EP3110 - Midterm exam solution

Q1

a) $k_0 = \omega\sqrt{\mu_0 \epsilon_0} = \dfrac{10^7\pi}{3\times10^8} = \dfrac{\pi}{30} = 0.105$ rad/m.

at $t = 3\times10^{-3}$ s, we require

$$10^7\pi\left(3\times10^{-3}\right) - \frac{\pi}{30}y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2}, \quad n = 0, 1, 2, 3 \ldots$$

$$y = \pm 30\pi - 7.5 \quad (m)$$

[2 marks]

But $\lambda = \dfrac{2\pi}{k_0} = 60$ m

$$\therefore \boxed{y = 22.5 \pm \frac{n\lambda}{2} \quad m}$$

b) $\boxed{\vec{E} = \eta_0 \vec{H}\times\hat{y} = -\hat{x}\, 1.5\times10^{-3}\cos\left(10^7\pi t - \frac{\pi}{30}y + \frac{\pi}{4}\right)}$ [1 mark]

Q2

$f = 3\times10^{9}$ Hz, $\varepsilon_r = 25$, $\tan\delta_c = \frac{\sigma}{\omega\varepsilon} = 10^{-2}$.

a) $$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2}.$$ (2 marks).

$$= \omega\sqrt{\frac{\mu\varepsilon}{2}}\,\frac{1}{\sqrt{2}}\left(\frac{\sigma}{\omega\varepsilon}\right) = 0.497 \text{ Np/m}$$

$$e^{-\alpha x} = \frac{1}{2} \Rightarrow x = \frac{1}{\alpha}\ln 2 \; \boxed{= 1.395 \text{ m}}$$

b) $$\eta_c = \frac{1}{\sqrt{\varepsilon_r}}\left(\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1+j\frac{\sigma}{2\omega\varepsilon}\right)\right) = 238(1+j\,0.005)\ \Omega$$

$$\beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right] = 31.6\pi \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = 0.063 \text{ m}$$

$$u_p = \frac{\omega}{\beta} = 1.8973\times10^{8} \text{ m/s}$$

$$u_g = \frac{1}{d\beta/dw} = \frac{c}{\sqrt{\varepsilon_r}}\left[1 + \frac{1}{8}\left(\frac{\sigma}{w\varepsilon}\right)^2\right]$$

(3 marks)

$$u_g = 1.8975 \times 10^8 \text{ m/s.}$$

__Q3__

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{w}{\beta} \qquad \beta = 5 \text{ rad/m}, \; w = 10^9 \text{ rad/s.}$$

(3 marks)

$$\therefore \quad \frac{3\times10^8}{\sqrt{\varepsilon_r}} = \frac{10^9}{5} \qquad \therefore \sqrt{\varepsilon_r} = 1.5.$$

$$\boxed{\varepsilon_r = 2.25.}$$

$$\boxed{u_p = \frac{w}{\beta} = \frac{10^9}{5} = \underline{2\times10^8 \text{ m/s}}}$$

Q4

here $\sigma >> \omega \varepsilon$

$$4 >> 2\pi \times 10^{3} \times 8.85 \times 10^{-12} \times 80$$

$$\alpha = \sqrt{\pi f \mu \sigma} = 0.13 \text{ Np/m}.$$

(3 marks)

$$\frac{E \times 10^{-6}}{E_0} = e^{-\alpha x}.$$

$$\therefore x = \frac{6}{\alpha} \log 10 = \frac{13.8}{\alpha} = \underline{\underline{106 \text{ m}}}.$$

Q:5

$$S_{av} = \frac{E^2}{2Z} = \frac{1}{2} \frac{E_1^2 + E_2^2}{Z}$$

$$= \frac{1}{2} \frac{3^2 + 6^2}{377} = \frac{1}{2} \times \frac{45}{377}$$

$$\boxed{S_{av} = 60 \text{ mW m}^{-2}}$$

Q6

Critical angle $\theta_c = \sin^{-1}\sqrt{\frac{1}{81}} = 6.38$

since $\theta_i = 45^{\circ} > \theta_c$, the wave will be totally internally reflected.

$\sin\theta_t = \sqrt{\varepsilon_r}\,\sin\theta_i = \sqrt{81}\ 0.707 = 6.36$

$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-6.36^2}$

E_t and H_t vary spatially in accordance with the following factor $e^{-j\beta_2 \hat{n}_t \cdot r} = e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

for $\theta_i > \theta_c \Rightarrow \quad = e^{-\alpha_2 z}\, e^{-j\beta_{2x} x}$.

where $\alpha_2 = \beta_2\sqrt{(\varepsilon_1/\varepsilon_2)\sin^2\theta_i - 1}$

$\beta_{2x} = \beta_2\sqrt{\varepsilon_1/\varepsilon_2}\,\sin\theta_i$

$$\boxed{\vec{E}_t = \hat{z}\, \tau_{\perp} E_0\, e^{-\alpha_2 z}\, e^{-j\beta_{2x} x}}$$

Q7

Because $\beta_1 \cos\theta_i = 0.866$ and $\theta_i = 30^\circ$

$\beta_1 = 1$ rad/m. (1 mark)

In free space $\omega = c\beta_1 = 3\times10^8 \times 1 = 3\times10^8$ m/s

$\beta_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = 3$ rad/m.

$\eta_1 = 120\pi\ \Omega$

$\eta_2 = \frac{120\pi}{\sqrt{9}} = 40\pi\ \Omega$.

From Snell's law of refraction

$\sin\theta_2 = 0.167$

$\theta_2 = 9.594^\circ$

and $\cos\theta_2 = 0.986$.

$$\Gamma = \frac{40\pi \times 0.866 - 120\pi \times 0.986}{40\pi \times 0.866 + 120\pi \times 0.986} = -0.547$$

$$\tau_n = 1 + \Gamma = 0.453.$$

$\therefore$ $E_i = 377\, e^{-j0.866z}\, e^{-j0.5y}\, \hat{x}$ V/m.

$H_i = (-0.866\hat{y} + 0.5\hat{z})\, e^{-j0.866z}\, e^{-j0.5y}$ A/m

$E_r = -206.22\, e^{-j0.5y}\, e^{j0.866z}\, \hat{x}$ V/m.

$H_r = (0.474\hat{y} + 0.274\hat{z})\, e^{-j0.5y}\, e^{j0.866z}$ A/m.

$E_t = 170.78\, e^{-j2.958z}\, e^{-j0.5y}\, \hat{x}$ V/m

$H_t = -(-1.34\hat{y} + 0.227\hat{z})\, e^{-j2.958z}\, e^{-j0.5y}$ A/m.

$$\langle S \rangle = \frac{1}{2} \mathrm{Re}\{ E_t \times H_t \}.$$

$$= 114.4\,\hat{z} + 19.4\,\hat{y} \quad \text{W/m}^2.$$

Q8 Since the intrinsic impedance of the medium

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{60(2-j1)}{60(2-j1)}}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega \quad \text{free space.}$$

The wave enters the slab without any reflection.

$$\alpha = \mathrm{Re}\left[j\omega \sqrt{\mu\varepsilon \left(1 + \frac{\sigma}{j\omega\varepsilon}\right)} \right]$$

$$\alpha = \mathrm{Re}\left[\sqrt{-\omega^2\mu\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}\right]$$

$$= \mathrm{Re}\left[-\omega^2\mu\varepsilon + j\sigma\omega\mu\right]$$

$$= \mathrm{Re}\left[j\omega\sigma\mu - \omega^2\mu\varepsilon\right]$$

$$= \mathrm{Re}\left[j\omega\sigma\mu_0\, 60\,(2-j)\right]$$

$$\alpha = \frac{120\pi}{\lambda_0}$$

The attenuation $\alpha d = \frac{120\pi}{\lambda_0} d = \frac{120\pi \times 10 \times 10^{-3}}{3}$

$= 1.25$ Np

$= 11$ dB