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Assignment 2

EP3110:- Electro-magnetics and Applications

Answer to Question 1

Given,

$$\vec{E}(x,y,z,t) = 0.2\sin(10\pi y)\cos(6\pi 10^9 t - \beta z)\hat{x}$$

Phasor form,

$$\vec{E}(x,y,z) = \hat{x}0.2\sin(10\pi y)e^{-j\beta z}$$

We know,

$$\boxed{\vec{H} = -\frac{1}{j\omega\mu_o}\vec{\nabla}X\vec{E}} \tag{1}$$

$$\vec{H}(x,y,z) = -\frac{1}{j\omega\mu_o}\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)X(\hat{x}0.2\sin(10\pi y)e^{-j\beta z})$$

$$\vec{H}(x,y,z) = -\frac{1}{j\omega\mu_o}0.2e^{-j\beta z}\left(10\pi\cos(10\pi y)(-\hat{z}) - j\beta\sin(10\pi y)(\hat{y})\right)$$

$$\boxed{\vec{H}(x,y,z) = \frac{1}{j\omega\mu_o}0.2e^{-j\beta z}\left(j\beta\sin(10\pi y)\hat{y} + 10\pi\cos(10\pi y)\hat{z}\right)} \tag{2}$$

Answer to Question 2

Given,

$$\vec{E} = 30\pi e^{j(\omega t - \frac{4}{3}y)}\hat{z}(\frac{V}{m})$$

$$\vec{H} = 1.0e^{j(\omega t - \frac{4}{3}y)}\hat{x}(\frac{A}{m})$$

We know,

$$\boxed{\frac{\left|\vec{E}\right|}{\left|\vec{H}\right|} = \eta = \sqrt{\frac{\mu_r\mu_o}{\epsilon_r\epsilon_o}}} \tag{3}$$

$$\eta = \frac{30\pi}{1} = \sqrt{\frac{\mu_o(1)}{\epsilon_r\epsilon_o}}$$

$$\epsilon_r = \frac{\mu_o}{900\pi^2\epsilon_o} = 15.978 \approx 16$$

We know,

$$\boxed{\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_r\mu_o\epsilon_r\epsilon_o}}} \tag{4}$$

$$\frac{3\omega}{4} = \sqrt{\frac{1}{(1)\mu_o\left(\frac{\mu_o}{900\pi^2\epsilon_o}\right)\epsilon_o}}$$

$$\omega = 1.0X10^8$$

Answers:

$$\boxed{\epsilon_r \approx 16}$$

$$\boxed{\omega = 1.0X10^8 \text{ radians}}$$

Answer to Question 3

Given,

$$E(x,y,z) = E_o e^{-j(k_x x + k_y y + k_z z)}$$

Homogenous Helmholtz equation

$$\boxed{\nabla^2 E + \omega^2 \mu \epsilon E = 0} \tag{5}$$

$$\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right)(E_o e^{-j(k_x x + k_y y + k_z z)}) + \omega^2 \mu \epsilon E_o e^{-j(k_x x + k_y y + k_z z)} = 0$$

$$E_o(-k_x^2 - k_y^2 - k_z^2) e^{-j(k_x x + k_y y + k_z z)} + \omega^2 \mu \epsilon E_o e^{-j(k_x x + k_y y + k_z z)} = 0$$

$$\boxed{k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon} \tag{6}$$

Therefore, Electric field Intensity $E(x,y,z) = E_o e^{-j(k_x x + k_y y + k_z z)}$ satisfies the homogeneous Helmholtz equation provided that the condition $k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon$ is satisfied.

Answer to Question 4

Given,

$$\vec{E}(R) = \vec{E}_o e^{-j\vec{k}.\vec{R}}$$

Magnetic field $\vec{H}(R)$ can be written as

$$\vec{H}(R) = \vec{H}_o e^{-j\vec{k}.\vec{R}} = \vec{k} X \vec{E}_o e^{-j\vec{k}.\vec{R}}$$

$$\boxed{\vec{\nabla} X \vec{E} = \vec{\nabla}(e^{-j\vec{k}.\vec{R}}) X \vec{E}_o = -j\mu\omega\vec{H}} \tag{7}$$

$$\vec{\nabla}(e^{-j\vec{k}.\vec{R}}) = e^{-j\vec{k}.\vec{R}} \vec{\nabla}(-j\vec{k}.\vec{R}) = e^{-j(k_x\hat{x} + k_y\hat{y} + k_z\hat{z}).(x\hat{x} + y\hat{y} + z\hat{z})}[-j\vec{\nabla}(k_x x + k_y y + k_z z)]$$

$$= e^{-j\vec{k}.\vec{R}}[-j\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)(k_x x + k_y y + k_z z)]$$

$$\boxed{\vec{\nabla}(e^{-j\vec{k}.\vec{R}}) = e^{-j\vec{k}.\vec{R}}[-j\vec{k}]} \tag{8}$$

$$\vec{\nabla}(e^{-j\vec{k}.\vec{R}}) X \vec{E}_o = e^{-j\vec{k}.\vec{R}}[-j\vec{k}] X \vec{E}_o = -j\mu\omega\vec{H}$$

$$\boxed{\vec{k} X \vec{E} = \mu\omega\vec{H}}$$

using equation (8)

$$\vec{\nabla} X \vec{H} = \vec{\nabla}(e^{-j\vec{k}.\vec{R}}) X \vec{H}_o = j\omega\epsilon\vec{E}$$

$$\boxed{\vec{k} X \vec{H} = -\omega\epsilon\vec{E}}$$

$$\vec{\nabla}.\vec{E} = \vec{\nabla}(e^{-j\vec{k}.\vec{R}}).\vec{E}_o = 0$$

$$\boxed{\vec{k}.\vec{E} = 0}$$

$$\vec{\nabla}.\vec{H} = 0$$

$$\boxed{\vec{k}.\vec{H} = 0}$$

Answer to Question 5

Given,

$$\vec{E} = 2\cos(10^8 t - \frac{z}{\sqrt{3}})\hat{x} - \sin(10^8 t - \frac{z}{\sqrt{3}})\hat{y}$$

(a) we know,

$$\boxed{\omega = 2\pi f} \tag{9}$$

$$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} = 15.9MHz$$

$$\boxed{\lambda = \frac{2\pi}{k}} \tag{10}$$

$$\lambda = \frac{2\pi}{\frac{1}{\sqrt{3}}} = 10.88m$$

(b) we know,

$$\boxed{v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_r \mu_o \epsilon_r \epsilon_o}}} \tag{11}$$

$$\sqrt{3} \times 10^8 = \frac{1}{\sqrt{\mu_o \epsilon_r \epsilon_o}}$$

$$\epsilon_r = 3$$

(c)

$$E_x = 2\cos(10^8 - \frac{z}{\sqrt{3}}) \implies \cos(10^8 - \frac{z}{\sqrt{3}}) = \frac{E_x}{2}$$

$$E_y = -\sin(10^8 - \frac{z}{\sqrt{3}}) \implies \sin(10^8 - \frac{z}{\sqrt{3}}) = -E_y$$

$$\cos^2(10^8 - \frac{z}{\sqrt{3}}) + \sin^2(10^8 - \frac{z}{\sqrt{3}}) = 1$$

$$\frac{E_x^2}{4} + \frac{E_y^2}{1} = 1$$

Therefore, Elliptically polarized.

(d) we know,

$$\boxed{\vec{H} = \frac{\vec{k} X \vec{E}}{\eta}} \tag{12}$$

Here,

$$\vec{k} = \hat{z}$$

$$\vec{H} = \hat{z} X (2\cos(10^8 t - \frac{z}{\sqrt{3}})\hat{x} - \sin(10^8 t - \frac{z}{\sqrt{3}})\hat{y})$$

$$\vec{H} = \sqrt{\frac{\epsilon_r \epsilon_o}{\mu_r \mu_o}}(\sin(10^8 t - \frac{z}{\sqrt{3}})\hat{x} + 2\cos(10^8 t - \frac{z}{\sqrt{3}})\hat{y})$$

$$\vec{H} = 4.59 \times 10^{-3}(\sin(10^8 t - \frac{z}{\sqrt{3}})\hat{x} + 2\cos(10^8 t - \frac{z}{\sqrt{3}})\hat{y})\left(\frac{A}{m}\right)$$

Answers

1. $f = 15.9MHz$ and $\lambda = 10.88m$.

2. $\epsilon_r = 3$

3. Elliptically polarized.

4. $\vec{H} = 4.59 \times 10^{-3}(\sin(10^8 t - \frac{z}{\sqrt{3}})\hat{x} + 2\cos(10^8 t - \frac{z}{\sqrt{3}})\hat{y})\left(\frac{A}{m}\right)$

Answer to Question 6

Let $\alpha = \omega t - kz$
$\vec{E} = \hat{a}_x E_{10}\sin(\alpha) + \hat{a}_y E_{20}\sin(\alpha + \psi) = \hat{a}_x E_x + \hat{a}_y E_y$

$$\frac{E_x}{E_{10}} = \sin(\alpha), \frac{E_y}{E_{20}} = \sin(\alpha + \psi) = \sin(\alpha)\cos(\psi) + \cos(\alpha)\sin(\psi)$$

$$\frac{E_y}{E_{20}} = \frac{E_x}{E_{10}}\cos(\psi) + \sqrt{1 - \left(\frac{E_x}{E_{10}}\right)^2}\sin(\psi)$$

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2 + \left(\frac{E_x}{E_{10}\sin\psi}\right)^2 - 2\frac{E_x E_y}{E_{10}E_{20}}\frac{\cos\psi}{\sin^2\psi} = 1 \tag{13}$$

Coordinates transform

$$\left(\frac{E'_x}{a}\right)^2 + \left(\frac{E'_y}{b}\right)^2 = 1$$

$$E'_x = E_x\cos\theta + E_y\sin\theta$$

$$E'_y = -E_x\sin\theta + E_y\cos\theta$$

substituting and rearranging,

$$E_x^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) + E_y^2\left(\frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}\right) - 2E_x E_y\sin\theta\cos\theta(\frac{1}{b^2} - \frac{1}{a^2}) = 1 \tag{14}$$

Comparing equations 13 and 14.

$$\theta = \frac{1}{2}\tan^{-1}(\frac{2E_{10}E_{20}\cos\psi}{E_{10}^2 - E_{20}^2})$$

$$a = \sqrt{\frac{2}{\frac{1}{E_{10}^2}(1 + \sec 2\theta) + \frac{1}{E_{20}^2}(1 - \sec 2\theta)}}\sin\psi$$

$$b = \sqrt{\frac{2}{\frac{1}{E_{10}^2}(1 - \sec 2\theta) + \frac{1}{E_{20}^2}(1 + \sec 2\theta)}}\sin\psi$$