Note template

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Abstract

This is a note template, with all but minimal compilable files provided. Feel free to adjust for your usage.

Now let's start a simple demo for you to take fancy notes in LaTeX!

Contents

Chapter 1

Entropy

Definition 1.0.1 (Entropy). A measure of uncertainty of a physical system.

$$H(x) = H(p_1, p_2, \dots p_n) = -\sum_x p_x \log p_x$$

$$\lim_{p \to 0} p \log p = 0$$

X - Information we gain, on an average when we learn the value of X.

Example. Coin toss :- HHHH - H, if it gives only heads, Information gain is zero.

Operational interpretation of entropy

Entropy is tied to memory resources.

Example. X takes values (x_1, x_2, x_3, x_4) with probability $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$
encoding them with $(0, 10, 110, 111) \Rightarrow \frac{1}{2}[1] + \frac{1}{4}[2] + \frac{1}{8}[3] + \frac{1}{8}[3] = \frac{7}{4}$bits

$$-\sum_{x=1}^{4} p_x \log p_x = \frac{7}{4}\text{bits}$$

Example. For a coin $p_H = 1$ and $p_T = 0$ size of memory = 0

Entropy from intuitive axioms

1. I(p)
2. I(p) is smooth
3. I(pq) = I(p)+I(q)

Properties of Entropy

$$H_{bin}(p) = -p \log p - (1-p) \log(1-p)$$

get a quadratic curve

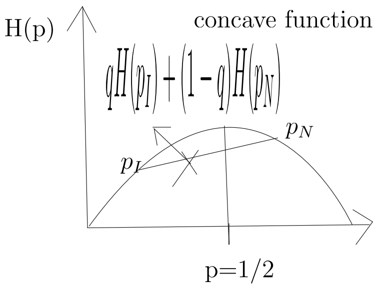

Figure 1.1: title

$$H(qp_I + (1-q)p_N) \geq qH(p_I) + (1-q)H(p_N)$$
$$f(px + (1-p)y) \geq pf(x) + (1-p)f(y)$$

Relative Entropy

Definition 1.0.2.

$$H(p(x) \mid\mid q(x)) = -\sum_{x=1}^{n} p(x) \log \frac{q(x)}{p(x)}$$

Theorem 1.0.1.

$$H(p(x) \mid\mid q(x)) = \sum p(x) \log \frac{p(x)}{q(x)} \text{is non-negative}$$
$$= 0 \text{ iff p(x) = q(x) for all x}$$

Appendix

Appendix A

Additional Proofs

A.1 Proof of ??

We can now prove **??**.

Proof of ??. See https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence. ■