IIT Madras
Department of Physics
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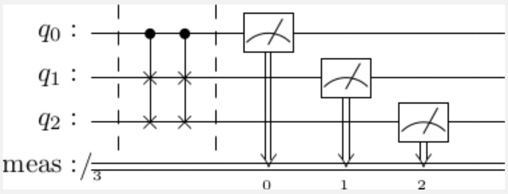
ID : EP20B012

Jul-Nov, 2022
Assignment 1

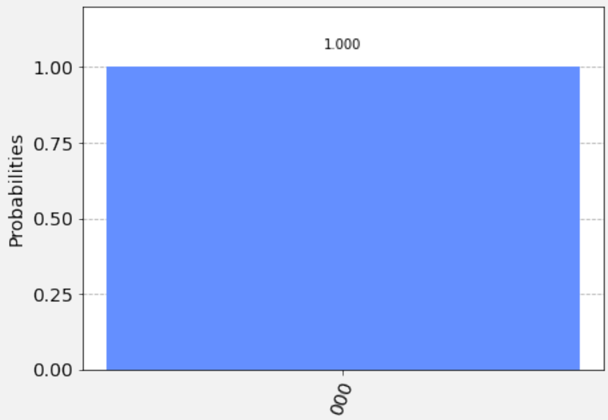
ID5841: Quantum Computing Lab

Answer to Question 1 a

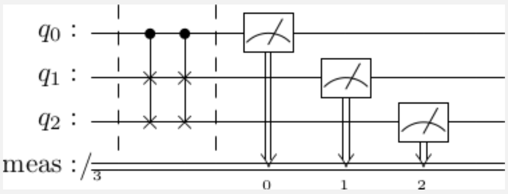
Circuit for 2 Fredkin gates on state 000



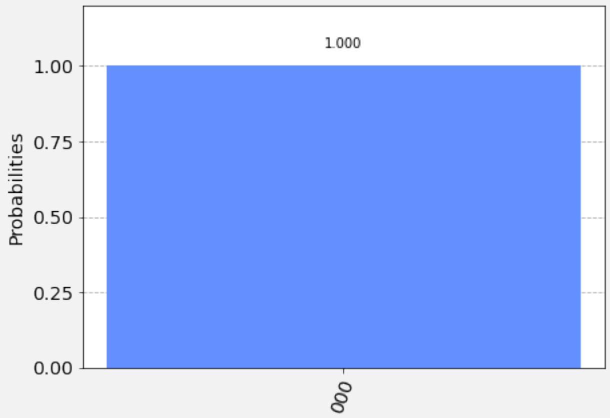
Output for input 000 is



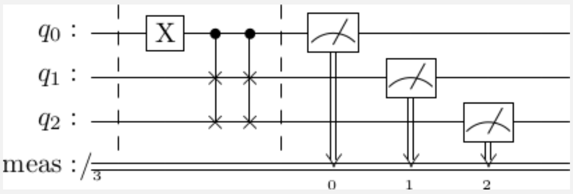
Circuit for 2 Fredkin gates on state 000



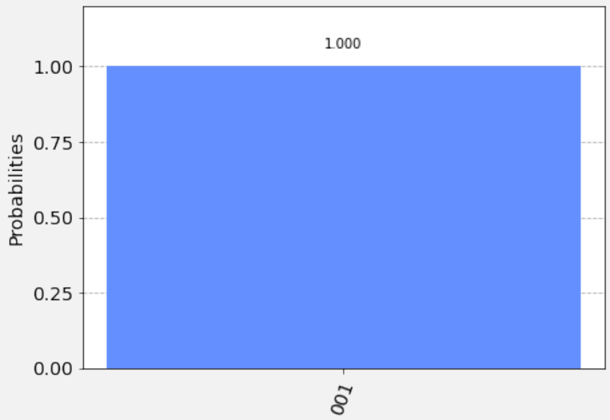
Output for input 000 is



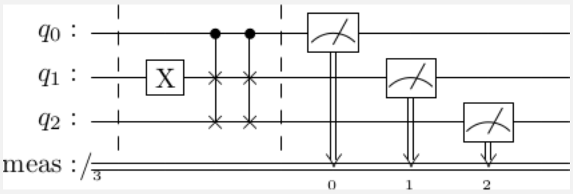
Circuit for 2 Fredkin gates on state 001



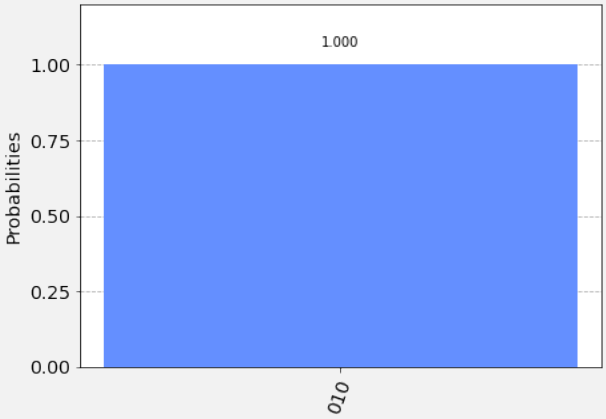
Output for input 001 is



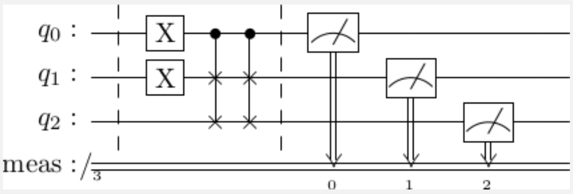
Circuit for 2 Fredkin gates on state 010



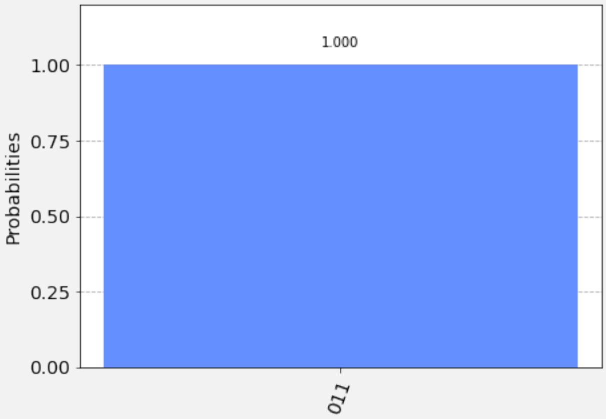
Output for input 010 is



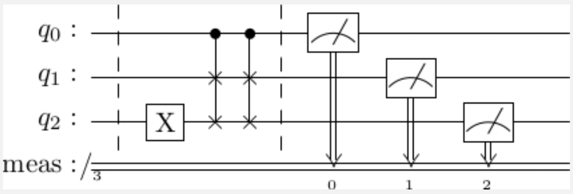
Circuit for 2 Fredkin gates on state 011



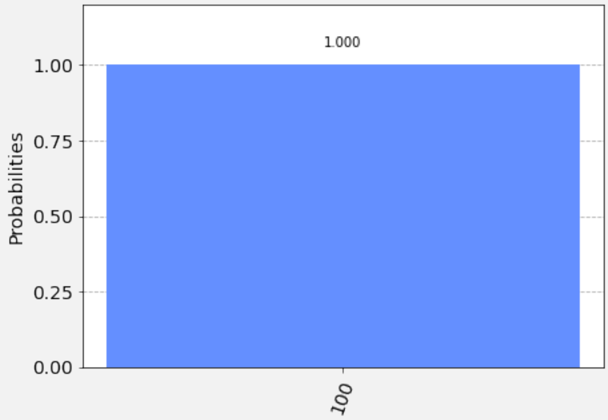
Output for input 011 is



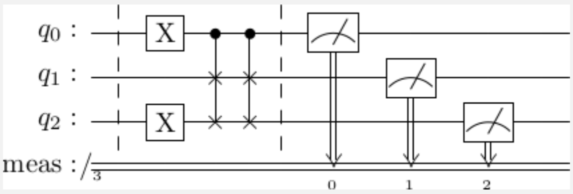
Circuit for 2 Fredkin gates on state 100



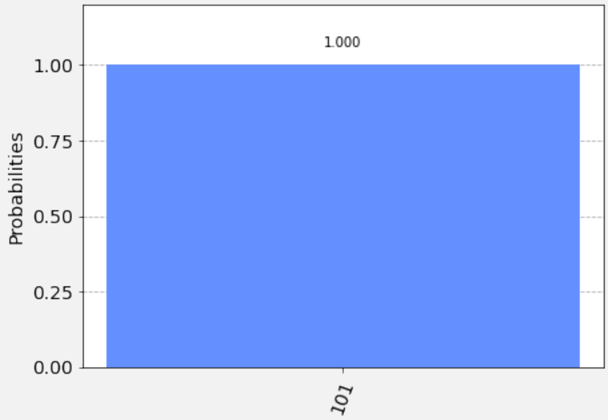
Output for input 100 is



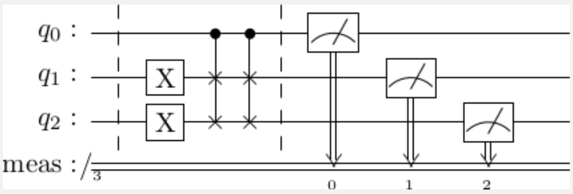
Circuit for 2 Fredkin gates on state 101



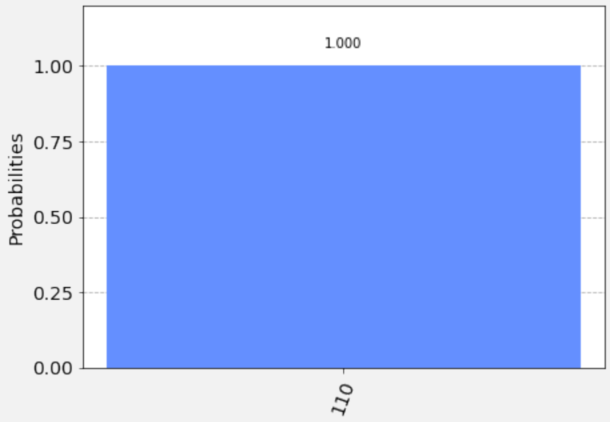
Output for input 101 is



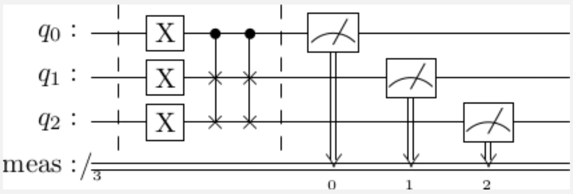
Circuit for 2 Fredkin gates on state 110



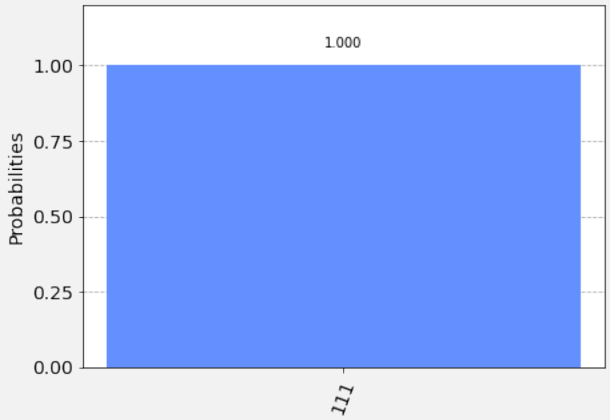
Output for input 110 is



Circuit for 2 Fredkin gates on state 111



Output for input 111 is

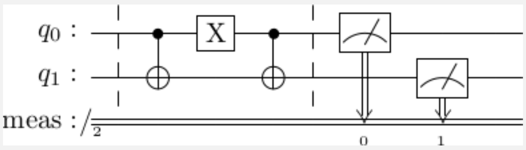


Input	Output
000	000
001	001
010	010
011	011
100	100
101	101
110	110
111	111

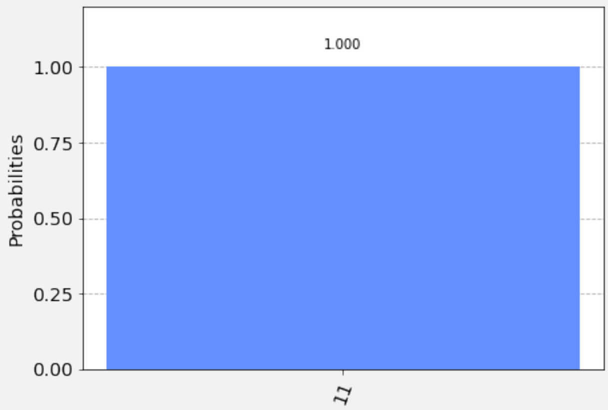
Since, input is equal to output we can clearly say Fredkin gate is reversible gate.

Answer to Question 1 b (a)

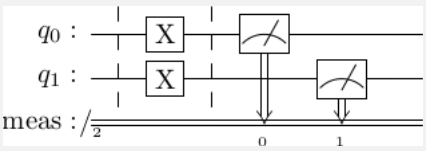
LHS Circuit for input 00



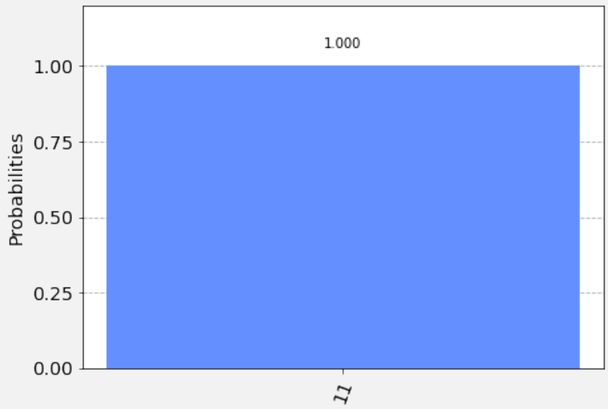
Output of LHS circuit for input 00



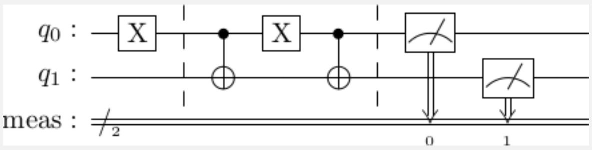
RHS Circuit for input 00



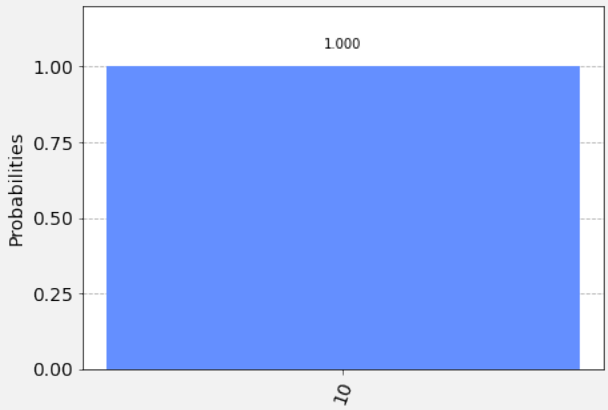
Output of RHS circuit for input 00



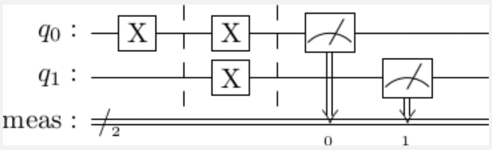
LHS Circuit for input 01



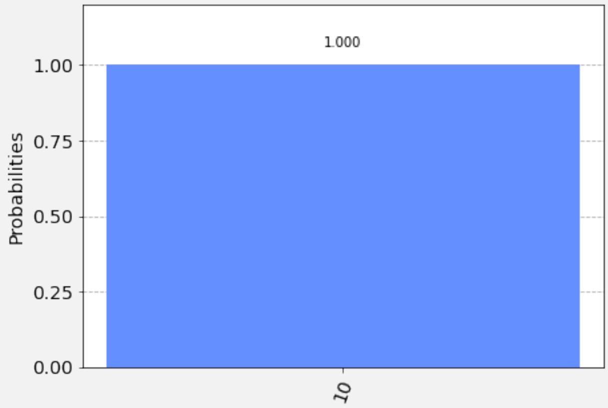
Output of LHS circuit for input 01



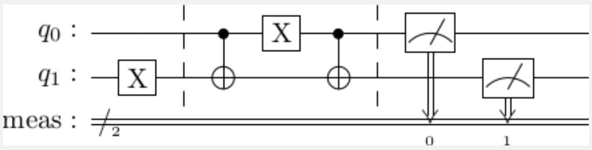
RHS Circuit for input 01



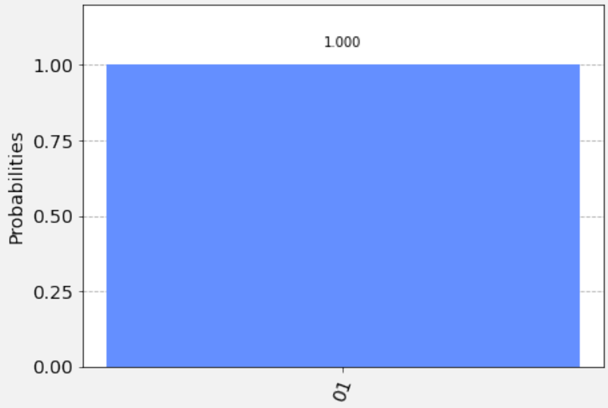
Output of RHS circuit for input 01



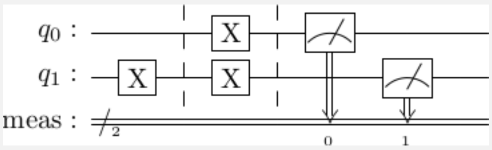
LHS Circuit for input 10



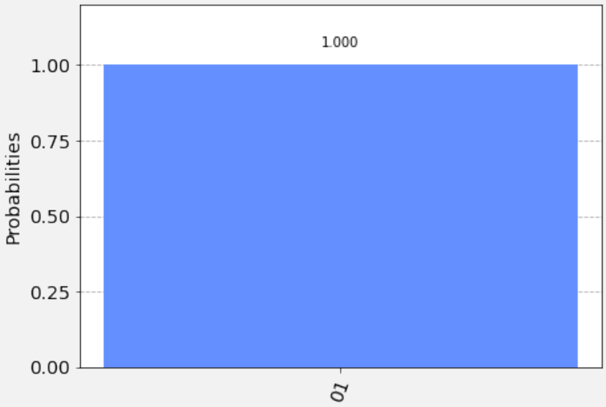
Output of LHS circuit for input 10



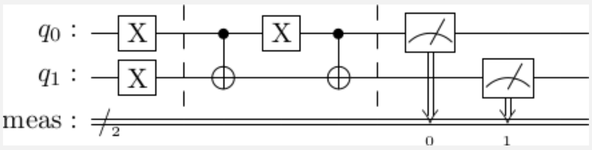
RHS Circuit for input 10



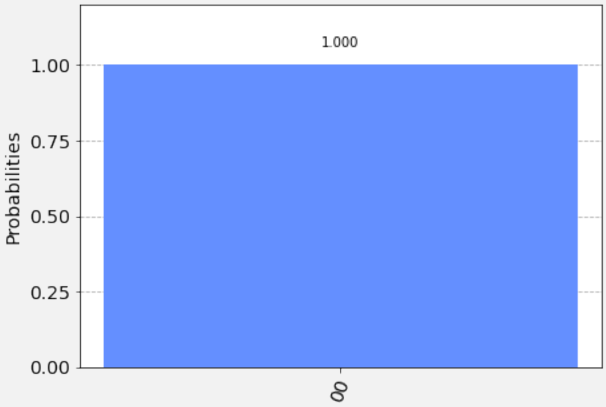
Output of RHS circuit for input 10



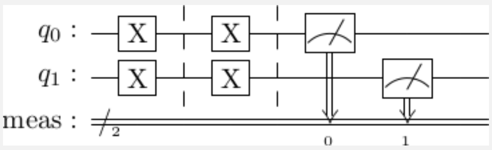
LHS Circuit for input 11



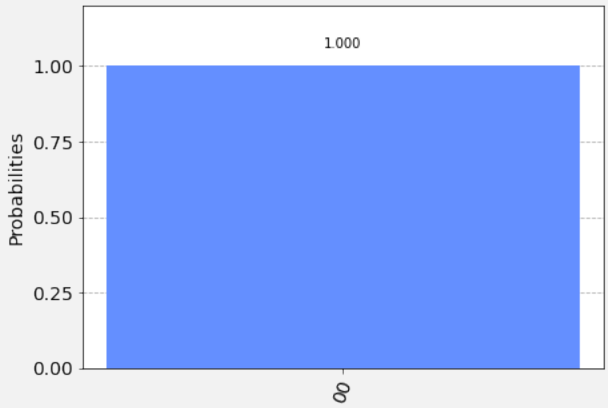
Output of LHS circuit for input 11



RHS Circuit for input 11



Output of RHS circuit for input 11



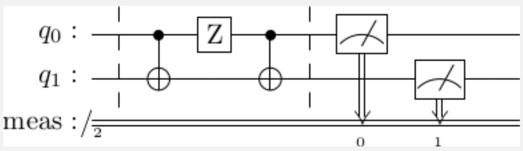
Truth table

Input	Output of LHS	Output of RHS
00	11	11
01	10	10
10	01	01
11	00	00

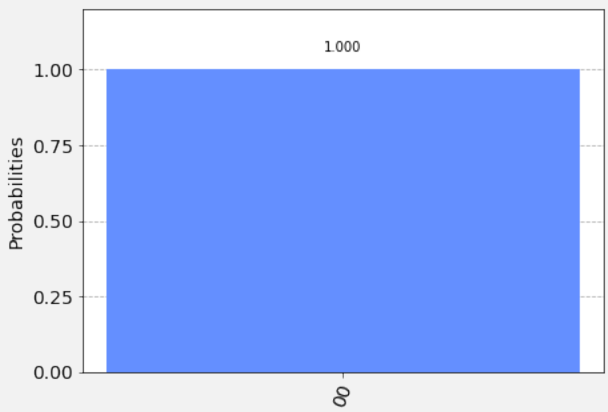
Since, truth table is same for both LHS and RHS both circuits are equivalent.

Answer to Question 1 b (b)

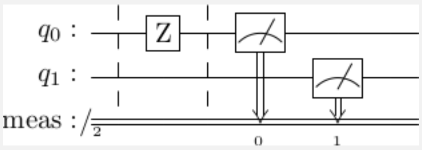
LHS Circuit for input 00



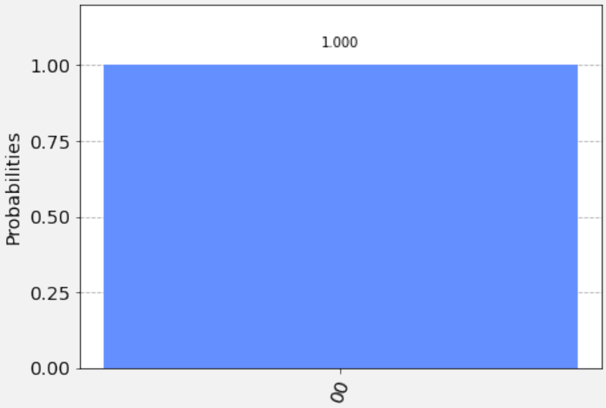
Output of LHS circuit for input 00



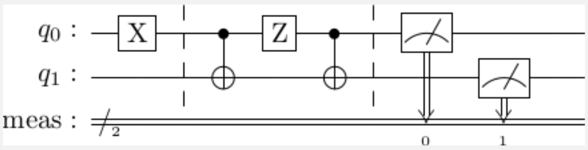
RHS Circuit for input 00



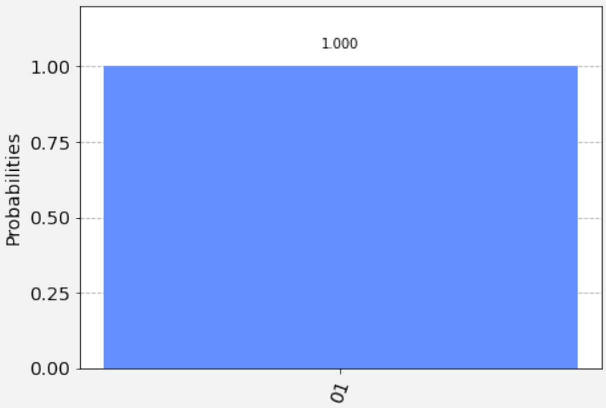
Output of RHS circuit for input 00



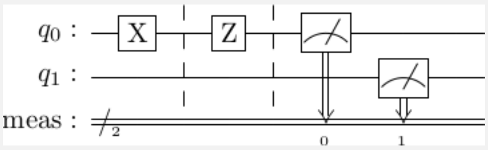
LHS Circuit for input 01



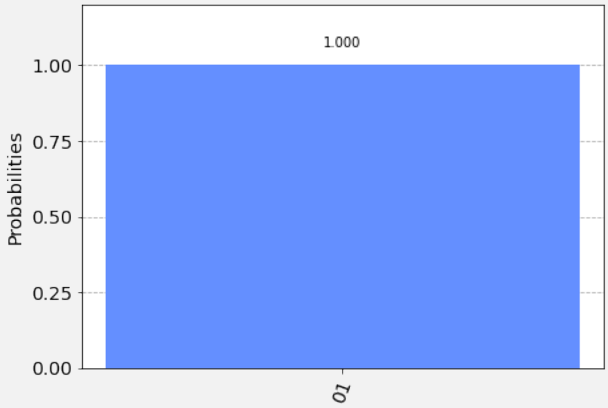
Output of LHS circuit for input 01



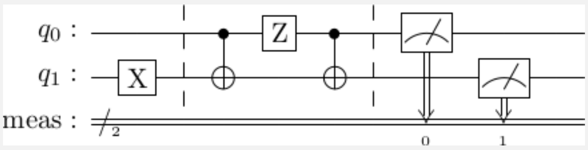
RHS Circuit for input 01



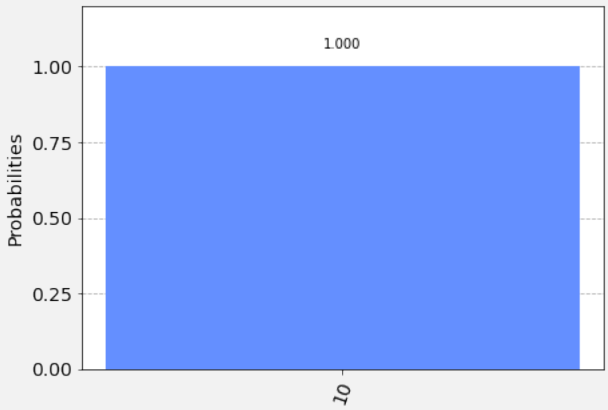
Output of RHS circuit for input 01



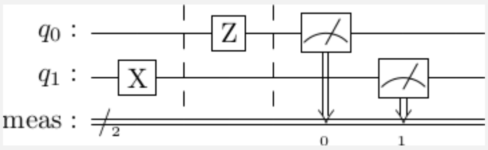
LHS Circuit for input 10



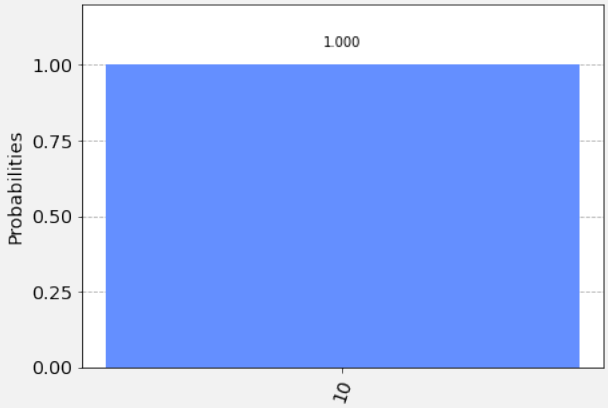
Output of LHS circuit for input 10



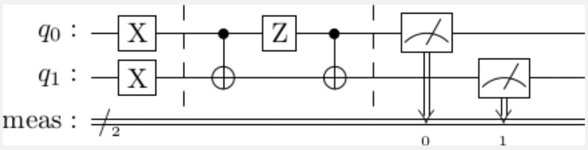
RHS Circuit for input 10



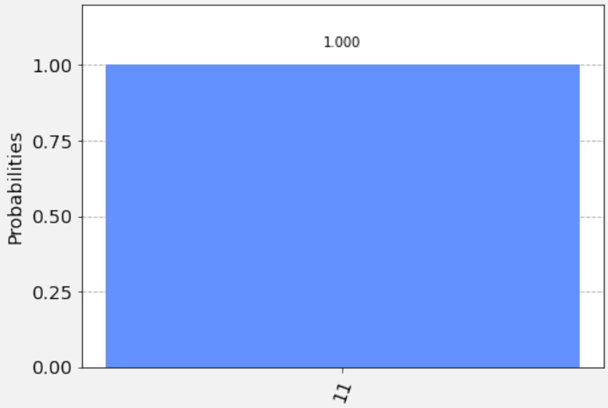
Output of RHS circuit for input 10



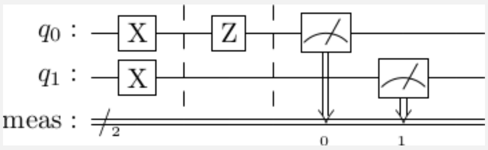
LHS Circuit for input 11



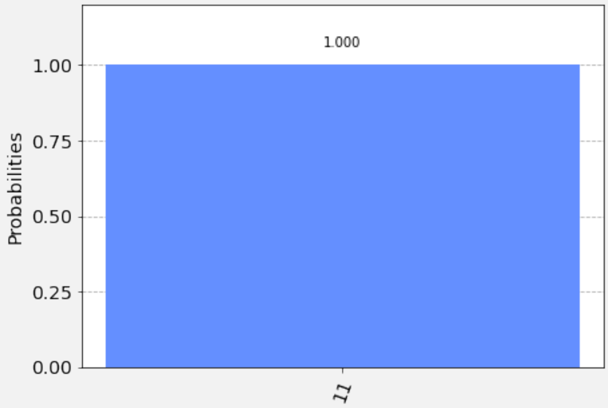
Output of LHS circuit for input 11



RHS Circuit for input 11



Output of RHS circuit for input 11



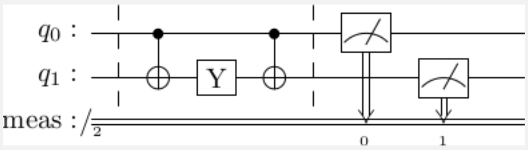
Truth table

Input	Output of LHS	Output of RHS
00	00	00
01	01	01
10	10	10
11	11	11

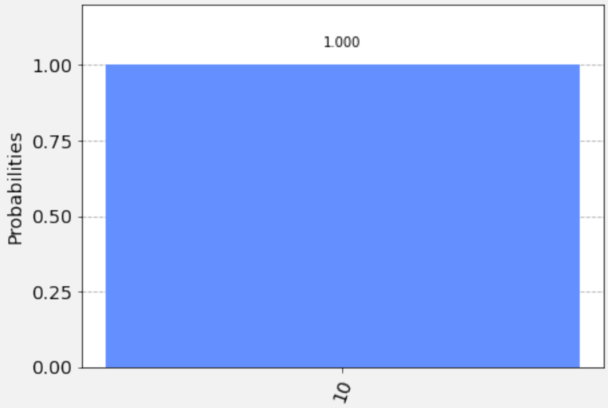
Since, truth table is same for both LHS and RHS both circuits are equivalent.

Answer to Qeusiton 1 b (c)

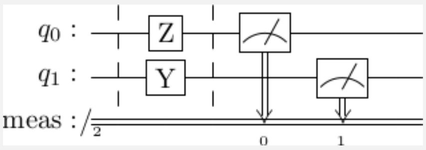
LHS Circuit for input 00



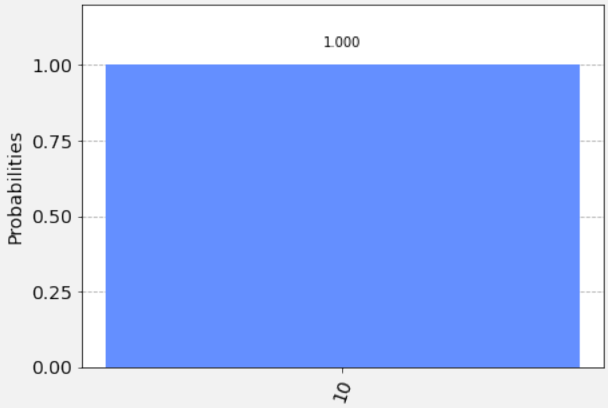
Output of LHS circuit for input 00



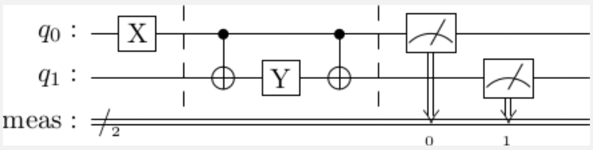
RHS Circuit for input 00



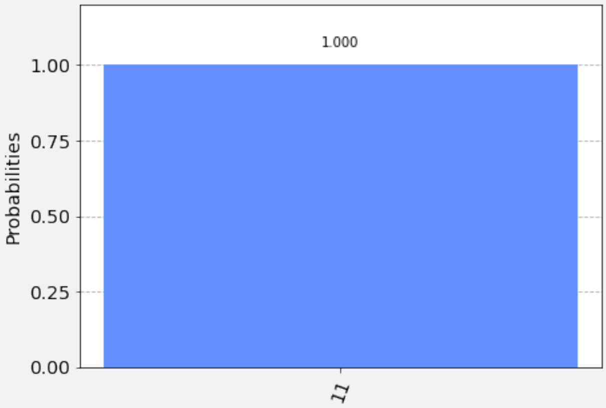
Output of RHS circuit for input 00



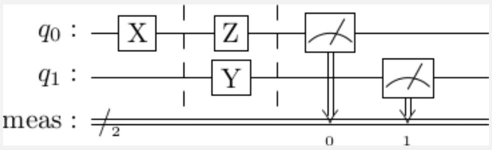
LHS Circuit for input 01



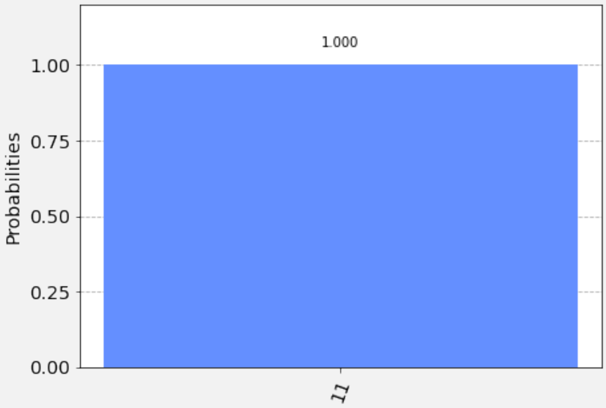
Output of LHS circuit for input 01



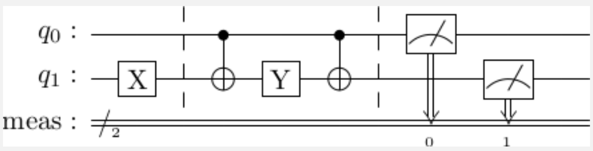
RHS Circuit for input 01



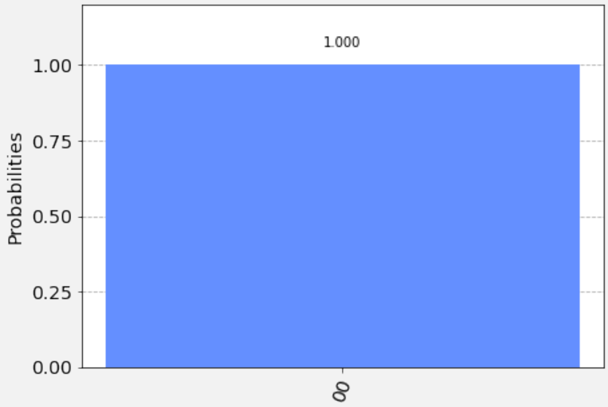
Output of RHS circuit for input 01



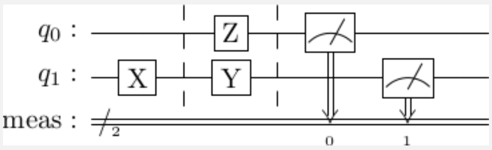
LHS Circuit for input 10



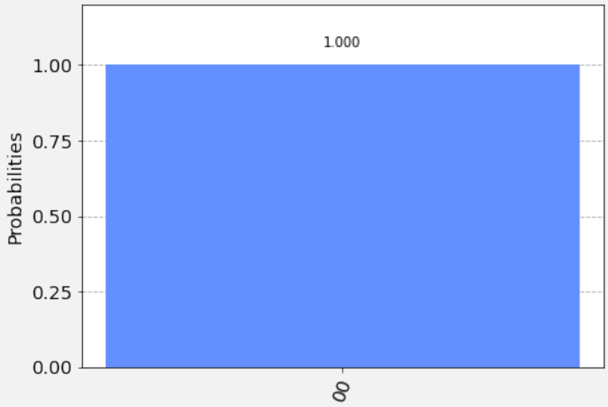
Output of LHS circuit for input 10



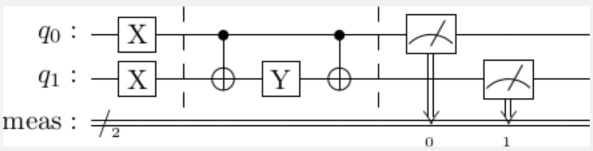
RHS Circuit for input 10



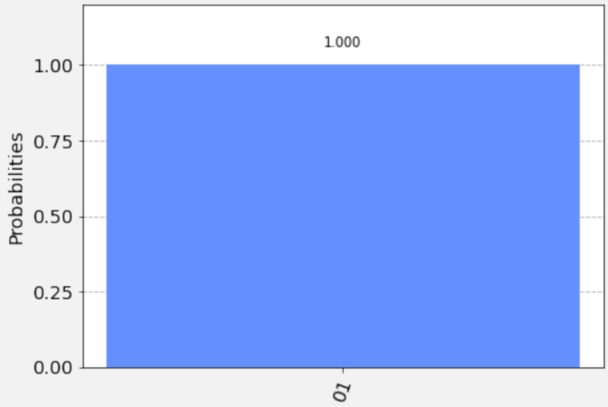
Output of RHS circuit for input 10



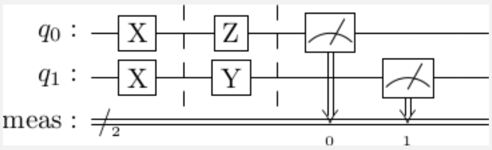
LHS Circuit for input 11



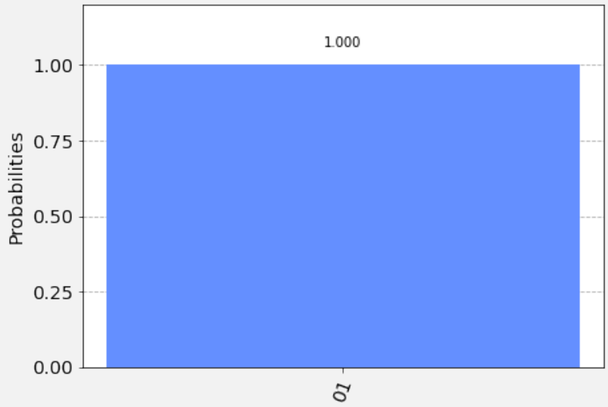
Output of LHS circuit for input 11



RHS Circuit for input 11



Output of RHS circuit for input 11



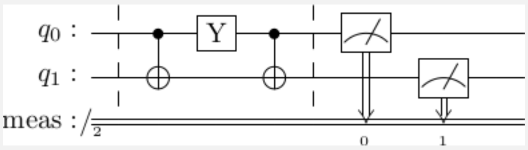
Truth table

Input	Output of LHS	Output of RHS
00	10	10
01	11	11
10	00	00
11	01	01

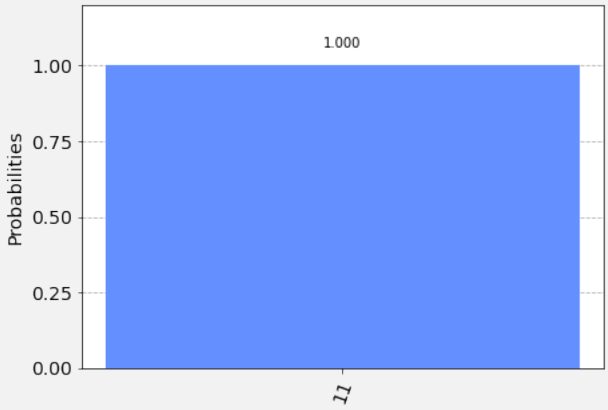
Since, truth table is same for both LHS and RHS both circuits are equivalent.

Answer to Question 1 b (d)

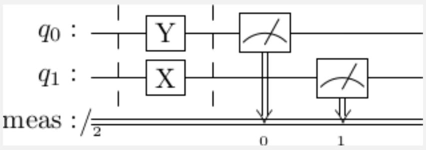
LHS Circuit for input 00



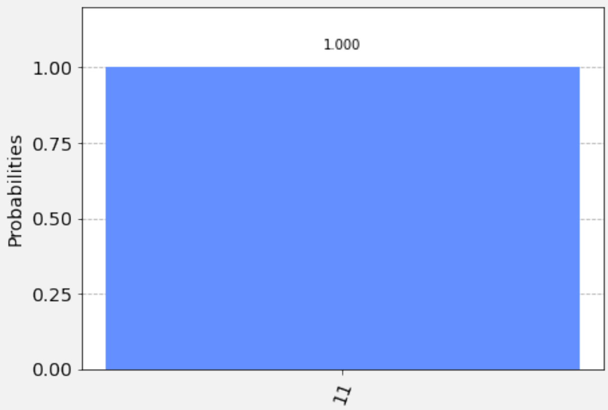
Output of LHS circuit for input 00



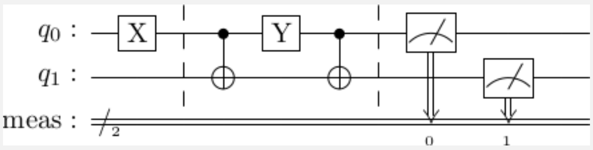
RHS Circuit for input 00



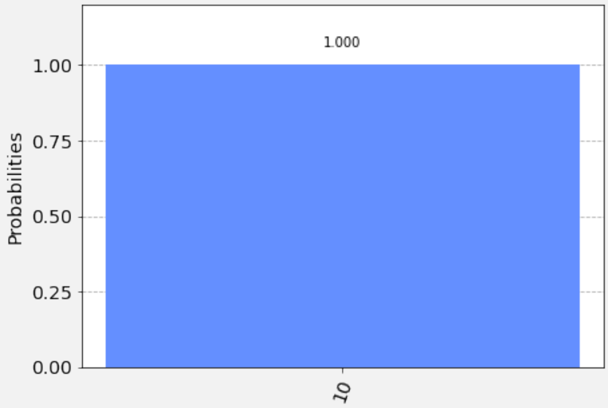
Output of RHS circuit for input 00



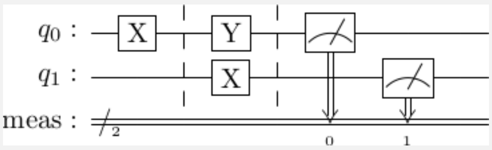
LHS Circuit for input 01



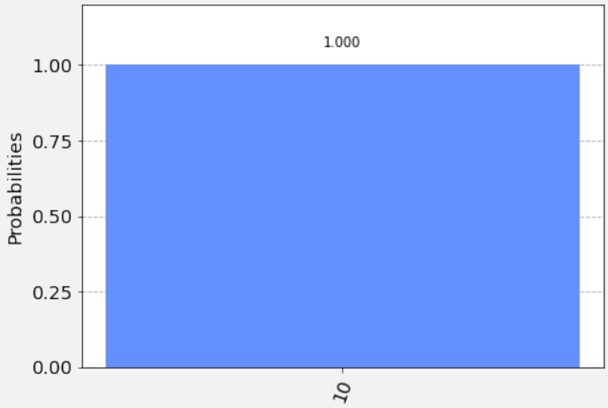
Output of LHS circuit for input 01



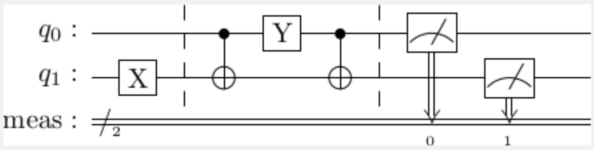
RHS Circuit for input 01



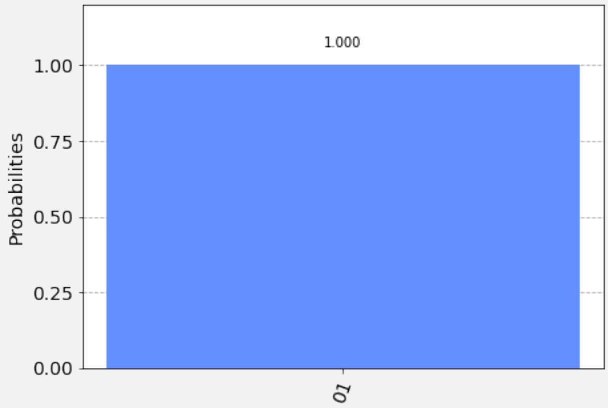
Output of RHS circuit for input 01



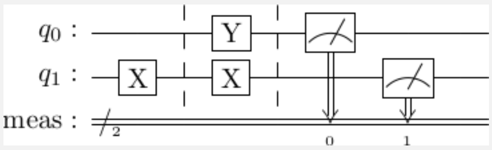
LHS Circuit for input 10



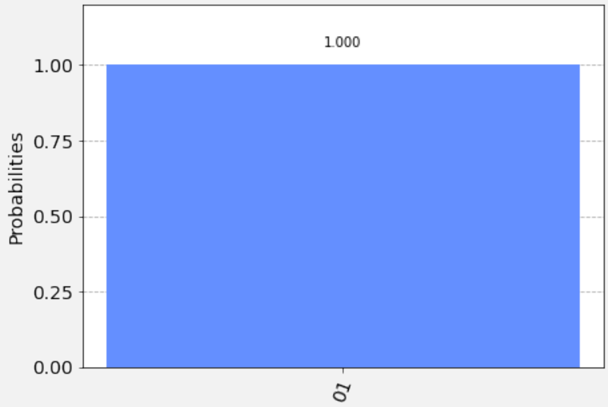
Output of LHS circuit for input 10



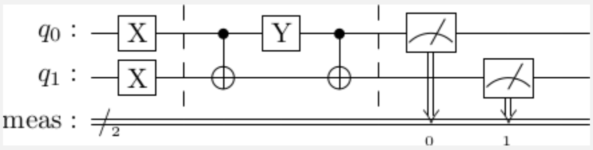
RHS Circuit for input 10



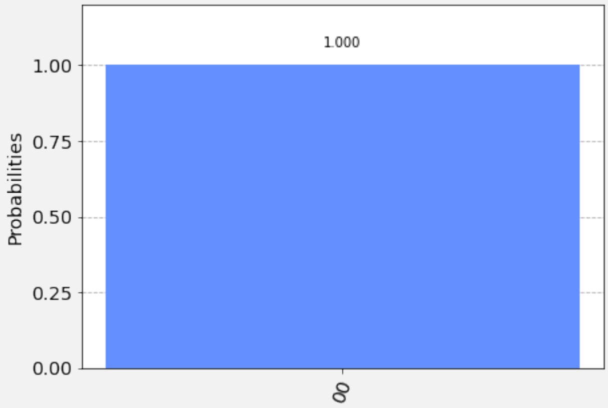
Output of RHS circuit for input 10



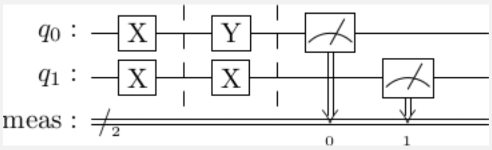
LHS Circuit for input 11



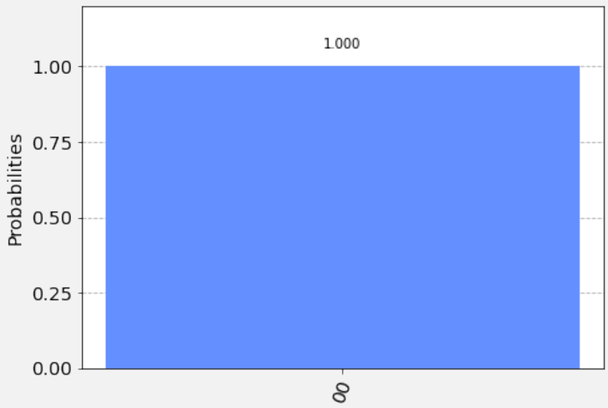
Output of LHS circuit for input 11



RHS Circuit for input 11



Output of RHS circuit for input 11



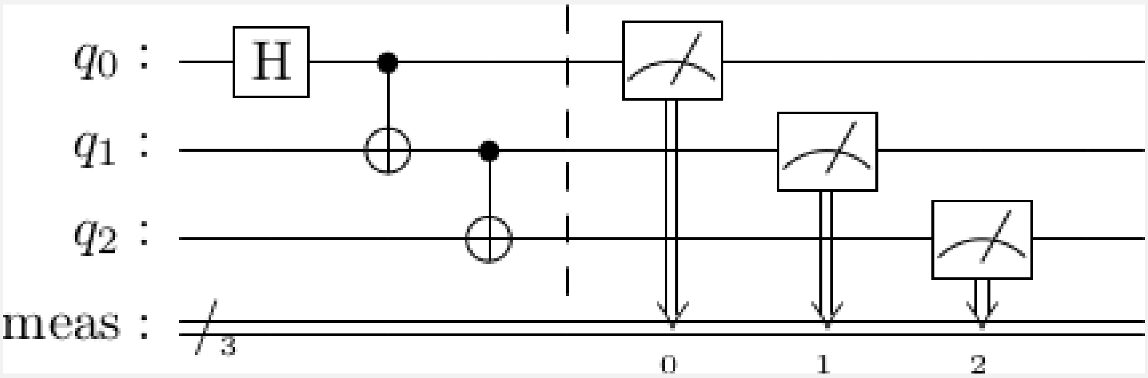
Truth table

Input	Output of LHS	Output of RHS
00	11	11
01	10	10
10	01	01
11	00	00

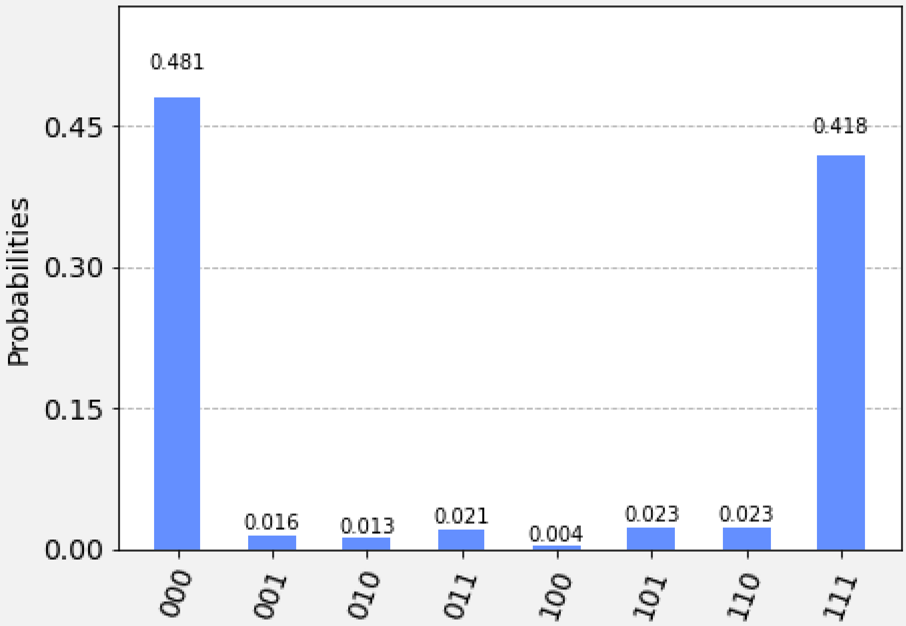
Since, truth table is same for both LHS and RHS both circuits are equivalent.

Answer to Question 2

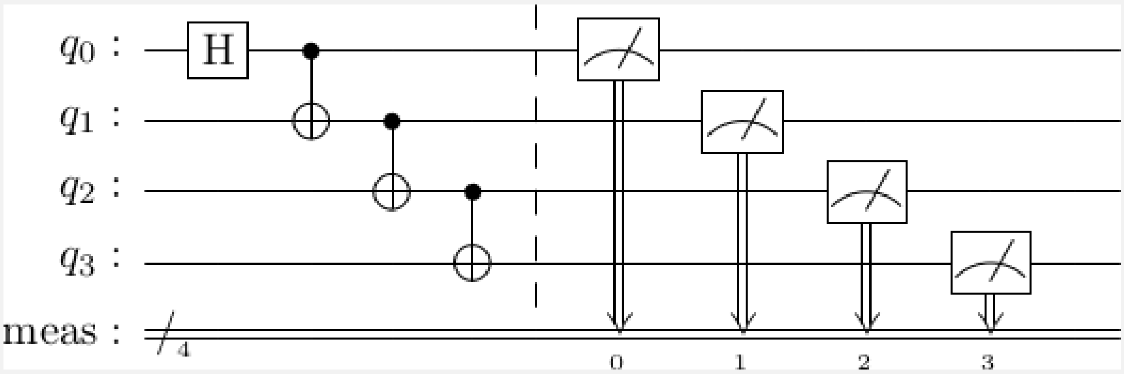
Circuit for 3 qubit GHZ state



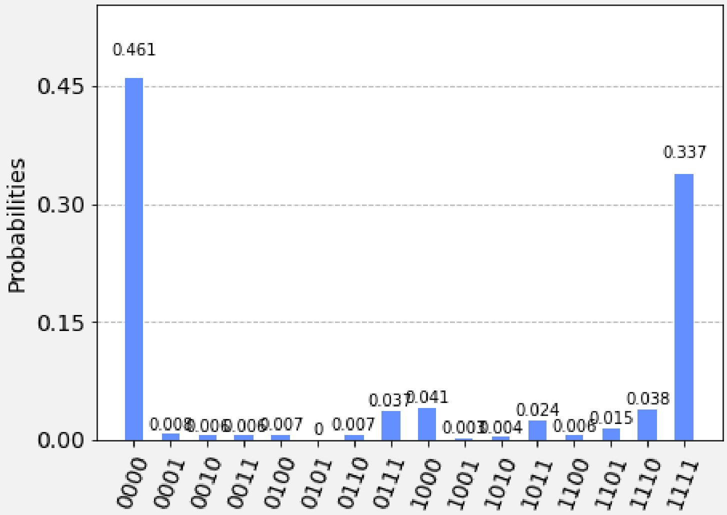
Output of IBMQ machine



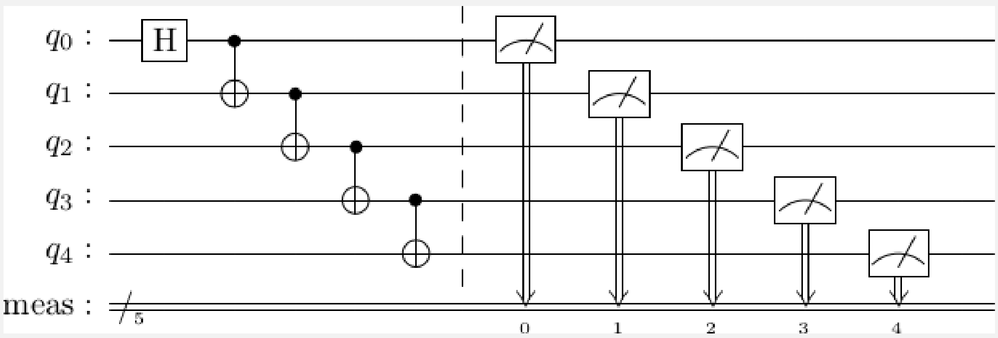
Circuit for 4 qubit GHZ state



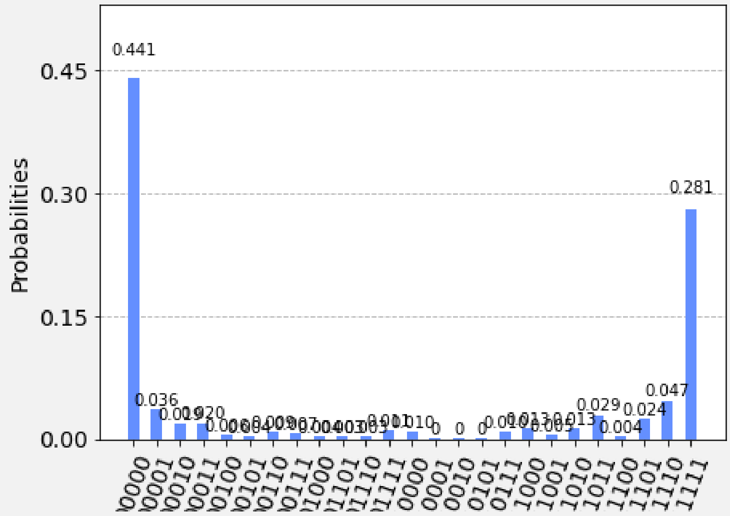
Output of IBM Q machine for 4 - qubit GHZ state



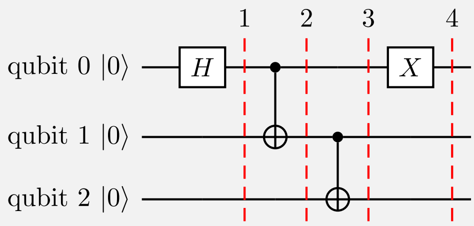
Circuit for 5 qubit GHZ state



Output of IBM Q machine for 5 - qubit GHZ state



Answer to Question 3 (a)



Initial State,

$$|\psi_0\rangle = |000\rangle \tag{1}$$

Applying Hadamard gate to qubit 0,

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |001\rangle) \tag{2}$$

CNOT gate with qubit 0 as control and qubit 1 as target,

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |011\rangle) \tag{3}$$

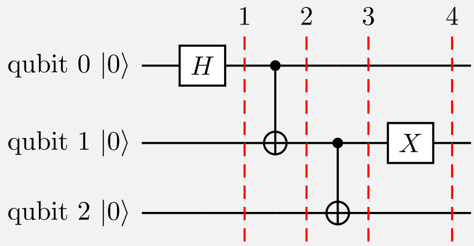
CNOT gate with qubit 1 as control and qubit 2 as target,

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{4}$$

Applying X gate on qubit 0,

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|001\rangle + |110\rangle) \tag{5}$$

Answer to Question 3 (b)



Initial State,

$$|\psi_0\rangle = |000\rangle \tag{6}$$

Applying Hadamard gate to qubit 0,

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |001\rangle) \tag{7}$$

CNOT gate with qubit 0 as control and qubit 1 as target,

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |011\rangle) \tag{8}$$

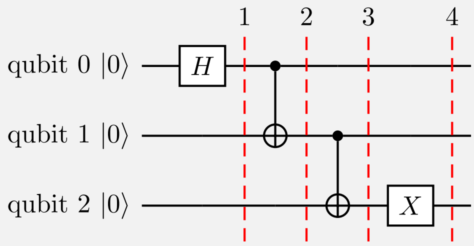
CNOT gate with qubit 1 as control and qubit 2 as target,

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{9}$$

Applying X gate on qubit 1,

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|010\rangle + |101\rangle) \tag{10}$$

Answer to Question 3 (c)



Initial State,

$$|\psi_0\rangle = |000\rangle \tag{11}$$

Applying Hadamard gate to qubit 0,

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |001\rangle) \tag{12}$$

CNOT gate with qubit 0 as control and qubit 1 as target,

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |011\rangle) \tag{13}$$

CNOT gate with qubit 1 as control and qubit 2 as target,

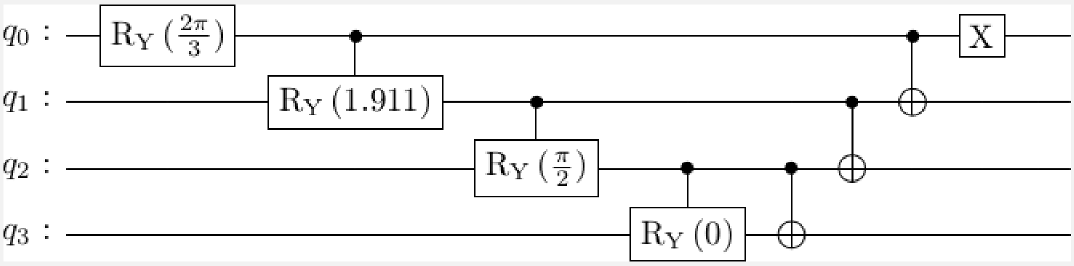
$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{14}$$

Applying X gate on qubit 2,

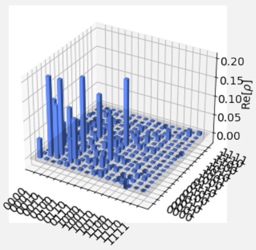
$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |011\rangle) \tag{15}$$

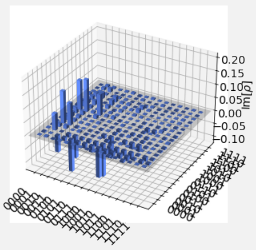
Answer to Question 4

Circuit for W-state



State tomography





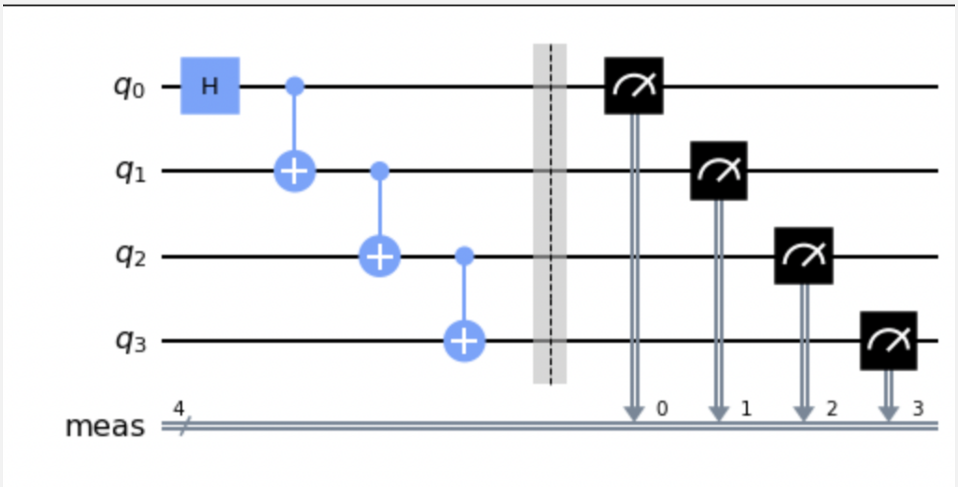
$$\boxed{\text{Entangled bipartite states } : \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)}$$

Answer to Question 5 a

Construction of a N - Qubit GHZ gate

```
N = int(input('Please enter no.of Qubits'))
                    => Number of qubits can be manually entered
qc = QuantumCircuit(N) => Making a quantum circuit of with N qubits.
qc.h(0) => Applying Hadamard gate to zeroth qubit.
for i in range(1,N):
    qc.cx(i-1,i) => CNOT gate is applied to every qubit as target
                    qubit and previous one as control qubit
qc.measure_all()
qc.draw()
```

A 4 qubit GHZ gate



Answer to Question 5 b

Construction of a N - Qubit Entangled bipartite state

```
N=int(input('Please enter no.of Qubits'))
                => Number of qubits can be manually entered
qc = QuantumCircuit(N) => Making a quantum circuit of with N qubits.
qc.ry(2*np.arccos(1/np.sqrt(N)),0)
        => Qubit 0 is Rotated 2arccos(1/sqrt(N)) times w.r.t y-axis
for i in range(1,N):
    qc.cry(2*np.arccos(1/np.sqrt(N-i)),i-1,i)
        => Qubit i is target qubit rotated 2arccos(1/sqrt(N-i))
            times w.r.t y-axis and i-1 qubit is control qubit

for i in range(1,N):
    qc.cx(N-i-1,N-i) => CNOT gate is applied on i th qubit
                        with i-1 th qubit as control qubit.
qc.x(0)=> NOT gate is applied on qubit 0.
qc.draw()
```

A 5 qubit Entangled bipartite state