Note template

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Abstract

This is a note template, with all but minimal compilable files provided. Feel free to adjust for your usage.

Now let's start a simple demo for you to take fancy notes in $\LaTeX$!

Contents

Chapter 1

Entropy

Definition 1.0.1 (Entropy). A measure of uncertainty of a physical system.

$$H(x) = H(p_1, p_2, \dots p_n) = -\sum_x p_x \log p_x$$

$$\lim_{p \to 0} p \log p = 0$$

X - Information we gain, on an average when we learn the value of X.

Example. Coin toss :- HHHH - H, if it gives only heads, Information gain is zero.

Operational interpretation of entropy

Entropy is tied to memory resources.

Example. X takes values (x_1, x_2, x_3, x_4) with probability $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$
encoding them with $(0, 10, 110, 111) \Rightarrow \frac{1}{2}[1] + \frac{1}{4}[2] + \frac{1}{8}[3] + \frac{1}{8}[3] = \frac{7}{4}$bits

$$-\sum_{x=1}^{4} p_x \log p_x = \frac{7}{4}\text{bits}$$

Example. For a coin $p_H = 1$ and $p_T = 0$ size of memory $= 0$

Entropy from intuitive axioms

1. I(p)
2. I(p) is smooth
3. I(pq) = I(p)+I(q)

Properties of Entropy

$$H_{bin}(p) = -p \log p - (1-p) \log(1-p)$$

get a quadratic curve

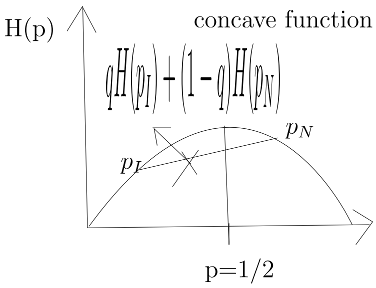

Figure 1.1: title

$$H(qp_I + (1-q)p_N) \geq qH(p_I) + (1-q)H(p_N)$$
$$f(px + (1-p)y) \geq pf(x) + (1-p)f(y)$$

Relative Entropy

Definition 1.0.2.

$$H(p(x) \mid\mid q(x)) = -\sum_{x=1}^{n} p(x) \log \frac{q(x)}{p(x)}$$

Theorem 1.0.1.

$$H(p(x) \mid\mid q(x)) = \sum p(x) \log \frac{p(x)}{q(x)} \text{is non-negative}$$
$$= 0 \text{ iff p(x) = q(x) for all x}$$

Chapter 2

Tensor products

There is a ball which can be red or blue the quantum state associated with it look like

$$|\psi_1\rangle = \alpha\,|r\rangle + \beta\,|b\rangle$$

what if I have 2 balls?

$$|\psi_2\rangle = \alpha\,|r,r\rangle + \beta\,|r,b\rangle + \gamma\,|b,r\rangle + \delta\,|b,b\rangle$$

Each vectors have their own vector spaces. Let the 2 vector spaces be V, W. where,

$$V = v_1, v_2, \dots v_n$$

$$V \otimes W = Tensorproductspace$$

$$Dimension(V \otimes W) = nm$$

Representation

$$a = (a_1, a_2, \dots a_n) \in V$$

$$b = (b_1, b_2, \dots b_n) \in W$$

$$a \otimes b = \begin{bmatrix} a_1 b_1 \\ a_1 b_2 \\ \vdots \\ a_n b_n \end{bmatrix} \tag{2.1}$$

$$\langle a \otimes b | c \otimes d \rangle = \langle a|c\rangle \langle b|d\rangle$$

$$A \in L(v) \dots B \in L(W) \Rightarrow L(V \otimes W)$$

$$(A \otimes B)(a \otimes b) = A\,|a\rangle \otimes B\,|b\rangle$$

$$\sum_{i,j} c_{ij}(A \otimes B)$$

$\boxed{C = UDV}$ where U and V are Unitary matrices and D is a diagonal matrix.

$$[c_{jk}] = \sum_{i,j,k} U_{ji} D_{ii} V_{ik}$$

$$|\psi_{AB}\rangle = |\psi'_A\rangle \otimes |\psi'_B\rangle \rightarrow \text{ Separable State}$$

$$\frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} \text{ Is a Separable state}$$

$|\psi\rangle$ is a state of the composite system AB.
$|i_A\rangle$ for A $|i_B\rangle$ for B

$$|\psi_{AB}\rangle = \sum_i \lambda_i |i_A\rangle |i_B\rangle \rightarrow \text{Schmidt decomposition / basis}$$

if λ is only one value then it is separable. or else it is entangled.

$$\sum_i \lambda_i (U |i_A\rangle) |i_B\rangle \text{ it is also a Schmidt decomposition.}$$

$$\psi_{AB} = \sum c_{jk} |j\rangle |k\rangle = \sum_{i,j,k} U_{ji} d_{ii} V_{ik} |j\rangle |k\rangle$$

$$\sum U_{ji} |j\rangle = |i_A\rangle$$

$$\sum V_{ik} |k\rangle = |i_B\rangle$$

Example. Consider 2 states, which is more entangled?

$$|\psi_1\rangle = \sqrt{0.99999} |0\rangle |0\rangle + \sqrt{0.00001} |1\rangle |1\rangle$$

$$|\psi_2\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$|\psi_2\rangle$ is more entangled state.

$$Entropy(|\psi\rangle) = -\sum_i \lambda_i^2 \log \lambda_i^2$$

Chapter 3

Density Matrix

$$\rho = \frac{I + \vec{r}.\vec{\sigma}}{2}$$
$$\vec{r} = \text{ 3d vector}$$
$$||\vec{r}|| \leq 1$$

3.1 The reduced density operator

1. deepest
2. elegant

$$\rho_{AB}$$
$$\rho^A = tr_B(\rho_{AB})$$

partial trace is a map from vector space of the composite system to the vector space of one of the subsystems.

Definition 3.1.1.

$$tr_B(|a_1\rangle \langle a_2| \otimes |b_1\rangle \langle b_2|) = |a_1\rangle \langle a_2| \langle b_2|b_1\rangle$$

Linear operation.

$$p(x) \rightarrow \hat{\rho}$$
$$\int p(x)dx = 1 \rightarrow Tr(\hat{\rho}) = 1$$
$$\int_y p(x,y)dx = p(x) \rightarrow Tr_B(\hat{\rho}_{AB}) = \hat{\rho}_A$$

$$\hat{\rho} = \hat{\rho}_A \otimes \hat{\rho}_B$$
$$Tr_B(\hat{\rho}_A \otimes \hat{\rho}_B) = \hat{\rho}_A$$

Bell state

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$\hat{\rho}_{Bell} = \frac{|00\rangle \langle 00| + |00\rangle \langle 11| + |11\rangle \langle 00| + |11\rangle \langle 11|}{2}$$
$$\rho' = Tr_2(\hat{\rho}_{Bell}) = \frac{|0\rangle \langle 0| + |1\rangle \langle 1|}{2} \text{ Complete Ignorance}$$

Schmidt decomposition

$$|\psi\rangle = \sum_i \lambda_i |i_A\rangle |i_B\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi| = \sum_{i,j} \lambda_i \lambda_j^* |i_A\rangle\langle j_A| \otimes |i_B\rangle\langle j_B|$$
$$\hat{\rho}_A = \sum_i |\lambda_i|^2 |i_A\rangle\langle i_A|$$
$$\hat{\rho}_B = \sum_i |\lambda_i|^2 |i_B\rangle\langle i_B|$$
$$Tr(\hat{\rho}_A^2) = \sum_i |\lambda_i|^4 < 1$$

$Tr(\hat{\rho}_A^2)$ is called purity.
Measure of metric for entanglement $1 - Tr(\hat{\rho}_A^2)$.
$1 - Tr(\hat{\rho}_A^2)$ is $\frac{1}{2}$ for Mixed states and 0 for separable states.

3.1.1 Purification

Suppose we are given a state $\hat{\rho}_A$ of the quantum system A. It is always possible to introduce another system R and define a pure state $|AR\rangle$ such that the reduced state $Tr_R(|AR\rangle\langle AR|) = \hat{\rho}_A$
$AR \rightarrow$ church of the larger hilbert space.

$$\rho_A = \sum p_i |i_A\rangle\langle i_A|$$
$$\rho_{AR} = \sum \sqrt{p_i} |i_A\rangle |i_R\rangle$$
$$(\text{ Purification }) |AR\rangle\langle AR| = \sum \sqrt{p_i p_j} |i_A\rangle\langle j_A| \otimes |i_R\rangle\langle j_R|$$
$$|AR\rangle\langle AR| = \sum \sqrt{p_i p_j} |i_A\rangle\langle j_A| \delta_{ij}$$

Example. There is a room and ball is in the room where you don't know. You assign a equal probability distribution to the ball. When you measure the ball position the probability distribution collapses to one point. Where is the ball before your measurement?

$$\frac{|{+}{+}\rangle + |{-}{-}\rangle}{\sqrt{2}}$$
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Bell states are locally and rotationally invariant.

3.2 EPR and Bell's Inequality

$$|\psi\rangle = \sum c_i |\psi_i\rangle$$
$$|c_i|^2 \rightarrow |\psi_i\rangle$$

Just before the measurement, what was the value of observable $\hat{A} = \sum_i |\psi_i\rangle\langle\psi_i|$.

Answer. **Copenhagen Interpretation** The particle doesn't have a position before measurement.

⊛

3.2.1 EPR paper

On the elements of reality.
"If, without in anyway disturbing a system, we can predict with certainty (with p = 1) the value of a physical quantity, then there exists an element of physical reality. corresponding to the physical quantity."

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}}$$
$$\vec{v}.\vec{\sigma} = \pm 1$$

3.3 Bell's Inequalities

Not about Quantum mechanics. forget qm.

3.3.1 Statistics of parameter estimation

$$V_{test} = V_{test}(M_1, M_2 \dots M_N) \to X$$

How do my measurements determine the parameter?

$$\sqrt{< (\Delta Y_{test})^2 >} = < Y_{test} - < Y_{test} >^2 > = Variance.$$

2. Estimator has different "units" from the parameter.

$$Mass \propto \Delta L$$

$$Ma_s = \lambda \Delta \Lambda$$

$$\frac{d < M_{test} >}{dl} = \lambda$$

Lecture 6: Interesting things

3.4 Quantum Circuit Model

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1. States represented as wires.
2. Gates represented as boxes.
3. Flow of time is from left to right.

We will see

1. Quantum Algorithms
2. Teleportation
3. Superdense coding

Classical Gates

$$\text{NOT gate } a \to \overline{a} \text{ Quantum gate exist}$$

$$\text{Identity gate } a \to a \text{ Quantum gate exist}$$

$$\text{Fanout gate} a R_1 R_2 \to aaa \text{ Quantum gate does not exist}$$

$$\text{SWAP } a, b \to b, a \text{ Quantum gate exist}$$

NoCloning Theorem

$$|\psi_1\rangle \otimes |R\rangle \xrightarrow{U}$$

Appendix

Appendix A

Additional Proofs

A.1 Proof of ??

We can now prove **??**.

Proof of ??. See https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence. ∎