EP3110 - Electro-magnetics and Applications

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Abstract

Instructor:P Murugavel (HSB 116B)
Lecture Timings: D Slot (Mon – 11am, Tue – 10am, Wed - 9am)
Venue: HSB 263
Course Outline: Learning objectives This is an intermediate level course in electromagnetic fields and assumes background in electrostatics, magneto-statics and introductory knowledge in electrodynamics. The main objective is to introduce electromagnetic fields with emphasis on analytical rigour and physical reasoning required for solving problems having direct application. The course will also provide sufficient background to motivate students to take up advanced levels courses such as, electromagnetic scattering, computational electrodynamics, etc. Learning outcomes Upon successful completion, the students will have learned i. the importance of constitutive properties of materials and their use in applications, ii. the effect of boundaries and be able to develop and analyze optical coatings, iii. time dependent formulation of potentials and fields, and fields dues to moving charges, iv. the fundamental ideas in electromagnetic scattering with relevance to applications and v) the concept of waveguides and propagation of guided electromagnetic waves. **Pre-requisite**: PH1020 - Physics II

Grading: Assignments - 10 marks Quiz I - 20 marks Quiz II - 20 marks End-semester - 50 marks

Contents

Chapter 1

Electrodynamics(Review)

Lecture 1: First Lecture

25 July 2022

1.1 Electrostatics(Review)

Two postulates(propositions) in Electrostatics

Proposition 1.1.1.

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_o}$$

$$\oint_s \vec{E}.\vec{ds} = \frac{Q_{en}}{\epsilon_o}$$

Proposition 1.1.2.

$$\vec{\nabla} X \vec{E} = \vec{0}$$

$$\oint_c \vec{E}.\vec{dl} = 0$$

Electric field around a closed path is 0.

Theorem 1.1.1. kirchhoff's voltage Law, Algebraic sum of voltage drop in a closed loop is zero.

Theorem 1.1.2 (Coloumbs Law). $\vec{E}$ due to a point charge "q" is

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{\left|\vec{R}\right|^3}\vec{R}$$

For a discrete distribution of charges

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\sum_{j=1}^{n}\frac{q_j}{\left|\vec{R}-\vec{R_j}\right|^3}(\vec{R}-\vec{R_j})$$

For a continuos distribution of charges

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\int \hat{R}\frac{\rho}{\vec{R}^2}dV'$$

since,

$$\vec{\nabla} X \vec{E} = \vec{0}$$

$$\vec{E} = -\vec{\nabla} V$$

Negative sign because potential decreases in the direction of electric field by convention.

Definition 1.1.1.

$$\vec{E} = -\vec{\nabla} V$$

$$V_2 - V_1 = \int_{P_1}^{P_2} \vec{E}.\vec{dl}$$

$$V(R) = \frac{1}{4\pi\epsilon_o} \int \frac{\rho}{R} dV'$$

prime indicates coordinate system w.r.t source.

Definition 1.1.2 (Dipole). A pair of equal and opposite charges separated by a distance.

$$V_p(R) = \frac{q}{4\pi\epsilon_o}[\frac{1}{R_+} - \frac{1}{R_-}]$$

$$V_p(R) = \frac{qd\cos\theta}{4\pi\epsilon_o R^2} = \frac{\vec{p}.\hat{R}}{4\pi\epsilon_o R^2}$$

$$\vec{E_p}(R) = -\vec{\nabla} V_p(R)$$

$$\vec{E_p}(R) = [-\hat{R}\frac{\partial}{\partial R} - \frac{\hat{\theta}}{R}\frac{\partial}{\partial\theta}]V_p(R)$$

$$\vec{E} = \frac{\vec{p}}{4\pi\epsilon_o R^3}[2\cos\theta\hat{R} + \sin\theta\hat{\theta}]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_o R^3}[3(\vec{p}.\hat{R})\hat{R} - \vec{p}]$$

Conductor

$\vec{E} = 0$ inside a conductor.

$$\oint_c \vec{E}.\vec{dl} = 0$$

$$\vec{E_{t1}}.\Delta\vec{w} - \vec{E_{t2}}\Delta\vec{w} = 0$$

$\vec{E_{t1}} = 0 \because$ inside a conductor.

$\therefore \vec{E_{t2}} = 0$.
$\therefore$ No tangential electric field.

$$\frac{\sigma_s \Delta s}{\epsilon_o} = [\vec{E_1}.\vec{n_1} + \vec{E_2}.\vec{n_2}]\Delta s$$

$$\vec{E_2} = 0$$

inside a conductor.

$$\vec{E_1} = \frac{\sigma_s}{\epsilon_o}\hat{n}$$

Normal electric field is $\vec{E_n} = \frac{\sigma_s}{\epsilon_o}\hat{n}$

Dielectric in a static $\vec{E}$

$$\vec{p} = q\vec{d}$$

$$\vec{\tau} = \vec{p} X \vec{E}$$

$$U = -\vec{p}.\vec{E}$$

Gauss law in differential form

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_o} = \frac{1}{\epsilon_o}[\rho_f + \rho_b]$$

Definition 1.1.3. The *polarization* of a medium P gives the electric dipole moment per unit volume of the material.

$$\vec{P} = \frac{n\vec{p}}{V}$$

Bound charges

$$\sigma_b = -\vec{P}.\hat{n}$$

$$\rho_b = -\vec{\nabla}.\vec{P}$$

Potential V,

$$V = \frac{1}{4\pi\epsilon_o}\oint\frac{\vec{P}.\hat{n}}{R}ds + \frac{1}{4\pi\epsilon_o}\int\frac{-\vec{\nabla}.\vec{P}}{R}dV$$

refer David J griffith's for detailed explanation.

For a Dielectric,

$$\vec{\nabla}.\vec{E} = \frac{1}{\epsilon_o}(\rho_f - \vec{\nabla}.\vec{P})$$

$$\vec{\nabla}.[\epsilon_o\vec{E} + \vec{P}] = \rho_f$$

Definition 1.1.4. Displacement vector, $\vec{D} = \epsilon_o\vec{E} + \vec{P}$

$$\vec{\nabla}.\vec{D} = \rho_f$$

Lecture 2: Second Lecture

26 July 2022

For a **Linear isotropic homogenous dielectric**

$$\vec{P} = \epsilon_o\chi_e\vec{E}$$

χ_e = electric susceptibility.

$$\vec{D} = \epsilon_o(1+\chi_e)\vec{E}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_o} = 1 + \chi_e$$

Boundary Conditions

Tangential Component

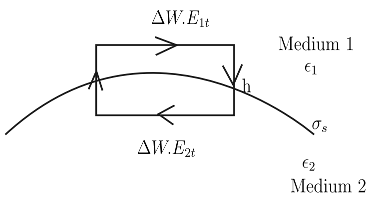

Figure 1.1: Tangential Component

$$\oint\vec{E}.\vec{dl} = 0 \Rightarrow E_{1t}\Delta\vec{W} - E_{2t}\Delta\vec{W} = 0$$

$$\vec{E_{1t}} = \vec{E_{2t}}$$

$$\frac{\vec{D_{1t}}}{\epsilon_1} = \frac{\vec{D_{2t}}}{\epsilon_2}$$

Normal component

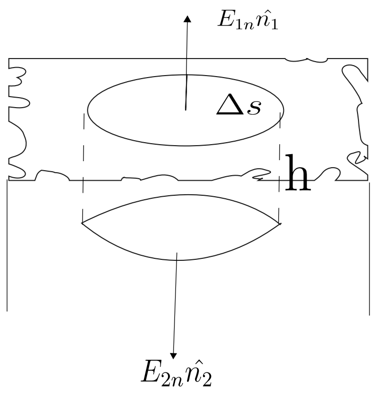

Figure 1.2: Normal Component

$$\vec{E_{1n}}\Delta s - \vec{E_{2n}}\Delta s = \frac{\Delta s \sigma_s}{\epsilon_o}$$

$$\frac{\vec{D_{1n}}}{\epsilon_1} - \frac{\vec{D_{2n}}}{\epsilon_2} = \frac{\sigma_s}{\epsilon_o}$$

Summary

$$\vec{E_{1t}} = \vec{E_{2t}}$$

$$\vec{E_{1n}} - \vec{E_{2n}} = \frac{\sigma_s}{\epsilon_o}$$

$$\vec{D} = \epsilon \vec{E}$$

Energy stored

$$W = \frac{\epsilon_o}{2}\int E^2 d\tau$$

Poisson's equation

$$\nabla^2 V = \frac{\rho}{\epsilon_o}$$

Laplace equation

$$\nabla^2 V = 0$$

1.2 Magnetostatics(review)

Definition 1.2.1.

$$\vec{\nabla}.\vec{B} = 0$$

Magnetic monopole doesn't exist Integral Form:

$$\oint_S \vec{B}.\vec{da} = 0$$

Definition 1.2.2.

$$\vec{\nabla} X \vec{B} = \mu_o \vec{J}$$

Integral form:

$$\oint \vec{B}.\vec{dl} = \mu_o I$$

Definition 1.2.3 (Bio-Savart's law).

$$\vec{B}(R) = \frac{\mu_o}{4\pi}\int I\frac{\vec{dl}'X\hat{R}}{R^2}$$

Definition 1.2.4 (Maganetic Vector Potential).

$$\vec{B} = \vec{\nabla}X\vec{A}$$

$$\vec{\nabla}.\vec{A} = ?$$

$$\vec{\nabla}X\vec{B} = \vec{\nabla}X(\vec{\nabla}X\vec{A}) = \mu_o\vec{J}$$

$$\vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A} = \mu_o\vec{J}$$

$$\vec{A} = \vec{A}_o + \vec{\nabla}\lambda$$

$$\vec{B} = \vec{\nabla}X\vec{A}_o + \vec{\nabla}X\vec{\nabla}\lambda$$

$$\vec{B} = \vec{\nabla}X\vec{A}_o$$

$$\vec{\nabla}.\vec{A} = \vec{\nabla}.\vec{A} + \nabla^2\lambda$$

let

$$\nabla^2\lambda = -\vec{\nabla}.\vec{A}_o \Rightarrow \vec{\nabla}.\vec{A} = 0$$

Solving using symmetry

$$\nabla^2\lambda = -\vec{\nabla}.\vec{A}_o \Rightarrow \vec{\nabla}.\vec{A} = 0$$

Poisson's equation

$$\nabla^2 V = -\frac{\rho}{\epsilon_o}$$

$$V = \frac{1}{4\pi\epsilon_o}\int\frac{\rho}{R}dV'$$

$$\lambda = \frac{1}{4\pi}\int\frac{\vec{\nabla}.\vec{A}_o}{R}dV'$$

$$\therefore \vec{A} = \vec{A}_o + \vec{\nabla}\lambda$$

$$\nabla^2\vec{A} = -\mu_o\vec{J}$$

Poisson's equation again!!!

$$\vec{A} = \frac{\mu_o}{4\pi}\int\frac{\vec{J}}{R}dV'$$

Definition 1.2.5 (**magnetic dipole moment**).

$$\vec{m} = I\int\vec{ds} = \vec{I}a(\because a = area)$$

$$\vec{A} = \frac{\mu_o\vec{m}X\hat{R}}{4\pi R^2}$$

Definition 1.2.6 (**Magnetization**).

$$\vec{M} = \frac{n\vec{m}}{V}$$

V = volume, n = number of dipoles.

$$\vec{J_b} = \vec{\nabla} X \vec{M}$$
$$\vec{k_b} = \vec{M} X \hat{n}$$

similar to σ_b and ρ_b

$$\vec{\nabla} X \vec{B} = \mu_o(\vec{J_f} + \vec{J_b})$$
$$\frac{1}{\mu_o} \vec{\nabla} X \vec{B} = \vec{J_f} + \vec{\nabla} X \vec{M}$$
$$\vec{\nabla} X (\frac{\vec{B}}{\mu_o} - \vec{M}) = \vec{J_f}$$

Definition 1.2.7. $\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M}$

$$\vec{\nabla} X \vec{H} = \vec{J_f}$$

Ampere's law in magnetic material.

Similar to Polarization $\vec{P} = \epsilon_o \chi_e$ we have Magnetization $\vec{M} = \frac{1}{\mu_o} \chi_m \vec{B}$ for a linear homogenous isotropic material, where χ_m is Magnetic susceptibility.

$$\bar{H}(\mu_o)(1 + \chi_m) = \vec{B}$$
$$\vec{B} = \mu \vec{H}$$
$$\frac{\mu}{\mu_o} = \mu_r = 1 + \chi_m$$

Lecture 3: Third Lecture

27 July 2022

Boundary Conditions

Normal Component

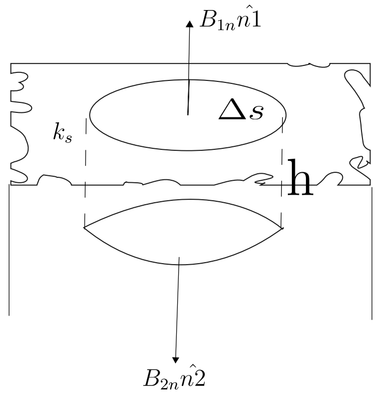

Figure 1.3: title

$$\oint \vec{B}.d\vec{s} = 0$$
$$B_{1n}\Delta S - B_{2n}\Delta S = 0 \Rightarrow B_{1n} = B_{2n}$$
$$\oint \vec{H}.d\vec{l} = I_f$$

Tangential Component

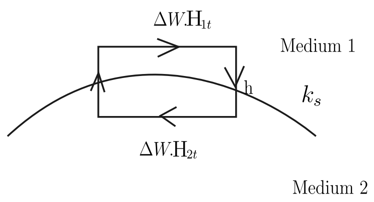

Figure 1.4: title

$$H_{1t}\Delta W - H_{2t}\Delta W = \Delta W k_s$$
$$H_{1t} - H_{2t} = k_s$$

Where, k_s = surface current density.

$$\bar{n}_2 X(\vec{H}_1 - \vec{H}_2) = \vec{k_s}$$

Ohms law

$$\vec{J} = \sigma\vec{E}$$
$$\epsilon = -\frac{dQ}{dt} = \oint \vec{E}.d\vec{l}$$

Faraday's laws of induction

$$\vec{\nabla}X\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Energy stored in a Magnetic Field

$$W = \frac{1}{2\mu_o}\int B^2 d\tau$$

Summary of Equations

Gauss law

$$\vec{\nabla}.\vec{D} = \rho_f$$

Magnetic monopole doesn't exist

$$\vec{\nabla}.\vec{B} = 0$$

Faraday's law

$$\vec{\nabla}X\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Ampere's law

$$\vec{\nabla}X\vec{H} = J_f$$

1.2.1 Correction in Ampere's law

$$\vec{\nabla}.\vec{\nabla}X\vec{H} = 0,$$

but

$$\vec{\nabla}.\vec{J_f} \neq 0$$
$$\vec{\nabla}.\vec{J} = -\frac{\partial\rho}{\partial t}$$
$$\vec{\nabla}X\vec{H} = \vec{J_f} + ?$$
$$\vec{\nabla}.\vec{\nabla}X\vec{H} = \vec{\nabla}.\vec{J_f} + \frac{\partial\rho_f}{\partial t}$$
$$\vec{\nabla}.\vec{\nabla}X\vec{H} = \vec{\nabla}.\vec{J_f} + \frac{\partial\vec{\nabla}.\vec{D}}{\partial t}$$

$$\vec{\nabla}.\vec{\nabla}X\vec{H} = \vec{\nabla}.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

Definition 1.2.8. Displacement current density $\frac{\partial \vec{D}}{\partial t}$

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$

Non-linear dielectric

$$\vec{P_i} = \Sigma_j \alpha_{ij} E_j + \Sigma_{jk} \alpha_{ijk} E_j E_k$$

Linear dielectric

$$\vec{P_i} = \epsilon_o \chi_{ij} \vec{E_{jk}}$$

Example.

$$P_x = \epsilon_o(\chi_{xx}E_x + \chi_{xy}E_y + \chi_{xz}E_z)$$
$$P_y = \epsilon_o(\chi_{yx}E_x + \chi_{yy}E_y + \chi_{yz}E_z)$$
$$P_z = \epsilon_o(\chi_{zx}E_x + \chi_{zy}E_y + \chi_{zz}E_z)$$

Linear Isotropic Dielectric

$\chi \rightarrow$ independent of direction of electric field.

Linear Homogenous Isotropic Dielectric

$\chi \rightarrow$ Same everywhere.

Potential formulation

$$\vec{B} = \vec{\nabla}X\vec{A}\left(\because \vec{\nabla}.\vec{B} = 0\right)$$

Faraday's law

$$\vec{\nabla}X\vec{E} = \frac{\partial \vec{B}}{\partial t} = \frac{\partial(\vec{\nabla}X\vec{A})}{\partial t}$$
$$\vec{\nabla}X\vec{E} = -\vec{\nabla}X\frac{\partial \vec{A}}{\partial t}$$
$$\vec{\nabla}X\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0$$
$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla}V$$
$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla}X\vec{A}$$

Solution for time-independent can be given by poisson's equations, but for time dependent we cannot use poisson's equations.

Conditions

1. At low frequency :- changes very slowly
2. Near to source :- quasi-static condition

Lecture 4: Fourth Lecture

Wave equation for potentials

01/08/2022

$$\vec{E} = -\vec{\nabla} - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} X \vec{A}$$

$$\vec{\nabla} X \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla} X \vec{B} = \mu \vec{J}_f + \mu\epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} X (\vec{\nabla} X \vec{A}) = \mu \vec{J}_f + \mu\epsilon \frac{\partial \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \right)}{\partial t}$$

$$\vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}_f - \mu\epsilon \frac{\partial(\vec{\nabla} V)}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$-\nabla^2 \vec{A} + \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t} = \mu \vec{J}_f - \vec{\nabla}(\vec{\nabla}.\vec{A}) - \vec{\nabla}(\mu\epsilon \frac{\partial V}{\partial t})$$

$$-\nabla^2 \vec{A} + \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t} = \mu \vec{J}_f - \vec{\nabla}(\vec{\nabla}.\vec{A} + \mu\epsilon \frac{\partial V}{\partial t})$$

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t} = -\mu \vec{J}_f + \vec{\nabla}(\vec{\nabla}.\vec{A} + \mu\epsilon \frac{\partial V}{\partial t})$$

applying **Lorentz gauge**

$$\vec{\nabla}.\vec{A} = -\mu\epsilon \frac{\partial V}{\partial t}$$

$$\nabla^2 A - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

Lecture 15

1.3 Oblique Incidence

05 sept 2022

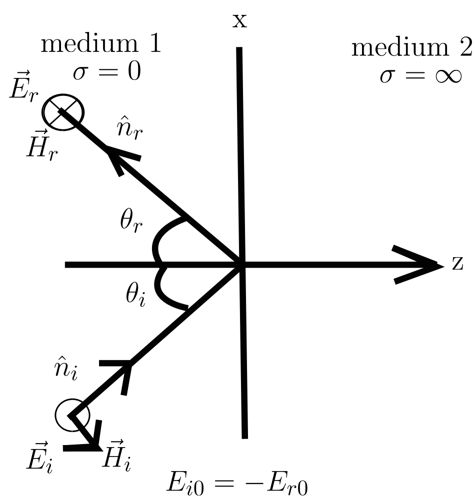

Figure 1.5: title

$$\vec{E}_r = -\hat{y} E_{io} e^{-j\beta_1 \hat{n}_r.\vec{R}}$$

$$\vec{E}_1(x,z) = -\hat{y}j2E_{io}\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i}$$

$$\vec{H}_1(x,z) = -2\frac{E_{ro}}{\eta_1}[\hat{x}\cos\theta_i\cos(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i} + \hat{z}\sin\theta_i\sin(\beta_1 z\cos\theta_i)e^{-j\beta_1 x\sin\theta_i}]$$

1.3.1 along z - direction

$$\beta_1\cos\theta_i = \beta_{1z}$$

$$S_{av} = \frac{1}{2}Re[\vec{E}_y X\vec{H}^*{}_x] = 0.$$

no propagation of EM wave along z-direction.

1.3.2 along x - direction

Surface wave

1. Plane wave - x-direction
2. Non uniform plane wave.

Electric field becomes "0" at

$$\beta_1 z\cos\theta_i = -n\pi$$

$$z = -\frac{n\pi}{\beta_1\cos\theta_i}$$

$$\boxed{z = -\frac{n\lambda_1}{2\cos\theta_i}}$$

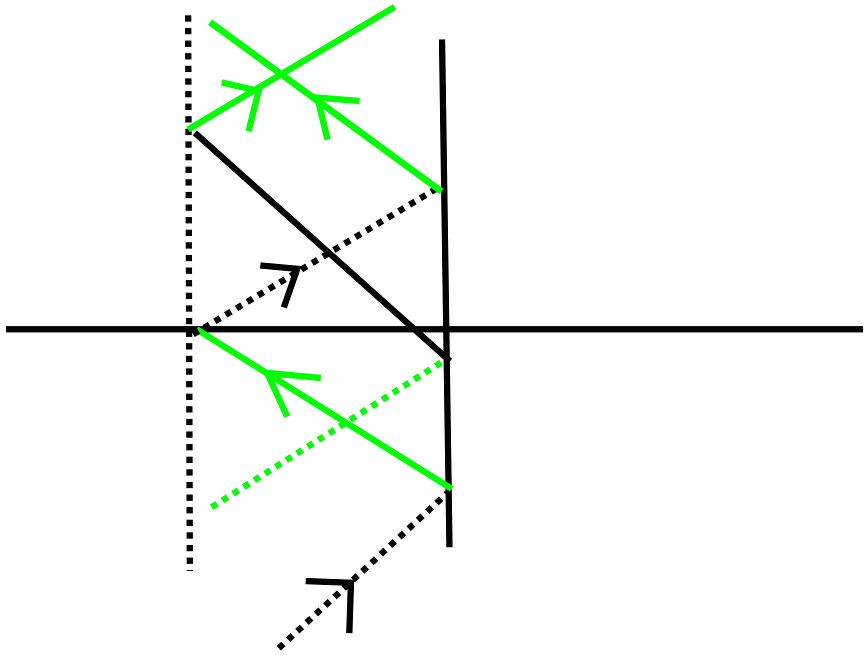

Figure 1.6: title

$$\vec{E}_i = (\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)e^{-j\beta_1\hat{n}_i.\vec{R}}$$

$$\vec{H}_i = \frac{\hat{n}_i X \vec{E}_i}{\eta_1}$$

$$\hat{n}_i = \hat{x}\sin\theta_i + \hat{z}\cos\theta_i$$

$$\hat{n}_r = \hat{x}x\sin\theta_i - \hat{z}\cos\theta_i$$

$$\boxed{E_{10} = -E_{ro} \text{ and } \theta_i = \theta_r}$$

Example. Given θ_i and ω, Find out $\vec{J}_s(x)$

Answer.

$$E_1(x,0) = 0, \quad E_2 = 0$$

$$H_1(x,0) = ? \quad H_2 - 0$$

$$\hat{n}_2 X \vec{H}_1 = \vec{J}_s(x)$$

$$\vec{H}_1(x,0) = \frac{E_{io}}{\eta_o}(\hat{x}2\cos\theta_i)e^{-j\beta_o x\sin\theta_i}$$

$$\vec{J}_s(x) = \hat{n}_2 X \vec{H}_1(x,0)$$

$$\vec{J}_s = -\hat{y}\frac{E_{io}}{\eta_o}2\cos\theta_i e^{-j\frac{\omega}{c}x\sin\theta_i}$$

⊛

Appendix

Appendix A

Additional Proofs

A.1 Proof of ??

We can now prove **??**.

Proof of ??. See https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence. ■