = 9xy$$

4. $$4y^2 - 3y^2 + 15xy - 3x = (4 - 3)y^2 + 15xy - 3x \\ = y^2 + 15xy - 3x$$

5. $$11x^2y^3 - 10x^3y^2 - x^2y^3 + 2x^3y^2 = (11x^2y^3 - x^2y^3) + (-10x^3y^2 + 2x^3y^2) \\ = (11 - 1)x^2y^3 + (-10 + 2)x^3y^2 \\ = 10x^2y^3 - 8x^3y^2$$

Did you get all the correct answers? If you did, very good! Now, we can start discussing how to subtract trinomials and multinomials.



Let’s Learn

Subtracting Trinomials and Multinomials

The steps you should follow in subtracting trinomials and multinomials are almost the same as those in adding them.

Look at the following example.

- Subtract $28x^2 + 20$ from $35x^2 - 22x + 30$.
- $35x^2 - 22x + 30$ is the minuend and $28x^2 - 15x + 20$ is the subtrahend.

STEP 1 Write the expressions in such a way that the terms and operation signs are in the proper columns. Fill in missing terms.

$$\begin{array}{rrrrrrr} & 35x^2 & - & 22x & + & 30 & \\ - & (28x^2 & - & 0x & + & 20) & \\ \hline \end{array}$$

STEP 2 Distribute the negative sign so that the operation becomes addition.

- Remember that if we distribute the negative sign in **1 – (–a + b)**, the answer would be **1 + (–(–a) + (–b))**, which is also equal to **1 + (a – b)** and the operation becomes addition.

$$\begin{array}{rrrrrr} & 35x^2 & - & 22x & + & 30 \\ \Rightarrow \quad + & -28x^2 & + & 0x & - & 20 \\ \hline \end{array}$$

STEP 3 Now, we can add the terms and apply what we learned in the earlier section.

$$\begin{array}{rrrrrr} & 35x^2 & - & 22x & + & 30 \\ + & -28x^2 & + & 0x & - & 20 \\ \hline \end{array}$$

$$(35 - 28)x^2 + (-22 + 0)x + (30 - 20)$$

$$= 7x^2 - 22x + 10$$

Let's Review

Subtract the following polynomials.

1. $(4x^2 - 10x + 3) - (5x^2 + 2x + 1)$
2. $(2x^2 + 1) - (x^2 + x - 1)$
3. $(5x^4 + 3x - 4) - (x^2 + 14)$
4. $(3y^3 + 2) - (3y^2 + 2y^3 - 5)$
5. $(5y^2 - 15) - (2y^2 - y + 10)$

Compare your answers with those in the *Answer Key* on pages 49 and 50. Did you get all the correct answers? If you did, very good! If you didn't, review the items you missed before continuing with the rest of the module.

Let's See What You Have Learned

A. Add and/or subtract the following polynomials.

1. $12x^3 + 40x^3 + 13x^3$
2. $14x^2 + 3y^3 - 8y^3 + 9x^2$
3. $56x^3 - 18x^3 - 13x^3$
4. $-67xy + 13x^2 + 3xy - 15y^3$
5. $14x^3y^2 + 3x^2y^3 - 9x^3y^2 - 7x^2y^3$
6. $(6x^2 + 5x^3 + 2x) + (x^3+1)$
7. $(5xy^2 + 2y) + (7xy^2 + 3y)$
8. $(11y^3 - 16) + (4y^3 + 5)$
9. $(12z^4 - 13z^3) - (9z^4 + 2z^3)$
10. $(x^6 - 5) + (3x^5 + 4x^2 - 5)$

Compare your answers with those in the *Answer Key* on page 50. If you got all the correct answers, very good! If you did not, review the items you missed first before proceeding to the next lesson.

Let's Remember

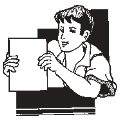
- Term having the same literal coefficients are said to be **similar.**
- To add/subtract similar polynomials, we add/subtract their numerical coefficients and retain their literal coefficients.

Performing Multiplication and Division on Polynomials

In Lesson 2, you learned how to add and subtract polynomials. In this lesson you will learn how to multiply and divide polynomials. You will also learn how to use synthetic division on polynomials.

After studying this lesson, you should be able to:

- multiply polynomials using the law of exponents and the distributive property of multiplication over addition;
- divide polynomials using the law of exponents and the algorithm for division; and
- divide multinomials using synthetic division.



Let's Learn

Before we study how to multiply polynomials, let us first review what exponents are. x^4 has 4 as its **exponent** and x as its **base.** The exponent tells us how many times the base is multiplied by itself. In the monomial x^4, x is multiplied four times by itself, so we can also write x^4 as $x \cdot x \cdot x \cdot x$. On the other hand, $(3z)(3z)(3z)$ can be written in exponential form as $(3z)^3$, while $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ can be written as 2^5.

Multiplying Monomials With One Variable

Knowing these concepts, let us try to multiply monomials with only one variable.

- Multiply x^3 by x^4.

$$\begin{aligned} x^3 \cdot x^4 &= (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) \\ &= (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \\ &= x^7 \end{aligned}$$

- Multiply y^4 by y^2.

$$\begin{aligned} y^4 \cdot y^2 \cdot y^3 &= (y \cdot y \cdot y \cdot y)(y \cdot y)(y \cdot y \cdot y) \\ &= (y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y) \\ &= y^9 \end{aligned}$$

In the examples, we can see that if we multiply terms with the same variable, the exponent of their product is equal to the sum of the exponents of their factors.

- $x^3 \cdot x^4 = x^{(3+4)} = x^7$
- $y^4 \cdot y^2 \cdot y^3 = y^{(4+2+3)} = y^9$

In multiplying monomials with numerical coefficients, we multiply the numerical coefficients first to get the numerical coefficient of the product. Then we multiply their literal coefficients to get the literal coefficient of the product.

Look at the following examples.

- Multiply $-3x^3$ by $2x^2$.

  $-3x^3 \cdot 2x^2 = (-3 \cdot 2)y^{(3+2)}$

  $= -6x^5$

- Find the product of $4y^3$, $-5y^2$ and $-2y^4$.

  $4y^3 \cdot -5y^2 \cdot -2y^4 = (4 \cdot -5 \cdot -2)y^{(3+2+4)}$

  $= 40y^9$

Let's Try This

Multiply the following monomials.

1. $x^3 \cdot x^6$
2. $y^3 \cdot y^2 \cdot y^7$
3. $6x^4 \cdot 8x^2$
4. $5y^6 \cdot -7y^3$
5. $-3y^5 \cdot 2y^6 \cdot -y^2$

Compare your answers with mine below.

1. $x^3 \cdot x^6 = x^{(3+6)}$

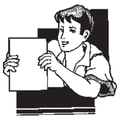
   $= x^9$
2. $y^3 \cdot y^2 \cdot y^7 = y^{(3+2+7)}$

   $= y^{12}$
3. $6x^4 \cdot 8x^2 = (6 \cdot 8)x^{(4+2)}$

   $= 48x^6$
4. $5y^6 \cdot -7y^3 = (5 \cdot -7)y^{(6+3)}$

   $= -35y^9$
5. $-3y^5 \cdot 2y^6 \cdot -y^2 = (-3 \cdot 2 \cdot -1)y^{(5+6+2)}$

   $= 6y^{13}$



Let’s Learn

Multiplying Monomials With More Than One Variable

Now, let’s try to multiply monomials with more than one variable. Like in the previous examples, all you need to do is to group terms with the same variable. Study the following examples.

- Multiply $5x^2y^3$ by $-4xy^2$.

  $5x^2y^3 \cdot -4xy^2 = (5 \cdot -4)(x^2 \cdot x)(y^3 \cdot y^2)$

  $= -20x^3y^5$

- Find the product of $-2x^2y$, $3xy$ and $-5x^3y^2$.

  $-2x^2y \cdot 3xy \cdot -5x^3y^2 = (-2 \cdot 3 \cdot -5)(x^2 \cdot x \cdot x^3)(y \cdot y \cdot y^2)$

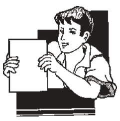
  $= 30x^6y^4$

Let’s Review

Before we continue learning how to multiply polynomials, let’s see what you have learned so far. Multiply the following.

1. $4x^2y^2 \cdot x^3y$
2. $-5x^3y^3 \cdot 4x^2z^2$
3. $3x^7y \cdot 2y^4$
4. $2xy^3 \cdot 4x^3y^2$
5. $-5x^2y^3 \cdot 3x^3y^2 \cdot -2xy$

Compare your answers with those in the *Answer Key* on page 51. If all your answers are correct, very good. You can proceed to the next section of this module. If they are not, review the items you missed then proceed to the next section.



Let’s Learn

Multiplication of Binomials

Using the following examples, let us learn how to multiply a binomial by a monomial step by step.

- Multiply x^2 by $x + 1$.

  a. Distribute x^2 to the binomial $x + 1$.

     $x^2(x + 1) = (x^2 \cdot x) + (x^2 \cdot 1)$

b. Multiply.

$(x^2 \cdot x) + (x^2 \cdot 1) = x^3 + x^2$

♦ Multiply $3xy$ by $2xy^3 - x^2y$.

a. Distribute $3xy$ to the binomial $2xy^3 - x^2y$.

$$3xy(2xy^3 - x^2y) = (3xy \cdot 2xy^3) - (3xy \cdot x^2y)$$

b. Multiply.

$(3xy \cdot 2xy^3) - (3xy \cdot x^2y) = 6x^2y^4 - 3x^3y^2$

Now, let us learn how to multiply a binomial with another binomial. The following examples will illustrate how to do so step by step.

♦ Multiply $(x + 1)$ by $(x + y^2)$.

a. The distributive property of multiplication over addition tells that $(a + 1)(b + 2)$ is equal to $a(b + 2) + 1(b + 2)$.

Using it in this example, we can rename the equation into:

$$(x + 1)(x + y^2) = x(x + y^2) + 1(x + y^2)$$

b. Then distribute x and 1.

$$x(x + y^2) + 1(x + y^2) = (x \cdot x + x \cdot y^2) + (1 \cdot x + 1 \cdot y^2)$$

c. The last step is to multiply.

$(x \cdot x + x \cdot y^2) + (1 \cdot x + 1 \cdot y^2) = x^2 + xy^2 + x + y^2$

♦ Multiply $(2x^2 - y)$ by $(x^4 + 3y^2)$.

a. Distribute $2x^2 - y$ using the distributive property of multiplication over addition.

$$(2x^2 - y)(x^4 + 3y^2) = 2x^2(x^4 + 3y^2) - y(x^4 + 3y^2)$$

b. Distribute $2x^2$ and $-y$.

$$2x^2(x^4 + 3y^2) - y(x^4 + 3y^2) = (2x^2 \cdot x^4 + 2x^2 \cdot 3y^2) + (-y \cdot x^4 + -y \cdot 3y^2)$$

c. The last step is to multiply.

$$\begin{aligned}(2x^2 \cdot x^4 + 2x^2 \cdot 3y^2) + ((-y)x^4 + (-y)3y^2) &= 2x^6 + 6x^2y^2 + (-yx^4) + (-3y^3)\\ &= 2x^6 + 6x^2y^2 - x^4y - 3y^3\end{aligned}$$

Let's Try This

Multiply the following polynomials.

1. $(x + 1)(x - 1)$
2. $(2y^3 - 3)(z + y)$
3. $(3xy - 1)(2xy + 2)$
4. $(4x^3 + 9y^2)(6x^2 - 10y)$
5. $(5xy + x^2y)(xy^3 - 2)$

Compare your answers with mine below.

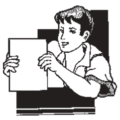
1. $$\begin{aligned}(x + 1)(x - 1) &= x(x - 1) + 1(x - 1)\\ &= (x^2 - x) + (x - 1)\\ &= x^2 - 1\end{aligned}$$

2. $$\begin{aligned}(2y^3 - 3)(z + y) &= 2y^3(z + y) - 3(z + y)\\ &= 2y^3z + 2y^4 - 3z - 3y\end{aligned}$$

3. $$\begin{aligned}(3xy - 1)(2xy + 2) &= 3xy(2xy + 2) - 1(2xy + 2)\\ &= (3xy \cdot 2xy + 3xy \cdot 2) - 2xy - 2\\ &= 6x^2y^2 + 6xy - 2xy - 2\end{aligned}$$

4. $$\begin{aligned}(4x^3 + 9y^2)(6x^2 - 10y) &= 4x^3(6x^2 - 10y) + 9y^2(6x^2 - 10y)\\ &= (4x^3 \cdot 6x^2 - 4x^3 \cdot 10y) + (9y^2 \cdot 6x^2 - 9y^2 \cdot 10y)\\ &= 24x^5 - 40x^3y + 54x^2y^2 - 90y^3\end{aligned}$$

5. $$\begin{aligned}(5xy + x^2y)(xy^3 - 2) &= 5xy(xy^3 - 2) + x^2y(xy^3 - 2)\\ &= (5xy \cdot xy^3 - 5xy \cdot 2) + (x^2y \cdot xy^3 - x^2y \cdot 2)\\ &= 5x^2y^4 - 10xy + x^3y^4 - 2x^2y\end{aligned}$$



Let's Learn

Multiplication of Trinomials

You have already learned how to multiply a monomial by another monomial, a monomial by a binomial and a binomial by another binomial. Now, let's learn how to multiply trinomials. An example, which gives a step-by-step solution, is given below. Study the problem carefully.

EXAMPLE 1 Aling Ising bought a rectangular pond with an area of $9x^2 - 2x + 3$ m^2. She found out that the pond has a height of $x + 3$ m. What polynomial represents the volume of the pond? (**Hint:** Volume of rectangular pond = Area × Height)

SOLUTION

STEP 1 List down the given. The problem is asking for the volume of the rectangular pond. To find the polynomial that represents this, you need to know the area and height of the pond first.

Area $= 9x^2 - 2x + 3 \text{ m}^2$

Height $= x + 3 \text{ m}$

STEP 2 Write the polynomial that will represent the volume.

$$
\begin{aligned}
\text{Volume of the rectangular pond} &= \text{Area} \times \text{Height} \\
&= (9x^2 - 2x + 3)(x + 3) \\
&= (x + 3)(9x^2 - 2x + 3)
\end{aligned}
$$

STEP 3 As in the example earlier, distribute $x + 3$ to the trinomial $9x^2 - 2x + 3$ using the distributive property of multiplication over addition.

$$(x + 3)(9x^2 - 2x + 3) = x(9x^2 - 2x + 3) + 3(9x^2 - 2x + 3)$$

STEP 4 Then distribute x and 3.

$$x(9x^2 - 2x + 3) + 3(9x^2 - 2x + 3) = (9x^3 - 2x^2 + 3x) + (27x^2 - 6x + 9)$$

STEP 5 Group together similar terms.

$$(9x^3 - 2x^2 + 3x) + (27x^2 - 6x + 9) = 9x^3 + (-2x^2 + 27x^2) + (3x - 6x) + 9$$

STEP 6 Add similar terms.

$$9x^3 + (-2x^2 + 27x^2) + (3x - 6x) + 9 = 9x^3 + 25x^2 - 3x + 9$$

STEP 7 Make a conclusion.

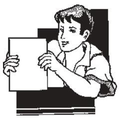
The volume of the pond can be expressed as $9x^3 + 25x^2 - 3x + 9 \text{ m}^3$.

Let's Review

Multiply the following polynomials.

1. $(x + 2)(2x^2 - 5x + 4)$
2. $(x + 1)(5y^3 - 3x + 8)$
3. $(x^2 + 3)(3xy + 2x - 1)$
4. $(x + y)(4x + xy + 2)$
5. $(x + y + 1)(x + y + 3)$

Compare your answers with those in the *Answer Key* on page 51. Did you get all the correct answers? If you did, then you may proceed to the next part of the lesson. If you did not, don't worry, just review the parts you did not understand very well and answer the exercises again.



Let's Learn

Division is the inverse operation of multiplication. If $x^9x^6 = x^{15}$, then $x^{15} + x^9 = x^6$ and $x^{15} + x^6 = x^9$.

As in multiplying polynomials, we also use the law of exponents in dividing polynomials. In dividing powers of the same base, where the exponent of the dividend is greater than the exponent of the divisor, the quotient's exponent is equal to the exponent of the dividend minus the exponent of the divisor.

$$x^a - x^b = x^{a-b} \text{ where } a > b$$

a = exponent of the dividend
b = exponent of the divisor
x = literal coefficient of the dividend and the divisor

The law of exponents for division is used this way:

- Divide x^8 by x^2.

$$\begin{aligned} x^8 \div x^2 &= \frac{x^8}{x^2} \\ &= x^{(8-2)} \\ &= x^6 \end{aligned}$$

- Divide x^{15} by x^{15}.

$$\begin{aligned} x^{15} \div x^{15} &= \frac{x^{15}}{x^{15}} \\ &= x^{(15-15)} \\ &= x^0 \\ &= 1 \end{aligned}$$

Remember that any number raised to zero is always equal to 1.

Any number divided by itself is equal to zero.

Dividing Monomials

When dividing a monomial by another monomial, divide the numerical coefficient of the dividend by that of the divisor to get the numerical coefficient of the quotient. To get the literal coefficient of the quotient, use the law of exponents for division.

To further understand how to divide polynomials, a sample problem is provided below. Study it carefully.

EXAMPLE 1 Lily bought a rectangular shaped parlor. The parlor has a land area of $24x^2$ sq. m. If one side of the parlor is expressed as $2x$ m wide, what polynomial then represents the length of the parlor?

SOLUTION

STEP 1 Determine what is being asked for in the problem.

What polynomial represents the length of the parlor?

STEP 2 List down the given.

Area = $24x^2$ m^2

Width = $2x$ m

STEP 3 Solve for the unknown.

Using the formula for solving the area of the rectangle, we will be able to get the formula for the length of the parlor.

$$
\begin{aligned}
\text{Area} &= \text{Length} \times \text{Width} \\
\text{Length} &= \text{Area} \div \text{Width} \\
\text{Length} &= 24x^2 \div 2x \\
&= (24 \div 2)x^{(2-1)} \\
&= 12x \text{ m}
\end{aligned}
$$

STEP 4 Make a conclusion.

The length of the parlor is $12x$ m.

Let's Try This

Determine the following polynomials.

1. $x^4 \div x^2$
2. $4x^2 \div 2x$
3. $x^3y^2 \div xy$
4. $10x^4y^3 \div 5x^2y$
5. $116x^5y^4 \div 116x^3y^4$

Compare your answers with mine below.

1. $x^4 \div x^2 = x^{(4+2)}$
   $= x^6$
2. $4x^2 \div 2x = (4 \div 2)x^{(2-1)}$
   $= 2x$
3. $x^3y^2 \div xy = x^{(3-1)}y^{(2-1)}$
   $= x^2y$
4. $10x^4y^3 \div 5x^2y = (10 \div 5)x^{(4-2)}y^{(3-1)}$
   $= 2x^2y^2$
5. $116x^5y^4 \div 116x^3y^4 = (116 \div 116)x^{(5-3)}y^{(4-4)}$
   $= (1)x^2y^0$
   $= x^2$

Did you get all the correct answers? Make sure that you understood the law of exponents on division very well. You will use this knowledge throughout the following sections.

Let's Learn

Dividing Binomials

To divide a binomial by a monomial, divide each term of the binomial by the monomial. The example given shows you how to do this step by step.

- Divide $24x^2 + 36x$ by $12x$.

  Doing so is the same as multiplying the first term (dividend) by the reciprocal of the second term (divisor), which is $\frac{1}{12x}$.

STEP 1 Turn division into multiplication by getting the reciprocal of the divisor.

$$(24x^2 + 36x) \div 12x = (24x^2 + 26x)\left(\frac{1}{12x}\right)$$

STEP 2 Since the operation was already turned into multiplication, we can now distribute.

$$(24x^2 + 36x)\left(\frac{1}{12x}\right) = \frac{12x^2}{1}\left(\frac{1}{12x}\right) + \left(\frac{36x}{1}\right)\frac{1}{12x}$$

STEP 3 Multiply.

$$\left(\frac{12x^2}{1}\right)\left(\frac{1}{12x}\right) + \left(\frac{36x}{1}\right)\left(\frac{1}{12x}\right) = \frac{24x^2}{12x} + \frac{36x}{12x}$$

STEP 4 Divide.

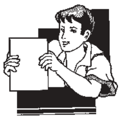
$$\begin{aligned}\frac{24x^2}{12x} + \frac{36x}{12x} &= (24 + 12)x^{(2-1)} + (36 + 12)x^{(1-1)} \\ &= 2x + 3x^0 \\ &= 2x + 3\end{aligned}$$

Let's Review

Divide the following polynomials.

1. $(6x^3 - 2x) - 2x$
2. $(14y^5 + 21y^3) - 7y^2$
3. $(9z^7 - 81z^7) - 9z^7$
4. $(3x^2y^5 - 9x^4y^3) - 3xy$
5. $(16x^4y^3 - 8x^2y^8) - 4x^2y^2$

Compare your answers with those in the *Answer Key* on page 52.



Let's Learn

Dividing Trinomials and Multinomials

In dividing trinomials and multinomials, we follow the same procedure as in dividing monomials and binomials. Only this time, the solution is very long. To make the process easier, we use the algorithm or shortcut method for dividing whole numbers by two or three digits.

Examples to explain the algorithm that we will use for dividing trinomials and multinomials are given below.

EXAMPLE 1 Boyet bought a cylindrical water container. It has a volume of $x^2 + 6x + 8$ m^2. If the height of this container is $x + 2$ m, what is its area? (**Hint:** Volume of the cylinder = Area of the circular base × Height)

SOLUTION

STEP 1 Determine what is being asked for in the problem.

What is the area of its circular base?

STEP 2 Write the equation that will be used to compute for the area.

$$\text{Volume} = \text{Area} \times \text{Height}$$

$$\text{Area} = \frac{\text{Volume}}{\text{Height}}$$

$$\text{Area} = \frac{x^2 + 6x + 8}{x + 2}$$

$$x + 2 \overline{\smash{\big)}\, x^2 + 6x + 8}$$

STEP 3 Solve for the area of the circular base.

a. Divide the first term of the dividend by the first term of the divisor.

$$x^2 - x = x$$

$$\begin{array}{r} x \\ x + 2 \overline{\smash{\big)}\, x^2 + 6x + 8} \end{array}$$

b. Multiply the divisor by the quotient and subtract their product from the dividend.

$$x(x + 2) = x^2 + 2x$$

$$\begin{array}{r} x \\ x + 2 \overline{\smash{\big)}\, x^2 + 6x + 8} \\ \underline{-\left(x^2 + 2x\right)} \end{array}$$

c. Subtract.

$$\begin{array}{r} x \\ x+2\overline{)x^2+6x+8} \\ -\left(x^2+2x\right) \\ \hline 4x+8 \end{array}$$

d. $4x + 8$, called a **partial dividend** should then be divided by $x + 2$. Repeat steps one to three until the last partial dividend is divided by the divisor.

$$\begin{array}{r} x+4 \\ x+2\overline{)x^2+6x+8} \\ -\left(x^2+2x\right) \\ \hline 4x+8 \\ -\left(4x+8\right) \\ \hline 0 \end{array}$$

STEP 4 Make a conclusion.

The area of the circular bases is $x + 4$ sq. m.

EXAMPLE 2 Mang Andres bought a rectangular shaped land with a land area of $x^2 - 7x + 30$ m^2. If the length of the land is expressed as $x + 4$ m, find the width of the land. (**Hint:** Area of rectangle = Length × Width)

SOLUTION

STEP 1 Determine what is being asked for in the problem.

Find the width of the land.

STEP 2 Write the equation that will be used to get the width of the land.

$$\frac{x^2-7x+30}{x+4}$$

STEP 3 Solve for the width of the land.

a. Divide the first term of the dividend by the first term of the divisor.

$x^2 - x = x$

$$\begin{array}{r} x \\ x+4\overline{)x^2-7x+30} \end{array}$$

b. Multiply the divisor by the quotient and subtract their product from the dividend.

$x(x + 4) = x^2 + 4x$

$$\begin{array}{r} x \\ x+4\overline{)x^2-7x+30} \\ -\left(x^2+4x\right) \\ \hline \end{array}$$

c. Subtract.

$$
\begin{array}{r}
x \\
x+4\overline{)\,x^2-7x+30} \\
-(x^2+4x) \qquad \\
\hline
-3x+30
\end{array}
$$

d. $-3x+30$, a partial dividend, should then be divided by $x+4$. Repeat steps one to three until the last partial dividend is divided by the divisor.

$$
\begin{array}{r}
x-3 \\
x+4\overline{)\,x^2-7x+30} \\
-(x^2+4x) \qquad \\
\hline
-3x+30 \\
-(-3x-12) \\
\hline
42
\end{array}
$$

- 42 is the remainder of the operation. If we want to include it in the quotient, we need to specify that it still has to be divided by $x+4$. So we write:

$$x-3+\frac{42}{x+4}$$

STEP 4 Make a conclusion.

The width of the land is $x-3+\dfrac{42}{x+4}$ m.

Let's Try This

Divide the following trinomials using the algorithm for division.

1. $(x^2-x-20) \div (x+4)$
2. $(x^2+4x-22) \div (x-3)$
3. $(x^2+11x+30) \div (x+6)$
4. $(3x^2+2x+1) \div (x-5)$
5. $(-4x^2+9x-2) \div (-x+2)$

Compare your answers with mine below.

1. $(x^2-x-20) \div (x+4)$

$$
\begin{array}{r}
x-5 \\
x+4\overline{)\,x^2-x-20} \\
-(x^2+4x) \qquad \\
\hline
-5x-20 \\
-(-5x-20) \\
\hline
0
\end{array}
$$

$= x-5$

2. $\left(x^2+4x-22\right)\div(x-3)$

$$\begin{array}{r} x+7 \\ x-3\overline{)\,x^2+4x-22} \\ -(x^2-3x) \\ \hline 7x-22 \\ -(7x-21) \\ \hline 1 \end{array}$$

$$=(x+7)+\left(\frac{1}{x-3}\right)$$

3. $(x^2 + 11x + 30) \div (x + 6)$

$$\begin{array}{r} x+5 \\ x+6\overline{)\,x^2+11x+30} \\ -(x^2+6x) \\ \hline 5x+30 \\ -(5x+30) \\ \hline 0 \end{array}$$

$$= x + 5$$

4. $\left(3x^2+2x+1\right)\div(x-5)$

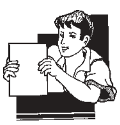
$$\begin{array}{r} 3x+17 \\ x-5\overline{)\,3x^2+2x+1} \\ -(3x^2-15x) \\ \hline 17x+1 \\ -(17x-85) \\ \hline 86 \end{array}$$

$$=(3x+17)+\left(\frac{86}{x-5}\right)$$

5. $(-4x^2 + 9x - 2) \div (-x + 2)$

$$\begin{array}{r} 4x-1 \\ -x+2\overline{)\,-4x^2+9x-2} \\ -(-4x^2+8x) \\ \hline x-2 \\ x-2 \\ \hline 0 \end{array}$$

$$= 4x - 1$$



Let's Learn

Synthetic Division of Polynomials

Synthetic division is another method used to divide polynomials. It is the best method that we can use to divide multinomials. We can divide multinomials by using the algorithm that we just learned but the solution we'll come up with would be very long. On the other hand, in synthetic division, only the numerical coefficients of the divisor and the dividend are written. It is an easier way of dividing multinomials.

The literal coefficient of the remainder should follow the last term of the constant dividend. The remainder would not have a literal coefficient then.

Synthetic division can be used to divide a higher degree polynomials written in descending powers of x by a binomial form $x - r$. For example, $x^4 + x^3 - x^2 + x + 5$ can be divided by $x - 1$ using synthetic division.

If the divisor is of the form $mx - r$, then the divisor will become $x - \frac{r}{m}$. For example, if the divisor is $3x^2 - 4$, you can also write it as $x^2 - \frac{4}{3}$.

To further understand synthetic division, an example with its step-by-step solution is provided. Study it carefully.

EXAMPLE 1 Divide $3x^4 + 13x^2 + 30x - 40$ by $x + 2$.

SOLUTION

STEP 1 Write the dividend in standard form. Fill in the missing terms.

$$3x^4 + 0x^3 + 13x^2 + 30x - 40$$

STEP 2 Form line 1. Line 1 should include all the numerical coefficients of the dividend.

$$\begin{array}{rrrrrl} 3 & 0 & 13 & 30 & -40 & \text{(line 1)} \\ & & & & & \text{(line 2)} \\ \hline & & & & & \text{(line 3)} \end{array}$$

STEP 3 Write the divisor $x + 2$ in the form $x - r$.

$$x + 2 = x - (-2)$$

STEP 4 The number r is the synthetic divisor.

$$\begin{array}{rrrrr|ll} 3 & 0 & 13 & 30 & -40 & -2 & \text{(line 1)} \\ & & & & & & \text{(line 2)} \\ \hline & & & & & & \text{(line 3)} \end{array}$$

STEP 5 Bring the first coefficient down line 3.

$$\begin{array}{rrrrr|ll} 3 & 0 & 13 & 30 & -40 & -2 & \text{(line 1)} \\ & & & & & & \text{(line 2)} \\ \hline 3 & & & & & & \text{(line 3)} \end{array}$$

STEP 6 Multiply the first coefficient on line 3 by the synthetic divisor and write the product under the second coefficient of line 1.

$$\begin{array}{rrrrr|ll} 3 & 0 & 13 & 30 & -40 & -2 & \text{(line 1)} \\ & -6 & & & & & \text{(line 2)} \\ \hline 3 & & & & & & \text{(line 3)} \end{array}$$

STEP 7 Add the coefficient and the product.

$$\begin{array}{rrrrrr|l} & 3 & 0 & 13 & 30 & -40 & \underline{-2} \quad \text{(line 1)} \\ + & & -6 & & & & \quad\;\; \text{(line 2)} \\ \hline & 3 & -6 & & & & \quad\;\; \text{(line 3)} \end{array}$$

STEP 8 Multiply the sum by the synthetic divisor. Put the product under the third coefficient of line 1.

$$\begin{array}{rrrrrr|l} & 3 & 0 & 13 & 30 & -40 & \underline{-2} \quad \text{(line 1)} \\ + & & -6 & 12 & & & \quad\;\; \text{(line 2)} \\ \hline & 3 & -6 & & & & \quad\;\; \text{(line 3)} \end{array}$$

STEP 9 Add the coefficient and the product.

$$\begin{array}{rrrrrr|l} & 3 & 0 & 13 & 30 & -40 & \underline{-2} \quad \text{(line 1)} \\ + & & -6 & 12 & & & \quad\;\; \text{(line 2)} \\ \hline & 3 & -6 & 25 & & & \quad\;\; \text{(line 3)} \end{array}$$

STEP 10 Repeat the procedure until the last coefficient has been added to a product.

$$\begin{array}{rrrrrr|l} & 3 & 0 & 13 & 30 & -40 & \underline{-2} \quad \text{(line 1)} \\ + & & -6 & 12 & -50 & 40 & \quad\;\; \text{(line 2)} \\ \hline & 3 & -6 & 25 & -20 & 0 & \quad\;\; \text{(line 3)} \end{array}$$

The quotient is $3x^3 - 6x^2 + 25x - 20$.

EXAMPLE 2 Divide $6x^4 - 13x^3 + 8x^2 - x - 12$ by $2x - 3$.

SOLUTION

STEP 1 Write the polynomial in standard form. Fill in the missing terms.

$6x^4 - 13x^3 + 8x^2 - x - 12$

STEP 2 Form line 1.

$$\begin{array}{rrrrrl} 6 & -13 & 8 & -1 & -12 & \text{(line 1)} \\ & & & & & \text{(line 2)} \\ \hline & & & & & \text{(line 3)} \end{array}$$

STEP 3 The divisor is in the form $mx - r$ so we have to change it to the form

$x - \dfrac{r}{m}$.

$$2x - 3 = \frac{x - 3}{2}$$

STEP 4 The number r/m is the synthetic divisor.

$$\begin{array}{rrrrr|l l}
6 & -13 & 8 & -1 & -12 & \frac{3}{2} & \text{(line 1)} \\
 & & & & & & \text{(line 2)} \\
\hline
 & & & & & & \text{(line 3)}
\end{array}$$

STEP 5 Bring the first coefficient down line 3.

$$\begin{array}{rrrrr|l l}
6 & -13 & 8 & -1 & -12 & \frac{3}{2} & \text{(line 1)} \\
 & & & & & & \text{(line 2)} \\
\hline
6 & & & & & & \text{(line 3)}
\end{array}$$

STEP 6 Multiply the first coefficient by the synthetic divisor and put the product on line 2, under the second coefficient of line 2.

$$\begin{array}{rrrrr|l l}
6 & -13 & 8 & -1 & -12 & \frac{3}{2} & \text{(line 1)} \\
 & 9 & & & & & \text{(line 2)} \\
\hline
6 & & & & & & \text{(line 3)}
\end{array}$$

STEP 7 Add the coefficient and the product.

$$\begin{array}{rrrrrr|l l}
 & 6 & -13 & 8 & -1 & -12 & \frac{3}{2} & \text{(line 1)} \\
+ & & 9 & & & & & \text{(line 2)} \\
\hline
 & 6 & -4 & & & & & \text{(line 3)}
\end{array}$$

STEP 8 Multiply the sum by the synthetic divisor and put the product under the third coefficient. Add the third coefficient and the product.

$$\begin{array}{rrrrrr|l l}
 & 6 & -13 & 8 & -1 & -12 & \frac{3}{2} & \text{(line 1)} \\
+ & & 9 & -6 & & & & \text{(line 2)} \\
\hline
 & 6 & -4 & 2 & & & & \text{(line 3)}
\end{array}$$

STEP 9 Repeat the procedure until the last coefficient has been added to a product.

$$\begin{array}{rrrrrr|l l}
 & 6 & -13 & 8 & -1 & -12 & \frac{3}{2} & \text{(line 1)} \\
+ & & 9 & -6 & 3 & 3 & & \text{(line 2)} \\
\hline
 & 6 & -4 & 2 & 2 & -9 & & \text{(line 3)}
\end{array}$$

If the last coefficient in line 3 is not zero, then the operation has a remainder.

If we want to add the remainder –9 to the quotient, we have to specify that –9 should still be divided by $2x - 3$. So we write it as $-\dfrac{9}{2x-3}$.

In the example, the quotient is $6x^3 - 4x^2 + 2x + 2 - \dfrac{9}{2x-3}$.

Let's Try This

Divide the following multinomials using synthetic division.

1. $(x^3 + 2x^2 - 5x - 6) \div (x - 1)$
2. $(x^3 + 2x^2 - 13x + 10) \div (x - 2)$
3. $(y^3 - 12y - 16) \div (y + 2)$
4. $(z^4 + 9z^3 + 21z^2 + 6z + 5) \div (z + 5)$
5. $(2x^3 + 5x^2 - 4x - 3) \div (x + ½)$

Compare your answers with mine below.

1. $(x^3 + 2x^2 - 5x - 6) \div (x - 1)$

$$\begin{array}{rrrr|l} 1 & 2 & -5 & -6 & 1 \\ & 1 & 3 & -2 & \\ \hline 1 & 3 & -2 & -8 & \end{array}$$

$$= x^2 + 3x - 2 - \left(\frac{8}{x-1}\right)$$

2. $(x^3 + 2x^2 - 13x + 10) \div (x - 2)$

$$\begin{array}{rrrr|l} 1 & 2 & -13 & 10 & 2 \\ & 2 & 8 & -10 & \\ \hline 1 & 4 & -5 & 0 & \end{array}$$

$$= x^2 + 4x - 5$$

3. $(y^3 - 12y - 16) \div (y - 2)$

$$\begin{array}{rrrr|l} 1 & 0 & -12 & -16 & -2 \\ & -2 & 4 & 16 & \\ \hline 1 & -2 & -8 & 0 & \end{array}$$

$$= y^2 - 2y - 8$$

4. $(z^4 + 9z^3 + 21z^2 + 6z + 5) \div (z + 5)$

$$\begin{array}{rrrrr|l} 1 & 9 & 21 & 6 & 5 & -5 \\ & -5 & -20 & -5 & -5 & \\ \hline 1 & 4 & 1 & 1 & 0 & \end{array}$$

$$= z^3 + 4z^2 + z + 1$$

5. $(2x3 + 5x2 - 4x - 3) \div (x + ½)$

$$\begin{array}{rrrr|l} 2 & 5 & -4 & -3 & -\frac{1}{2} \\ & -1 & -2 & 3 & \\ \hline 2 & 4 & -6 & 0 & \end{array}$$

$$= 2x^2 + 4x - 6$$

Let's Review

A. Add/Subtract the following polynomials.

1. $3x^2 + 9x^2 + 7x^2$
2. $9x^2 + 3y^2 - 15x^3 - 2y^2$
3. $14y^3 + 16y^2 + 13y^3 - 20y^2$
4. $50xy - 12y^3 - 25xy - 13y^3$
5. $9x^2y^3 + 2x^3y^2 + 3xy - 14x^2y^3$

B. Multiply the following polynomials.

1. $(5x)(7x)$
2. $(9x^2)(-4x^3y^3)$
3. $(4x^2)(9x - 3)$
4. $(-5y)(x^2 + 2x - 1)$
5. $(x + 1)(x + 2x + 1)$

C. Divide the following polynomials.

1. $\dfrac{35x^2}{7x}$

2. $\dfrac{25y^3 - 40y^2}{5y^2}$

3. $\dfrac{4x^2 + 6x + 2}{x + 1}$

D. Divide the following using synthetic division.

$$\frac{2x^4 + 4x^3 - 7x^2 + 9}{x + 3}$$

E. Simplify the following polynomials.

1. $\dfrac{(x^2y)(x^3y^2) + (xy)(x^4y^2)}{2xy}$

2. $\dfrac{20x^3y^3 - (5x^2y)(xy^2)}{x^2y^2 + 2x^2y^2}$

Compare your answers with those in the *Answer Key* on pages 53 and 54.

Let's See What You Have Learned

A. Multiply the following polynomials.

1. $(13x)(2x)$
2. $(3x^2y)(-5x^3y^4)$
3. $(3x)(2x + 3)$
4. $(2x)(9x^2 + 3x + 2)$
5. $(3x^2 + 2x)(3x^2 + 2x + 1)$

B. Divide the following polynomials.

1. $\dfrac{81y^3}{9y^2}$

2. $\dfrac{36x^5 + 54x^3}{6x^3}$

3. $\dfrac{3x^2 - x^2 - 10}{x - 2}$

C. Solve the following using synthetic division.

$$\frac{-x^4 + 26x^2 + 60x + 80}{x + 4}$$

D. Simplify the following expressions.

1. $\dfrac{20x^2}{5x} + \dfrac{35y^4}{7y^2}$

2. $\dfrac{(4x^2)(5x^2)(5x^2)}{20x^3}$

Compare your answers with those in the *Answer Key* on pages 54 and 55.

Let's Remember

- To multiply/divide polynomials, we multiply/divide the numerical coefficients and combine the literal coefficients by using the laws of exponents.
- In multiplying binomials, trinomials and multinomials, we apply the distributive property of multiplication over addition.
- In dividing trinomials and multinomials, we can use the algorithm for division.
- In dividing multinomials, we can also use synthetic division.

Well, this is the end of the module! Congratulations for finishing it. Did you like it? Did you learn anything useful from it? A summary of its main points is given below to help you remember them better.

Let's Sum Up

This module tells us that:

- An algebraic expression is an expression made up of numbers, variables and grouping and operation symbols.
- Terms are parts of an algebraic expression separated by plus or minus signs and made up of numerical and literal coefficients.
- A polynomial is an algebraic expression that involves only addition and multiplication of numbers and variables.
- A monomial is a polynomial with only one term.
- A binomial is a polynomial with two terms.
- A trinomial is a polynomial with three terms.
- A multinomial is a polynomial with four or more terms.
- A constant polynomial is a polynomial that does not have a variable.
- The degree of a polynomial is determined by the highest exponent of any of the variables in an algebraic expression.
- To evaluate a polynomial, we should replace the variable/s with the given numerical value/s.
- To add/subtract similar terms in a polynomial, we add/subtract the numerical coefficients and retain the literal coefficients.
- To multiply/divide polynomials, we multiply/divide the numerical coefficients and combine the literal coefficients by using the law of exponents.
- In multiplying polynomials, we apply the distributive property of multiplication over addition.
- In dividing trinomials and multinomials, we can use the algorithm for division and synthetic division.

What Have You Learned?

A. Classify the following polynomials according to degree and number of terms. The first one has been done as an example for you.

1. monomial of the fifth degree y^5
2. ______________________ $x^6 + x^4 + x^3 + 1$
3. ______________________ $y^3 + 7y^5 + 9$
4. ______________________ $9z^8 + 10z^4$
5. ______________________ 1200

B. Solve the following problem.

Sandy bought a science book and a math book. The science book costs $6x^2 + 5y$ pesos while the math book costs $10y^2 + 5x$ pesos. If $x = 3$ pesos and $y = 2$ pesos, how much did she pay for the books in all?

C. Add/Subtract the following polynomials.

1. $(x^4 + 2x^3 – x + 2) – (x^2 + 3x^4 – 2)$
2. $(5y^6 + 3y^7 + 6y) + (4y^6 + 9y)$
3. $(5x^3 – 2x^2 + x – 1) – (4x^2 + 5x – 4)$

D. Multiply the following expressions.

1. $(x + 2)(x – 3)$
2. $(3x + 2)(2x + 1)$
3. $(x + 1)(3x^2 – x + 1)$

E. Divide the following polynomials using the algorithm for division.

1. $15y^5 \div 15y^5$
2. $(12x^2 – 6x) \div 3x$
3. $(3x^2 – 5x – 2) \div (x – 2)$

F. Solve the following using synthetic division.

$(3x^4 – 7x^3 – 2x^2 – 11x – 3) \div (x – 3)$

Compare your answers with those in the *Answer Key* on page 56.

Answer Key

A. Let's See What You Already Know *(pages 2–3)*

A. 1. An algebraic expression involving only addition and multiplication of numbers and variables

2. A polynomial with only one term

3. A polynomial with two terms

4. A polynomial with three terms

5. A polynomial with four or more terms

B. 1. fifth degree

2. sixth degree

3. third degree

C. Area of poultry farm = $8x^2 + 97x + 12\ \text{m}^2$

If $x = 2$:

$$8(2)^2 + 97(2) + 12 = 32 + 194 + 12$$
$$= 238\ \text{m}^2$$

The land area of the poultry farm is 238 m^2 if x equals 2 m.

D. 1. $(2x^2 + 3x) - (5x^2 + x) = -3x^2 + 2x$

2. $(4x^2 + 5x^3 - x) + (4x^3 + 2x^2 - x) = 9x^3 + 6x^2 - 2x$

3. $(x^4 + 3x^2 + x - 5) - (3x^4 + 2x^3 - x^2 + 3) = -2x^4 - 2x^3 + 4x^2 + x - 8$

E. 1. $(x + 1)(x - 1) = x^2 - x + x - 1$
$= x^2 - 1$

2. $(x + 2)(2x + 1) = 2x^2 + x + 4x + 2$
$= 2x^2 + 5x + 2$

3. $(x + 1)(x + 1) = x^2 + x + x + 1$
$= x^2 + 2x + 1$

F. 1. $17x^5 \div x^2 = 17x^{(5-2)}$
$= 17x^3$

2. $(16y^5 - 8y^4) \div 4y^3 = (16 \div 4)y^{(5-3)} - (8 \div 4)y^{(4-3)}$
$= 4y^2 - 2y$

3. $(4x^2 - 7x - 2) \div (x - 2) = 4x + 1$

$$\begin{array}{r|l} & 4x+1 \\ \hline x-2 & 4x^2-7x-2 \\ & -(4x^2-8x) \\ \hline & x-2 \\ & -(x-2) \\ \hline & 0 \end{array}$$

G. $(x^4 - 134x^2 - 100x + 240) \div (x + 2)$

$$\begin{array}{rrrrr|l} 1 & 0 & -134 & -100 & 240 & -2 \\ & -2 & 4 & 260 & -320 & \\ \hline 1 & -2 & -130 & 160 & 80 & \end{array}$$

$$= x^3 - 2x^2 - 130x + 160 + \frac{80}{x+2}$$

B. Lesson 1

Let's Review (pages 8–9)

A. 1. 11
xy^2

2. 1
$a^2b^2c^2$

3. 36
z^2

B. 1. $\boxed{4}$

2. $\boxed{8}$

3. $\boxed{4}$

C. 1. monomial

2. monomial

3. trinomial

4. multinomial

5. binomial

D. 1. third degree

2. zero degree

3. seventh degree

Let's See What You Have Learned (page 12)

A. 1. $\underline{3}\underline{\underline{x}}^2 + \underline{9}\underline{\underline{y}}^3$

2. $\underline{12}\underline{\underline{x}}^2 + \underline{5}\underline{\underline{x}} + \underline{1}$

3. $\underline{\underline{y}}^3 - \underline{3}$

B. 1. $\boxed{8}$

2. $\boxed{4}$

3. $\boxed{8}$

C. 1. monomial of the third degree
2. multinomial of the fifth degree
3. constant polynomial of zero degree
4. binomial of the eighth degree
5. trinomial of the ninth degree

D. $4x^3 + 9x^2 - 5x - 10$

Substitute 2 for *x:*

$4(2)^3 + 9(2)^2 - 5(2) - 10$
$= 4(8) + 9(4) - 10 - 10$
$= 32 + 36 - 10 - 10$
$= 48$
Mang Isko harvested 48 bushels of corn.

C. Lesson 2

Let's Review (page 20)

A. 1. $(5x^3 + 6x^2 + 2x + 0) + (0x^3 + x^2 + 0x - 1)$

$$\begin{array}{rl} & 5x^3 + 6x^2 + 2x + 0 \\ + & 0x^3 + x^2 + 0x - 1 \\ \hline \end{array}$$

$(5 + 0)x^3 + (6 + 1)x^2 + (2 + 0)x + (0 - 1)$

$= 5x^3 + 7x^2 + 2x - 1$

2. $(x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 3) + (3x^5 - 6x^4 + 3x^3 - 2x^2 + x - 13)$

$$\begin{array}{rl} & x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 3 \\ + & 3x^5 - 6x^4 + 3x^3 - 2x^2 + x - 13 \\ \hline \end{array}$$

$(1 + 3)x^5 + (0 - 6)x^4 + (0 + 3)x^3 + (0 - 2)x^2 + (0 + 1)x + (3 - 13)$

$= 4x^5 - 6x^4 + 3x^3 - 2x^2 + x - 10$

3. $(0y^4 + 11y^3 + 0y^2 + 0y - 19) + (4y^4 + 0y^3 + 0y^2 + 0y + 1)$

$$\begin{array}{rl} & 0y^4 + 11y^3 + 0y^2 + 0y - 19 \\ + & 4y^4 + 0y^3 + 0y^2 + 0y + 1 \\ \hline \end{array}$$

$(0 + 4)y^4 + (11 + 0)y^3 + (0 + 0)y^2 + (0 + 0)y + (-19 + 1)$

$= 4y^4 + 11y^3 - 19$

4. $(9y^4 + 12y^3 - 10y^2 + 36y - 7) + (12y^4 + 9y^3 + 8y^2 - y + 1)$

$$\begin{array}{rl} & 9y^4 + 12y^3 - 10y^2 + 36y - 7 \\ + & 12y^4 + 9y^3 + 8y^2 - y + 1 \\ \hline \end{array}$$

$(9 + 12)y^4 + (12 + 9)y^3 + (-10 + 8)y^2 + (36 - 1)y + (-7 + 1)$

$= 21y^4 + 21y^3 - 2y^2 + 35y - 6$

5. $(-13z^6 + 13z^5 + 0z^4 + 0z^3 + 0z^2 + 0z + 0) + (0z^6 + 0z^5 + 9z^4 + 0z^3 + z^2 - z + 1)$

$$
\begin{array}{rrrrrrrr}
 & -13z^6 & +\ 13z^5 & +\ 0z^4 & +\ 0z^3 & +\ 0z^2 & +\ 0z & +\ 0 \\
+ & 0z^6 & +\ 0z^5 & +\ 9z^4 & +\ 0z^3 & +\ z^2 & -\ z & +\ 1 \\
\hline
\end{array}
$$

$(-13 + 0)z^6 + (13 + 0)z^5 + (0 + 9)z^4 + (0 + 0)z^3 + (0 + 1)z^2 + (0 - 1)z + (0 + 1)$

$= -13z^6 + 13z^5 + 9z^4 + z^2 - z + 1$

B. **STEP 1** How many pieces of art paper does Atoy have in all?

STEP 2 a. number of red art paper = $9x^2 + 16x^4 - 5x^3 + 10$

b. number of green art paper = $2x^4 - 10x^2 - 17 + 13x$

STEP 3 $(16x^4 - 5x^3 + 9x^2 + 0x + 10) + (2x^4 + 0x^3 - 10x^2 + 13x - 17)$

$$
\begin{array}{rrrrrr}
 & 16x^4 & -\ 5x^3 & +\ 9x^2 & +\ 0x & +\ 10 \\
+ & 2x^4 & +\ 0x^3 & -\ 10x^2 & +\ 13x & -\ 17 \\
\hline
\end{array}
$$

$(16 + 2)x^4 + (-5 + 0)x^3 + (9 - 10)x^2 + (0 + 13)x + (10 - 17)$

$= 18x^4 - 5x^3 - x^2 + 13x - 7$

Atoy has $18x^4 - 5x^3 - x^2 + 13x - 7$ pieces of art paper in all.

Let's Review (page 23)

1. $(4x^2 - 10x + 3) - (5x^2 + 2x + 1) = (4x^2 - 10x + 3) + (-5x^2 - 2x - 1)$

$$
\begin{array}{rrrr}
 & 4x^2 & -\ 10x & +\ 3 \\
+ & -\ 5x^2 & -\ 2x & -\ 1 \\
\hline
\end{array}
$$

$(4 - 2)x^2 + (-10 - 2)x + (3 - 1)$

$= 2x^2 - 12x + 2$

2. $(2x^2 + 0x + 1) - (x^2 + x - 1) = (2x^2 + 0x + 1) + (-x^2 - x + 1)$

$$
\begin{array}{rrrr}
 & 2x^2 & +\ 0x & +\ 1 \\
+ & -\ x^2 & -\ x & +\ 1 \\
\end{array}
$$

$(2 - 1)x^2 + (0 - 1)x + (1 + 1)$

$= x^2 - x + 2$

3. $(5x^4 + 0x^3 + 0x^2 + 3x - 4) - (0x^4 + 0x^3 + x^2 + 0x + 14)$

$= (5x^4 + 0x^3 + 0x^2 + 3x - 4) + (-0x^4 - 0x^3 - x^2 - 0x - 14)$

$$
\begin{array}{rrrrrr}
 & 5x^4 & +\ 0x^3 & +\ 0x^2 & +\ 3x & -\ 4 \\
+ & -0x^4 & -\ 0x^3 & -\ x^2 & -\ 0x & -\ 14 \\
\hline
\end{array}
$$

$(5 - 0)x^4 + (0 - 0)x^3 + (0 - 1)x^2 + (3 - 0)x + (-4 - 14)$

$= 5x^4 - x^2 + 3x - 18$

4. $(3y^3 + 0y^2 + 0y + 2) – (2y^3 + 3y^2 + 0y – 5)$

$= (3y^3 + 0y^2 + 0y + 2) + (–2y^3 – 3y^2 – 0y + 5)$

$$\begin{array}{rl} & 3y^3 + 0y^2 + 0y + 2 \\ + & –2y^3 – 3y^2 – 0y + 5 \\ \hline \end{array}$$

$(3 – 2)y^3 + (0 – 3)y^2 + (0)\, y + (2 + 5)$

$= y3 – 3y^2 + 7$

5. $(5y^2 + 0y – 15) – (2y^2 – y +10) = (5y^2 + 0y – 15) + (–2y^2 + y – 10)$

$$\begin{array}{rl} & 5y^2 + 0y – 15 \\ + & – 2y^2 + y – 10 \\ \hline \end{array}$$

$(5 – 2)y^2 + (0 + 1)y + (–15 – 10)$

$= 3y^2 + y – 25$

Let's See What You Have Learned (page 24)

A. 1. $12x^3 + 40x^3 + 13x^3 = (12 + 40 + 13)x^3$
$= 65x^3$

2. $14x^2 + 3y^3 – 8y^3 + 9x^2 = (14 + 9)x^2 + (3 – 8)y^3$
$= 23x^2 – 5y^3$

3. $56x^3 – 18x^3 – 13x^3 = (56 – 18 – 13)x^3$
$= 25x^3$

4. $–67xy + 13x^2 + 3xy – 15y^3 = (–67 + 3)xy + 13x^2 – 15y^3$
$= –64xy + 13x^2 – 15y^3$

5. $14x^3y^2 + 3x^2y^3 – 9x^3y^2 –7x^2y^3 = (14 – 9)x^3y^2 + (3 – 7)x^2y^3$
$= 5x^3y^2 – 4x^2y^3$

6. $(6x^2 + 5x^3 + 2x) + (x^3 + 1) = 6x^2 + (5 + 3)x^3 + 2x + 1$
$= 6x^2 + 8x^3 + 2x + 1$

7. $(5xy^2 + 2y) + (7xy^2 + 3y) = (5 + 7)xy^2 + (2 + 3)y$
$= 12xy^2 + 5y$

8. $(11y^3 – 16) + (4y^3 + 5) = (11 + 4)y^3 + (–16 + 5)$
$= 15y^3 – 11$

9. $(12z^4 – 13z^3) – (9z^4 + 2z^3) = (12 – 9)z^4 – (–13 + 2)z^3$
$= 3z^4 – (–11z^3)$
$= 3z^4 + 11z^3$

10. $(x^6 – 5) + (3x^5 + 4x^2 – 5) = x^6 + 3x^5 + 4x^2 – 10$

D. Lesson 3

Let's Review (page 27)

1. $4x^2y^2 \cdot x^3y = 4x^{(2+3)}y^{(2+1)}$
   $= 4x^5y^3$

2. $-5x^3y^3 \cdot 4x^2z^2 = (-5 \cdot 4)x^{(3+2)}y^3z^2$
   $= -20x^5y^3z^2$

3. $3x^7y \cdot 2y^4 = (3 \cdot 2)x^7y^{(1+4)}$
   $= 6x^7y^5$

4. $2xy^3 \cdot 4x^3y^2 = (2 \cdot 4)x^{(1+3)}y^{(3+2)}$
   $= 8x^4y^5$

5. $-5x^2y^3 \cdot 3x^3y^2 \cdot -2xy = (-5 \cdot 3 \cdot -2)x^{(2+3+1)}y^{(3+2+1)}$
   $= 30x^6y^6$

Let's Review (page 30)

1. $(x+2)(2x^2-5x+4) = x(2x^2-5x+4) + 2(2x^2-5x+4)$
   $= (2x^3-5x^2+4x) + (4x^2-10x+8)$
   $= 2x^3 + (-5+4)x^2 + (4-10)x + 8$
   $= 2x^3 - x^2 - 10x + 8$

2. $(x-1)(5y^3-3x+8) = x(5y^3-3x+8) - 1(5y^3-3x+8)$
   $= (5xy^3-3x^2+8x) + (-5y^3+3x-8)$
   $= 5xy^3 - 5y^3 - 3x^2 + (8+3)x - 8$
   $= 5xy^3 - 5y^3 - 3x^2 + 11x - 8$

3. $(x^2+3)(3xy+2x-1) = x^2(3xy+2x-1) + 3(3xy+2x-1)$
   $= (3x^3y+2x^3-x^2) + (9xy+6x-3)$
   $= 3x^3y + 2x^3 - x^2 + 9xy + 6x - 3$

4. $(x-y)(4x+xy+2) = x(4x+xy+2) - y(4x+xy+2)$
   $= (4x^2+x^2y+2x) + (-4xy-xy^2-2y)$
   $= 4x^2 + x^2y + 2x - 4xy - xy^2 - 2y$

5. $(x+y+3)(x+y+3) = x(x+y+3) + y(x+y+3) + 1(x+y+3)$
   $= (x^2+xy+3x) + (xy+y^2+3y) + (x+y+3)$
   $= x^2 + (1+1)xy + (3+1)x + y^2 + (3+1)y + 3$
   $= x^2 + 2xy + 4x + y^2 + 4y + 3$

Let's Review (page 33)

1. $$\left(6x^3-2x\right)\div 2x=\left(6x^3-2x\right)\left(\frac{1}{2x}\right)$$
$$=\left(\frac{6x^3}{2x}\right)-\left(\frac{2x}{2x}\right)$$
$$=(6-2)x^{(3-1)}-(2-2)x^{(1-1)}$$
$$=3x^2-(1)x^0$$
$$=3x^2-1$$

2. $$\left(14y^5-21y^3\right)\div 7y^2=\left(14y^5+21y^3\right)\left(\frac{1}{7y^2}\right)$$
$$=\left(\frac{14y^5}{7y^2}\right)+\left(\frac{21y^3}{7y^2}\right)$$
$$=(14+7)y^{(5-2)}+(21+7)y^{(3-1)}$$
$$=2y^3+3y^2$$

3. $$\left(9z^7-81z^7\right)\div 9z^7=\left(9z^7-\ 81z^7\right)\left(\frac{1}{9z^7}\right)$$
$$=\frac{\left(9z^7\right)}{9z^7}-\frac{\left(81z^7\right)}{9z^7}$$
$$=(9+9)z^{(7-7)}-\ (81+9)z^{(7-7)}$$
$$=(1)z^0-\ 9z^0$$
$$=1-9$$
$$=-8$$

4. $$\left(3x^2y^5-9x^4y^3\right)\div 3xy=\left(3x^2y^5-9x^4y^3\right)\left(\frac{1}{3xy}\right)$$
$$=\left(\frac{3x^2y^5}{3xy}\right)-\left(\frac{9x^4y^3}{3xy}\right)$$
$$=(3+3)x^{(2-1)}y^{(5-1)}-(9+3)x^{(4-1)}y^{(3-1)}$$
$$=xy^4-3x^3y^2$$

5. $$\left(16x^4y^3-8x^2y^8\right)\div 4x^2y^2=\left(16x^4y^3-8x^2y^8\right)\left(\frac{1}{4x^2y^2}\right)$$
$$=\left(\frac{16x^4y^3}{4x^2y^2}\right)-\left(\frac{8x^2y^8}{4x^2y^2}\right)$$
$$=(16+4)x^{(4-2)}y^{(3-2)}-(8+4)x^{(2-2}$$
$$=4x^2y-2x^0y^6$$
$$=4x^2y-2x^6$$

Let's Review (page 42)

A. 1. $3x^2 + 9x^2 + 7x^2 = (3 + 9 + 7)x^2$
$= 19x^2$

2. $9x^2 + 3y^2 - 15x^3 - 2y^2 = 9x^2 - 15x^3 + (3 - 2)y^2$
$= 9x^2 - 15x^3 + y^2$

3. $14y^3 + 16y^2 + 13y^3 - 20y^2 = (14 + 13)y^3 + (16 - 20)y^2$
$= 17y^3 - 4y^2$

4. $50xy - 12y^3 - 25xy - 13y^3 = (-12 - 13)y^3 + (50 - 25)xy$
$= -25y^3 + 25xy$

5. $9x^2y^3 + 2x^3y^2 + 3xy - 14x^2y^3 = (9 - 14)x^2y^3 + 2x^3y^2 + 3xy$
$= -5x^2y^3 + 2x^3y^2 + 3xy$

B. 1. $(5x)(7x) = (5 \cdot 7)x^{(1+1)}$
$= 35x^2$

2. $(9x^2)(-4x^3y^3) = (9 \cdot -4)x^{(2+3)}y^3$
$= -36x^5y^3$

3. $(4x^2)(9x - 3) = 4x^2 \cdot 9x - 4x^2 \cdot 3$
$= (4 \cdot 9)x^{(2+1)} - (4 \cdot 3)x^2$
$= 36x^3 - 12x^2$

4. $(-5y)(x^2 + 2x - 1) = (-5y \cdot x^2) + (-5y \cdot 2x) + (-5y \cdot -1)$
$= -5x^2y - 10xy + 5y$

5. $(x + 1)(x + 2x + 1) = x(x + 2x + 1) + 1(x + 2x + 1)$
$= (x \cdot x + x \cdot 2x + x \cdot 1) + (1 \cdot x + 1 \cdot 2x + 1 \cdot 1)$
$= (x^2 + 2x^2 + x) + (x + 2x + 1)$
$= (x^2 + 2x^2) + (x + x + 2x) + 1$
$= 3x^2 + 4x + 1$

C. 1. $\dfrac{35x^2}{7x} = (35 \div 7)x^{(2-1)}$

2. $\dfrac{25y^3 - 40y^2}{5y^2} = \dfrac{25y^3}{5y^2} - \dfrac{40y^2}{5y^2}$
$= (25 \div 5)y^{(3-2)} - (40 \div 5)y^{(2-2)}$
$= 5y - 8$

3. $\dfrac{4x^2 + 6x + 2}{x + 1}$

$$\begin{array}{r@{}l} & \;\;4x+2 \\ x+1\,\big) & \overline{4x^2+6x+2} \\ & -(4x^2+4x) \\ \hline & \;\;2x+2 \\ & -(2x+2) \\ \hline & \;\;0 \end{array}$$

D. $\dfrac{2x^4+4x^3-7x^2+9}{x+3}$

$$\begin{array}{rrrrr|r} 2 & 4 & -7 & 0 & 9 & -3 \\ & -6 & 6 & 3 & -9 & \\ \hline 2 & -2 & -1 & 3 & 0 & \end{array}$$

$= 2x^3 - 2x^2 - x + 3$

E. 1. $$\frac{(x^2y)(x^3y^2)+(xy)(x^4y^2)}{2xy} = \frac{x^{(2+3)}y^{(1+2)}+x^{(1+4)}y^{(1+2)}}{2xy}$$
$$= \frac{x^5y^3+x^5y^3}{2xy}$$
$$= \frac{(1+1)x^5y^3}{2xy}$$
$$= \frac{2x^5y^3}{2xy}$$
$$= (2\div 2)x^{(5-1)}y^{(3-1)}$$
$$= x^4y^2$$

2. $$\frac{20x^3y^3-(5x^2y)(xy^2)}{x^2y^2+2x^2y^2} = \frac{20x^3y^3-5x^{(2+1)}y^{(1+2)}}{(1+2)x^2y^2}$$
$$= \frac{20x^3y^3-5x^3y^3}{3x^2y^2}$$
$$= \frac{(20-5)x^3y^3}{3x^2y^2}$$
$$= \frac{15x^3y^3}{3x^2y^2}$$
$$= (15\div 3)x^{(3-2)}y^{(3-2)}$$
$$= 5xy$$

Let's See What You Have Learned (page 43)

A. 1. $(13x)(2x) = (13 \cdot 2)x^{(1+1)} = 26x^2$

2. $(3x^2y)(-5x^3y^4) = (3 \cdot -5)x^{(2+3)}y^{(1+4)}$
$= -15x^5y^5$

3. $(3x)(2x + 3) = (3x \cdot 2x) + (3x \cdot 3)$
$= 6x^2 + 3x^3$

4. $(2x)(9x^2 + 3x + 2) = (2x \cdot 9x^2) + (2x \cdot 3x) + (2x \cdot 2)$
$= 18x^3 + 6x^2 + 4x$

5. $(3x^2 + 2x)(3x^2 + 2x + 1)$
$= [(3x^2 \cdot 3x^2) + (3x^2 \cdot 2x) + (3x^2 \cdot 1)] + [(2x \cdot 3x^2) + (2x \cdot 2x) + (2x \cdot 1)]$
$= (9x^4 + 6x^3 + 3x^2) + (6x^3 + 4x^2 + 2x)$
$= 9x^4 + 12x^3 + 7x^2 + 2x$

B. 1. $\dfrac{81y^3}{9y^2} = (81 \div 9)y^{(3-2)}$

$= 9y$

2. $\dfrac{36x^5 + 54x^3}{6x^3} = \dfrac{36x^5}{6x^3} + \dfrac{54x^3}{6x^3}$

$= (36 \div 6)x^{(5-3)} + (54 \div 6)x^{(3-3)}$

$= 6x^2 + 9$

3. $\dfrac{3x^2 - x - 10}{x-2}$

$$\begin{array}{r}
3x+5 \\
x-2\,\big)\,\overline{3x^2 - x + 10} \\
-(3x^2 + 6x) \\
\hline
5x + 10 \\
-(5x + 10) \\
\hline
0
\end{array}$$

C. $\dfrac{-x^4 + 26x^2 + 60x + 80}{x+4}$

	–1	0	26	60	80	–4
+		4	–16	–40	–80	
	–1	4	10	20	0	

$= -x^2 + 4x^2 + 10x + 20$

D. 1. $\dfrac{20x^2}{5x} + \dfrac{35y^4}{7y^2} = (20 \div 5)x^{(2-1)} + (35 \div 7)y^{(4-2)}$

$= 4x + 5y^2$

2. $\dfrac{(4x^2)(5x^2)(5x^2)}{20x^3} = \dfrac{(4 \cdot 5 \cdot 5)x^{(2+2+2)}}{20x^3}$

$= \dfrac{100x^6}{20x^3}$

$= (100 \div 20)x^{(6-3)}$

$= 5x^3$

E. **What Have You Learned?** *(page 45)*

A. 1. monomial of the fifth degree

2. multinomial of the sixth degree

3. trinomial of the fifth degree

4. binomial of the eighth degree

5. constant polynomial (zero degree)

B. Total cost = $6x^2 + 5y + 10y^2 + 5x$

If $x = 3$, $y = 2$:

$6(3)^2 + 5(2) + 10(2)^2 + 5(3) = 54 + 10 + 40 + 15$

$= 119$

C. 1. $(x^4 + 2x^3 - x + 2) - (x^2 + 3x^4 - 2) = -2x^4 + 2x^3 - x^2 - x + 4$

2. $(5y^6 + 3y^7 + 6y) + (4y^6 + 9y) = 9y^6 + 3y^7 + 15y$

3. $(5x^3 - 2x^2 + x - 1) - (4x^2 + 5x - 4) = 5x^3 - 6x^2 - 4x + 3$

D. 1. $(x + 2)(x - 3) = x^2 - 3x + 2x - 6$
$= x^2 - x - 6$

2. $(3x + 2)(2x + 1) = 6x^2 + 3x + 4x + 2$
$= 6x^2 + 7x + 2$

3. $(x + 1)(3x^2 - x + 1) = 3x^3 - x^2 + x + 3x^2 - x + 1$

$= 3x^3 + 2x^2 + 1$

E. 1. $15y^5 \div 15y^5 = 1$

2. $(12x^2 - 6x) \div 3x = (12 \div 3)x^{(2-1)} - (6 \div 3)x^{(1-1)}$
$= 4x - 2$

3. $(3x^2 - 5x - 2) \div (x - 2)$

$$\begin{array}{r@{}l}
 & \;\;3x+1 \\
x-2 & \big)\overline{3x^2-5x-2} \\
 & -\left(3x^2-6x\right) \\
\hline
 & \qquad\quad x-2 \\
 & \qquad -\left(x-2\right) \\
\hline
 & \qquad\qquad 0
\end{array}$$

E. $(3x^4 - 7x^3 - 2x^2 - 11x - 3) \div (x - 3)$

$$\begin{array}{rrrrr|l}
3 & -7 & -2 & -11 & -3 & \;3 \\
 & 9 & 6 & 12 & 3 & \\
\hline
3 & 2 & 4 & 1 & 0 &
\end{array}$$

$= 3x^3 + 2x^2 + 4x + 1$

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