EP3110: Assignment # 2 — 1st September, 2022.

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Q1: $\vec{E} = 0.2\sin(10\pi y)\cos(6\pi\cdot10^9 t - \beta z)\hat{x}$. $\vec{H} = ?$ $\beta = ?$

$$\tilde{\vec{E}} = 0.2\sin(10\pi y)e^{-j\beta z}\hat{x}. \quad -(1)$$

Now, $\vec{\nabla}\times\tilde{\vec{E}} = -\mu j\omega\tilde{\vec{H}}$

$$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{pmatrix} = \hat{x}(0-0) - \hat{y}\left(0 - \frac{\partial E_x}{\partial z}\right) + \hat{z}\left(0 - \frac{\partial}{\partial y}E_x\right)$$

$$= \frac{\partial E_x}{\partial z}\hat{y} - \frac{\partial E_x}{\partial y}\hat{z}$$

$$= (0.2)\sin(10\pi y)e^{-j\beta z}(-j\beta)\hat{y} - (0.2)\cos(10\pi y)(10\pi)e^{-j\beta z}\hat{z}$$

$$= (0.2)e^{-j\beta z}(-1)\left(j\beta\sin(10\pi y)\hat{y} + 10\pi\cos(10\pi y)\hat{z}\right) = \cancel{-}\mu j\omega\tilde{\vec{H}}$$

$$\boxed{\therefore \tilde{\vec{H}} = \frac{(0.2)e^{-j\beta z}}{\mu j\omega}\left(j\beta\sin(10\pi y)\hat{y} + 10\pi\cos(10\pi y)\hat{z}\right)} \quad -(2).$$

Now, $\vec{\nabla}\times\tilde{\vec{H}} = \cancel{\vec{J_f}} + \varepsilon j\omega\tilde{\vec{E}}$

source free region $\Rightarrow \vec{J_f} = 0$.

$$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & H_z \end{pmatrix} = \hat{x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \hat{y}\left(\underbrace{\cancel{\frac{\partial H_z}{\partial x}}}_{=0} - 0\right) + \hat{z}\left(\underbrace{\cancel{\frac{\partial H_y}{\partial x}}}_{=0} - 0\right)$$

$$= \hat{x}\left(\frac{(0.2)e^{-j\beta z}}{\mu j\omega}(10\pi)^2(-\sin(10\pi y)) + \frac{(0.2)}{\mu j\omega}(j\beta)\sin(10\pi y)e^{-j\beta z}(-j\beta)\right)$$

$$= \frac{(0.2)e^{-j\beta z}\sin(10\pi y)}{\mu j\omega}\left(-\beta^2 - (10\pi)^2\right)\hat{x}$$

$$\therefore -\frac{(0.2)\sin(10\pi y)}{\mu j\omega}e^{-j\beta z}\left(\beta^2 + (10\pi)^2\right)\hat{x} = \varepsilon j\omega(0.2)\sin(10\pi y)e^{-j\beta z}\hat{x}$$

$$\therefore \boxed{\cancel{-}(\beta^2 + (10\pi)^2) = \mu\varepsilon\cancel{(-1)}\omega^2} \quad \text{and } \omega = 6\pi\cdot10^9$$

Now, $\frac{1}{\sqrt{\mu\varepsilon}} = c_{air} \approx 3\times10^8 \Rightarrow \mu\varepsilon = \frac{1}{9\times10^{16}}$

$$\therefore \beta^2 + 100\pi^2 = \frac{\cancel{36}^{4}\,\pi^2\,(\cancel{10^{16}})(100)}{\cancel{9}\times\cancel{10^{16}}}$$

$$\therefore \beta^2 = 400\pi^2 - 100\pi^2 = 300\pi^2 \Rightarrow \boxed{\beta = \sqrt{3}\,(10\pi) \approx 54.41 \text{ rad/sec}}$$

Now, $$\vec{\tilde{H}} = \frac{0.2e^{-j\beta z}}{\mu j\omega}\left(j\beta\sin(10\pi y)\,\hat{y} + 10\pi\cos(10\pi y)\,\hat{z}\right)$$

$$= \frac{0.2}{\mu j\omega}e^{-j\beta z}\left(10\pi\cos(10\pi y)\right)\hat{z} + \frac{0.2e^{-j\beta z}}{\mu\omega}\beta\sin(10\pi y)\,\hat{y}$$

$$= \cancel{0.2(54.41)}$$

$$= \left(\frac{\cancel{0.2}^{2}\sqrt{3}\,(10)\cancel{\pi}}{4\cancel{\pi}\times10^{-7}\,(6\pi\times10^{9})}\sin(10\pi y)\,\hat{y} - \frac{\cancel{0.2}^{2}\,j\,\cancel{10}\cancel{\pi}}{4\cancel{\pi}\cdot10^{-7}\cdot(6\pi\cdot10^{9})}\cos(10\pi y)\,\hat{z}\right)e^{-j\beta z}$$

$$\vec{\tilde{H}} = \left(0.046\times10^{-2}\sin(10\pi y)\,\hat{y} - j0.027\times10^{-2}\cos(10\pi y)\,\hat{z}\right)e^{-j\beta z}$$

$$\boxed{\begin{aligned}\therefore \vec{H} &= 4.6\times10^{-4}\sin(10\pi y)\cos(6\pi\cdot10^{9}t - 54.41z)\,\hat{y}\\ &\quad + 2.7\times10^{-4}\cos(10\pi y)\sin(6\pi\cdot10^{9}t - 54.41z)\,\hat{z}\\ \text{and } \beta &= 10\sqrt{3}\,\pi \approx 54.41 \text{ rad/sec.}\end{aligned}}$$

Q2: $\mu_r = 1$; $\vec{E} = 30\pi e^{j(\omega t - \frac{4}{3}y)}\hat{z}$ and $\vec{H} = e^{j(\omega t - \frac{4}{3}y)}\hat{x}$.

$\varepsilon_r = ?$; $\omega = ?$

Using Maxwell's equations...

$\vec{\tilde{E}} = 30\pi e^{-j\frac{4y}{3}}\hat{z}$ and $\vec{\tilde{H}} = e^{-j\frac{4}{3}y}\hat{x}$

$\therefore \underbrace{\vec{\nabla}\times\vec{\tilde{E}}} = -\mu j\omega\vec{\tilde{H}}$

$$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{pmatrix} = \hat{x}\left(\frac{\partial E_z}{\partial y} - 0\right) - \hat{y}\left(\underset{=0}{\frac{\partial E_z}{\partial x}} - 0\right) + \hat{z}(0-0)$$

$$= 30\pi e^{-j\frac{4}{3}y}\left(-j\frac{4}{3}\right)\hat{x}$$

$$\therefore 30\pi\left(+\cancel{j}\frac{4}{3}\right)\cancel{e^{-j\frac{4}{3}y}}\cancel{\hat{x}} = +(\mu_0\mu_r)\cancel{j}\omega\, \cancel{e^{-j\frac{4}{3}y}}\cancel{\hat{x}}$$

$$\therefore 40\pi = \mu_0\omega \Rightarrow \boxed{\omega = \frac{40\pi}{\mu_0} = \frac{40\pi}{4\pi\times10^{-7}} = 10^8}$$

and $\underbrace{\vec{\nabla}\times\vec{\tilde{H}}} = \underset{=0}{\vec{J_f}} + \varepsilon j\omega\vec{\tilde{E}}$

$$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & 0 & 0 \end{pmatrix} = \hat{x}(0-0) - \hat{y}\left(0 - \underset{=0}{\frac{\partial H_x}{\partial z}}\right) + \hat{z}\left(0 - \frac{\partial H_x}{\partial y}\right)$$

$$= -e^{-j\frac{4}{3}y}\left(-j\frac{4}{3}\right)\hat{z}$$

$$\therefore \frac{4}{3}\cancel{j}\cancel{e^{-j\frac{4}{3}y}} = \varepsilon_r\varepsilon_0 j\omega\left(30\pi e^{-j\frac{4}{3}y}\right)$$

$$\therefore \varepsilon_r = \left(\frac{4}{3}\right)\left(\frac{1}{\varepsilon_0}\right)\left(\frac{1}{\omega}\right)\left(\frac{1}{30\pi}\right) = \left(\frac{16}{3}\right)\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{1}{\omega}\right)\left(\frac{1}{30}\right)$$

$$= \left(\frac{16}{3}\right)\frac{(9\times10^9)}{10^8}\left(\frac{1}{30}\right) = 16.$$

$$\boxed{\therefore \varepsilon_r = 16 \text{ and } \omega = 10^8 \text{ rad/sec.}}$$

#3

Q3 : $\vec{\tilde{E}}(x,y,z) = E_0 e^{-j(k_x x + k_y y + k_z z)}\hat{n}$ where $\underbrace{\hat{n} = n_x\hat{x} + n_y\hat{y} + n_z\hat{z}}_{\text{Polarization Vector.}}$

Helmholtz eq : $\nabla^2\vec{\tilde{E}} = -k^2\vec{\tilde{E}}$ where k is $\bar{\nu}$ (wavenumber)

$$\therefore \nabla^2\vec{E} = \underbrace{\nabla^2 E_x\hat{x}} + \nabla^2 E_y\hat{y} + \nabla^2 E_z\hat{z}$$

$$\hookrightarrow \nabla^2(E_0 e^{-j(k_x x + k_y y + k_z z)} n_x)$$

$$= \frac{\partial^2}{\partial x^2}(E_0 e^{-j(k_x x + k_y y + k_z z)} n_x) + \frac{\partial^2}{\partial y^2}(E_0 e^{-j(k_x x + k_y y + k_z z)} n_x) + \cdots$$

$$= E_0 n_x \frac{\partial}{\partial x}\left(e^{-j(k_x x + k_y y + k_z z)}(-jk_x)\right) + E_0 n_x \frac{\partial}{\partial y}\left(e^{-j(k_x x + k_y y + k_z z)}(-jk_y)\right) + \cdots$$

$$= E_0 n_x (e^{-j(k_x x + k_y y + k_z z)})((-jk_x)^2 + (-jk_y)^2 + (-jk_z)^2)$$

$$= E_0 n_x (e^{-j(k_x x + k_y y + k_z z)})(-1)(k_x^2 + k_y^2 + k_z^2)$$

$$\therefore \nabla^2\vec{\tilde{E}} = (-1)\{(k_x^2 + k_y^2 + k_z^2)\}\{E_0 e^{-j(k_x x + k_y y + k_z z)}\}(n_x\hat{x} + n_y\hat{y} + n_z\hat{z})$$

$$= (-1)(k_x^2 + k_y^2 + k_z^2)(E_0 e^{-j(k_x x + k_y y + k_z z)}\hat{n})$$

$$\nabla^2\vec{\tilde{E}} = (-1)(k_x^2 + k_y^2 + k_z^2)\vec{\tilde{E}}$$

if $k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\varepsilon$.

$$\therefore \nabla^2\vec{\tilde{E}} = -(\omega^2\mu\varepsilon)\vec{\tilde{E}} \Rightarrow \boxed{\nabla^2\vec{\tilde{E}} = -(\sqrt{\omega^2\mu\varepsilon})^2\vec{\tilde{E}}}$$

Hence, $\vec{\tilde{E}}$ does satisfy the Helmholtz equation.

(further, $\underset{(\text{freq.})^2}{\omega^2 = 4\pi^2 f^2}$ and $\mu\varepsilon = \frac{1}{c^2}$ $(\text{velocity})^2$

$\therefore$ unit of $\sqrt{\omega^2\mu\varepsilon}$ is $\frac{(\cancel{s^{-1}})}{m(\cancel{s^{-1}})} = m^{-1}$

which is the same dimension as wavenumber $\bar{\nu}$.)

Q4: $\vec{\tilde{E}}(\vec{R}) = E_0 e^{-j(\vec{K}\cdot\vec{R})}\hat{n}$ where $\hat{n} = n_x\hat{x} + n_y\hat{y} + n_z\hat{z}$, the polarization vector.

Now, using $\underbrace{\vec{\nabla}\times\vec{\tilde{E}}} = -\mu j\omega\vec{\tilde{H}}\ldots$

$\therefore$

$$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{pmatrix} = \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right)\hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{z}$$

$$= \hat{x}\left(\frac{\partial E_0 e^{-j(\vec{K}\cdot\vec{R})} n_z}{\partial y} - \frac{\partial E_0 e^{-j(\vec{K}\cdot\vec{R})} n_y}{\partial z}\right) + \ldots$$

since $\vec{K} = K_x\hat{x} + K_y\hat{y} + K_z\hat{z}$ and $\vec{R} = x\hat{x} + y\hat{y} + z\hat{z}\ldots$

$$= \hat{x}\left((-j)n_z E_0 e^{-j(\vec{K}\cdot\vec{R})} K_y - n_y E_0 e^{-j(\vec{K}\cdot\vec{R})}(-j)K_z\right) + \ldots$$

$$= (-j)E_0 e^{-j(\vec{K}\cdot\vec{R})}(n_z K_y - n_y K_z)\hat{x} + \ldots$$

$$= (-j)E_0 e^{-j(\vec{K}\cdot\vec{R})}\left\{(n_z K_y - n_y K_z)\hat{x} - (n_z K_x - n_x K_z)\hat{y} + (n_y K_x - n_x K_y)\hat{z}\right\}.$$

$$= (-j)E_0 e^{-j(\vec{K}\cdot\vec{R})}(\vec{K}\times\hat{n}) = -\mu j\omega\vec{\tilde{H}}$$

$$\therefore \mu\omega\vec{\tilde{H}} = E_0 e^{-j(\vec{K}\cdot\vec{R})}(\vec{K}\times\hat{n})$$

$$= \left(\vec{K}\times\left(E_0 e^{-j(\vec{K}\cdot\vec{R})}\hat{n}\right)\right) = \vec{K}\times\vec{\tilde{E}}$$

$$\boxed{\therefore \vec{K}\times\vec{\tilde{E}} = \mu\omega\vec{\tilde{H}}} \quad -(i)$$

Similarly, using $\underbrace{\vec{\nabla}\times\vec{\tilde{H}}} = \vec{J}_f + \varepsilon\omega j\vec{\tilde{E}}$

source free region $\Rightarrow \vec{J}_f = 0.$

$$\cancel{\vec{\nabla}\times\vec{\tilde{H}} = \vec{\nabla}\times\left(\frac{\vec{K}\times\vec{\tilde{E}}}{\mu\omega}\right) = \vec{\nabla}\times\left(\frac{E_0}{\mu\omega}e^{-j(\vec{K}\cdot\vec{R})}\vec{K}\times\hat{n}\right)}$$

$$\vec{K}\times\vec{\tilde{E}} = \mu\omega\vec{\tilde{H}} \Rightarrow \vec{\tilde{H}} = \frac{E_0}{\mu\omega}e^{-j(\vec{K}\cdot\vec{R})}(\vec{K}\times\hat{n}).$$

let $\vec{K}\times\hat{n} = \vec{h}$. $\therefore \vec{\tilde{H}} = \frac{E_0}{\mu\omega}e^{-j(\vec{K}\cdot\vec{R})}\vec{h}$.

$$\vec{h} = h_x\hat{x} + h_y\hat{y} + h_z\hat{z}.$$

$$\therefore \vec{\nabla}\times\vec{\tilde{H}} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{pmatrix} = \hat{x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \hat{y}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= \hat{x}\left(\frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}(-j)k_y h_z - \frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}(-j)k_z h_y\right) + \cdots$$

$$= \frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}(-j)\left\{ (k_y h_z - k_z h_y)\hat{x} + (k_x h_z - k_z h_x)\hat{y} + (k_x h_y - k_y h_x)\hat{z} \right\}.$$

$$= \frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}(-j)(\vec{K}\times\hat{n})$$

$$= \frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}\cancel{(-j)}(\vec{K}\times(\vec{K}\times\hat{n})) = \cancel{-j}(\omega\varepsilon\vec{\tilde{E}})$$

$$\therefore -\omega\varepsilon\vec{\tilde{E}} = \frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}(\vec{K}\times(\vec{K}\times\hat{n}))$$

$$= \left(\vec{K}\times\left(\frac{E_0}{\mu\omega} e^{-j(\vec{K}\cdot\vec{R})}(\vec{K}\times\hat{n})\right)\right) = \vec{K}\times\vec{\tilde{H}}$$

$$\therefore \boxed{\vec{K}\times\vec{\tilde{H}} = -\omega\varepsilon\vec{\tilde{E}}} \quad -(ii).$$

Now, $\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\varepsilon_0} = 0$ source-free region $\Rightarrow \rho_f = 0$.

$$\therefore \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\cdot\left(E_0 e^{-j(\vec{K}\cdot\vec{R})}\hat{n}\right) = 0.$$

$$\therefore \frac{\partial}{\partial x}\left(n_x E_0 e^{-j(\vec{K}\cdot\vec{R})}\right) + \frac{\partial}{\partial y}\left(n_y E_0 e^{-j(\vec{K}\cdot\vec{R})}\right) + \cdots = 0$$

$$\therefore n_x E_0 e^{-j(\vec{K}\cdot\vec{R})}(-j)k_x + n_y E_0 e^{-j(\vec{K}\cdot\vec{R})}(-j)k_y + \cdots = 0.$$

$$\therefore E_0 e^{-j(\vec{K}\cdot\vec{R})}(-j)(n_x k_x + n_y k_y + n_z k_z) = 0.$$

$$\therefore \cancel{(-j)} E_0 e^{-j(\vec{K}\cdot\vec{R})}(\hat{n}\cdot\vec{K}) = 0.$$

$$\therefore \vec{K}\cdot(E_0 e^{-j(\vec{K}\cdot\vec{R})}\hat{n}) = 0 \Rightarrow \boxed{\vec{K}\cdot\vec{\tilde{E}} = 0} \quad -(iii)$$

Finally, $\vec{K}\cdot\vec{\tilde{H}} = \vec{K}\cdot\left(\frac{\vec{K}\times\vec{\tilde{E}}}{\mu\omega}\right) = 0 \Rightarrow \boxed{\vec{K}\cdot\vec{\tilde{H}} = 0} \quad -(iv)$

Hence, we get: $\vec{K}\times\vec{\tilde{E}} = \omega\mu\vec{\tilde{H}}$, $\vec{K}\times\vec{\tilde{H}} = -\omega\varepsilon\vec{\tilde{E}}$, $\vec{K}\cdot\vec{\tilde{E}} = 0$, and $\vec{K}\cdot\vec{\tilde{H}} = 0$.

Q5: $\vec{E}(t,z) = 2\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right)\hat{x} - \sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)\hat{y}$

$\therefore$ According to the given equation, this is a wave travelling in the $\hat{z}$ direction, and polarized in the XY plane.

comparing with $\vec{E}(\vec{r},t) = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta)\,\hat{n}$ ...

$\therefore k_x = k_y = 0$; $k_z = \frac{1}{\sqrt{3}}$ and $\omega = 10^8$ rad/sec.

$\therefore k = k_z = \frac{1}{\sqrt{3}}$ ~~$\frac{k}{\omega} = \frac{1}{v}$~~

Now, $2\pi f = \omega = 10^8 \Rightarrow \boxed{f = \frac{1}{2\pi} * 10^8 \approx 1.59 * 10^7 \text{ Hz.}}$

and $\frac{2\pi}{\lambda} = k \Rightarrow \boxed{\lambda = \frac{2\pi}{k} = 2\sqrt{3}\pi \approx 10.88\text{m.}}$ ← (a)

Now, $\frac{k}{\omega} = \frac{1}{v} \Rightarrow v = \frac{\omega}{k} = \sqrt{3} * 10^8$ m/s.

since $c = 3 * 10^8$ m/s and $v = \sqrt{3} * 10^8$ m/s ...

$\therefore$ ~~$v = \epsilon c$~~. $v = \frac{c}{\sqrt{\epsilon_r}} \Rightarrow \sqrt{3} * 10^8 = \frac{3 * 10^8}{\sqrt{\epsilon_r}} = \boxed{\epsilon_r = 3}$

$\boxed{\therefore \text{Dielectric constant for medium is } 3\epsilon_0 \text{ ; } \epsilon_r = 3.}$ – (b)

again, comparing with $\vec{E}(\vec{r},t) = E_0 \cos(\vec{k}\cdot\vec{r} - \omega t + \delta)\,\hat{n}$.

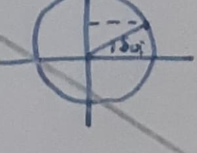
$= E_0 \cos\left(\frac{z}{\sqrt{3}} - 10^8 t + \delta\right)\hat{n}$ $(k_x = k_y = 0;\ k_z = \frac{1}{\sqrt{3}};\ \omega = 10^8)$.

$= E_0 \left\{\cos\left(\frac{z}{\sqrt{3}} - 10^8 t\right)\cos(\delta) - \sin\left(\frac{z}{\sqrt{3}} - 10^8 t\right)\sin(\delta)\right\}\hat{n}$

$= E_0 \left\{\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right)\cos(\delta) + \sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)\sin(\delta)\right\}\hat{n}$

$\Updownarrow$

$\sqrt{5}\left\{\frac{2}{\sqrt{5}}\cos\left(10^8 t - \frac{z}{\sqrt{3}}\right)\hat{x} - \frac{1}{\sqrt{5}}\sin\left(10^8 t - \frac{z}{\sqrt{3}}\right)\hat{y}\right\}$.



$\Rightarrow \cos(\delta) = \frac{2}{\sqrt{5}}$ and $\sin(\delta) = -\frac{1}{\sqrt{5}}$

For a wave travelling in the $\hat{z}$ direction, polarized in $\hat{n}$ ~~such that~~ ~~$\hat{n} = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$, θ being the polarization angle~~, we can consider it to be a superposition of 2 waves:

$\vec{E_x} = E_1 \cos(10^8 t - \frac{z}{\sqrt{31}})\hat{x}$ and $\vec{E_y} = E_2 \cos(10^8 t - z/\sqrt{31} + \underbrace{\phi}_{\text{phase difference}})\hat{y}$

$$\therefore \vec{E} = \vec{E_x} + \vec{E_y} = E_1 \cos(10^8 t - \frac{z}{\sqrt{31}})\hat{x} + E_2 \cos(10^8 t - \frac{z}{\sqrt{31}} + \phi)\hat{y}$$

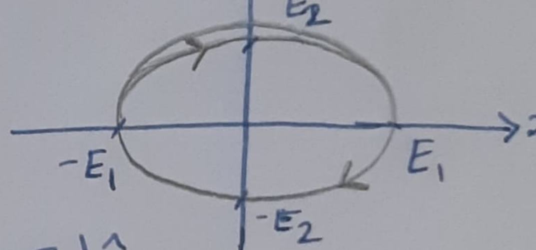
$$\Updownarrow$$

$$2\cos(10^8 t - \frac{z}{\sqrt{31}})\hat{x} - \sin(10^8 t - \frac{z}{\sqrt{31}})\hat{y}$$

$\therefore E_1 = 2$; $E_2 = 1$; $\phi = \frac{3\pi}{2}$.

since $E_1 \neq E_2$, we say that the polarization is elliptical!

"(c).



Finally, we need to determine $\vec{H}$.

$$\vec{E}(t,z) = 2\cos(10^8 t - z/\sqrt{31})\hat{x} - \sin(10^8 t - \frac{z}{\sqrt{31}})\hat{y}$$

$$\therefore \tilde{\vec{E}} = (2\hat{x} + j\hat{y})\, e^{-jz/\sqrt{31}}$$

$$\therefore \vec{\nabla} \times \tilde{\vec{E}} = -j\mu\omega \tilde{\vec{H}} = \cancel{\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2e^{-jz/\sqrt{31}} & & \end{pmatrix}} \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2e^{-jz/\sqrt{31}} & je^{-jz/\sqrt{31}} & 0 \end{pmatrix}$$

$$= \hat{x}\left(0 - je^{-jz/\sqrt{31}}\left(\frac{-j}{\sqrt{31}}\right)\right) - \hat{y}\left(0 - 2e^{-jz/\sqrt{31}}(-j/\sqrt{31})\right) + \hat{z}(0).$$

$$= \hat{x}\left(\frac{-e^{-jz/\sqrt{31}}}{\sqrt{31}}\right) - \hat{y}\left(\frac{2j}{\sqrt{31}} e^{-jz/\sqrt{31}}\right) = \frac{e^{-jz/\sqrt{31}}}{\sqrt{31}}(-\hat{x} - 2j\hat{y}).$$

$$\therefore \left(\frac{e^{-jz/\sqrt{31}}}{\sqrt{31}}\right)(\cancel{-1})(\hat{x} + 2j\hat{y}) = \cancel{-}j\mu\omega \tilde{\vec{H}}$$

$$\therefore \tilde{\vec{H}} = \frac{e^{-jz/\sqrt{31}}}{\sqrt{31}\,\mu_0\omega}(-j\hat{x} + 2\hat{y}) = \frac{e^{-jz/\sqrt{31}}}{(\sqrt{31})(4\pi \times 10^{-7})(10^8)}(-j\hat{x} + 2\hat{y})$$

$$\tilde{\vec{H}} = \frac{e^{-jz/\sqrt{31}}}{40\sqrt{31}\,\pi}(-j\hat{x} + 2\hat{y})$$

$$\therefore \vec{H} = \mathrm{Re}\left[\left(\frac{1}{40\sqrt{3}\pi}\right) e^{-j(z/\sqrt{3} - 10^8 t)} (-j\hat{x} + 2\hat{y})\right]$$

$$= \left(\frac{1}{40\sqrt{3}\pi}\right) \left\{ 2\cos\left(-(z/\sqrt{3} - 10^8 t)\right) \hat{y} + \sin\left(10^8 t - z/\sqrt{3}\right) \hat{x} \right\}.$$

$$\boxed{\therefore \vec{H} = \left(\frac{1}{40\sqrt{3}\pi}\right)\left(2\cos\left(10^8 t - z/\sqrt{3}\right) \hat{y} + \sin\left(10^8 t - z/\sqrt{3}\right) \hat{x}\right).}$$

(d)

Q6: $\vec{E}(z,t) = E_{10}\sin(wt-kz)\hat{x} + E_{20}\sin(wt-kz+\psi)\hat{y}$

$\sin(wt-kz+\psi) = \sin(wt-kz)\cos\psi + \cos(wt-kz)\sin\psi.$

$$\cancel{\therefore \tilde{\vec{E}} = (-E_{10}j\hat{x} + (-E_{20}j\cos\psi)\hat{y})e^{-j(kz)}}$$

$$\cancel{\hookrightarrow \text{Since } \vec{E}(z,t) = Re[\tilde{\vec{E}}e^{wt}] =}$$

$$\cancel{Re[(-E_{10}j\hat{x} + (-E_{20}j\cos\psi\,\hat{y}))(\cos(wt-kz) + j\sin(wt-kz))]}$$

$$\cancel{= Re[(-E_{10}j\cos(wt-kz) + E_{10}\sin(wt-kz))\hat{x} + (-E_{20}j\cos\psi\cos(wt-kz)}$$

$$\cancel{+ E_{20}\cos\psi\sin}$$

$$\therefore \tilde{\vec{E}} = (-E_{10}j\hat{x} + (-E_{20}j\cos\psi + E_{20}\sin\psi)\hat{y})e^{-j(kz)}$$

$\hookrightarrow$ since $\vec{E}(z,t) = Re[\tilde{\vec{E}}e^{wt}]$

$$= Re[(\cancel{-E_{10}j\cos(wt-kz)} + E_{10}\sin(wt-kz))\hat{x}$$
$$+ (\cancel{-E_{20}j\cos\psi\cos(wt-kz)} + E_{20}\cos\psi\sin(wt-kz)$$
$$+ E_{20}\sin\psi\cos(wt-kz) + \cancel{E_{20}\sin\psi j\sin(wt-kz)})\hat{y}]$$

$$= E_{10}\sin(wt-kz)\hat{x} + E_{20}(\cos(wt-kz)\sin\psi + \sin(wt-kz)\cos\psi)\hat{y}$$

which is the given $\vec{E}(z,t)$.

$$\therefore \tilde{\vec{E}} = (-E_{10}j\hat{x} + (-E_{20}j\cos\psi + E_{20}\sin\psi)\hat{y})e^{-jkz}$$
$$= (-j)\{E_{10}\hat{x} + (E_{20}\cos\psi - jE_{20}\sin\psi)\hat{y}\}e^{-jkz}$$
$$= (-j)\{E_{10}\hat{x} + E_{20}e^{-j\psi}\hat{y}\}e^{-jkz}$$
$$= \underbrace{\{E_{10}e^{j(3\pi/2)}\hat{x} + E_{20}e^{j(3\pi/2-\psi)}\hat{y}\}}_{\text{This is the polarization vector (at least the direction)}}e^{-jkz}$$

$$\boxed{\therefore \hat{n} = \frac{E_{10}e^{j(3\pi/2)}\hat{x} + E_{20}e^{j(3\pi/2-\psi)}\hat{y}}{\sqrt{|E_{10}|^2 + |E_{20}|^2}} \text{ is the polarization vector.}}$$

Assuming $|E_{10}| \neq |E_{20}| \ldots \Rightarrow \vec{E}(z,t)$ is elliptically Polarized.

the relative Phase difference is $3/2\pi - \psi - 3\pi/2 = -\psi.$

Plotting this...