High Energy Physics PH5211 - Problem Set 1

To be submitted in class on Wednesday 10th August 2022

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Suggested references:

- Krane, Chapter 3

1. Consider elastic scattering of an electron from a nucleus that is treated as an extended spherical charge distribution. From Fermi's golden rule the rate at which the scattering occurs will be proportional to the matrix element squared $|V'_{if}|^2$, where

$$V'_{if} = \int \psi_f^* V(r) \psi_i d\tau \ ,$$

$\psi_{i(f)}$ are the initial (final) state wavefunction of the electron $e^{i\vec{\mathbf{k}}_{i(f)}\cdot\vec{\mathbf{r}}}$ with momentum $\hbar\vec{\mathbf{k}}_{i(f)}$ and $V(r)$ is the potential due to the nucleus. The potential is

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0}\int \frac{\rho_e(r')}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} d\tau'$$

where $\vec{\mathbf{r}}$ is the position of the electron and $\vec{\mathbf{r}}'$ is the position of an infinitesimal piece of the charge distribution $\rho_e(r')$. Here $\int \rho_e(r')d\tau' = 1$. Introducing $\vec{\mathbf{q}} = \vec{\mathbf{k}}_i - \vec{\mathbf{k}}_f$ and $\vec{\mathbf{R}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}'$ show that after two angular integrations

$$V'_{if} = -\frac{Ze^2 F(\vec{\mathbf{q}})}{2i\epsilon_0 q}\int_0^\infty \left(e^{iqR} - e^{-iqR}\right) dR \ ,$$

where $F(\vec{\mathbf{q}}) = \int \rho_e(r') e^{i\vec{\mathbf{q}}\cdot\vec{\mathbf{r}}'} d\tau'$ is the Fourier transform of the charge distribution known as the form factor.

Perform the R integral by inserting a factor $e^{-\mu R}$ in the integrand, then taking $\mu \to 0$ in the result, to show that

$$V'_{if} \propto -\frac{F(\vec{\mathbf{q}})}{q^2} \ .$$

$F(\vec{\mathbf{q}}) = (1 + q^2/0.71)^{-2}$ is a good approximation of the proton-electron scattering form factor, where q^2 is measured in $(\mathrm{GeV}/c)^2$. Take the definition of the form factor given above, then consider small values of q^2 to estimate the mean square radius of the proton. *Hints: (i) write $\vec{\mathbf{q}} \cdot \vec{\mathbf{r}}' = qr' \cos\theta'$ then perform the angular integrations. (ii) The mean square radius is $\int r'^2 \rho_e d\tau'$. (iii) You will need $\hbar c = 0.197$ GeV fm.*

2. What is meant by the 'size' of the atomic nucleus? Describe briefly (one paragraph) a method other than scattering to measure the radius. Define R_0 and state how the radius varies with atomic mass number A. What does that imply about the nucleur force?

In the semi-empirical mass formula, the mass of the nucleus is given by

$$M = NM_n + ZM_p - a_v A + a_s A^{2/3} + a_{sym}\frac{(N-Z)^2}{A} + a_c \frac{Z(Z-1)}{A^{1/3}} + \delta(A, Z) \ .$$

Explain the origin of each term and derive an expression for a_c in terms of $e^2/4\pi\epsilon_0$ and R_0.

In the decay ${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + e^+ + \nu_e$ the maximum kinetic energy of the positron is 1.69 MeV. Use this result to estimate R_0. [$m_p c^2 = 938.280$ MeV, $m_n c^2 = 939.573$ MeV, $m_e c^2 = 0.511$ MeV and the neutrino is considered massless.]

3. The figure shows a plot of the average nuclear binding energy per nucleon as a function of mass number A. Also indicated are several specific nuclei with proton number Z and neutron number N

   (a) Define the nuclear binding energy B and show how the mass of a nucleus may be calculated from this plot, taking as an example that labelled $Z = N = 20$.

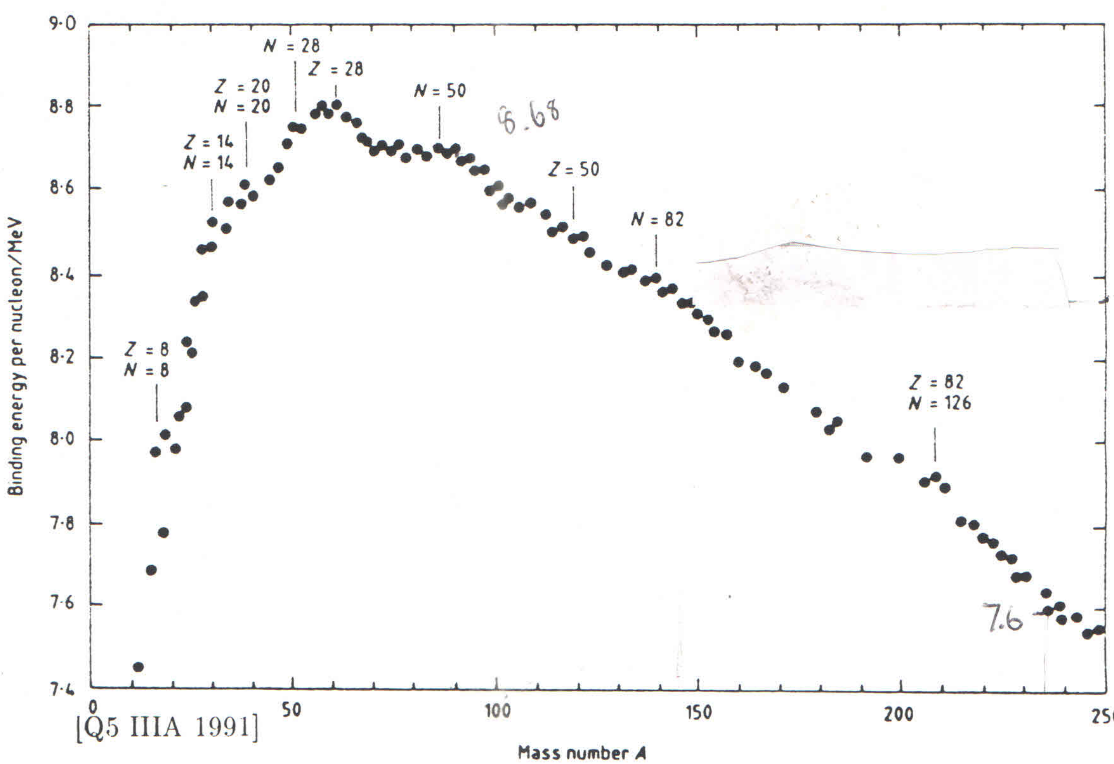
   (b) Offer an explanation for the general steep upward trend of the plot in the region of light nuclei.

   (c) How do you explain the fact that, for points shown with A larger than 10, the average binding energy per nucleon varies by no more than $\pm 10\%$.

   (d) Make a rough sketch of how you would expect the nuclear mass to vary with respect to Z for a constant $A \sim 100$. Indicate regions of instability and what sort of decay processes you would expect.

   (e) Suggest an explanation for the fact that on Earth and in stellar spectra, nuclei of mass higher than iron ($A = 56,\ Z = 26$) are relatively rare.

   (f) Estimate the energy released when a nucleus of ${}^{235}_{92}U$ undergoes fission into the fragments ${}^{87}_{35}\mathrm{Br}$ and ${}^{145}_{57}\mathrm{La}$, along with three prompt neutrons.



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