Student Information

Full Name : Mustafa Ozan ALPAY
Id Number : 2309615

Exercise 3.10

Question: Every day, the number of traffic accidents has the probability mass function

x	0	1	2	more than 2
$P(x)$	0.6	0.2	0.2	0

independently of other days.What is the probability that there are more accidents on Friday than on Thursday?

Solution: The possible cases for having more accidents on Friday than on Thursday are;

1. Having more than 2 accidents on Friday, and 2, 1 or 0 accidents on Thursday
2. Having exactly 2 accidents on Friday, and having less than 2 accidents on Thursday
3. Having exactly 1 accident on Friday, and having 0 accidents on Thursday

By summing the probabilities of these events, we can find the total probability of having more accidents on Friday than on Thursday.

$$\begin{aligned}
P(1) &= 0 \cdot (0.2 + 0.2 + 0.6) &&= 0.00 \\
P(2) &= 0.2 \cdot (0.2 + 0.6) &&= 0.16 \\
P(3) &= 0.2 \cdot 0.6 &&= 0.12 \\
\sum_{n=1}^{3} P(n) &= 0 + 0.16 + 0.12 &&= 0.28
\end{aligned}$$

Exercise 3.15

Question: Let X and Y be the number of hardware failures in two computer labs in a given month. The joint distribution of X and Y is given in the table below.

$P(x,y)$		x		
		0	1	2
y	0	0.52	0.20	0.04
	1	0.14	0.02	0.01
	2	0.06	0.01	0

(a) Compute the probability of at least one hardware failure.

(b) From the given distribution, are X and Y independent? Why or why not?

Solution:

(a) By using the **Complement Rule**, we can say that the probability of having at least one hardware failure can be calculated by subtracting the probability of having no hardware failure from 1. Since $P(0,0) = 0.52$, the probability of at least one hardware failure is 0.48.

(b) By summing row-wise and column-wise, we can get the marginal pmf's;

$P(x,y)$		x			$P_y(Y)$
		0	1	2	
y	0	0.52	0.20	0.04	0.76
	1	0.14	0.02	0.01	0.17
	2	0.06	0.01	0	0.07
$P_x(X)$		0.72	0.23	0.05	1

To decide on the independence of X and Y, we can check if their joint pmf factors into the product of marginal pmf's.

$$\begin{aligned} P_x(0) \cdot P_y(0) &= 0.72 \cdot 0.76 \\ &= 0.5472 \end{aligned}$$

When we look at the table, we can see that $P(0,0)$ is 0.52, and since $P_x(0) \cdot P_y(0) \neq P(0,0)$, we can conclude that the events are dependent.

Exercise 3.19

Question: A and B are two competing companies. An investor decides whether to buy

(a) 100 shares of A, or

(b) 100 shares of B, or

(c) 50 shares of A and 50 shares of B.

- A profit made on 1 share of A is a random variable X with the distribution $P(X = 2) = P(X = -2) = 0.5$.
- A profit made on 1 share of B is a random variable Y with the distribution $P(Y = 4) = 0.2, P(Y = -1) = 0.8$.

If X and Y are independent, compute the expected value and variance of the total profit for strategies (a), (b), and (c).

Solution: To calculate the expected values, we can simply use the formula;

$$\mu = E(X) = \sum_x xP(x)$$

(a) $E(A) = 2 \cdot 0.5 + (-2) \cdot 0.5 = 0$

(b) $E(B) = 4 \cdot 0.2 + (-1) \cdot 0.8 = 0$

(c) $E(A + B) = E(A) + E(B) = 0 + 0 = 0$

To calculate the variance of the total profit, we can use the following formula;

$$\sigma^2 = \text{Var}(X) = \sum_x (x - \mu)^2 P(x)$$

(a) The variance of the total profit for strategy (a) is,

$$\begin{aligned}
\text{Var}(100 \cdot A) &= 100^2 \cdot \text{Var}(A) \\
&= 10^4 \cdot \sum_x (x - 0)^2 P(x) \\
&= 10^4 \cdot [(2 - 0)^2 \cdot 0.5 + (-2 - 0)^2 \cdot 0.5] \\
&= 4 \cdot 10^4
\end{aligned}$$

(b) The variance of the total profit for strategy (b) is,

$$\begin{aligned}
\mathrm{Var}(100 \cdot B) &= 100^2 \cdot \mathrm{Var}(B) \\
&= 10^4 \cdot \sum_x (x-0)^2 P(x) \\
&= 10^4 \cdot [(4-0)^2 \cdot 0.2 + (-1-0)^2 \cdot 0.8] \\
&= 4 \cdot 10^4
\end{aligned}$$

(c) The variance of the total profit for strategy (c) is,

$$\mathrm{Var}(50 \cdot A + 50 \cdot B) = 50^2 \cdot \mathrm{Var}(A) + 50^2 \cdot \mathrm{Var}(B) + 2 \cdot 50 \cdot 50 \cdot \mathrm{Cov}(A, B)$$

Since A and B are independent, their covariance is 0.

$$\begin{aligned}
&= 50^2 \cdot 4 + 50^2 \cdot 4 + 0 \\
&= 2 \cdot 10^4
\end{aligned}$$

Exercise 3.26

Question: After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files.

(a) Compute the probability that at least 5 of the first 20 files are damaged.

(b) Compute the probability that the manager has to check at least 6 files in order to find 3 undamaged files.

Solution:

(a) Let X be the number of files that are damaged by the virus. From the 20 files, each one of them are either damaged or not, therefore X is the number of successful Bernouilli trials. Since we are interested in the probability of having at least 5 of the files damaged, and since X has a Binomial Distribution with $n = 20, x = 4, p = 0.2$, by checking the Binomial Distribution Table, we can see that $P(X \leq 4) = 0.63$. By applying the **Complement Rule**, we can say that $P(X \geq 5) = 1 - P(X \leq 4) = 0.37$

(b) Let X be the number of files to checked until 3 undamaged ones are found. It is a number of trials needed to see 3 successes, hence X has Negative Binomial distribution with $k = 6$ and $p = 0.80$.

Virtually any Negative Binomial problem can be solved by a Binomial distribution. Although X is not Binomial at all, the probability $P\{X > 5\}$ can be related to some Binomial variable, such as;

$$\begin{aligned} P\{X > 5\} &= P\{\text{more than 5 trials needed to get 3 successes }\} \\ &= P\{\text{5 trials are not sufficient }\} \\ &= P\{\text{there are fewer than 3 successes in 5 trials }\} \\ &= P\{Y < 3\} \end{aligned}$$

where Y is the number of successes (non-damaged files) in 5 trials, which is a Binomial variable with parameters n = 5 and p = 0.8.

By using the Binomial Distribution Table, we can conclude that

$$\begin{aligned} P\{X > 5\} &= P\{Y < 3\} \\ &= P\{Y \leq 2\} \\ &= F(2) \\ &= 0.058 \end{aligned}$$