Physics 521 Fall 2014

Problem Set #1 Mathematical Foundations

Cohen-Tannoudji et al. vol. I, Chap. II

Due: Tuesday Sept. 2, 2014 @ 5PM

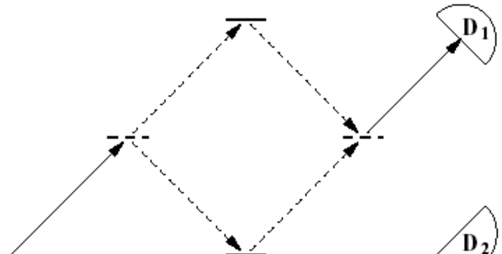
Problem 1: "Interaction-Free" measurement, or: How I learned to stopped worrying about quantum mechanics and love the bomb! (10 points)

Imagine the following scenario (no political statement intended):

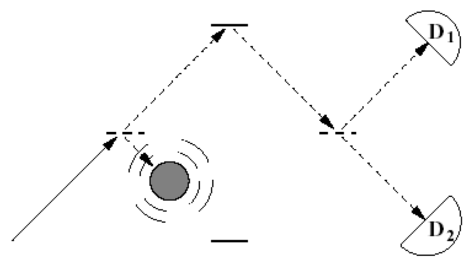
In our current environment of high-tech security, the government has called upon physicists to design the ultimate sensor to detect the presence of bombs without setting them off. Our diabolical foes have designed bombs so sensitive that they absorb photons with unit efficiency, and even a *single* photon will cause them to explode. These bombs are so deadly, that even if we could catch a fraction of them, it would be worthwhile. These bombs do have one defect – some are duds. If the bomb is a dud it is completely transparent.

How can we proceed? If we shine light on the bomb and it is not a dud we are guaranteed to blow it up. Luckily quantum mechanics, with counter-factual reasoning, can come to our rescue, due to an idea by Elitzur and Vaidman.

Consider a perfectly balanced Mach-Zender interferomter so that all photons reach detector D1



Now one of the super bombs (or possible duds) is placed in one arm of the interferometer,



Explain how with 25% probability we can use this interferometer to detect the presence of an active bomb without setting it off? (Hint: What happens if D2 goes "click").

PH 5100

Quiz 1

Department of Physics, IIT Madras

Concepts in Quantum Mechanics

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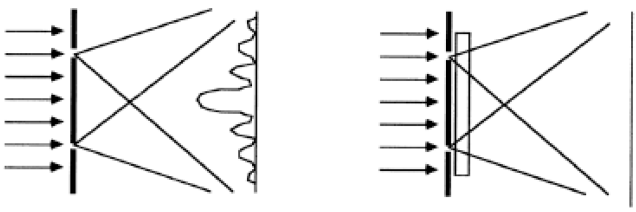
1. Problem 1
   (10 marks)

   The left half of the figure below depicts a double slit diffraction experiment. If the amplitude emerging from the top hole is $\psi_1(x)$ and the amplitude emerging from the bottom hole is $\psi_2(x)$, then the probability density for detecting a particle at a point on the screen where the two amplitudes overlap is $|\psi_1(x) + \psi_2(x)|^2 = |\psi_1(x)|^2 + |\psi_2(x)|^2 + [\psi_1(x)]^* \psi_2(x) + \psi_1(x)[\psi_2(x)]^*$. The last two terms are responsible for the interference pattern.



   In the right half of the figure, the experiment is modified by the presence, to the right of the holes, of a device whose state is altered by the passage of a particle, but which does not otherwise affect the propagation of the amplitudes. From the state change we may infer (though perhaps not with certainty) which hole a particle has passed through. How will the interference pattern be affected by the presence of this device?

Sketch the interference pattern for -

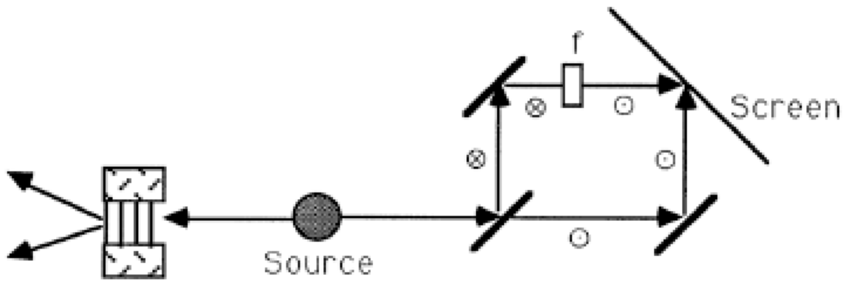
1) When our device gives us complete information about which hole the particle passed through.

2) When our device works only 50 percent of the times and rest of the 50 percent of the time the particles go undetected. (You may want to draw two sketches for this part. One for the particles which were detected as coming from a specific hole, and other for the undetected particles.).

The answer has to be "qualitative" sketches that can explain the basic physics behind this.

2. Problem 2
(10 marks)

The figure below depicts a novel proposal for an interference experiment using particles of spin $s = \frac{1}{2}$. The source produces correlated pairs of particles, one of which enters an interferometer on the right, and the other enters a Stern–Gerlach magnet on the left.



Let us first consider only the right side of the diagram. The first mirror transmits particles whose spin is up (in the z direction) and reflects particles whose spin is down. The other reflectors are of the ordinary spin-independent variety. The upper (spin down) beam passes through a spin-flipper (f|), so that both beams have spin up when they reach the screen.

Let $|\uparrow\rangle$ and $|\downarrow\rangle$ be the eigenstates of the S_z operator (which forms a complete basis to express any quantum state in this Hilbert Space).

Answer the following questions (consider only the right half of the diagram.)

1) A particle takes only the upper path through the interferometer. Determine the state of the particle.

2) A particle takes only the lower path through the interferometer. Determine the state of the particle.

3) A particle takes a path that has an amplitude on both paths of the interferometer. What is the general state of such a particle?