Experiment Number 4
State Variable Filter
EP3290

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EP20B012

August 16, 2022

Date Performed: August 02, 2022

1 Objective

Design a State Variable Filter which has a corner (natural undamped) frequency, f_C of 2.5 kHz and a quality factor, Q of 10. Assume both the frequency determining resistors and capacitors are equal. Determine the filters DC gain and draw the resulting circuit and Bode plot.

1.1 Definitions

State Variable Filter The state variable filter is a type of multiple-feedback filter circuit that can produce all three filter responses, Low Pass, High Pass and Band Pass simultaneously from the same single active filter design.

1.2 State Variable Filter Block Diagram

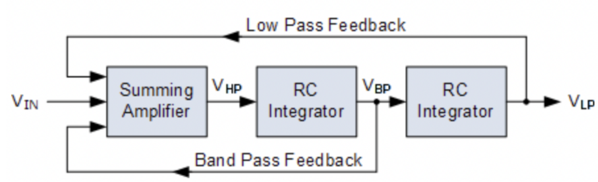

Figure 1: State Variable Filter Block Diagram

1.3 State Variable Filter Circuit

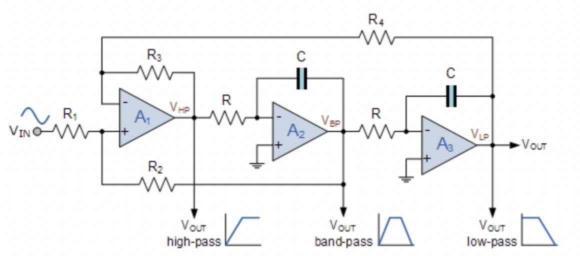

Figure 2: State Variable Filter Circuit

1.4 Normalized response of a State Variable Filter

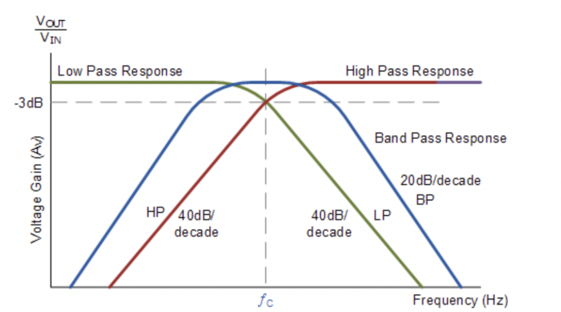

Figure 3: Normalized response of a State Variable Filter

2 Calculations

2.1 Op-amp Integrator Circuit

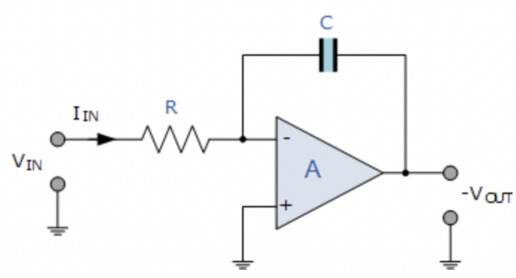

Figure 4: Op-amp Integrator Circuit

$$V_{out} = -\frac{1}{RC}\int_0^t V_{IN} dt \tag{1}$$

$$V_{out} = -\frac{1}{2\pi f_c RC} V_{IN} \tag{2}$$

The output voltage V_{out} is a constant 1/RC times the integral of the input voltage Vin with respect to time. Integrators produce a phase lag with the minus sign (-) indicating a 180° phase shift because the input signal is connected directly to the inverting input terminal of the op-amp. In the case of op-amp A2 above, its input signal is connected to the output of the proceeding op-amp, A1 so its input is given as V_{HP} and its output as V_{BP}. Then from above, the expression for op-amp, A2 can be written as:

$$V_{BP} = -\frac{1}{2\pi f_c RC} V_{HP} \tag{3}$$

2.2 Op-amp A2 Transfer Function

$$\frac{V_{out}}{V_{IN}} = \frac{V_{BP}}{V_{HP}} = -\frac{1}{2\pi f_c RC} \tag{4}$$

2.3 Op-amp A3 Transfer Function

Exactly the same assumption can be made as above to find the transfer function for the other op-amp integrator, A3

$$\frac{V_{out}}{V_{IN}} = \frac{V_{LP}}{V_{BP}} = -\frac{1}{2\pi f_c RC} \tag{5}$$

So the two op-amp integrators, A2 and A3 are connected together in cascade, so the output from the first (V_{BP}) becomes the input of the second. So we can see that the band pass response is created by integrating the high pass response and the low pass response is created by integrating the band pass response. Therefore the transfer function between V_{HP} and V_{LP} is given as:

$$\frac{V_{LP}}{V_{HP}} = -\frac{1}{2\pi f_c RC} X - \frac{1}{2\pi f_c RC} = \frac{1}{(2\pi f_c RC)^2} \tag{6}$$

2.4 Amplifier Summing Circuit

Operational amplifier, A1 is connected as an adder-subtracter circuit. That is it sums the input signal, V_{IN} with the V_{BP} output of op-amp A2 and the subtracts from it the V_{LP} output of op-amp A3, thus

$$i_1 = \frac{V_{IN} - (+V)}{R_1} + \frac{V - V_{BP}}{R_2} = 0 \tag{7}$$

$$\implies V = \frac{V_{IN} R_2 + V_{BP} R_1}{R_1 + R_2} \tag{8}$$

and

$$i_2 = \frac{V_{HP} - (-V)}{R_3} + \frac{-V - V_{LP}}{R_4} = 0 \tag{9}$$

$$\implies -V = \frac{V_{LP} R_3 + V_{HP} R_4}{R_3 + R_4} \tag{10}$$

From equations (5), (6), (8), (10)

$$\boxed{\frac{V_{OUT}}{V_{IN}} = \frac{V_{LP}}{V_{IN}} = \frac{\frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} X \frac{1}{RC}}{\frac{R_3}{R_4 RC} + \left(\frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} X \frac{1}{RC}\right) + \left(\frac{1}{2\pi f RC}\right)^2}} \tag{11}$$

2.5 Normalized 2nd-order Transfer Function

$$\boxed{\frac{V_{OUT}}{V_{IN}} = \frac{A_o\left(\frac{f}{f_o}\right)}{1 + 2\zeta \frac{f}{f_o} + \left(\frac{f}{f_o}\right)^2}} \tag{12}$$

2.6 State Variable Filter Corner Frequency

$$f_{c(HP)} = f_{c(BP)} = f_{cLP} = \frac{1}{2\pi RC} \tag{13}$$

2.7 Q-Factor of a state variable Filter

$$Q = \frac{1}{2\zeta} = \frac{R_4(R_1 + R_2)}{R_1(R_3 + R_4)} \sqrt{\frac{R_3}{R_4} X \frac{RC}{RC}} \tag{14}$$

Objective $f_c = 2.5kHz$ and $Q = 10$, using equation (13) and fixing C = 10nF, to find R...

$$R = \frac{1}{2\pi f_c C} = \frac{1}{2\pi X 2500 X 10nF} = 6.33k\Omega \tag{15}$$

Taking $R_3 = R_4 = 10k\Omega$ and using equation (14), to find R_1 and R_2...

$$10 = \frac{10(R_1 + R_2)}{R_1(10+10)} \implies \frac{R_2}{R_1} = 19. \tag{16}$$

Taking $R_1 = 1k\Omega$ and $R_2 = 19k\Omega$

3 State Variable Filter Design

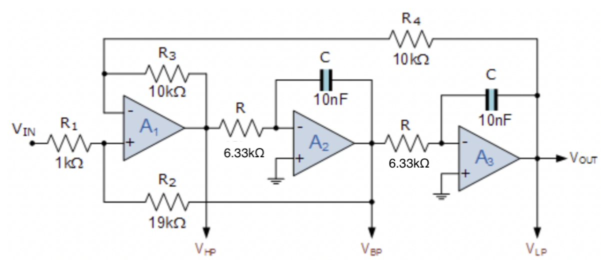

Figure 5: State Variable Filter Design

4 Experimental Data

Frequency	V_{OUT} Low Pass	Gain Low Pass	V_{OUT} High Pass	Gain High Pass	V_{OUT} Band Pass	Gain Band Pass
1.2	4.80	1.600000	0.40	0.133333	0.80	0.266667
1.4	6.40	2.133333	1.00	0.333333	2.20	0.733333
1.6	8.20	2.733333	3.00	1.000000	4.00	1.333333
1.8	12.80	4.266667	7.40	2.466667	8.40	2.800000
2.0	20.00	6.666667	18.60	6.200000	15.40	5.133333
2.1	20.20	6.733333	19.80	6.600000	19.20	6.400000
2.2	5.40	1.800000	9.52	3.173333	6.40	2.133333
2.3	3.20	1.066667	7.68	2.560000	4.20	1.400000
2.4	2.16	0.720000	6.52	2.173333	3.60	1.200000
2.5	1.62	0.540000	6.28	2.093333	2.60	0.866667
2.6	1.42	0.473333	6.12	2.040000	2.60	0.866667
2.7	1.26	0.420000	6.20	2.066667	2.40	0.800000
2.8	1.14	0.380000	6.08	2.026667	2.20	0.733333
2.9	1.04	0.346667	5.96	1.986667	2.20	0.733333
3.0	0.94	0.313333	5.88	1.960000	2.00	0.666667
3.2	0.84	0.280000	5.76	1.920000	2.00	0.666667
3.4	0.78	0.260000	5.68	1.893333	1.82	0.606667
3.6	0.72	0.240000	5.60	1.866667	1.74	0.580000
3.8	0.68	0.226667	5.56	1.853333	1.66	0.553333
4.0	0.62	0.206667	5.56	1.853333	1.58	0.526667

Figure 6: Experimental Data

5 Results and Conclusions

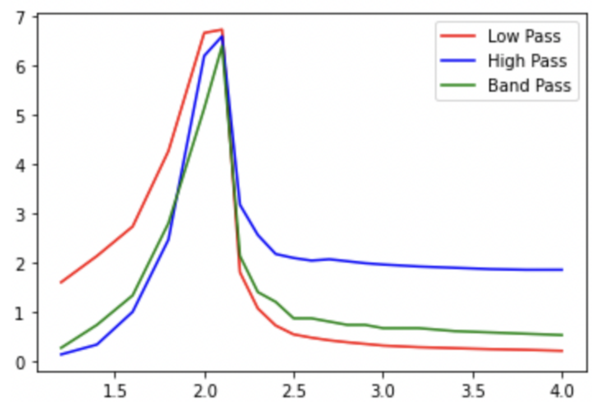

Figure 7: Bode Plot of Gain Vs Frequency

The cut-off frequency of Low-Pass, Band-Pass and High-Pass are approximately equal. The frequency is near to 2.5kHz but not exactly equal due to some fault in resistor values and capacitor values etc.