AE5093 — Scientific Applications of Deep Learning

Homework 3

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1 Introduction

In this assigment, a Feed Forward Neural Network (FFNN) is trained and informed using a physical model of a system. The network will be trained on two PDEs: the 1D viscous Burgers equation and the 2D wave equation. The goal is to demonstrate the ability of the FFNN to learn the underlying physics of the system and make predictions based on the model. For both cases, it will be fed a series of boundary conditions and interior points to learn.

2 1D Burgers Equation

The 1D viscous Burgers equation is a fundamental partial differential equation (PDE) that describes the motion of a viscous fluid. It is given by:

$$u_t + u * u_x = \nu u_{xx} \tag{1}$$

where u is the velocity field, t is time, x is the spatial coordinate, and ν is the kinematic viscosity.

2.1 Problem Setup

For this problem, the domain is defined as $x \in [-1, 1]$ and $t \in [0, 1]$. With the following boundary conditions:

$$u(x, 0) = -\sin(\pi x) \tag{2}$$

$$u(-1, t) = u(1, t) = 0 \tag{3}$$

where,

$$\nu \in \{\frac{0.01}{\pi}, \frac{0.0001}{\pi}, 0.0\}$$

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For each viscous case, the following parameters are used:

- Epochs = 5000
- LearningRate = 1000
- Neurons = 50
- $N_i = 5000$ (interior points)
- $N_b = 256$ (boundary points)
- $N_{ic} = 256$ (initial condition points)

The NN will be trained on the interior points, boundary points and initial conditions. The total loss function is defined by:

$$L = L_{pde} + L_{ic} + L_{bc} \tag{4}$$

where,

$$L_{pde} = u_t + uu_x - \nu u_{xx} \tag{5}$$

$$L_{ic} = \frac{1}{N} \sum_{i=1}^{N} (\theta(u|x_{ic}, t_{ic}) - u_{ic})^2 \tag{6}$$

$$L_{bc} = L_{ic} = \frac{1}{N} \sum_{i=1}^{N} (\theta(u|x_{bc}, t_{bc}) - u_{bc})^2 \tag{7}$$

2.2 Results - No LR Annealing

2.3 Results - LR Annealing