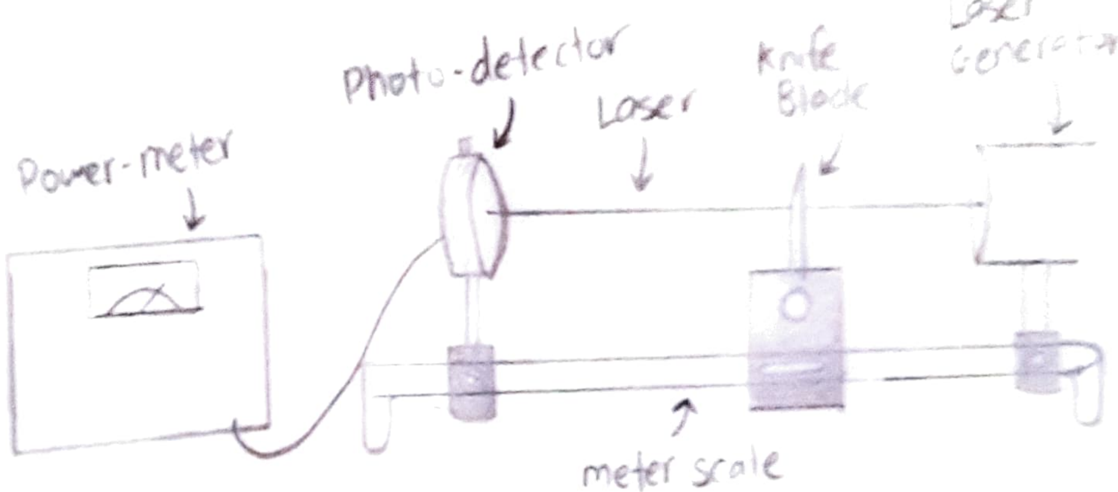
Experiment #1 Laser Beam Parameters

Sumedh Kshitij Ghude — EP20B015

Aim To determine the various parameters of a Laser Beam, specifically Divergence (θ_0), Beam Radius (w_0), and Location of Beam Radius (z)

Apparatus Laser Beam Generator, meter scale, detector, knife blade, Photo detector, lens, Power Meter, etc

Diagram



Theory The technique that will be used to determine Power distribution, spot size, divergence, etc is as follows:

We shine the laser in the direction of a knife edge that is slowly inserted into the path of the laser. Further, the laser is assumed to be oscillating in TEM_{00} mode so that the spatial distribution $I(x,y)$ of the beam is Gaussian. Hence, the irradiance distribution $I(x,y)$ is

$$I(x,y) = \frac{2P_0}{\pi w_0^2} \exp\left(-\frac{2(x^2+y^2)}{w_0^2}\right)$$

where P_0 Total Power of the beam
w_0 (Spot size of beam)/2

Power transmitted past a knife edge blocking of all points $x < a$

$$P = \int_{y=-\infty}^{y=\infty}\int_{x=a}^{x=\infty} I(x,y)\,dx\,dy = \frac{2P_0}{\pi w_0^2}\int_{y=-\infty}^{y=\infty}\int_{x=a}^{x=\infty} \exp\left(-\frac{2(x^2+y^2)}{w_0^2}\right)dx\,dy$$

$$= \frac{P_0}{2}\,\mathrm{erfc}\left(\frac{a\sqrt{2}}{w_0}\right)$$

Now, $\frac{I(x,y)}{P_0} = \underbrace{\frac{1}{2\pi(w')^2} \exp\left(-\frac{(x^2+y^2)}{2w'^2}\right)}_{\text{Normalized Gaussian Form}}$ w': standard deviation

and $\frac{P}{P_0} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a}{\sqrt{2}\,w'}\right)$

It is easy to show that the points with 25% and 75% of relative powers are located at distances equal to the probable error e_p ($e_p \approx 0.6745\,w'$) on either side of the Gaussian Distribution's maximum. Hence, w' can be obtained from the relative power vs knife-edge position curve and beam spot size ($2w_0 = 4w'$) can be calculated.

To determine the Divergence of the laser beam, we once again assume that the laser is oscillating in TEM_{00} mode, and it's output is a Gaussian beam as before. It can be shown that:

$$w(z) = w_0\left(1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right)^{1/2}$$

where $w(z)$: Radius of the beam
z: axis along the direction of propogation
λ: wavelength of laser radiation
w_0: (spot size of beam)/2.

The gradient $\frac{dw(z)}{dz} = \theta(z)$ of the beam radius at z is:

$$\theta(z) = \left(\frac{\lambda}{\pi w_0}\right)^2 \frac{z}{w_0}\left(1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right)^{-1/2}$$

when $z \to \infty$, $(\theta(z) \longrightarrow \theta_0)$ where $\theta_0 = \frac{\lambda}{\pi w_0} = \frac{1}{2}$ Divergence

$w^2(z) = w_0^2 + \theta_0^2 z^2$. Here, w_0, θ_0, and z are all unknown

we choose 3 z values: z, $z+D$, $z+2D$ so that we get:

$$\left.\begin{aligned} w_1^2 &= w_0^2 + \theta_0^2 z^2 \\ w_2^2 &= w_0^2 + \theta_0^2 (z+D)^2 \\ w_3^2 &= w_0^2 + \theta_0^2 (z+2D)^2 \end{aligned}\right\}$$

Form a set of equations which we can solve to find w_0, θ_0, and z.

From these equations, we obtain:

$$2\theta_0^2 D^2 = w_3^2 - 2w_2^2 + w_1^2 \Rightarrow \theta_0 = \frac{1}{\sqrt{2}D}\left(w_3^2 - 2w_2^2 + w_1^2\right)^{1/2}$$

from this θ_0, $w_0 = \frac{\lambda}{\pi\theta_0}$ and $z = \frac{(w_1^2 - w_0^2)^{1/2}}{\theta_0}$.

In this way, Divergence, Beam Spot Size, and Location of Beam Radius, can be calculated.

Observations:

(i) Set of measurements with $z = 300$ mm: (To determine w_1):

S. No.	Position of Knife (mm)	Power Meter Reading (mW)	Normalized
1.	3.4	4.97	1.0
2.	3.3	4.90	0.986
3.	3.2	4.75	0.956
4.	3.1	4.34	0.873
5.	3.0	3.78	0.761
6.	2.9	2.88	0.579
7.	2.8	1.90	0.382
8.	2.7	1.09	0.219
9.	2.6	0.057	0.011
10.	2.5	0.027	0.005
11.	2.4	0.010	0.002
12.	2.3	0.003	0.0006
13.	2.2	0.001	0.0002
14.	2.1	0	0

(ii) Set of readings with $z = z + D = 400mm$: (To determine ω_2)

S. No.	Position of Knife (mm)	Power-Meter Reading (mW)	Normalized.
1.	6.2	4.94	1.0
2.	6.1	4.86	0.984
3.	6.0	4.58	0.927
4.	5.9	4.10	0.830
5.	5.8	3.34	0.676
6.	5.7	2.51	0.508
7.	5.6	1.69	0.342
8.	5.5	0.97	0.196
9.	5.4	0.46	0.093
10.	5.3	0.17	0.034
11.	5.2	0.066	0.013
12.	5.1	0.030	0.0061
13.	5.0	0.015	0.003
14.	4.9	0.010	0.002
15.	4.8	0.0	0

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(iii) Set of readings with $z = z + 2D = 500$ mm (To determine w_3)

S.No.	Position of knife (mm)	Power-Meter Reading (mW)	Normalized
1.	11.4	4.96	1.0
2.	11.3	4.89	0.986
3.	11.2	4.70	0.948
4.	11.1	4.34	0.875
5.	11.0	3.75	0.756
6.	10.9	2.91	0.588
7.	10.8	2.19	0.441
8.	10.7	1.42	0.286
9.	10.6	0.93	0.187
10.	10.5	0.44	0.090
11.	10.4	0.14	0.028
12.	10.3	0.05	0.010
13.	10.2	0.02	0.004
14.	10.1	0.01	0.002
15.	10.0	0.005	0.001
16.	9.9	0.0	0.0

Calculations:

(i) For $z = 300mm$: 75% power = 0.405 mm
25% power = 0.681 mm

$\therefore 2 * \text{error} = |\,75\% \text{ power} - 25\% \text{ power}\,| = |0.405 - 0.681|$

$\Rightarrow \text{error} \approx \frac{0.276}{2} \approx 0.138$

$$\boxed{\therefore \underset{(300mm)}{w_1} = \frac{0.138}{0.6745} \approx 0.205 \text{ mm}}$$

(ii) For $z = 400mm$: 75% power = 0.350 mm
25% power = 0.660 mm

$\therefore 2 * \text{error} = |\,75\% \text{ power} - 25\% \text{ power}\,| = |0.350 - 0.660|$

$\therefore \text{error} \approx \frac{0.31}{2} \approx 0.155$.

$$\boxed{\therefore \underset{(400mm)}{w_2} = \frac{0.155}{0.6745} \approx 0.230 \text{ mm}}$$

(iii) For $z = 500mm$: 75% power = 0.395 mm
25% power = 0.744 mm

$\therefore 2 * \text{error} = |\,75\% \text{ power} - 25\% \text{ power}\,| = |0.395 - 0.744|$

$\Rightarrow \text{error} \approx \frac{0.349}{2} \approx 0.1745$

$$\boxed{\therefore \underset{(500mm)}{w_3} = \frac{0.1745}{0.6745} \approx 0.259 \text{ mm}}$$

Now, Θ_0 (Divergence of ~~light~~ laser beam) is given by:

$$\Theta_0 = \frac{1}{\sqrt{2}D}\left(w_3^2 - 2w_2^2 + w_1^2\right)^{1/2} \quad \text{where } D = 100mm$$

$$= \frac{1}{\sqrt{2}(100)}\left((0.259)^2 - 2(0.23)^2 + (0.205)^2\right)^{1/2}$$

$$\therefore \theta_0 = \frac{1}{\sqrt{200}} \left(0.0671 - 0.\cancel{106}^{1058} + 0.042\right)^{1/2}$$

$$\approx \frac{1}{\sqrt{200}} \sqrt{0.003325} \approx \sqrt{1.6625 \times 10^{-5}} \approx 0.0041.$$

$$\boxed{\therefore \theta_0 \approx 4.1 \times 10^{-3}.\ \text{Divergence of Laser Beam}}.$$

Now, $w_0 = \dfrac{\lambda}{\pi\theta_0}$ where λ: wavelength of the laser used (He-Ne lasers are of $\lambda = 632.8\,nm$).

$$\boxed{\therefore w_0 \approx \frac{632.8\,nm}{(3.1416)(4.1)(10^{-3})} \approx 4.913 \times 10^{4}\ nm \approx 49.13\,\mu m}.$$

and finally, $z = \dfrac{(w_1^2 - w_0^2)^{1/2}}{\theta} = \dfrac{((0.205)^2 - (0.049)^2)^{1/2}}{4.1 \times 10^{-3}}$

$$\boxed{z \approx \frac{0.199}{4.1 \times 10^{-3}} \approx 48.537\,mm}.$$

Error Analysis:

We measure w_1, w_2, and w_3 with the help of position of knife edge (with L.C = 0.02mm) and Power-Meter readings (with L.C = 0.001 mW

~~and $(0.6745)\,w\,(2) = P_{25\%} - P_{75\%}$ $\quad P_{25\%}$: position with 25% Power.~~

~~$\Rightarrow (0.6745)(2)\,dw = dP_{25\%} + dP_{75\%}$~~

~~Now $Power_{25\%} = Power_{100\%} (1/2)\, erfc\left(\frac{P_{25\%}}{\sqrt{2}\,w}\right)$~~

~~$\therefore dPower_{25\%} = Power_{100\%} (1/2) \left(\frac{-2}{\sqrt{\pi}} +\right.$~~

Error in measuring w is given like so:

$$(0.6745)\,w\,(2) = x_{25\%} - x_{75\%}$$

$$\Rightarrow 2(0.6745)\,dw = 2\,dx \Rightarrow dw = \frac{1}{0.6745}\,dx$$

$$\boxed{\therefore \frac{dw}{w} = \frac{0.02\,mm}{0.6745(w)}}.$$

$$\therefore \frac{dw_1}{w_1} = 0.145 \qquad \frac{dw_2}{w_2} = 0.129 \qquad \frac{dw_3}{w_3} = 0.114.$$

Hence, $$\frac{d\theta_0}{\theta_0} = \frac{1}{\theta_0} * \frac{1}{\sqrt{2D}}\left(\tfrac{1}{2}\right)\frac{1}{(\omega_3^2 - 2\omega_2^2 + \omega_1^2)^{1/2}} * 2\left(\frac{d\omega_3}{\omega_2} + \frac{2d\omega_2}{\omega_2} + \frac{d\omega_1}{\omega_1}\right)$$

$$= \frac{1}{\cancel{\theta_0}} * \frac{1}{2D}\left(\frac{1}{2}\right)\cancel{\theta_0} * 2\,(0.114 + 0.258 + 0.145)$$

$$= \frac{0.517}{200} \approx \underline{0.259\,\%}$$

and $$\frac{d\omega_0}{\omega_0} = \left(\underbrace{\frac{d\lambda}{\lambda}}_{=0} + \frac{d\theta_0}{\theta_0}\right)\left(\frac{1}{\pi}\right) = \underline{0.082\,\%}.$$

and finally, $$\frac{dz}{z} = \frac{1}{z}\,d\left(\frac{1}{\theta_0}\cdot\sqrt{\omega_1^2 - \omega_0^2}\right)$$

$$= \frac{1}{z}\left(\frac{(-1)}{\theta_0^2}\,d\theta_0\sqrt{\omega_1^2-\omega_0^2} + \frac{1}{\theta_0} * \frac{1}{2\sqrt{\omega_1^2-\omega_0^2}}\,2\left(\frac{d\omega_1}{\omega_1} + \frac{d\omega_0}{\omega_0}\right)\right)$$

$$= \frac{1}{z} * \frac{\frac{1}{\theta_0}}{\sqrt{\omega_1^2-\omega_0^2}}\left(\frac{d\theta_0}{\theta_0^2}(\omega_1^2 - \omega_0^2) + \frac{d\omega_1}{\omega_1} + \frac{d\omega_0}{\omega_0}\right)$$

$$= ((0.259)(0.0396) + 0.082 + 14.5)\,\%$$

$$= \underline{14.59\,\%}$$

Conclusion: (i) In this way, various parameters of the He-Ne laser are measured.

(ii) Divergence of Laser (θ_0) $= 4.1 * 10^{-3} \pm 0.259\,\%$

Beam Radius (ω_0) $= 49.13\,\mu m \pm 0.082\,\%$

Distance of Beam Radii (z) $= 48.537\,mm \pm 14.59\,\%$

Sources of Error (& Precautions): (i) Care should be taken to ensure that the laser beam, after reflection off of the detector does not enter into the laser apparatus or human eyes.

(ii) Massive error is propogated due to ω_1, ω_2, and ω_3. Ensure that the Power v's knife-edge distance graphs are sufficiently accurate.

(iii) Ensure that the laser, knife-edge, and detector are perfectly aligned with respect to one another so that they form a straight line