

# EP3110 - Electro-magnetics and Applications

froufroujaguar24964

August 2, 2022

### **Abstract**

**Instructor:** P Murugavel (HSB 116B)

**Lecture Timings:** D Slot (Mon – 11am, Tue – 10am, Wed - 9am)

**Venue:** HSB 263

**Course Outline:** Learning objectives This is an intermediate level course in electromagnetic fields and assumes background in electrostatics, magneto-statics and introductory knowledge in electrodynamics. The main objective is to introduce electromagnetic fields with emphasis on analytical rigour and physical reasoning required for solving problems having direct application. The course will also provide sufficient background to motivate students to take up advanced levels courses such as, electromagnetic scattering, computational electrodynamics, etc. Learning outcomes Upon successful completion, the students will have learned i. the importance of constitutive properties of materials and their use in applications, ii. the effect of boundaries and be able to develop and analyze optical coatings, iii. time dependent formulation of potentials and fields, and fields due to moving charges, iv. the fundamental ideas in electromagnetic scattering with relevance to applications and v) the concept of waveguides and propagation of guided electromagnetic waves. **Pre-requisite:** PH1020 - Physics II

**Grading:** Assignments - 10 marks Quiz I - 20 marks Quiz II - 20 marks End-semester - 50 marks

# Contents

<b>1</b>	<b>Electrodynamics(Review)</b>	<b>2</b>
1.1	Electrostatics(Review) . . . . .	2
1.2	Magnetostatics(review) . . . . .	5
<b>A</b>	<b>Additional Proofs</b>	<b>12</b>
A.1	Proof of ?? . . . . .	12

# Chapter 1

## Electrodynamics(Review)

### Lecture 1: First Lecture

#### 1.1 Electrostatics(Review)

25 July 2022

Two postulates(propositions) in Electrostatics

**Proposition 1.1.1.**

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$
$$\oint_s \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\epsilon_o}$$

**Proposition 1.1.2.**

$$\vec{\nabla} \times \vec{E} = \vec{0}$$
$$\oint_c \vec{E} \cdot d\vec{l} = 0$$

Electric field around a closed path is 0.

**Theorem 1.1.1.** kirchhoff's voltage Law, Algebraic sum of voltage drop in a closed loop is zero.

**Theorem 1.1.2** (Coloumbs Law).  $\vec{E}$  due to a point charge "q" is

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \frac{q}{|\vec{R}|^3} \vec{R}$$

For a discrete distribution of charges

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \sum_{j=1}^n \frac{q_j}{|\vec{R} - \vec{R}_j|^3} (\vec{R} - \vec{R}_j)$$

For a continuos distribution of charges

$$\vec{E} = \frac{1}{4\pi\epsilon_o} \int \hat{R} \frac{\rho}{R^2} dV'$$

since,

$$\vec{\nabla} \times \vec{E} = \vec{0}$$

$$\vec{E} = -\vec{\nabla} V$$

Negative sign because potential decreases in the direction of electric field by convention.

**Definition 1.1.1.**

$$\vec{E} = -\vec{\nabla}V$$

$$V_2 - V_1 = \int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$$

$$V(R) = \frac{1}{4\pi\epsilon_o} \int \frac{\rho}{R} dV'$$

prime indicates coordinate system w.r.t source.

**Definition 1.1.2 (Dipole).** A pair of equal and opposite charges separated by a distance.

$$V_p(R) = \frac{q}{4\pi\epsilon_o} \left[ \frac{1}{R_+} - \frac{1}{R_-} \right]$$

$$V_p(R) = \frac{qd \cos \theta}{4\pi\epsilon_o R^2} = \frac{\vec{p} \cdot \hat{R}}{4\pi\epsilon_o R^2}$$

$$\vec{E}_p(R) = -\vec{\nabla}V_p(R)$$

$$\vec{E}_p(R) = \left[ -\hat{R} \frac{\partial}{\partial R} - \frac{\hat{\theta}}{R} \frac{\partial}{\partial \theta} \right] V_p(R)$$

$$\vec{E} = \frac{\vec{p}}{4\pi\epsilon_o R^3} [2 \cos \theta \hat{R} + \sin \theta \hat{\theta}]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_o R^3} [3(\vec{p} \cdot \hat{R}) \hat{R} - \vec{p}]$$

## Conductor

$\vec{E} = 0$  inside a conductor.

$$\oint_c \vec{E} \cdot d\vec{l} = 0$$

$$\vec{E}_{t1} \cdot \Delta \vec{w} - \vec{E}_{t2} \cdot \Delta \vec{w} = 0$$

$\vec{E}_{t1} = 0 \because$  inside a conductor.

$\therefore \vec{E}_{t2} = 0.$

$\therefore$  No tangential electric field.

$$\frac{\sigma_s \Delta s}{\epsilon_o} = [\vec{E}_1 \cdot \vec{n}_1 + \vec{E}_2 \cdot \vec{n}_2] \Delta s$$

$$\vec{E}_2 = 0$$

inside a conductor.

$$\vec{E}_1 = \frac{\sigma_s}{\epsilon_o} \hat{n}$$

Normal electric field is  $\vec{E}_n = \frac{\sigma_s}{\epsilon_o} \hat{n}$

## Dielectric in a static $\vec{E}$

$$\vec{p} = q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Gauss law in differential form

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o} = \frac{1}{\epsilon_o} [\rho_f + \rho_b]$$

**Definition 1.1.3.** The *polarization* of a medium  $\vec{P}$  gives the electric dipole moment per unit volume of the material.

$$\vec{P} = \frac{n\vec{p}}{V}$$

Bound charges

$$\sigma_b = -\vec{P} \cdot \hat{n}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

Potential  $V$ ,

$$V = \frac{1}{4\pi\epsilon_o} \oint \frac{\vec{P} \cdot \hat{n}}{R} ds + \frac{1}{4\pi\epsilon_o} \int \frac{-\vec{\nabla} \cdot \vec{P}}{R} dV$$

refer David J Griffith's for detailed explanation.

For a Dielectric,

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_o} (\rho_f - \vec{\nabla} \cdot \vec{P})$$

$$\vec{\nabla} \cdot [\epsilon_o \vec{E} + \vec{P}] = \rho_f$$

**Definition 1.1.4.** Displacement vector,  $\vec{D} = \epsilon_o \vec{E} + \vec{P}$

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

## Lecture 2: Second Lecture

For a **Linear isotropic homogenous dielectric**

26 July 2022

$$\vec{P} = \epsilon_o \chi_e \vec{E}$$

$\chi_e$  = electric susceptibility.

$$\vec{D} = \epsilon_o (1 + \chi_e) \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} = 1 + \chi_e$$

## Boundary Conditions

### Tangential Component

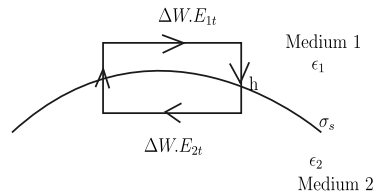


Figure 1.1: Tangential Component

$$\oint \vec{E} \cdot d\vec{l} = 0 \Rightarrow E_{1t} \Delta \vec{W} - E_{2t} \Delta \vec{W} = 0$$

$$E_{1t} = E_{2t}$$

$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

## Normal component

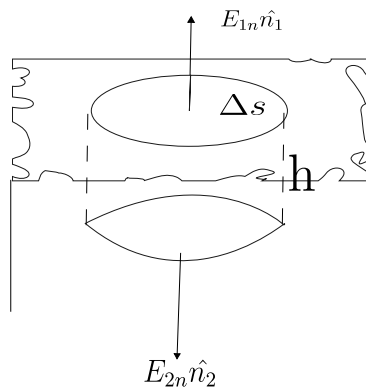


Figure 1.2: Normal Component

$$E_{1n} \Delta s - E_{2n} \Delta s = \frac{\Delta s \sigma_s}{\epsilon_o}$$

$$\frac{D_{1n}}{\epsilon_1} - \frac{D_{2n}}{\epsilon_2} = \frac{\sigma_s}{\epsilon_o}$$

## Summary

$$E_{1t} = E_{2t}$$

$$E_{1n} - E_{2n} = \frac{\sigma_s}{\epsilon_o}$$

$$\vec{D} = \epsilon \vec{E}$$

Energy stored

$$W = \frac{\epsilon_o}{2} \int E^2 d\tau$$

Poisson's equation

$$\nabla^2 V = \frac{\rho}{\epsilon_o}$$

Laplace equation

$$\nabla^2 V = 0$$

## 1.2 Magnetostatics(review)

### Definition 1.2.1.

$$\vec{\nabla} \cdot \vec{B} = 0$$

Magnetic monopole doesn't exist Integral Form:

$$\oint_S \vec{B} \cdot d\vec{a} = 0$$

### Definition 1.2.2.

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J}$$

Integral form:

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I$$

**Definition 1.2.3** (Bio-Savart's law).

$$\vec{B}(R) = \frac{\mu_o}{4\pi} \int I \frac{d\vec{l}' \times \hat{R}}{R^2}$$

**Definition 1.2.4** (Magnetic Vector Potential).

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot \vec{A} = ?$$

$$\vec{\nabla} \times \vec{B} = \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_o \vec{J}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_o \vec{J}$$

$$\vec{A} = \vec{A}_o + \vec{\nabla} \lambda$$

$$\vec{B} = \vec{\nabla} \times \vec{A}_o + \vec{\nabla} \times \vec{\nabla} \lambda$$

$$\vec{B} = \vec{\nabla} \times \vec{A}_o$$

$$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot \vec{A}_o + \nabla^2 \lambda$$

let

$$\nabla^2 \lambda = -\vec{\nabla} \cdot \vec{A}_o \Rightarrow \vec{\nabla} \cdot \vec{A} = 0$$

Solving using symmetry

$$\nabla^2 \lambda = -\vec{\nabla} \cdot \vec{A}_o \Rightarrow \vec{\nabla} \cdot \vec{A} = 0$$

Poisson's equation

$$\nabla^2 V = -\frac{\rho}{\epsilon_o}$$

$$V = \frac{1}{4\pi\epsilon_o} \int \frac{\rho}{R} dV'$$

$$\lambda = \frac{1}{4\pi} \int \frac{\vec{\nabla} \cdot \vec{A}_o}{R} dV'$$

$$\therefore \vec{A} = \vec{A}_o + \vec{\nabla} \lambda$$

$$\nabla^2 \vec{A} = -\mu_o \vec{J}$$

Poisson's equation again!!!

$$\vec{A} = \frac{\mu_o}{4\pi} \int \frac{\vec{J}}{R} dV'$$

**Definition 1.2.5** (magnetic dipole moment).

$$\vec{m} = I \int d\vec{s} = \vec{I}a (\because a = \text{area})$$

$$\vec{A} = \frac{\mu_o \vec{m} \times \hat{R}}{4\pi R^2}$$

**Definition 1.2.6** (Magnetization).

$$\vec{M} = \frac{n\vec{m}}{V}$$

V = volume, n = number of dipoles.



$$\vec{J}_b = \vec{\nabla} X \vec{M}$$

$$\vec{k}_b = \vec{M} X \hat{n}$$

similar to  $\sigma_b$  and  $\rho_b$

$$\vec{\nabla} X \vec{B} = \mu_o (\vec{J}_f + \vec{J}_b)$$

$$\frac{1}{\mu_o} \vec{\nabla} X \vec{B} = \vec{J}_f + \vec{\nabla} X \vec{M}$$

$$\vec{\nabla} X \left( \frac{\vec{B}}{\mu_o} - \vec{M} \right) = \vec{J}_f$$

**Definition 1.2.7.**  $\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M}$

$$\vec{\nabla} X \vec{H} = \vec{J}_f$$

Ampere's law in magnetic material.

Similar to Polarization  $\vec{P} = \epsilon_o \chi_e \vec{E}$  we have Magnetization  $\vec{M} = \frac{1}{\mu_o} \chi_m \vec{B}$  for a linear homogenous isotropic material, where  $\chi_m$  is Magnetic susceptibility.

$$\vec{H}(\mu_o)(1 + \chi_m) = \vec{B}$$

$$\vec{B} = \mu \vec{H}$$

$$\frac{\mu}{\mu_o} = \mu_r = 1 + \chi_m$$

## Lecture 3: Third Lecture

### Boundary Conditions

27 July 2022

### Normal Component

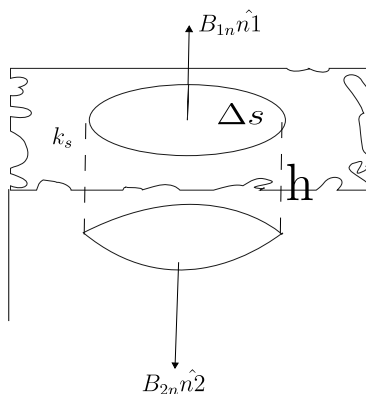


Figure 1.3: title

$$\oint \vec{B} \cdot d\vec{s} = 0$$

$$B_{1n} \Delta S - B_{2n} \Delta S = 0 \Rightarrow B_{1n} = B_{2n}$$

$$\oint \vec{H} \cdot d\vec{l} = I_f$$

## Tangential Component

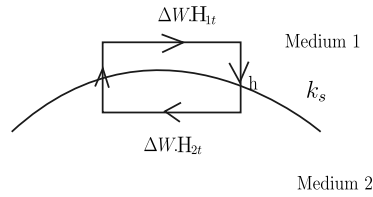


Figure 1.4: title

$$H_{1t}\Delta W - H_{2t}\Delta W = \Delta W k_s$$

$$H_{1t} - H_{2t} = k_s$$

Where,  $k_s$  = surface current density.

$$\vec{n}_2 \times (\vec{H}_1 - \vec{H}_2) = \vec{k}_s$$

## Ohms law

$$\vec{J} = \sigma \vec{E}$$

$$\epsilon = -\frac{dQ}{dt} = \oint \vec{E} \cdot d\vec{l}$$

Faraday's laws of induction

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Energy stored in a Magnetic Field

$$W = \frac{1}{2\mu_o} \int B^2 d\tau$$

## Summary of Equations

Gauss law

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

Magnetic monopole doesn't exist

$$\vec{\nabla} \cdot \vec{B} = 0$$

Faraday's law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere's law

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

### 1.2.1 Correction in Ampere's law

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = 0,$$

but

$$\vec{\nabla} \cdot \vec{J}_f \neq 0$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + ?$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{J}_f + \frac{\partial \rho_f}{\partial t}$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{J}_f + \frac{\partial \vec{\nabla} \cdot \vec{D}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

**Definition 1.2.8.** Displacement current density  $\frac{\partial \vec{D}}{\partial t}$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

### Non-linear dielectric

$$\vec{P}_i = \sum_j \alpha_{ij} E_j + \sum_{jk} \alpha_{ijk} E_j E_k$$

### Linear dielectric

$$\vec{P}_i = \epsilon_o \chi_{ij} \vec{E}_{jk}$$

**Example.**

$$P_x = \epsilon_o (\chi_{xx} E_x + \chi_{xy} E_y + \chi_{xz} E_z)$$

$$P_y = \epsilon_o (\chi_{yx} E_x + \chi_{yy} E_y + \chi_{yz} E_z)$$

$$P_z = \epsilon_o (\chi_{zx} E_x + \chi_{zy} E_y + \chi_{zz} E_z)$$

### Linear Isotropic Dielectric

$\chi \rightarrow$  independent of direction of electric field.

### Linear Homogenous Isotropic Dielectric

$\chi \rightarrow$  Same everywhere.

### Potential formulation

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (\because \vec{\nabla} \cdot \vec{B} = 0)$$

Faraday's law

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} = \frac{\partial (\vec{\nabla} \times \vec{A})}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \frac{\partial \vec{A}}{\partial t}$$

$$\vec{\nabla} \times \left( \vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla} V$$

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Solution for time-independent can be given by poisson's equations, but for time dependent we cannot use poisson's equations.

#### Conditions

1. At low frequency :- changes very slowly
2. Near to source :- quasi-static condition

---

## Lecture 4: Fourth Lecture

### Wave equation for potentials

01/08/2022

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{J}_f + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu \vec{J}_f + \mu \epsilon \frac{\partial \left( -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t} \right)}{\partial t}$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}_f - \mu \epsilon \frac{\partial(\vec{\nabla}V)}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$-\nabla^2 \vec{A} + \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = \mu \vec{J}_f - \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}(\mu \epsilon \frac{\partial V}{\partial t})$$

$$-\nabla^2 \vec{A} + \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = \mu \vec{J}_f - \vec{\nabla}(\vec{\nabla} \cdot \vec{A} + \mu \epsilon \frac{\partial V}{\partial t})$$

$$\nabla^2 \vec{A} - \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J}_f + \vec{\nabla}(\vec{\nabla} \cdot \vec{A} + \mu \epsilon \frac{\partial V}{\partial t})$$

applying **Lorentz gauge**

$$\vec{\nabla} \cdot \vec{A} = -\mu \epsilon \frac{\partial V}{\partial t}$$

$$\nabla^2 A - \mu \epsilon \frac{\partial^2 A}{\partial t^2}$$

# Appendix

# Appendix A

## Additional Proofs

### A.1 Proof of ??

We can now prove ??.

**Proof of ??.** See [https://en.wikipedia.org/wiki/Mass%E2%80%93energy\\_equivalence](https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence). ■