

Chaganti Kamaraja Siddhartha

EP20B012

ASSIGNMENT I — JULY-NOV 2022

📅 Due date: August 11th, before midnight 📅

Contents

1 Potential, Field and Charge distribution	2
2 Dipole	3
3 Lorentz Condition and Equation of Continuity	4

1 Potential, Field and Charge distribution

Given

$$V(r) = A \frac{e^{-\lambda r}}{r} \quad (1)$$

Electric field, \vec{E} :

$$\vec{E} = -\vec{\nabla}V \quad (2)$$

$$\vec{E} = -\left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r\sin\phi}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{1}{r}\frac{\partial}{\partial\phi}\hat{\phi}\right)V(r) \quad (3)$$

$$\vec{E} = -\frac{dV(r)}{dr}\hat{r} \quad (4)$$

$$\boxed{\vec{E} = A \frac{e^{-\lambda r}}{r^2}(1 + \lambda r)\hat{r}} \quad (5)$$

Charge distribution, ρ :

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o} \quad (6)$$

$$\vec{\nabla} \cdot \left(Ae^{-\lambda r}(1 + \lambda r)\frac{\hat{r}}{r^2}\right) = \frac{\rho}{\epsilon_o} \quad (7)$$

$$Ae^{-\lambda r}(1 + \lambda r)\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) + \frac{\hat{r}}{r^2} \cdot \vec{\nabla}(Ae^{-\lambda r}(1 + \lambda r)) = \frac{\rho}{\epsilon_o} \quad (8)$$

$$Ae^{-\lambda r}(1 + \lambda r)(4\pi\delta^3(r)) + \frac{\hat{r}}{r^2} \cdot (Ae^{-\lambda r}(-\lambda^2 r)\hat{r}) = \frac{\rho}{\epsilon_o} \quad (9)$$

$$\left(\because \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(r)\right)$$

$$\boxed{\rho = A\epsilon_o \left(4\pi\delta^3(r) - \lambda^2 \frac{e^{-\lambda r}}{r}\right)} \quad (\because f(x)\delta(x) = f(0)\delta(x)) \quad (10)$$

Total Charge, Q :

$$Q = \int_{-\infty}^{+\infty} \rho d\tau \quad (11)$$

$$Q = \int_{-\infty}^{+\infty} A\epsilon_o \left(4\pi\delta^3(r) - \lambda^2 \frac{e^{-\lambda r}}{r}\right) d\tau \quad (12)$$

$$Q = A\epsilon_o \int_{-\infty}^{+\infty} 4\pi\delta^3(r) d\tau - A\epsilon_o \int_{-\infty}^{+\infty} \lambda^2 \frac{e^{-\lambda r}}{r} d\tau \quad (13)$$

$$Q = A\epsilon_o(4\pi) - A\epsilon_o\lambda^2 4\pi\left(\frac{1}{\lambda^2}\right) \quad (14)$$

$$\boxed{Q = 0} \quad (15)$$

Answers:- $\vec{E} = A \frac{e^{-\lambda r}}{r^2}(1 + \lambda r)\hat{r}$, $\rho = A\epsilon_o \left(4\pi\delta^3(r) - \lambda^2 \frac{e^{-\lambda r}}{r}\right)$, $Q = 0$.

2 Dipole

Considering Proton above $Z=0$ and electron below $Z=0$, $10^{-11}m \ll 13m$, so we can consider this charge distribution as dipole with dipole moment $\vec{p} = ed\hat{k}$

$$V(R) = \frac{q}{4\pi\epsilon_o} \left(\frac{1}{R_+} - \frac{1}{R_-} \right) \quad (16)$$

Law of cosines,

$$R_{\pm}^2 = R^2 + \left(\frac{d}{2}\right)^2 \mp R d \cos \phi = R^2 \left(1 \mp \frac{d}{R} \cos \phi + \frac{d^2}{4R^2}\right) \quad (17)$$

$$\frac{1}{R_{\pm}} \approx \frac{1}{R} \left(1 \mp \frac{d}{R} \cos \phi\right)^{-\frac{1}{2}} \approx \frac{1}{R} \left(1 \pm \frac{d}{2R} \cos \phi\right) \quad (18)$$

$$\Rightarrow \frac{1}{R_+} - \frac{1}{R_-} \approx \frac{d}{R^2} \cos \phi \quad (19)$$

$$V(R) \cong \frac{1}{4\pi\epsilon_o} \frac{qd \cos \phi}{R^2} \quad (20)$$

$$\boxed{V(R) = \frac{1}{4\pi\epsilon_o} \frac{\vec{p} \cdot \hat{R}}{R^2}} \quad (21)$$

Electric Field, $\vec{E}(R)$ is

$$\vec{E}(R) = -\vec{\nabla} V(R) \quad (22)$$

$$\vec{E}(R) = -\left(\frac{\partial}{\partial R} \hat{R} + \frac{1}{R \sin \phi} \frac{\partial}{\partial \theta} + \frac{1}{R} \frac{\partial}{\partial \phi}\right) \left(\frac{1}{4\pi\epsilon_o} \frac{qd \cos \phi}{R^2}\right) \quad (23)$$

$$\vec{E}(R) = \frac{qd}{4\pi\epsilon_o R^3} \left(2 \cos \phi \hat{R} + \sin \phi \hat{\phi}\right) \quad (24)$$

$$\boxed{\vec{E}(R) = \frac{1}{4\pi\epsilon_o R^3} (3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p})} \quad (25)$$

Here, $\vec{p} = 10^{-11}e\hat{k}\text{Cm}$ and $\vec{R} = (3\hat{i} + 4\hat{j} + 12\hat{k})m$

$$V(R) = \frac{1}{4\pi\epsilon_o} \frac{12 \times 10^{-11}e}{13^2} V = 6.05 \times 10^{-24} V \quad (26)$$

$$\vec{E}(R) = \frac{1}{4\pi\epsilon_o 13^3} \left(3\left(\frac{12}{13} \times 10^{-11}e\right) \frac{3\hat{i} + 4\hat{j} + 12\hat{k}}{13} - 10^{-11}e\hat{k}\right) \quad (27)$$

$$\vec{E}(R) = (4.188 \times 10^{-24}\hat{i} + 5.585 \times 10^{-24}\hat{j} + 1.019 \times 10^{-23}\hat{k}) NC^{-1} \quad (28)$$

Answers:- $\boxed{V(R) = 6.05 \times 10^{-24} V, \vec{E}(R) = (4.188 \times 10^{-24}\hat{i} + 5.585 \times 10^{-24}\hat{j} + 1.019 \times 10^{-23}\hat{k}) NC^{-1}}$

3 Lorentz Condition and Equation of Continuity

A **corollary** of **Helmholtz Decomposition theorem** says that all physically realistic scalar fields obey a continuity equation. The theorem states that for any reasonable scalar field S and Vector field \mathbf{C} there exists a vector field \mathbf{F} such that $\vec{\nabla} \cdot \mathbf{F} = S$ and $\vec{\nabla} \times \mathbf{F} = \mathbf{C}$. [ref](#)

Lorentz Gauge:

$$\vec{\nabla} \cdot \vec{A} = -\frac{1}{c^2} \frac{\partial \phi}{\partial t} \quad (29)$$

from definition of Magnetic Vector potential, \vec{A}

$$\vec{\nabla} \times \vec{A} = \vec{B} \quad (30)$$