Introduction to nuclear and particle physics PH5211 - Problem Set 4

To be submitted in class on Tuesday 30th of August 2022

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Suggested references:

- Problem 1: Krane 10.1-10.4
- Problem 2: Perkins 2.11 and Krane 11.8
- Problem 3: Krane 8.4
- Problem 4: Krane 9.1 - 9.4

1. Discuss the role of angular momentum and parity in the γ decay of nuclei. Describe how the changes in nuclear spin and parity lead to a classification of the transitions and indicate how these spin changes influence the decay rate.

2. The partial wave expansion of a plane wave $\psi_{\rm inc} = Ae^{ikz} = Ae^{ikr\cos\theta}$ when $kr >> l$ is

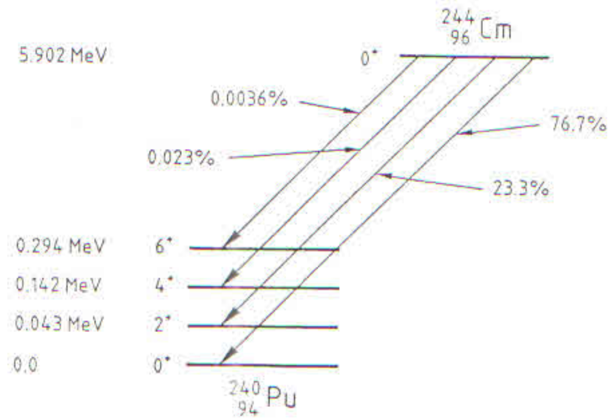
$$\psi_{\rm inc} = \frac{A}{2kr}\sum_{l=0}^{\infty} i^{l+1}\left(2l+1\right)\left[e^{-i(kr-l\pi/2)} - e^{+i(kr-l\pi/2)}\right]P_l\left(\cos\theta\right) \ ,$$

where l is the angular momentum associated with a component and $P_l(\cos\theta)$ are Legendre polynomials normalised such that $\int P_l P_{l'} d\Omega = 4\pi\delta_{ll'}/(2l+1)$. Introduce a complex factor η_l to represent the effect of the scattering centre for each partial wave to show that for a particular angular momentum l:

- $\sigma_{\rm el} = \pi\lambda\!\!\!^{-2}\left(2l+1\right)\left|1-\eta_l\right|^2$,
- $\sigma_{\rm inel} = \pi\lambda\!\!\!^{-2}\left(2l+1\right)\left(1-\left|\eta_l\right|^2\right)$, and
- $\sigma_{\rm max} = 4\pi\lambda\!\!\!^{-2}(2l+1)$,

where $\sigma_{\rm el}$, $\sigma_{\rm inel}$ and $\sigma_{\rm max}$ are the elastic, inelastic and maximum cross sections, respectively. Here $k = \lambda\!\!\!^{--1}$.

3. Describe briefly the physical process occurring in alpha decay. Without detailed calculation, give a qualitative explanation of the dependence of the transition rate on the Z of the daughter nucleus and the energy released in the transition Q.



The figure shows the α-decay scheme of $^{244}_{96}$Curium to $^{240}_{94}$Plutonium. The transitions are marked with the branching fraction in percent. According to a simple formula the transition rate, λ, for the ground state to ground state transition is given by $\ln\lambda = C - DZ/Q^{1/2}$, where $C = 132.8$ and $D = 3.97$ (MeV$^{1/2}$ when λ is in s^{-1}. Calculate the mean lifetime of $^{244}_{96}$Cm.

If the same C and D are assumed to apply to the transition from the ground state of the $^{244}_{96}$Cm to the 6^+ level of $^{240}_{94}$Pu, show the formula overestimates the rate for that transition and suggest a reason for the discrepancy.

4. The ground state of ^{14}O decays by β^+ emission predominantly (99.4%) to an excited state in ^{14}N, the maximum kinetic energy of the β^+ being 1.810 MeV. The ^{14}N excited state decays to the ^{14}N ground state emitting a γ ray of energy 2.313 MeV. The ground state of ^{14}C decays by β^- emission to the ^{14}N ground state, the maximum β^- kinetic energy being 0.156 MeV.

   (a) Assuming that the nuclear radius R is given in terms of the nuclear mass number A by $R = 1.4 \times A^{1/3}$ fm, explain why the above facts are consistent with the hypothesis that the ^{14}O ground state, the ^{14}C ground state and the ^{14}N excited state all have approximately the same binding energy due to the strong interaction.

   (b) Suggest spin and parity assignments for the ^{14}O and ^{14}C ground states. Given that the γ ray decay in ^{14}N is an M1 transition, assign spins and parities to the ground state and to the excited state in ^{14}N.

   (c) Discuss what your results in parts (a) and (b) imply concerning the nature of the strong internucleon potential.

   (d) Indicate qualitatively why the ^{14}O β^+ decay to the ^{14}N ground state is so suppressed (0.6%) relative to the β^+ decay to the ^{14}N excited state.