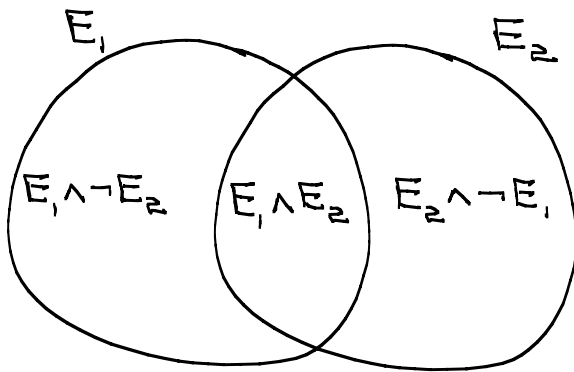


Venn diagram for events  $E_1$  and  $E_2$  ( $E_1 \wedge E_2 \neq \phi$ )



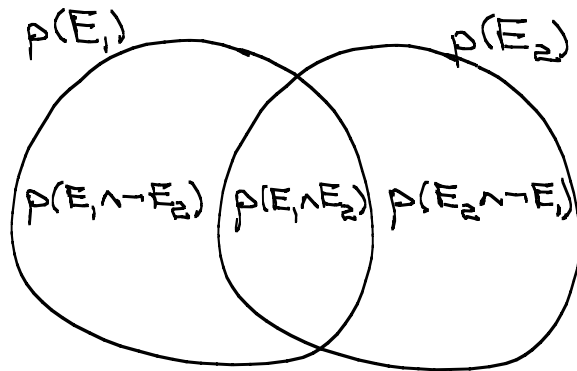
$E_1$  and  $E_2$  are not exclusive

Both circles:  $E_1 \vee E_2$

Complement of both circles:

$$\neg(E_1 \vee E_2) = \neg E_1 \wedge \neg E_2$$

That probabilities are a measure on the space of events means that we can regard the area occupied by an event as representing its probability.



Both circles:  $p(E_1 \vee E_2)$

Complement of both circles:

$$p(\neg(E_1 \vee E_2)) = p(\neg E_1 \wedge \neg E_2)$$

The diagram practically shows out how to show what we want:

$$\begin{aligned} (E_1 \wedge E_2) \wedge (E_1 \wedge \neg E_2) &= \phi \\ (E_1 \wedge E_2) \vee (E_1 \wedge \neg E_2) &= E_1 \end{aligned} \xrightarrow{\text{axiom 3}} p(E_1) = p(E_1 \wedge E_2) + p(E_1 \wedge \neg E_2)$$

$$\begin{aligned} E_2 \wedge (E_1 \wedge \neg E_2) &= \phi \\ E_2 \vee (E_1 \wedge \neg E_2) &= E_1 \vee E_2 \end{aligned} \xrightarrow{\text{axiom 3}} p(E_1 \vee E_2) = p(E_2) + p(E_1 \wedge \neg E_2)$$

Subtract to get

$$p(E_1 \vee E_2) - p(E_1) = p(E_2) - p(E_1 \wedge E_2)$$