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Monte Carlo Study and Simulation Size

To conduct the Monte Carlo Study, first we need to determine the size of the Monte Carlo simulation. Since it is given that we can use Normal approximation, we can use the following formula;

$$N \quad 0.25 \quad \frac{Z_{\alpha/2}}{\epsilon} \; ^{2}$$

Since $Z_\alpha = \Phi^{-1}(1 \quad \alpha)$, we can easily see that $\Phi(Z_\alpha) = 1 \quad \alpha$. Since we are interested in finding $Z_{\alpha/2}$ for $\alpha = 0.01$, by looking at the **Standard Normal Distribution** table of the book, we can see that the Φ value that equals to $(1 \quad \alpha/2) = (1 \quad 0.005) = 0.995$ is approximately 2.575, which tells us that our $Z_{\alpha/2}$ value is 2.575. By replacing the values in our formula, we get

$$N \quad 0.25 \quad \frac{2.575}{0.02} \; ^{2}$$
$$4144.14$$

By taking the ceiling of the N value, we get 4145 as our Monte Carlo Study size.

Since the vehicles have Poisson distribution, we can use the Algorithm 5.1 of the book to generate a Poisson variable with parameter λ. The sample MATLAB code for motorcycles that has $\lambda = 40$ can be seen below;

```
lambda = 40;
U = rand;
i = 0;
F = exp( lambda);
while (U≥F);
    i=i+1;
    F = F+exp( lambda)*lambda^i/gamma(i+1);
end;
```

To estimate the total probability of having the total weight of vehicles that pass over the bridge in a day more than 220 tonnes, we need to use the Gamma variables formula, since the weight of each vehicle is a Gamma distributed random variable.

```
X = sum(  1/lambda * log(rand(alpha,1)) )
```

Since this variable is valid for only 1 occurrence, we need to sum it i times that was the total number of times that the while loop run while calculating the Poisson variable.
With the attached MATLAB code (which is available as Appendix A), the estimated probability of having the total weight of all the vehicles that pass over the bridge in a day more than 220 tonnes is **0.379344**.

Estimation of Total Weight

Based on the Monte Carlo study, the estimated total weight of all the vehicles that pass over the bridge in a day is **208447.422677** kilograms.

Estimation of Std(X) and Accuracy of Estimator X

Based on the Monte Carlo study, the Standard deviation of our study is **38200.740513**.
Since we picked our Monte Carlo study size with the $\alpha = 0.01$ and $\epsilon = 0.02$, we can say that our study yields to accurate results within the error margin of 0.02, 99% of the time.
Since $\text{Std}(X) = \sigma/\sqrt{N}$, to have estimators with higher accuracy we can use larger study sizes and decrease our Std(X) value; which, as expected, yields to more accurate estimations.

Appendices

A MATLAB Code for the Monte Carlo Study

```
N=4145; % size of Monte Carlo Simulation with alpha = 0.01 and error = 0.02
TotalWeight=zeros(N,1); % vector to keep the total weight of vehicles for ...
    each monte carlo run
for k=1:N;
    weight = 0; % the total weight for this monte carlo run

    % motorcycles
    lambda = 40;
    U = rand; i = 0;
    F = exp( lambda);
    while (U≥F);
        i=i+1;
        F = F+exp( lambda)*lambda^i/gamma(i+1);
    end;
    Y = i; % total number of motorcycles
    for f=1:Y; % summing total weight of motorcycles
        F = sum( 1/0.15 * log(rand(16,1)));
        weight = weight + F;
    end;

    % automobiles
    lambda = 30;
    U = rand; i = 0;
    F = exp( lambda);
    while (U≥F);
        i=i+1;
        F = F+exp( lambda)*lambda^i/gamma(i+1);
    end;
    Y = i; % total number of automobiles
    for f=1:Y;  % summing total weight of automobiles
        F = sum( 1/0.05 * log(rand(60,1)));
        weight = weight + F;
    end;


    % trucks
    lambda = 20;
    U = rand; i = 0;
    F = exp( lambda);
    while (U≥F);
        i=i+1;
        F = F+exp( lambda)*lambda^i/gamma(i+1);
    end;
    Y = i; % total number of trucks
    for f=1:Y; % summing total weight of trucks
        F = sum( 1/0.01 * log(rand(84,1)));
        weight = weight + F;
    end;

    TotalWeight(k) = weight;

end;

p_est = mean(TotalWeight>220000);
expectedWeight = mean(TotalWeight);
stdWeight = std(TotalWeight);

fprintf('Estimated probability = %fnn',p_est);
fprintf('Expected weight = %fnn',expectedWeight);
fprintf('Standard deviation = %fnn',stdWeight);
```