

EP3110:- Electro-magnetics and Applications

Answer to Question 1

Given,

$$\vec{E}(x, y, z, t) = 0.2 \sin(10\pi y) \cos(6\pi 10^9 t - \beta z) \hat{x}$$

Phasor form,

$$\vec{E}(x, y, z) = \hat{x} 0.2 \sin(10\pi y) e^{-j\beta z}$$

We know,

$$\vec{H} = -\frac{1}{j\omega\mu_o} \vec{\nabla} \times \vec{E} \quad (1)$$

$$\begin{aligned} \vec{H}(x, y, z) &= -\frac{1}{j\omega\mu_o} \left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \times (\hat{x} 0.2 \sin(10\pi y) e^{-j\beta z}) \\ \vec{H}(x, y, z) &= -\frac{1}{j\omega\mu_o} 0.2 e^{-j\beta z} (10\pi \cos(10\pi y) (-\hat{z}) - j\beta \sin(10\pi y) (\hat{y})) \end{aligned}$$

$$\vec{H}(x, y, z) = \frac{1}{j\omega\mu_o} 0.2 e^{-j\beta z} (j\beta \sin(10\pi y) \hat{y} + 10\pi \cos(10\pi y) \hat{z}) \quad (2)$$

Answer to Question 2

Given,

$$\vec{E} = 30\pi e^{j(\omega t - \frac{4}{3}y)} \hat{z} \left(\frac{V}{m} \right)$$

$$\vec{H} = 1.0 e^{j(\omega t - \frac{4}{3}y)} \hat{x} \left(\frac{A}{m} \right)$$

We know,

$$\frac{|\vec{E}|}{|\vec{H}|} = \eta = \sqrt{\frac{\mu_r \mu_o}{\epsilon_r \epsilon_o}} \quad (3)$$

$$\begin{aligned} \eta &= \frac{30\pi}{1} = \sqrt{\frac{\mu_o(1)}{\epsilon_r \epsilon_o}} \\ \epsilon_r &= \frac{\mu_o}{900\pi^2 \epsilon_o} = 15.978 \approx 16 \end{aligned}$$

We know,

$$\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_r \mu_o \epsilon_r \epsilon_o}} \quad (4)$$

$$\begin{aligned} \frac{3\omega}{4} &= \sqrt{\frac{1}{(1)\mu_o \left(\frac{\mu_o}{900\pi^2 \epsilon_o} \right) \epsilon_o}} \\ \omega &= 1.0 \times 10^8 \end{aligned}$$

Answers:

$$\epsilon_r \approx 16$$

$$\omega = 1.0 \times 10^8 \text{ radians}$$

Answer to Question 3

Given,

$$E(x, y, z) = E_o e^{-j(k_x x + k_y y + k_z z)}$$

Homogenous Helmholtz equation

$$\nabla^2 E + \omega^2 \mu \epsilon E = 0 \quad (5)$$

$$\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2} \right) (E_o e^{-j(k_x x + k_y y + k_z z)}) + \omega^2 \mu \epsilon E_o e^{-j(k_x x + k_y y + k_z z)} = 0$$
$$E_o (-k_x^2 - k_y^2 - k_z^2) e^{-j(k_x x + k_y y + k_z z)} + \omega^2 \mu \epsilon E_o e^{-j(k_x x + k_y y + k_z z)} = 0$$

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon \quad (6)$$

Therefore, Electric field Intensity $E(x, y, z) = E_o e^{-j(k_x x + k_y y + k_z z)}$ satisfies the homogeneous Helmholtz equation provided that the condition $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon$ is satisfied.

Answer to Question 4

Given,

$$\vec{E}(R) = \vec{E}_o e^{-j\vec{k} \cdot \vec{R}}$$

Magnetic field $\vec{H}(R)$ can be written as

$$\vec{H}(R) = \vec{H}_o e^{-j\vec{k} \cdot \vec{R}} = \vec{k} X \vec{E}_o e^{-j\vec{k} \cdot \vec{R}}$$

$$\vec{\nabla} X \vec{E} = \vec{\nabla} (e^{-j\vec{k} \cdot \vec{R}}) X \vec{E}_o = -j\mu\omega \vec{H} \quad (7)$$

$$\vec{\nabla} (e^{-j\vec{k} \cdot \vec{R}}) = e^{-j\vec{k} \cdot \vec{R}} \vec{\nabla} (-j\vec{k} \cdot \vec{R}) = e^{-j(k_x \hat{x} + k_y \hat{y} + k_z \hat{z}) \cdot (x\hat{x} + y\hat{y} + z\hat{z})} [-j\vec{\nabla} (k_x x + k_y y + k_z z)]$$

$$= e^{-j\vec{k} \cdot \vec{R}} [-j \left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) (k_x x + k_y y + k_z z)]$$

$$\vec{\nabla} (e^{-j\vec{k} \cdot \vec{R}}) = e^{-j\vec{k} \cdot \vec{R}} [-j\vec{k}] \quad (8)$$

$$\vec{\nabla} (e^{-j\vec{k} \cdot \vec{R}}) X \vec{E}_o = e^{-j\vec{k} \cdot \vec{R}} [-j\vec{k}] X \vec{E}_o = -j\mu\vec{H}$$

$$\vec{\nabla} X \vec{H} = \vec{\nabla} (e^{-j\vec{k} \cdot \vec{R}}) X \vec{H}_o = j\omega\epsilon \vec{E}$$

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} (e^{-j\vec{k} \cdot \vec{R}}) \cdot \vec{E}_o = 0$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

Answer to Question 5

Given,

$$\vec{E} = 2 \cos(10^8 t - \frac{z}{\sqrt{3}}) \hat{x} - \sin(10^8 t - \frac{z}{\sqrt{3}}) \hat{y}$$

(a) we know,

$$\omega = 2\pi f \quad (9)$$

$$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} = 15.9MHz$$

$$\boxed{\lambda = \frac{2\pi}{k}} \quad (10)$$

$$\lambda = \frac{2\pi}{\frac{1}{\sqrt{3}}} = 10.88m$$

(b) we know,

$$\boxed{v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_r \mu_o \epsilon_r \epsilon_o}}} \quad (11)$$

$$\sqrt{3} \times 10^8 = \frac{1}{\sqrt{\mu_o \epsilon_r \epsilon_o}}$$

$$\epsilon_r = 3$$

(c)

$$E_x = 2 \cos(10^8 - \frac{z}{\sqrt{3}}) \implies \cos(10^8 - \frac{z}{\sqrt{3}}) = \frac{E_x}{2}$$

$$E_y = -\sin(10^8 - \frac{z}{\sqrt{3}}) \implies \sin(10^8 - \frac{z}{\sqrt{3}}) = -E_y$$

$$\cos^2(10^8 - \frac{z}{\sqrt{3}}) + \sin^2(10^8 - \frac{z}{\sqrt{3}}) = 1$$

$$\frac{E_x^2}{4} + \frac{E_y^2}{1} = 1$$

Therefore, Elliptically polarized.

(d) we know,

$$\boxed{\vec{H} = \frac{\vec{k} \times \vec{E}}{\eta}} \quad (12)$$

Here,

$$\vec{k} = \hat{z}$$

$$\vec{H} = \hat{z} \times (2 \cos(10^8 t - \frac{z}{\sqrt{3}}) \hat{x} - \sin(10^8 t - \frac{z}{\sqrt{3}}) \hat{y})$$

$$\vec{H} = \sqrt{\frac{\epsilon_r \epsilon_o}{\mu_r \mu_o}} (\sin(10^8 t - \frac{z}{\sqrt{3}}) \hat{x} + 2 \cos(10^8 t - \frac{z}{\sqrt{3}}) \hat{y})$$

$$\vec{H} = 4.59 \times 10^{-3} (\sin(10^8 t - \frac{z}{\sqrt{3}}) \hat{x} + 2 \cos(10^8 t - \frac{z}{\sqrt{3}}) \hat{y}) \left(\frac{A}{m} \right)$$

Answers

1. $f = 15.9MHz$ and $\lambda = 10.88m$.
2. $\epsilon_r = 3$
3. Elliptically polarized.
4. $\vec{H} = 4.59 \times 10^{-3} (\sin(10^8 t - \frac{z}{\sqrt{3}}) \hat{x} + 2 \cos(10^8 t - \frac{z}{\sqrt{3}}) \hat{y}) \left(\frac{A}{m} \right)$