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Formulae Sheet

EP3110:- Electro-magnetics and Applications

$$\frac{\omega}{k} = \frac{1}{\sqrt{\mu_r \mu_o \epsilon_r \epsilon_o}} = velocity; \;\; (\lambda = \frac{2\pi}{k}, \omega = 2\pi f) \tag{1}$$

$$\frac{\left|\vec{E}\right|}{\left|\vec{H}\right|} = \eta = \sqrt{\frac{\mu_r \mu_o}{\epsilon_r \epsilon_o}} \quad (\vec{E} = \eta \vec{H} X \hat{k} \text{ and } \vec{H} = \frac{\hat{k} X \vec{E}}{\eta}) \tag{2}$$

Maxwell Equations

$$\vec{\nabla}.\vec{D} = \rho \tag{3}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{4}$$

$$\vec{\nabla} X \vec{E} = -\frac{\partial B}{\partial t} \tag{5}$$

$$\vec{\nabla} X \vec{H} = \vec{J} + \frac{\partial D}{\partial t} \tag{6}$$

$$\vec{\nabla} X \vec{E} = -j\mu\omega\vec{H} \implies \vec{k} X \vec{E} = \omega\mu\vec{H} \tag{7}$$

$$\vec{\nabla} X \vec{H} = j\omega\epsilon\vec{E} \implies \vec{k} X \vec{H} = -\omega\epsilon\vec{E} \tag{8}$$

$$\vec{\nabla}.\vec{E} = 0 \implies \vec{k}.\vec{E} = 0 \tag{9}$$

$$\vec{\nabla}.\vec{H} = 0 \implies \vec{k}.\vec{H} = 0 \tag{10}$$

Helmholtz

$$\nabla^2 \vec{E} + \omega^2 \mu\epsilon \vec{E} = 0 \tag{11}$$

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu\epsilon \tag{12}$$

poynting vector

$$\vec{S} = \vec{E} X \vec{H} (W/m^2) \tag{13}$$

$$\vec{S}(z,t) = Re[\vec{E}(z)e^{j\omega t}] X Re[\vec{H}(z)e^{j\omega t}] \tag{14}$$

$$\vec{S}(z,t) = \frac{1}{2} Re[\vec{E}(z) X \vec{H}^*(z) + \vec{E}(z) X \vec{H}(z) e^{2j\omega t}] \tag{15}$$

$$\vec{S}_{avg}(Z) = \frac{1}{2} Re[\vec{E}(z) X \vec{H}^*(z)] = \frac{E^2}{2\eta} \tag{16}$$

$$Power_{avg} = \oint_S S_{avg}.dS = \int_0^{2\pi} \int_0^{\pi} S_{avg}(R,\theta) R^2 \sin^2\theta \, d\theta d\phi \tag{17}$$

Dipole

$$\vec{E}(R) = \frac{1}{4\pi\epsilon_o R^3} \left(3(\vec{p}.\hat{R})\hat{R} - \vec{p}\right) \tag{18}$$

$$V(R) = \frac{1}{4\pi\epsilon_o} \frac{\vec{p}.\hat{R}}{R^2} \tag{19}$$

Boundary Conditions

$$E_{1t} = E_{2t} \tag{20}$$

$$D_{1n} - D_{2n} = \sigma_f \implies \epsilon_1 E_{1n} - \epsilon_2 E_{2n} = \sigma_f \tag{21}$$

$$B_{1n} = B_{2n} \tag{22}$$

$$H_{1t} - H_{2t} = k_s \implies \frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = k_s \tag{23}$$

Properties of $\vec{\nabla}$

$$\text{Cylindrical} \quad \vec{\nabla}.\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\text{Cylindrical} \quad \vec{\nabla} X \vec{A} = (\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z})\hat{r} + (\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r})\hat{\phi} + (\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi})\hat{z}$$

$$\text{Spherical} \quad \vec{\nabla}.\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial A_\theta \sin\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla} X \vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial A_\phi \sin\theta}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\hat{\phi}$$

Quantity	Any medium	Loss Less	Low loss	Conductor
α- attenuation factor	$\omega\sqrt{\frac{\mu\epsilon'}{2}[\sqrt{1+\frac{\epsilon"}{\epsilon'}}-1]}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sqrt{\pi f\mu\sigma}$
$\beta = \frac{2\pi}{\lambda}$	$\omega\sqrt{\frac{\mu\epsilon'}{2}[\sqrt{1+\frac{\epsilon"}{\epsilon'}}+1]}$	$\omega\sqrt{\mu\epsilon}$	$\omega\sqrt{\mu\epsilon}[1+\frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2]$	$\sqrt{\pi f\mu\sigma}$
η-intrinsic impedance	$\sqrt{\frac{\mu}{\epsilon'}}(1-j\frac{\epsilon"}{\epsilon'})^{-\frac{1}{2}}$	$\sqrt{\frac{\mu}{\epsilon}}$	$\sqrt{\frac{\mu}{\epsilon}}(1+j\frac{\sigma}{2\omega\epsilon})$	$(1+j)\ \frac{\alpha}{\sigma}$
u_p-Phase velocity	$\frac{\omega}{\beta}$	$\frac{1}{\sqrt{\mu\epsilon}}$	$\frac{1}{\sqrt{\mu\epsilon}}[1-\frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2]$	$\sqrt{\frac{4\pi f}{\mu\sigma}}$
u_g - group velocity	$\frac{1}{\frac{d\beta}{d\omega}}$	$\frac{1}{\sqrt{\mu\epsilon}}$	$\frac{1}{\sqrt{\mu\epsilon}}[1+\frac{1}{8}\left(\frac{\sigma}{\omega\epsilon}\right)^2]$	$4\sqrt{\frac{\pi f}{\mu\sigma}}$

$$\boxed{\begin{aligned} \tan(\delta_c) = \frac{\epsilon"}{\epsilon'} = \frac{\sigma}{\omega\epsilon}; \quad \epsilon' = \epsilon = \epsilon_0\epsilon_r \qquad &(24)\\ u_g = \frac{u_p}{1-\frac{\omega}{u_p}\frac{du_p}{d\omega}} \qquad &(25)\\ \frac{E(x)}{E_o} = e^{-\alpha x} \qquad &(26)\end{aligned}}$$

Normal Incidence

$$\boxed{\begin{aligned} \vec{E_1}(z) = \vec{E_i}(z) + \vec{E_j}(z) = a_x E_i(e^{-j\beta z} - e^{j\beta z}) \qquad &(27)\\ \vec{E_1}(0) = 0 \qquad &(28)\\ \vec{H_1}(z) = \frac{1}{\eta}(\hat{a}_z X(E_i) + (-\hat{a}_z)XE_r) = \frac{1}{\eta}E_i(e^{-j\beta z} + e^{j\beta z}) \qquad &(29)\\ \hat{a}_n X\vec{H_1}(0) = \vec{J_s} \qquad &(30)\end{aligned}}$$

Lorentz gauge:- $\vec{\nabla}.\vec{A} = -\mu\epsilon\frac{\partial V}{\partial t}$ **Poisson equations**

$$\boxed{\begin{aligned} \nabla^2 V = -\frac{\rho}{\epsilon_o} \qquad &(31)\\ V = \frac{1}{4\pi\epsilon_o}\int\frac{\rho}{R}dV' \qquad &(32)\\ \nabla^2\vec{A} = -\mu_o\vec{J} \qquad &(33)\\ \vec{A} = \frac{\mu_o}{4\pi}\int\frac{\vec{J}}{R}dV' \qquad &(34)\end{aligned}}$$