Deriving Lines of Symmetry for j=4,5

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1 Introduction

$\underline{f(z) = z^5 + c}$

$(kz)^4 - 1 = 0$
$(kz)^4 = 1 = e^{2\pi i}$

$kz_0 = e^{2\pi i/4} = i$
$kz_1 = e^{4\pi i/4} = -1$
$kz_2 = e^{4\pi i/4} = -i$
$kz_3 = e^{8\pi i/4} = 1$
$\rightarrow$
$z_0 = \frac{i}{k}; (0, \frac{1}{k})$
$z_1 = -\frac{1}{k}; (-\frac{1}{k}, 0)$
$z_2 = -\frac{i}{k}; (0, -\frac{1}{k})$
$z_3 = \frac{1}{k}; (\frac{1}{k}, 0)$

$m_1 : (0, \frac{1}{k}), (\frac{1}{k}, 0)$
$\frac{0+\frac{1}{k}}{2} = \frac{1}{2k}, \frac{\frac{1}{k}+0}{2} = \frac{1}{2k}; (\frac{1}{2k}, \frac{1}{2k})$
$y_1 : (\frac{1}{2k}, \frac{1}{2k}), (0, 0)$
$m = \frac{0-\frac{1}{2k}}{0-\frac{1}{2k}} = 1$
$y_1 = x_1$

$m_2 : (0, \frac{1}{k}), (-\frac{1}{k}, 0)$
$\frac{0+(-\frac{1}{k})}{2} = -\frac{1}{2k}, \frac{\frac{1}{k}+0}{2} = \frac{1}{2k}; (-\frac{1}{2k}, \frac{1}{2k})$
$y_2 : (-\frac{1}{2k}, \frac{1}{2k}), (0, 0)$
$m = \frac{0-\frac{1}{2k}}{0-(-\frac{1}{2k})} = -1$
$y_2 = -x_2$

$y_3 : (-\frac{1}{k}, 0), (\frac{1}{k}, 0)$
$m = \frac{0-0}{\frac{1}{k}-(-\frac{1}{k})} = 0$
$y_3 = 0$

$y_4 : (0, \frac{1}{k}), (0, -\frac{1}{k})$
$m = \frac{-\frac{1}{k} - \frac{1}{k}}{0-0}$ DNE
$x = 0$

$\underline{f(z) = z^6 + c}$

$(kz)^5 - 1 = 0$
$(kz)^5 = 1 = e^{2\pi i}$

$kz_0 = e^{2\pi i/5}; (\cos(\frac{2\pi}{5}), \sin(\frac{2\pi}{5}))$
$kz_1 = e^{4\pi i/5}; (\cos(\frac{4\pi}{5}), \sin(\frac{4\pi}{5}))$
$kz_2 = e^{4\pi i/5}; (\cos(\frac{6\pi}{5}), \sin(\frac{6\pi}{5}))$
$kz_3 = e^{8\pi i/5}; (\cos(\frac{8\pi}{5}), \sin(\frac{8\pi}{5}))$
$kz_3 = e^{10\pi i/5}; (\cos(\frac{10\pi}{5}), \sin(\frac{10\pi}{5}))$
$\rightarrow$
$z_0 : (\frac{\cos(\frac{2\pi}{5})}{k}, \frac{\sin(\frac{2\pi}{5})}{k})$
$z_1 : (\frac{\cos(\frac{4\pi}{5})}{k}, \frac{\sin(\frac{4\pi}{5})}{k})$
$z_2 : (\frac{\cos(\frac{6\pi}{5})}{k}, \frac{\sin(\frac{6\pi}{5})}{k})$
$z_3 : (\frac{\cos(\frac{8\pi}{5})}{k}, \frac{\sin(\frac{8\pi}{5})}{k})$
$z_4 : (\frac{\cos(\frac{10\pi}{5})}{k}, \frac{\sin(\frac{10\pi}{5})}{k})$

$y_0 : (\frac{\cos(\frac{2\pi}{5})}{k}, \frac{\sin(\frac{2\pi}{5})}{k}), (0, 0)$
$m = \frac{0 - \frac{\sin(\frac{2\pi}{5})}{k}}{0 - \frac{\cos(\frac{2\pi}{5})}{k}} = \tan(\frac{2\pi}{5})$
$y_0 = \tan(\frac{2\pi}{5}) x_0$

$y_1 : (\frac{\cos(\frac{4\pi}{5})}{k}, \frac{\sin(\frac{4\pi}{5})}{k}), (0, 0)$
$m = \frac{0 - \frac{\sin(\frac{4\pi}{5})}{k}}{0 - \frac{\cos(\frac{4\pi}{5})}{k}} = \tan(\frac{4\pi}{5})$
$y_1 = \tan(\frac{4\pi}{5}) x_1$

$y_2 : (\frac{\cos(\frac{6\pi}{5})}{k}, \frac{\sin(\frac{6\pi}{5})}{k}), (0, 0)$
$m = \frac{0 - \frac{\sin(\frac{6\pi}{5})}{k}}{0 - \frac{\cos(\frac{6\pi}{5})}{k}} = \tan(\frac{6\pi}{5})$
$y_2 = \tan(\frac{6\pi}{5}) x_2$

$y_3 : (\frac{\cos(\frac{8\pi}{5})}{k}, \frac{\sin(\frac{8\pi}{5})}{k}), (0,0)$
$m = \frac{0 - \frac{\sin(\frac{8\pi}{5})}{k})}{0 - \frac{\cos(\frac{8\pi}{5})}{k}} = \tan(\frac{8\pi}{5})$
$y_3 = \tan(\frac{8\pi}{5})x_3$

$y_4 : (\frac{\cos(\frac{10\pi}{5})}{k}, \frac{\sin(\frac{10\pi}{5})}{k}), (0,0)$
$m = \frac{0 - \frac{\sin(\frac{10\pi}{5})}{k})}{0 - \frac{\cos(\frac{10\pi}{5})}{k}} = \tan(\frac{10\pi}{5})$
$y_4 = \tan(\frac{10\pi}{5})x_4$