# Formal Security Analysis of Neural Networks using Symbolic Intervals

Shiqi Wang et. al.

杨宇清

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Existence of Problems

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#### **Existence of Problems**

- Existing adversarial testing models:
  - No guarantee of non-existence of adversarial examples
  - My conjecture:
    - Tend to overestimate
    - The example might not be applied to real life
- High overhead of SMT
  - especially for non-linear, non-convex function

Goal

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Goal

### Goal:

A system for formally checking security properties of Relu-based DNNs

- High efficiency: "200 times on average"
- High accuracy: "a variety of optimizations to improve accuracy"

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Target

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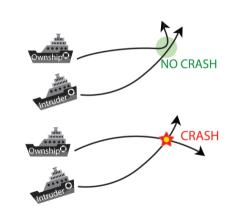
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#### Target

- Target system: ACAS Xu
- Security property: input-output-based

  To security property
- Attacker model: similar to adversarial examples:

given a computer vision DNN f, the attacker solves following optimization problem:  $min(L_p(x'-x))$  such that  $f(x) \neq f(x')$ , where  $L_p(\cdot)$  denotes the p-norm and x'-x is the perturbation applied to original input x. In other words, the security property of a vision DNN being robust against adversarial perturbations can be defined as: for any x' within a L-distance ball of x in the input space, f(x) = f(x').



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Overview

- Main method: interval analysis
- Optimization<sub>1</sub>: symbolic Interval
- Optimization<sub>2</sub>: iterative refinement: (existence of Lipschitz Consistency)

A Working Example

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A Working Example

### A Working Example: aiming to verify whether safe or not

Distance: x,

Approaching angle : y

Safe property :  $x \in [4, 6] y \in [1, 5]$ 

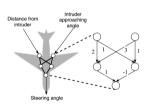
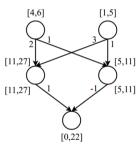


Figure 2: Running example to demonstrate our techniques.



(a) Naive interval propagation

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### **Dependency error**

- Naively computing output intervals in this way suffers from high errors as it computes extremely loose bounds.
- Only a highly conservative estimation of the output range, too wide to be useful for checking any safety property.

### **Symbolic Interval and Iterative Refinement**

- Symbolic interval propagation
  - explicitly represent the intermediate computations of each neuron in terms of the symbolic intervals that encode the interdependency of the inputs to minimize overestimation
- Iterative refinement
  - The dependency error for Lipschitz continuous functions decreases as the width of intervals decreases
  - Therefore, we can bisect the input interval by evenly dividing the interval into the union of two consecutive sub-intervals and reduce the overestimation

### **Symbolic Interval and Iterative Refinement**

- Symbolic interval propagation (algebraic operand preservation)
- Iterative refinement (even interval division)

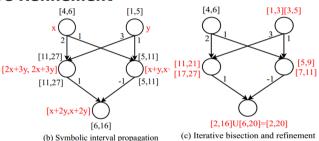
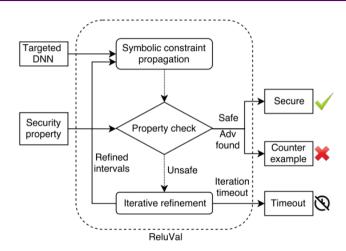


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#### Statistics

Source	Properties	Networks	Reluplex Time (sec)	ReluVal Time (sec)	Speedup
	$\phi_1$	45	>443,560.73*	14,603.27	>30×
	$\phi_2$	34* <sup>2</sup>	123,420.40	117,243.26	$1 \times$
	$\phi_3$	42	35,040.28	19,018.90	$2 \times$
Committee	$\phi_4$	42	13,919.51	441.97	32×
Security Properties from [25]	$\phi_5$	1	23,212.52	216.88	107×
	$\phi_6$	1	220,330.82	46.59	4729×
	$\phi_7$	1	>86400.0*	9,240.29	>9×
	$\phi_8$	1	43,200.01	40.41	1069×
	$\phi_9$	1	116,441.97	15,639.52	7×
	$\phi_{10}$	1	23,683.07	10.94	2165×
	$\phi_{11}$	1	4,394.91	27.89	158×
Additional	$\phi_{12}$	1	2,556.28	0.104	24580×
Security	$\phi_{13}$	1	>172,800.0*	148.21	>1166×
Properties	$\phi_{14}$	2	>172,810.86*	288.98	>598×
-	$\phi_{15}$	2	31,328.26	876.80	36×

<sup>\*</sup> Reluplex uses different timeout thresholds for different properties.

Table 1: ReluVal's performance at verifying properties of ACAS Xu compared with Reluplex.  $\phi_1$  to  $\phi_{10}$  are the properties proposed in Reluplex [25].  $\phi_{11}$  to  $\phi_{15}$  are our additional properties.

#### Statistics

# Seeds	CW	CW Miss	ReluVal	ReluVal Miss
50	24/40	40.0%	40/40	0%
40	21/40	47.5%	40/40	0%
30	17/40	58.5%	40/40	0%
20	10/40	75.0%	40/40	0%
10	6/40	85.0%	40/40	0%

Table 2: The number of adversarial inputs CW can find compared to ReluVal on 40 adversarial ACAS Xu properties. The third column shows the percentage of adversarial properties CW failed to find

P	Adv Range	Adv	Timeout	Non-adv
$S_1$	[6402.36, 10000]	98229	1	163915
$S_2$	[-0.2, -0.186] and $[-0.103, 0]$	18121	2	14645
C-	[_0.1.0.0085]	17738	1	15020

Table 3: The second column shows the input ranges containing at least one adversarial input, while the rest of ranges are found by ReluVal to be non-adversarial. The last three columns show the number of total sub-intervals checked by ReluVal with a precision of e-6.

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#### Adversarial machine learning

- Lack of Guarantee: None of the existing attacks can provide any provable guarantees about the non-existence of adversarial examples
- ReluVal can provide a provable security analysis of given input ranges, systematically narrowing down and detecting all adversarial ranges

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Customized SMT solvers

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#### **Customized STM solvers**

- **Significant overhead**: Customized SMT solvers for verifying security properties of DNNs are mostly limited by the scalability of the solver
- ReluVal uses interval-based techniques and significantly outperforms the state-of-the-art solver-based systems like Reluplex

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Convex-problem transformation

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Model

#### Convex-problem transformation

- Lack of Concrete Results: Focus on simply over-approximating the total number of potential adversarial violations without trying to find concrete counterexamples problem
- ReluVal can find concrete counterexamples as well as verify security properties of pre-trained DNNs

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Discreteness analysis

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Verivis: Transform the verification problem into a convex optimization problem using relaxations to over-approximate the outputs of ReLU nodes

- Lack of Verification: Leveraging the discreteness of image pixels cannot verify non-existence of norm-based adversarial examples
- ReluVal can.

### An example of Security Property Definition

**Property**  $\phi_1$ : If the intruder is distant and is significantly slower than the ownship, the score of a COC advisory will always be below a certain fixed threshold.

Tested on: all 45 networks.

Input ranges:  $\rho \ge 55947.691$ ,  $v_{own} \ge 1145$ ,  $v_{int} \le 60$ .

Desired output: the output of COC is at most 1500.

**Property**  $\phi_2$ : If the intruder is distant and is significantly slower than the ownship, the score of a COC advisory will never be maximal.

Tested on: model\_x\_y,  $x \ge 2$ , except model\_5\_3 and model\_4\_2

Input ranges:  $\rho \ge 55947.691$ ,  $v_{own} \ge 1145$ ,  $v_{int} \le 60$ .

Desired output: the score for COC is not the maximal score.

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