CENG 384 - Signals and Systems for Computer Engineers
Spring 2020
Written Assignment 1

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1. (a)

$$z = x + yj, \qquad \bar{z} = x - yj, \qquad z + 1 = j - 3\bar{z}$$

$$(x + yj) + 1 = j - 3(x - yj)$$
$$(x + 1) + j(y) = (-3x) + j(3y + 1)$$

$$x + 1 = -3x, \qquad y = 3y + 1$$
$$4x = -1, \qquad 2y = -1$$
$$x = -\frac{1}{4}, \qquad y = -\frac{1}{2}$$

$$\begin{aligned} |z|^2 &= x^2 + y^2 \\ &= \left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{2}\right)^2 \\ &= \frac{1}{16} + \frac{1}{4} \\ &= \frac{5}{16} \end{aligned}$$

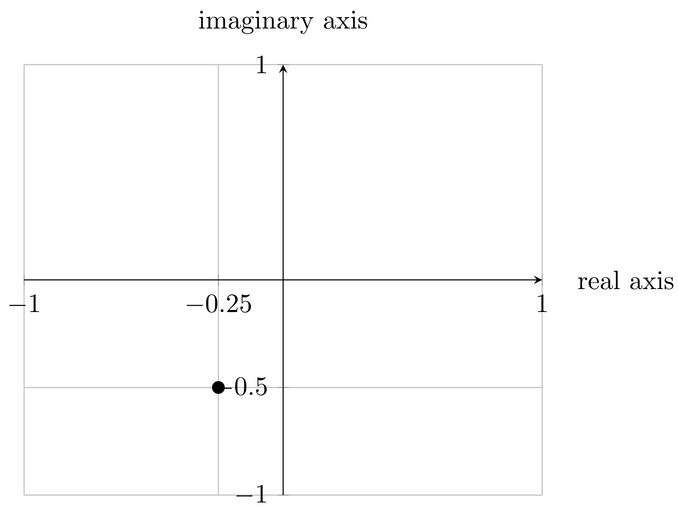

Figure 1: $z = -0.25 - j0.5$.

(b)

$$\begin{aligned} z &= re^{j\theta} \\ z^2 &= r^2 e^{j2\theta} && = 25j \\ &= r^2(cos(2\theta) + jsin(2\theta)) && = 25j \end{aligned}$$

$$r^2 = 25, \qquad cos(2\theta) + jsin(2\theta) = j$$

$$r = 5, \qquad \theta = \frac{\pi}{4}$$

$$z = 5e^{j\pi/4}$$

(c)

$$z = \frac{(1+j)(1-\sqrt{3}j)}{1-j} = \frac{(1+j)^2(1-\sqrt{3}j)}{(1-j)(1+j)} = \frac{2j(1-\sqrt{3}j)}{2} = j(1-\sqrt{3}j) = \sqrt{3}+j$$

$$\text{magnitude} = r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$$

$$\text{angle} = \theta = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

(d)

$$z = je^{-j\pi/2} = e^{j\pi/2}e^{-j\pi/2} = e^{j(\pi/2-\pi/2)} = e^{j0} (= 1)$$

2. When we look at the $y(t)$ signal, we can observe the followings:

- The amplitude of $y(t)$ is half the amplitude of $x(t)$ due to the $\frac{1}{2}$ constant.
- The $y(t)$ signal is scaled by a factor of 2 (compression).
- The $y(t)$ signal is shifted by 1 second (delay).

$$x(t) = \begin{cases} 0 & t < 1 \\ t-1 & 1 \leq t < 3 \\ 2 & 3 \leq t < 5 \\ 7-t & 5 \leq t < 7 \\ 0 & 7 \leq t \end{cases}$$

By replacing t with $2t-2$ we obtain

$$x(2t-2) = \begin{cases} 0 & 2t-2 < 1 \\ (2t-2)-1 & 1 \leq 2t-2 < 3 \\ 2 & 3 \leq 2t-2 < 5 \\ 7-(2t-2) & 5 \leq 2t-2 < 7 \\ 0 & 7 \leq 2t-2 \end{cases}$$

$$x(2t-2) = \begin{cases} 0 & t < 3/2 \\ 2t-3 & 3/2 \leq t < 5/2 \\ 2 & 5/2 \leq t < 7/2 \\ 9-2t & 7/2 \leq t < 9/2 \\ 0 & 9/2 \leq t \end{cases}$$

Finally, after multiplying by $\frac{1}{2}$, we get

$$y(t) = \frac{1}{2}x(2t-2) = \begin{cases} 0 & t < 3/2 \\ t-3/2 & 3/2 \leq t < 5/2 \\ 1 & 5/2 \leq t < 7/2 \\ 9/2-t & 7/2 \leq t < 9/2 \\ 0 & 9/2 \leq t \end{cases}$$

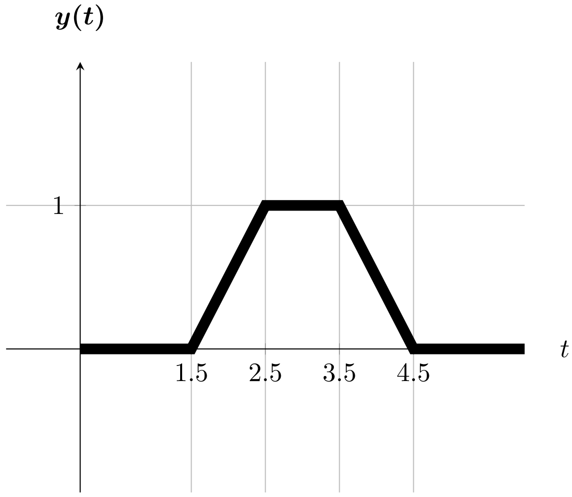

Figure 2: t vs. $y(t)$.

3. (a) See Figure 3.

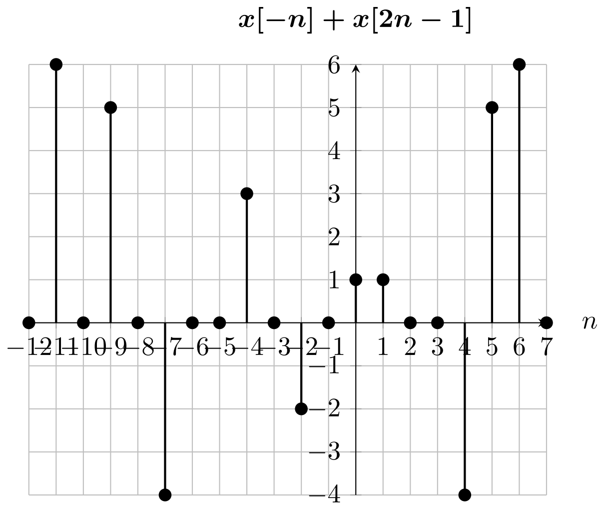

Figure 3: n vs. $x[-n] + x[2n-1]$.

(b)

$$\begin{aligned} x[-n] + x[2n-1] = \delta[n+11]\cdot 6 + \delta[n+9]\cdot 5 + \delta[n+7]\cdot(-4) + \delta[n+4]\cdot 3 \\ + \delta[n+2]\cdot(-2) + \delta[n]\cdot 1 + \delta[n-1]\cdot 1 \\ + \delta[n-4]\cdot(-4) + \delta[n-5]\cdot 5 + \delta[n-6]\cdot 6 \end{aligned}$$

4. (a) Magnitude and phase does not affect the period, so we need to find phases of $\sin[\frac{5\pi}{8}n]$ and $\cos[\frac{2\pi}{3}n]$.
$\sin[\frac{5\pi}{8}n]$: $\quad \frac{2\pi}{\frac{5\pi}{8}}k = N_{sin} \implies \frac{16}{5}k = N_{sin} \implies k = 5, N_{sin} = 16$
$\cos[\frac{2\pi}{3}n]$: $\quad \frac{2\pi}{\frac{2\pi}{3}}k = N_{cos} \implies \frac{2}{3}k = N_{cos} \implies k = 3, N_{cos} = 3$
Fundamental period N_0 of $x[n]$ is least common multiple of N_{sin} and N_{cos}.
$N_0 = \text{lcm}(16, 3) = 48 \in \mathbf{N}$
Hence $x[n]$ is periodic with fundamental period 48.

(b) Like **4(a)**, we need to find the period of $\cos[5n]$.
$\cos[5n]$: $\quad \frac{2\pi}{5}k = N_0$
There is no $k \in \mathbf{N}$ such that N_0 is an integer. Hence $x[n]$ is not periodic.

(c) This time the signal is continuous. Again, like **4(a)** we find the fundamental period of $\sin(5\pi t)$.
$\sin(5\pi t)$: $\quad \frac{2\pi}{5\pi} = T_0 \implies \frac{2}{5} = T_0$
Hence the signal is periodic with fundamental period $\frac{2}{5}$.

(d)

$$x(t) = je^{j2t} = e^{j\pi/2}e^{j2t} = e^{j(2t+\pi/2)} = cos(2t+\pi/2) + jsin(2t+\pi/2)$$

To determine the period, we need to find periods of $cos(2t)$ and $sin(2t)$.

cos($2t$): $\frac{2\pi}{2} = T_{cos} \implies \pi = T_{cos}$

sin($2t$): $\frac{2\pi}{2} = T_{sin} \implies \pi = T_{sin}$

Therefore, $x(t)$ is periodic with fundamental period $T_0 = \pi$.

5. A continuous signal is even if $x(t) = x(-t)$ holds. Similarly, a continuous signal is odd if $x(t) = -x(-t)$ holds.

When we check the graph of signal 1, we can see that the signal does not have any symmetry in regards to any axis. Therefore the requirements for even or odd signal do not hold.

Any signal can be decomposed into its Even and Odd parts by the following method:

$$x(t) = Even\{x(t)\} + Odd\{x(t)\}$$
$$Odd\{x(t)\} = \frac{1}{2}\left(x(t) - x(-t)\right)$$
$$Even\{x(t)\} = \frac{1}{2}\left(x(t) + x(-t)\right)$$

By plotting the $x(-t)$ (Figure 4), we can draw the Odd (Figure 5) and Even (Figure 6) parts of $x(t)$.

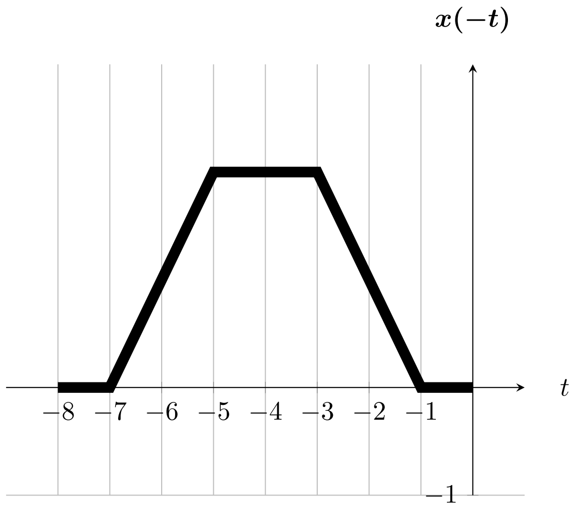

Figure 4: t vs. $x(-t)$.

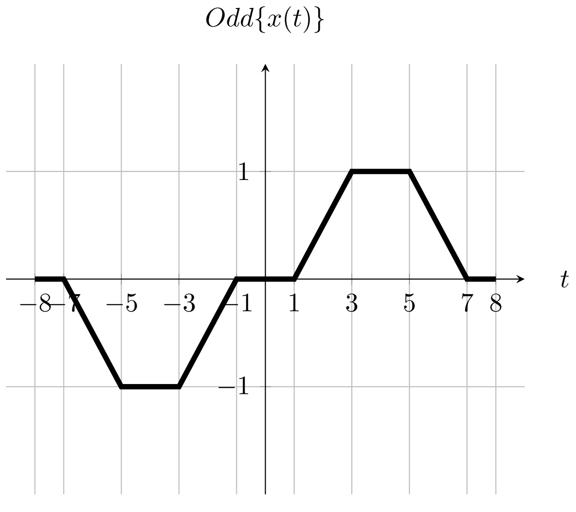

Figure 5: $Odd\{x(t)\}$.

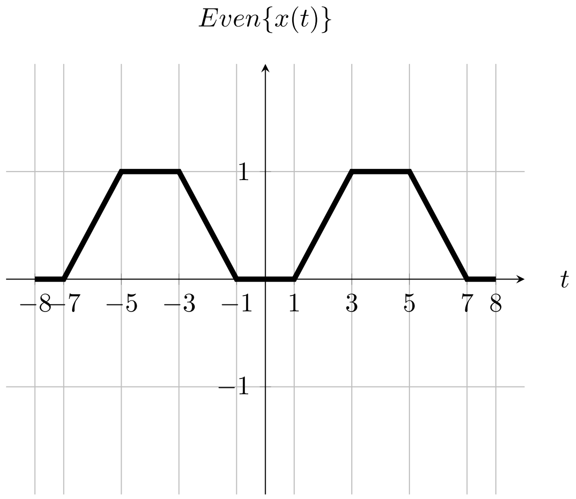

Figure 6: $Even\{x(t)\}$.

6. (a) $x(t) = u\,(t-1)\; - 3u\,(t-3)\; +\; 4u\,(t-4)$

   (b)

$$\begin{aligned}\frac{dx(t)}{dt} &= \frac{d(u(t-1) - 3u(t-3) + 4u(t-4))}{dt} \\ &= \frac{du(t-1)}{dt} - 3\frac{du(t-3)}{dt} + 4\frac{du(t-4)}{dt}\end{aligned}$$

$$\text{Since } \delta(t) = \frac{du(t)}{dt}$$

$$\begin{aligned}\frac{dx(t)}{dt} &= \frac{d(u(t-1) - 3u(t-3) + 4u(t-4))}{dt} \\ &= \delta(t-1) - 3\delta(t-3) + 4\delta(t-4)\end{aligned}$$

The plot of $\frac{dx(t)}{dt}$ can be seen at Figure 7.

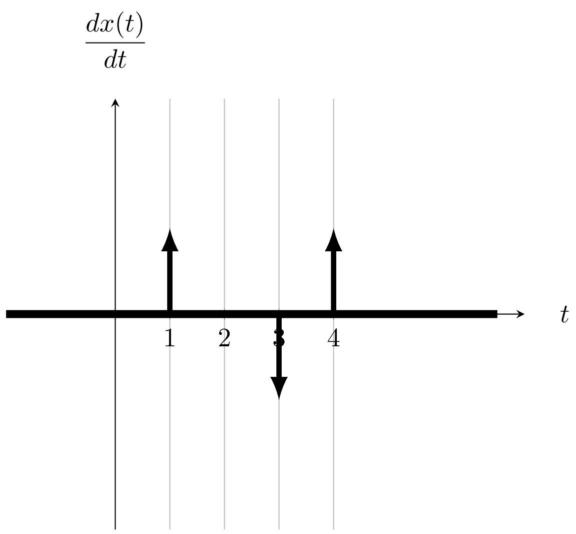

Figure 7: t vs. $\frac{dx(t)}{dt}$.