EMA - Assignment - 2

(2) Given, $\mu_r = 1$ need to find ε_r and ω

$\vec{E} = 30\pi e^{j(\omega t - 4/3 y)} \hat{z}$ $\vec{H} = 1 \cdot e^{j(\omega t - 4/3 y)} \hat{x}$

Now, $\frac{|\vec{E}|}{|\vec{H}|} = 30\pi = \sqrt{\frac{\mu}{\varepsilon}}$

$\Rightarrow 30\pi = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_r}{\varepsilon_r}}$

$\Rightarrow 30\pi = 120\pi \sqrt{\frac{1}{\varepsilon_r}}$

$\Rightarrow \frac{1}{4} = \frac{1}{\sqrt{\varepsilon_r}} \Rightarrow \boxed{\varepsilon_r = 16.}$

k = propogation constant $= 4/3$

$k = \omega\sqrt{\mu\varepsilon} \Rightarrow \frac{k}{\sqrt{\mu\varepsilon}} = \omega$

$\omega = \frac{k \times c}{\sqrt{\mu_r \varepsilon_r}} \Rightarrow \frac{\frac{4}{3} \times 3 \times 10^8}{\sqrt{16}}$ m/s

$= 1 \times 10^8$

$\therefore \omega = 1 \times 10^8$ rad.

(4) $\vec{E}(R) = E_0 e^{-j\vec{k} \cdot \vec{R}}$

Now, we need to derive the four maxwells equations for uniform plane wave in source free region.

* Harmonic time dependence : $e^{j\omega t}$

Phasors : $\vec{E}(r) = \vec{E}_0 e^{-j\vec{k}\cdot\vec{r}}$

$\bar{H}(r)$ can be written as $\bar{H}_0 e^{-j\bar{k}\cdot\vec{R}}$

Now, with time dependence; $\bar{H}_0 = \frac{\bar{E}}{\eta}$

$\bar{E} = \underline{E_0} e^{-j\bar{k}\cdot\bar{r}}$ $\quad H = \underline{H_0} e^{-j\bar{k}\cdot\bar{r}}$

constant vectors

$\bar{\nabla}\times\bar{E} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{r}})\times\vec{E_0} = -j\omega\mu H$

$\bar{\nabla}\times\bar{H} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})\times\bar{H_0} = j\omega\varepsilon E$

$\bar{\nabla}\cdot\bar{E} = \bar{\nabla}(e^{-j\bar{k}\cdot\bar{R}})\bar{E}_0 = 0$

$= \bar{\nabla}(e^{-j\bar{k}\cdot\bar{r}})\cdot\bar{H_0} = 0$

$\nabla(e^{-j\bar{k}\cdot\bar{R}}) = e^{-j\bar{k}\cdot\bar{R}}\bar{\nabla}(-j\bar{k}\cdot\bar{R})$

$= e^{-j\bar{k}\cdot\bar{R}}[-j\bar{\nabla}(k_x x + k_y y + k_z z)]$

$= -j(\bar{a}_x k_x + \bar{a}_y k_y + \bar{a}_z k_z)e^{-j\bar{k}\cdot\bar{r}}$

$= -j\bar{k}e^{-j\bar{k}\cdot\bar{r}}$

∴ Maxwell's equations become,

$\bar{k}\times\bar{E} = \omega\mu H$, $\bar{k}\times\bar{H} = -\omega\varepsilon\bar{E}$

$\bar{k}\cdot\bar{E} = 0$, $\bar{k}\cdot\bar{H} = 0$

⑥ Plane wave with an instantaneous expression for electric field.

$E(z,t) = E_0 \sin(\omega t - kz)\hat{x} + E_0 \sin(\omega t - kz + \psi)$

is elliptically polarized. need to find polarization.

at, $z = 0$,

$\vec{E}(0,t) = E_0 \sin(\omega t)\hat{x} + E_{20}\sin(\omega t + \psi)\hat{y}$

for simplicity lets assume $\psi = \pi/2$

$\vec{E}(0,t) = E_{10}\sin(\omega t)\hat{x} + E_{20}\cos(\omega t)\hat{y}$.

$\cos\omega t = \dfrac{E_2(0,t)}{E_{20}}$

$\sin\omega t = \dfrac{E_1(0,t)}{E_{10}}$

$\cos^2\omega t + \sin^2\omega t = 1$

$\Rightarrow \left(\dfrac{E_1(0,t)}{E_{10}}\right)^2 + \left(\dfrac{E_2(0,t)}{E_{20}}\right)^2 = 1$

$\Rightarrow \dfrac{E_1^2(0,t)}{E_{10}^2} + \dfrac{E_2^2(0,t)}{E_{20}^2} = 1$ ⇒ Ellipse equation.

or with som and ψ,

$\vec{E}(0,t) = E_{10}\sin(\omega t)\hat{x} + E_{20}\sin(\omega t + \psi)\hat{y}$

$\vec{E}(0,t) = E_{10}\sin(\omega t)\hat{x} + E_{20}(\sin\omega t\cos\psi + \cos\omega t\sin\psi)\hat{y}$

$\vec{E}(0,t) = (E_{10}\hat{x} + E_{20}\cos\psi)\, E_{20}\sin\omega t$

$+ E_{20}\sin\psi\cos\omega t\,\hat{y}$

$\vec{E}(0,t) = (E_{10}\hat{x} + E_{20}\cos 2\psi\hat{y})\sin\omega t$

$+ (E_{20}\sin 2\psi\hat{y})\cos\omega t$

$\vec{E}(0,t) = E_{10}\sin\omega t\,\hat{x} + E_{20}(\cos 2\psi\sin\omega t$
$+ \sin 2\psi\cos\omega t)\hat{y}$

$E_x = E_{10}\sin\omega t$

$E_y = (\cos 2\psi\sin\omega t + \sin 2\psi\cos\omega t)E_{20}$

$E_{10}E_y - E_{20}E_x = E_{20}E_{10}\cos 2\psi\sin\omega t$

$E_x = E_{10}\sin\omega t$

$$\left(\frac{E_{10}E_y - E_{20}E_x}{\cos 2\psi}\right)^2 + (E_x E_{20})^2 = E_{20}^2 E_{10}^2$$

$$\Rightarrow E_x^2\left(\frac{E_{20}^2}{\cos^2 2\psi} + E_{20}^2\right) + E_y^2\left(\frac{E_{10}^2}{\cos^2 2\psi}\right) - 2E_xE_y\left(\frac{E_{20}E_{10}}{\cos 2\psi}\right)$$
$$= E_{20}^2 E_{10}^2$$

which is an ellipse.

depending on ψ it can be left or rig
or linearly polarized.

(5) $\vec{E}(t,z) = 2\cos(10^8 t - z/\sqrt{3})\hat{x} - \sin(10^8 t - z/\sqrt{3})\hat{y}$

we know,

$$\omega = 2\pi f = 10^8$$

(a) $\therefore$ $\boxed{f = \frac{10^8}{2\pi} = 0.159 \times 10^8 \text{ Hz}}$

$k = 1/\sqrt{3}$

$$\boxed{\lambda = \frac{2\pi}{k} = \frac{2\pi}{1/\sqrt{3}} = 2\sqrt{3}\pi = 10.88 \text{ m}}$$

(b) $v_p = \frac{\omega}{k} = \frac{10^8}{1/\sqrt{3}} = \sqrt{3} \times 10^8 \text{ m/s}$

$v_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}} = \frac{c}{\sqrt{\varepsilon_r}}$

$\sqrt{3} \times 10^8 = \frac{3 \times 10^8}{\sqrt{\varepsilon_r}} \Rightarrow \boxed{\varepsilon_r = 3}$

$\therefore$ dielectric constant $= 3$

(c) the polarization is linear polarization

$\vec{E}(t,z) = 2\cos(10^8 t - z/\sqrt{3})\hat{x} - \sin(10^8 t - z/\sqrt{3})\hat{y}$

$\cos^2(10^8 t - z/\sqrt{3}) + \sin^2(10^8 t - z/\sqrt{3}) = \frac{E_x^2}{4} + \frac{E_y^2}{1}$

$\therefore \frac{E_x^2}{4} + \frac{E_y^2}{1} = 1$

$\vec{H} = \dfrac{\nabla \times \vec{E}}{-j\omega\mu}$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2\cos(10^8 t - 3/\sqrt{3})^2 & (\quad) & 0 \end{vmatrix}$$

$$\vec{H} = \frac{1}{-j\omega\mu}\Big[\hat{a}_x\Big[-\frac{\partial}{\partial z}(-\sin(10^8 t - 3/\sqrt{3})\Big]$$

$$- \hat{a}_y\Big[-\frac{\partial}{\partial z}(2\cos(10^8 t - 3/\sqrt{3})\Big]$$

$$\vec{H} = \frac{1}{-j\omega\mu}\Big[\hat{x}\Big[\cos(10^8 t - 3/\sqrt{3}) \times \frac{-1}{\sqrt{3}}\Big]$$

$$+ \hat{y}\Big[-2\sin(10^8 t - 3/\sqrt{3}) \times -1/\sqrt{3}\Big]$$

$$\vec{H} = \frac{j}{\omega\mu}\Big[-\frac{\hat{x}}{\sqrt{3}}\cos(10^8 t - 3/\sqrt{3}) + \frac{2\hat{y}}{\sqrt{3}}\sin(10^8 t - 3/\sqrt{3}$$

$$\vec{H} = \frac{1}{\omega\mu} = \cancel{\frac{1}{\omega \eta_0 \sqrt{\varepsilon_r}}} = \cancel{\frac{1}{10^8 \times 120\pi \times \sqrt{3}}} = \cancel{\frac{1}{}}$$

$\cancel{\eta_0/2 \sqrt{\frac{\mu}{\varepsilon}}}$ $\cancel{\frac{1}{10^8}}$

$\sqrt{\frac{\mu}{\varepsilon}} = 3\times10^8$ $\sqrt{\mu\varepsilon}$

$\cancel{\mu = 9\times10^{16}\,\varepsilon}$ $\mu = \eta$ $\sqrt{\frac{\mu}{\varepsilon}} = \eta$

$$\frac{1}{\sqrt{3}\omega\mu} \qquad \frac{1}{\sqrt{3}} = k$$

$$\Rightarrow \frac{k}{\omega\mu} = \frac{\omega\sqrt{\varepsilon\mu}}{\omega\mu} = \sqrt{\frac{\varepsilon}{\mu}} = \frac{\sqrt{3}}{120\pi}$$

$\frac{\mu_0}{\varepsilon_0\varepsilon_r} = \eta_0^2$ $\frac{\varepsilon\sqrt{\varepsilon_r}}{\eta_0}$ $\frac{\mu_0 = \eta^2\varepsilon_r}{\varepsilon_0}$

$$\vec{H} = \frac{\sqrt{3}}{120\pi} j\Big[-\hat{x}\cos(10^8 t - 3/\sqrt{3}) + 2\hat{y}\sin(10^8 t - 3/\sqrt{3})\Big]$$

(3) $\nabla \times \nabla (\nabla^2 + \omega^2 \mu \varepsilon) E = 0$:- helmholtz

$\nabla^2 E + \omega^2 \mu \varepsilon E = 0$

$\nabla^2 E = -\omega^2 \mu \varepsilon E$

$\nabla^2 (E_0 e^{-j(k_x x + k_y y + k_z z)}) = -\omega^2 \mu \varepsilon E$

$(-j)^2 (k_x^2 + k_y^2 + k_z^2) \cancel{E} = -\omega^2 \mu \varepsilon \cancel{E}$

$$\boxed{k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon} \quad (1)$$

∴ this vector satisfies helmholtz only for the condition (1)

(1) $\bar{E} = 0.2 \sin(10\pi y) \cos(6\pi 10^9 t - \beta z)\,\hat{x}$

$= 0.2 \sin(10\pi y) e^{-j\beta z} \hat{x}$ (phasor)

$\bar{E} = \frac{1}{2} \times 0.2 [\sin(10\pi y + 6\pi 10^9 t - \beta z)$

$+ \sin(10\pi y - 6\pi \times 10^9 t + \beta z)] \hat{x}$

$H(R) = \frac{-1}{j\omega\mu} \nabla \times E(R)$

$= \frac{-1}{j\omega\mu} 0.2 \times [\nabla \times (0.2 \sin(10\pi y) \cos(6\pi 10^9 t - \beta z)\,\hat{i})$

$= \frac{-0.2}{j\omega\mu} [-\hat{y} \cdot (-(-\sin(6\pi \times 10^9 t - \beta z)(-\beta) \sin(10\pi y))$

$+ \hat{z} \cdot (-\cos(10\pi y) 10\pi \cdot \cos(6\pi 10^9 t - \beta z)$

$= \frac{-0.2}{j\omega\mu} [\hat{y} \cdot \sin(6\pi \times 10^9 t - \beta z) \sin(10\pi y)\,\beta$

$- \hat{z} \cdot \cos(10\pi y) \cos(6\pi \times 10^9 t - \beta z) 10\pi$

for β, we can substitute E into

helmholtz

$$\nabla^2 E + \frac{\omega^2}{c^2} E = 0$$

$$\frac{\partial^2}{\partial y^2} E + \frac{\partial^2}{\partial z^2} E + \frac{\omega^2}{c^2} E = 0$$

$$-100\pi^2 E - \beta^2 E + \omega^2/c^2 \, E = 0$$

$$\omega^2 = \frac{\cancel{6\pi \times 10^9} \, \cancel{6\pi \times 10^9}}{\cancel{9 \times 10^{16}}}$$

$$\omega^2 = 4\pi^2 \times 100$$

$$\beta = \sqrt{\frac{\omega^2}{c^2} - 100\pi^2}$$

$$= \sqrt{400\pi^2 - 100\pi^2}$$

$$= \sqrt{3}\,\pi \times 10$$

$$= 54.14 \text{ rad/m}$$

substituting, β

$$H_x = \frac{-0.2}{j\omega\mu} \Big[\hat{y} \cdot \sin(6\pi \times 10^9 t - 54.14z) \sin 10\pi y \times 54.14$$

$$- \hat{z} \cos(10\pi y) \cos(6\pi \times 10^9 t - 54.14z) \times 10\pi \Big]$$