Note template

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Abstract

This is a note template, with all but minimal compilable files provided. Feel free to adjust for your usage.

Now let's start a simple demo for you to take fancy notes in LaTeX!

Contents

Chapter 1

Entropy

Definition 1.0.1 (Entropy). A measure of uncertainty of a physical system.

$$H(x) = H(p_1, p_2, \dots p_n) = -\sum_x p_x \log p_x$$

$$\lim_{p \to 0} p \log p = 0$$

X - Information we gain, on an average when we learn the value of X.

Example. Coin toss :- HHHH - H, if it gives only heads, Information gain is zero.

Operational interpretation of entropy

Entropy is tied to memory resources.

Example. X takes values (x_1, x_2, x_3, x_4) with probability $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$
encoding them with $(0, 10, 110, 111) \Rightarrow \frac{1}{2}[1] + \frac{1}{4}[2] + \frac{1}{8}[3] + \frac{1}{8}[3] = \frac{7}{4}$bits

$$-\sum_{x=1}^{4} p_x \log p_x = \frac{7}{4}\text{bits}$$

Example. For a coin $p_H = 1$ and $p_T = 0$ size of memory $= 0$

Entropy from intuitive axioms

1. I(p)
2. I(p) is smooth
3. I(pq) = I(p)+I(q)

Properties of Entropy

$$H_{bin}(p) = -p \log p - (1-p)\log(1-p)$$

get a quadratic curve

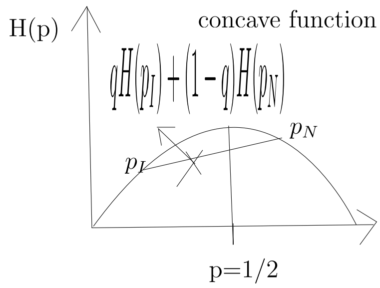

Figure 1.1: title

$$H(qp_I + (1-q)p_N) \geq qH(p_I) + (1-q)H(p_N)$$
$$f(px + (1-p)y) \geq pf(x) + (1-p)f(y)$$

Relative Entropy

Definition 1.0.2.

$$H(p(x) \mid\mid q(x)) = -\sum_{x=1}^{n} p(x) \log \frac{q(x)}{p(x)}$$

Theorem 1.0.1.

$$H(p(x) \mid\mid q(x)) = \sum p(x) \log \frac{p(x)}{q(x)} \text{is non-negative}$$
$$= 0 \text{ iff p(x) = q(x) for all x}$$

Chapter 2

Tensor products

There is a ball which can be red or blue the quantum state associated with it look like

$$|\psi_1\rangle = \alpha |r\rangle + \beta |b\rangle$$

what if I have 2 balls?

$$|\psi_2\rangle = \alpha |r, r\rangle + \beta |r, b\rangle + \gamma |b, r\rangle + \delta |b, b\rangle$$

Each vectors have their own vector spaces. Let the 2 vector spaces be V, W. where,

$$V = v_1, v_2, \dots v_n$$

$$V \otimes W = Tensorproductspace$$

$$Dimension(V \otimes W) = nm$$

Representation

$$a = (a_1, a_2, \dots a_n) \in V$$

$$b = (b_1, b_2, \dots b_n) \in W$$

$$a \otimes b = \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ \vdots \\ a_n b_n \end{bmatrix} \tag{2.1}$$

$$\langle a \otimes b | c \otimes d \rangle = \langle a | c \rangle \langle b | d \rangle$$

$$A \in L(v) \dots B \in L(W) \Rightarrow L(V \otimes W)$$

$$(A \otimes B)(a \otimes b) = A |a\rangle \otimes B |b\rangle$$

$$\sum_{i,j} c_{ij} (A \otimes B)$$

$\boxed{C = UDV}$ where U and V are Unitary matrices and D is a diagonal matrix.

$$[c_{jk}] = \sum_{i,j,k} U_{ji} D_{ii} V_{ik}$$

$$|\psi_{AB}\rangle = |\psi'_A\rangle \otimes |\psi'_B\rangle \rightarrow \text{ Separable State}$$

$$\frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} \text{ Is a Separable state}$$

$|\psi\rangle$ is a state of the composite system AB.
$|i_A\rangle$ for A $|i_B\rangle$ for B

$$|\psi_{AB}\rangle = \sum_i \lambda_i |i_A\rangle |i_B\rangle \rightarrow \text{Schmidt decomposition / basis}$$

if λ is only one value then it is separable. or else it is entangled.

$$\sum_i \lambda_i (U |i_A\rangle) |i_B\rangle \text{ it is also a Schmidt decomposition.}$$

$$\psi_{AB} = \sum c_{jk} |j\rangle |k\rangle = \sum_{i,j,k} U_{ji} d_{ii} V_{ik} |j\rangle |k\rangle$$

$$\sum U_{ji} |j\rangle = |i_A\rangle$$

$$\sum V_{ik} |k\rangle = |i_B\rangle$$

Example. Consider 2 states, which is more entangled?

$$|\psi_1\rangle = \sqrt{0.99999} |0\rangle |0\rangle + \sqrt{0.00001} |1\rangle |1\rangle$$

$$|\psi_2\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$|\psi_2\rangle$ is more entangled state.

$$Entropy(|\psi\rangle) = -\sum_i \lambda_i^2 \log \lambda_i^2$$

Chapter 3

Density Matrix

$$\rho = \frac{I + \vec{r}.\vec{\sigma}}{2}$$
$$\vec{r} = \text{ 3d vector}$$
$$||\vec{r}|| \leq 1$$

3.1 The reduced density operator

1. deepest
2. elegant

$$\rho_{AB}$$
$$\rho^A = tr_B(\rho_{AB})$$

partial trace is a map from vector space of the composite system to the vector space of one of the subsystems.

Definition 3.1.1.

$$tr_B(|a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|) = |a_1\rangle\langle a_2|\langle b_2|b_1\rangle$$

Linear operation.

$$p(x) \to \hat{\rho}$$
$$\int p(x)dx = 1 \to Tr(\hat{\rho}) = 1$$
$$\int_y p(x,y)dx = p(x) \to Tr_B(\hat{\rho}_{AB}) = \hat{\rho}_A$$

$$\hat{\rho} = \hat{\rho}_A \otimes \hat{\rho}_B$$
$$Tr_B(\hat{\rho}_A \otimes \hat{\rho}_B) = \hat{\rho}_A$$

Bell state

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$\hat{\rho}_{Bell} = \frac{|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$
$$\rho' = Tr_2(\hat{\rho}_{Bell}) = \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} \text{ Complete Ignorance}$$

Schmidt decomposition

$$|\psi\rangle = \sum_i \lambda_i |i_A\rangle |i_B\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi| = \sum_{i,j} \lambda_i \lambda_j^* |i_A\rangle\langle j_A| \otimes |i_B\rangle\langle j_B|$$

$$\hat{\rho}_A = \sum_i |\lambda_i|^2 |i_A\rangle\langle i_A|$$

$$\hat{\rho}_B = \sum_i |\lambda_i|^2 |i_B\rangle\langle i_B|$$

$$Tr(\hat{\rho}_A^2) = \sum_i |\lambda_i|^4 < 1$$

$Tr(\hat{\rho}_A^2)$ is called purity.
Measure of metric for entanglement $1 - Tr(\hat{\rho}_A^2)$.
$1 - Tr(\hat{\rho}_A^2)$ is $\frac{1}{2}$ for Mixed states and 0 for separable states.

3.1.1 Purification

Suppose we are given a state $\hat{\rho}_A$ of the quantum system A. It is always possible to introduce another system R and define a pure state $|AR\rangle$ such that the reduced state $Tr_R(|AR\rangle\langle AR|) = \hat{\rho}_A$
$AR \to$ church of the larger hilbert space.

$$\rho_A = \sum p_i |i_A\rangle\langle i_A|$$

$$\rho_{AR} = \sum \sqrt{p_i} |i_A\rangle |i_R\rangle$$

$$(\text{ Purification }) |AR\rangle\langle AR| = \sum \sqrt{p_i p_j} |i_A\rangle\langle j_A| \otimes |i_R\rangle\langle j_R|$$

$$|AR\rangle\langle AR| = \sum \sqrt{p_i p_j} |i_A\rangle\langle j_A| \delta_{ij}$$

Example. There is a room and ball is in the room where you don't know. You assign a equal probability distribution to the ball. When you measure the ball position the probability distribution collapses to one point. Where is the ball before your measurement?

$$\frac{|{+}{+}\rangle + |{-}{-}\rangle}{\sqrt{2}}$$

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Bell states are locally and rotationally invariant.

3.2 EPR and Bell's Inequality

$$|\psi\rangle = \sum c_i |\psi_i\rangle$$

$$|c_i|^2 \to |\psi_i\rangle$$

Just before the measurement, what was the value of observable $\hat{A} = \sum_i |\psi_i\rangle\langle\psi_i|$.

Answer. **Copenhagen Interpretation** The particle doesn't have a position before measurement. ⊛

3.2.1 EPR paper

On the elements of reality.
"If, without in anyway disturbing a system, we can predict with certainty (with p = 1) the value of a physical quantity, then there exists an element of physical reality. corresponding to the physical quantity."

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$\vec{v}.\vec{\sigma} = \pm 1$$

3.3 Bell's Inequalities

Not about Quantum mechanics. forget qm.

Appendix

Appendix A

Additional Proofs

A.1 Proof of ??

We can now prove **??**.

Proof of ??. See `https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence`. ■