Introduction to nuclear and particle physics PH5211 - Problem Set 3

To be submitted in class on Tuesday 23rd August 2022

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Suggested references:

- Krane, Chapters 5.1 and 11.3
- Perkins, 3.13 and 3.14, Appendix C

1. Define isospin and explain its physical origin with respect to the neutron and proton.

   Use isospin to explain why

   $$\sigma(pp \to d\pi^+) = 2\sigma(pn \to d\pi^0) \ ,$$

   here d is the deuteron.

   Derive the Clebsch-Gordon coefficients for the combined angular momentum states of a spin $\frac{1}{2}$ and a spin 1 state.

   Use the figure of $\sigma(\pi^+ p)$ and $\sigma(\pi^- p)$ given in class to assign a mass and an isospin to the state produced when $E_\pi \approx 600$ MeV.

2. Explain briefly why a model of the nucleus in which the nucleons are considered to be moving in a central potential predicts 'shell closure' effects in nuclear properties for neutron and proton numbers 2, 8, 20, 34, 40, 58, 92 and 138. The observed values at which closure occur are 2, 8, 20, 28, 50, 82 and 126 (neutrons only). Explain how a modification to the single-particle potential accounts for this observation.

   Assign spins and parities to the ground states of the nuclei

   $${}^{7}_{3}\mathrm{Li},\ {}^{17}_{8}\mathrm{O},\ {}^{20}_{10}\mathrm{Ne},\ {}^{27}_{13}\mathrm{Al},\ {}^{14}_{7}\mathrm{N}\ \text{ and }\ {}^{41}_{21}\mathrm{Sc}.$$

   State the assumptions you have made.

   What does the single-particle shell model predict for the magnetic dipole moment of the nucleus ${}^{3}_{2}\mathrm{He}$?

Please turn over

3. In a "deuteron stripping" experiment, the nuclear reaction

$$d + {}^{16}_{8}\mathrm{O} \rightarrow p + {}^{17}_{8}\mathrm{O}^{(*)}$$

takes place, where the final state nuclei may be excited. In such an experiment a beam of non-relativistic deuterons of kinetic energy E_d is directed at the target and protons with kinetic energy E_p are detected emerging at an angle θ to the deuteron beam. Ignoring nuclear binding energy, show that the kinetic energy of the recoiling nucleus is given approximately by

$$E = \left(2E_d + E_p - 2\sqrt{2E_pE_d}\cos\theta\right)/17\,.$$

When $E_d = 14.95$ MeV and $\theta = 19^\circ$, three groups of protons are observed with energy 16.62 MeV, 15.74 MeV and 11.97 MeV. Given the mass difference between the initial and final state is 1.93 MeV, show that the highest energy protons correspond to a reaction in which the recoiling ${}^{17}_{8}\mathrm{O}$ nucleus is left in its ground state. Calculate the excitation energies of the two levels in ${}^{17}_{8}\mathrm{O}$ that correspond to the other two proton groups.

The neutron separation energy of ${}^{17}_{8}\mathrm{O}$ is 4.15 MeV. Which of the ${}^{17}_{8}\mathrm{O}$ levels observed in the above stripping reactions would be observable via the scattering of neutrons on ${}^{16}_{8}\mathrm{O}$ and what neutron kinetic energy would be required in a frame where the ${}^{16}_{8}\mathrm{O}$ is at rest.