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Exercise 2.15

Question: A computer program consists of two blocks written independently by two different programmers. The first block has an error with probability 0.2. The second block has an error with probability 0.3. If the program returns an error, what is the probability that there is an error in both blocks?

Solution: Since the two blocks are written independently, the occurrence of error in one block does not affect the probability of error in the other block. For independent events,

$$P\{E_1 \cap E_2\} = P\{E_1\} \cdot P\{E_2\}$$

Let us call the probability of having an error in the first block $P\{E_1\}$, which is 0.2. By applying the **Complement Rule**, we can say that the probability of not having an error ($P\{\overline{E_1}\}$) is 0.8. Similarly, let us call the probability of having an error in the second block $P\{E_2\}$, which is 0.3. By applying the **Complement Rule**, we can say that the probability of not having an error ($P\{\overline{E_2}\}$) is 0.7.
In case of an error, there are 3 distinct cases that might have happened. Namely; the error might have happened on the first block only, the error might have happened on the second block only, and the error might have happened on both blocks. Since the blocks are independent, we can write the total probability of error as follows;

$$\begin{aligned} P\{error\} &= P\{E_1\} \cdot P\{\overline{E_2}\} + P\{\overline{E_1}\} \cdot P\{E_2\} + P\{E_1\} \cdot P\{E_2\} && (1) \\ &= 0.2 \cdot 0.7 + 0.8 \cdot 0.3 + 0.2 \cdot 0.3 && (2) \\ &= 0.44 && (3) \end{aligned}$$

The probability of having an error in both blocks is $P\{E_1\} \cdot P\{E_2\}$, which is 0.06.

To find the probability of having an error in both blocks, we can use the following division;

$$\frac{P\{E_1\} \cdot P\{E_2\}}{P\{E_1\} \cdot P\{\overline{E_2}\} + P\{\overline{E_1}\} \cdot P\{E_2\} + P\{E_1\} \cdot P\{E_2\}} = \frac{0.06}{0.44} \tag{4}$$

$$= 0.1364 \tag{5}$$

Exercise 2.20

Question: All athletes at the Olympic games are tested for performance-enhancing steroid drug use. The imperfect test gives positive results (indicating drug use) for 90% of all steroid-users but also (and incorrectly) for 2% of those who do not use steroids. Suppose that 5% of all registered athletes use steroids. If an athlete is tested negative, what is the probability that he/she uses steroids?

Solution: Let us denote the probability of an athlete using steroids as $P\{S\}$, and the probability of getting the test result negative as $P\{N\}$. The information given in the question can be denoted as the following;

- $P\{\overline{N}|S\} = 0.90$
- $P\{\overline{N}|\overline{S}\} = 0.02$
- $P\{S\} = 0.05$

By applying the **Complement Rule**, we can write the followings;

- $P\{N|S\} = 0.10$
- $P\{N|\overline{S}\} = 0.98$
- $P\{\overline{S}\} = 0.95$

By applying **Bayes Rule**,

$$P\{S|N\} = \frac{P\{N|S\} \cdot P\{S\}}{P\{N|S\} \cdot P\{S\} + P\{N|\overline{S}\} \cdot P\{\overline{S}\}} \tag{6}$$

$$= \frac{0.10 \cdot 0.05}{0.10 \cdot 0.05 + 0.98 \cdot 0.95} \tag{7}$$

$$= 0.005342 \tag{8}$$

Exercise 2.29

Question: An internet search engine looks for a keyword in 9 databases, searching them in a random order. Only 5 of these databases contain the given keyword. Find the probability that it will be found in at least 2 of the first 4 searched databases.

Solution: Since we are interested in finding the keyword in at least 2 of the first 4 searched databases, finding the keyword 2, 3 or 4 times would be acceptable by us. We can simply write this as combinations as follows;

$$= \sum_{n=2}^{4} \binom{5}{n} \binom{4}{4-n} \tag{9}$$

$$= \binom{5}{2}\binom{4}{2} + \binom{5}{3}\binom{4}{1} + \binom{5}{4}\binom{4}{0} \tag{10}$$

$$= 60 + 40 + 5 \tag{11}$$

By dividing this sum to $C(9,4)$, we can find the probability asked.

$$\frac{\sum_{n=2}^{4} \binom{5}{n}\binom{4}{4-n}}{\binom{9}{4}} = \frac{105}{126} \tag{12}$$

$$= 0.83 \tag{13}$$