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EP20B012

ASSIGNMENT II — JULY-NOV 2022

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Contents

1 Hyperfine Splitting

Splitting of Na ground and excited states

we know,

$$\vec{\mathbf{F}} = \vec{\mathbf{I}} + \vec{\mathbf{J}} \tag{1}$$

$$\Delta E = A\vec{\mathbf{I}}.\vec{\mathbf{J}} \tag{2}$$

$$\implies \frac{A}{2}\left(\left|\vec{F}\right|^2 - \left|\vec{I}\right|^2 - \left|\vec{J}\right|^2\right) \tag{3}$$

$$\implies \frac{A}{2}[F(F+1) - I(I+1) - J(J+1)] \tag{4}$$

For each $\vec{F}$ there is different levels in the range

$$|I - J| \leq F \leq I + J \tag{5}$$

If $I \geq J$ there are 2J+1 levels and if $J \geq I$ there are 2I+1 levels.
Ground state of Na($S_{\frac{1}{2}}$) has $I \geq J$ and J = 1/2. Therefore, 2J+1 = 2 states.
Second excited state of Na ($P_{\frac{3}{2}}$) has $I \geq J$ and J = 3/2. Therefore, 2J+1 = 4 states.

Constant of proportionality A

Ground state splitting,

$$h\Delta v = \frac{6.626X10^{-34}JsX1772X10^{6}Xs^{-1}}{1.6X10^{-19}\frac{J}{eV}} = 7.33\mu eV \tag{6}$$

$$\Delta E(F=2) - \Delta E(F=1) = h\Delta v \tag{7}$$

$$\implies \frac{A}{2}[2(3) - 1(2)] = 7.33\mu eV \tag{8}$$

$$\implies 2A = 7.33\mu eV \tag{9}$$

$$\implies A = 3.67\mu eV \tag{10}$$

First excited state splitting,

$$h\Delta v = \frac{6.626X10^{-34}JsX192X10^{6}Xs^{-1}}{1.6X10^{-19}\frac{J}{eV}} = 0.795\mu eV \tag{11}$$

$$\Delta E(F=2) - \Delta E(F=1) = h\Delta v \tag{12}$$

$$\implies \frac{A}{2}[2(3) - 1(2)] = 0.795\mu eV \tag{13}$$

$$\implies 2A = 0.795\mu eV \tag{14}$$

$$\implies A = 0.397\mu eV \tag{15}$$

Second excited state splitting, between F=0 and F=1,

$$h\Delta v = \frac{6.626X10^{-34}JsX17.1X10^{6}Xs^{-1}}{1.6X10^{-19}\frac{J}{eV}} = 0.07\mu eV \tag{16}$$

$$\Delta E(F=1) - \Delta E(F=0) = h\Delta v \tag{17}$$

$$\implies \frac{A}{2}[1(2) - 0(1)] = 0.07\mu eV \tag{18}$$

$$\implies A = 0.07\mu eV \tag{19}$$

Second excited state splitting, between F=1 and F=2,

$$h\Delta v = \frac{6.626X10^{-34}JsX36.6X10^{6}Xs^{-1}}{1.6X10^{-19}\frac{J}{eV}} = 0.152\mu eV \tag{20}$$

$$\Delta E(F=2) - \Delta E(F=1) = h\Delta v \tag{21}$$

$$\implies \frac{A}{2}[2(3) - 1(2)] = 0.152\mu eV \tag{22}$$

$$\implies 2A = 0.0152\mu eV \tag{23}$$

$$\implies A = 0.0757\mu eV \tag{24}$$

Second excited state splitting, between F=2 and F=3,

$$h\Delta v = \frac{6.626X10^{-34}JsX60.9X10^{6}Xs^{-1}}{1.6X10^{-19}\frac{J}{eV}} = 0.252\mu eV \tag{25}$$

$$\Delta E(F=3) - \Delta E(F=2) = h\Delta v \tag{26}$$

$$\implies \frac{A}{2}[3(4) - 2(3)] = 0.252\mu eV \tag{27}$$

$$\implies 3A = 0.252\mu eV \tag{28}$$

$$\implies A = 0.084\mu eV \tag{29}$$

2 Deuteron

3 Cross-section and Beam Intensity for an interaction

Cross-section for an interaction

It is the measure of the probability of a collision taking place between particles. It is expressed as an area.
The total rate W $\propto$ NI
where,
N = number of exposed targets
I = flux of incoming particles per unit area per unit time.
Therefore, $W = \sigma NI$
where σ is **cross-section** and the **constant of proportionality** with dimensions of area.

Beam Intensity

Assume a small length dx of solid perpendicular to the beam of area A. The change in intensity is -dI. This is equal to number of particles removed from the beam per unit area in length dx. Number of particles, N is

$$N = nAdx \because \text{n is number of particles per unit volume}$$

$$W = \sigma_t IN = -AdI \tag{30}$$

$$W = \sigma_t I(nAdx) = -AdI \tag{31}$$

$$\implies \frac{dI}{I} = -n\sigma_t dx \tag{32}$$

$$\implies I(x) = I_0 e^{-n\sigma_t x} \tag{33}$$

Thickness of Lead required

Given,

$$I(d) = \frac{I_o}{1000} \tag{34}$$

$$\rho(Pb) = 11300kgm^{-3} \tag{35}$$

$$m_{pb}(\text{mass of Pb}) = 207.21u \tag{36}$$

$$n = \frac{\rho}{m_{pb}} \tag{37}$$

from above 3 equations,

$$n = \frac{11300kgm^{-3}}{207.21X1.66X10^{-27}kg} = 3.28X10^{28}m^{-3} \tag{38}$$

$$\sigma_t = 2.6X10^3\text{barns} = 2.6X10^{-25}m^2 \tag{39}$$

$$\frac{I(d)}{I_o} = e^{-n\sigma_t d} \tag{40}$$

$$10^{-3} = e^{-n\sigma_t d} \tag{41}$$

$$3\ln(10) = 3.28X10^{28}X2.6X10^{-25}Xd \tag{42}$$

$$d = 0.81\text{mm} \tag{43}$$

Thickness of Aluminum required

Given,

$$I(d) = \frac{I_o}{1000} \tag{44}$$

$$\rho(Al) = 2700kgm^{-3} \tag{45}$$

$$m_{Al}(\text{mass of Al}) = 26.29u \tag{46}$$

$$n = \frac{\rho}{m_{Al}} \tag{47}$$

from above 3 equations,

$$n = \frac{2700kgm^{-3}}{26.29X1.66X10^{-27}kg} = 6.18X10^{28}m^{-3} \tag{48}$$

$$\sigma_t = 13\text{barns} = 13X10^{-28}m^2 \tag{49}$$

$$\frac{I(d)}{I_o} = e^{-n\sigma_t d} \tag{50}$$

$$10^{-3} = e^{-n\sigma_t d} \tag{51}$$

$$3\ln(10) = 6.18X10^{28}X13X10^{-28}Xd \tag{52}$$

$$d = 85.98\text{mm} \tag{53}$$

4 Elastic scattering, De-excitation, Fission

Attenuation rate

Given,

$$I_o = 10^5 s^{-1} \tag{54}$$
$$\rho x = 10^{-1} kgm^{-2} \tag{55}$$
$$\sigma_e = 20mb \tag{56}$$
$$\sigma_c = 70b \tag{57}$$
$$\sigma_f = 200b \tag{58}$$
$$A = 235 \tag{59}$$
$$N_A = \text{Avogadro's Number} \tag{60}$$

$$I(x) = I_o e^{-n\sigma_t x} \tag{61}$$
$$n = \frac{\rho}{m_U} \tag{62}$$
$$m_U = \frac{A}{N_A X 10^3} \text{ in Kg} = \frac{235}{6.023X10^26} = 3.90X10^{-25}kg \tag{63}$$
$$\sigma_t = \sigma_e + \sigma_c + \sigma_f = 270.02X10^{-28}m^2 \tag{64}$$
$$nx = \frac{\rho x}{m_U} = \frac{10^{-1}}{3.90X10^{-25}}m^{-2} = 2.56X10^{23}m^{-2} \tag{65}$$
$$n\sigma_t x = 6.92X10^{-3} \tag{66}$$
$$e^{-n\sigma_t x} = 0.9931 \tag{67}$$

$$\boxed{\text{Attenuation rate,} \frac{I(x)}{I_o} = 0.9931} \tag{68}$$

Number of fission reactions per second

Replacing σ_t with σ_f to get only Intensity decreased by fission reaction.

$$\frac{I_f(x)}{I_o} = e^{-n\sigma_f x} \tag{69}$$
$$\implies nx = 2.56X10^{23}m^{-2} \text{ and } \sigma_f = 200X10^{-28}m^2 \tag{70}$$
$$-n\sigma_f x = 5.12X10^{-3} \tag{71}$$
$$e^{-n\sigma_f x} = 0.9948 \tag{72}$$
$$\frac{I_f(x)}{I_o} = 0.9948 \tag{73}$$

Intensity decreased by fission is equal to $I_o - I_f(x)$,

$$I_o - I_f(x) = I_o(1 - e^{-n\sigma_f} x) = 10^5(1 - 0.9948) = 510s^{-1} \tag{74}$$

$$\boxed{\text{Number of fission reactions per second =} 510s^{-1}} \tag{75}$$

Elastic scattering

Attenuation due to elastic scattering,

$$\frac{I_e(x)}{I_o} = e^{-n\sigma_e x} \tag{76}$$

$$\implies nx = 2.56X10^{23}m^{-2} \text{ and } \sigma_e = 20X10^{-31}m^2 \tag{77}$$

$$-n\sigma_e x = 5.12X10^{-7} \tag{78}$$

$$e^{-n\sigma_e x} = 0.999999488 \tag{79}$$

$$\frac{I_e(x)}{I_o} = 0.999999488 \tag{80}$$

Number of scattered per second = $I_o(1 - \frac{I_e(x)}{I_o}) = 5.1X10^{-2}s^{-1}$.
Area of sphere at 10m = $400\pi m^2$

$$\boxed{\text{Flux} = \frac{I}{A} = \frac{5.1X10^{-2}}{400\pi}m^{-2}s^{-1} = 4.074X10^{-5}m^{-2}s^{-1}} \tag{81}$$

For gamma rays,

$$\frac{I_c(x)}{I_o} = e^{-n\sigma_c x} \tag{82}$$

$$\implies nx = 2.56X10^{23}m^{-2} \text{ and } \sigma_c = 70X10^{-28}m^2 \tag{83}$$

$$-n\sigma_c x = 1.792X10^{-3} \tag{84}$$

$$e^{-n\sigma_c x} = 0.9982 \tag{85}$$

$$\frac{I_c(x)}{I_o} = 0.9982 \tag{86}$$

Number of scattered per second = $I_o(1 - \frac{I_c(x)}{I_o}) = 179s^{-1}$.
Area of sphere at 10m = $400\pi m^2$

$$\boxed{\text{Flux} = \frac{I}{A} = \frac{179}{400\pi}m^{-2}s^{-1} = 0.1424m^{-2}s^{-1}} \tag{87}$$