

Lindenmayer systems - a C++ implementation

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Lindenmayer Systems, short L-systems, are the result of **research from Lindenmayer et al.**¹ about the geometric features of plants. L-systems are a concept to mathematically/formal describe and model the growth processes of plant development. They are not only restricted to the plant based developments, but can also be used to generate fractals.

L-systems have an initial state and use rules, like a formal grammar, to transform or rather rewrite the current state to create the next state of the development from a plant or a fractal. It is therefore possible to successive calculate each state of the development. Such a state of a L-system can be interpreted as commands for a turtle graphic, which creates the opportunity to draw the created fractals or plant states.

Goal of this paper is to design an architecture for L-systems, which includes an implementation for L-systems, their creation and an interface for a turtle graphic. The interface should enable the polymorphic use of different turtle graphic implementations and enable drawing of the L-system state.

¹ZITIEREN

1 Introduction

L-systems are a formal way to describe plant or fractal development and interpret the result as a graphic. In order to provide a general understanding of L-systems this paper is organised in several topics. Section 2 is a short introduction to the general idea, based around object rewriting, the grammar of L-systems and the interpretation of a L-system as graphic. After discussing the architecture and possible implementation steps, a final concept for an implementation is proposed. The code for this implementation is available via my github repository.²

Finally, there is a conclusion and an outlook for possible future extensions.

2 Lindenmayer systems

2.1 History

"[L-systems] were introduced in 1968 by Lindenmayer as a theoretical framework for studying the development of simple multicellular organisms [...]"³ and were later used in computer graphics to generate visuals of organisms and fractals.

On the beginning the focus of L-systems theory was based on larger plant parts and the graphical interpretation used chains of rectangles to display a L-system. Further research into L-system extended the interpretations, resulting in a interpretation of a L-system state with a LOGO-style Turtle. These extensions make it possible to model more complex plants and fractals and display them in a graphical way.⁴

2.2 General idea

The general idea of a L-system is the use of a rewriting system based on a formal grammar. The shape of a plant or a fractal consists of geometric pieces, for example a branch of a tree has several subbranches. "When each piece of a shape is geometrically similar to the whole, both the shape and the cascade that generate it are called self-similar."⁵ The self-similarity makes it possible to create a formal description for the plant or fractal generation as a formal grammar, further discussed in section 2.3. The rewriting uses this formal description to generate the different states of the development. "In general, rewriting is a technique for defining complex objects by successively replacing parts of a simple initial object using a set of rewriting rules or productions."⁶

For example this concept can be used to rewrite an initial string, called axiom, with defined rewriting rules. A simple example is the following grammar, which consists of only two nonterminals, A and B, and two production rules. The first rule is $A \rightarrow AB$, the second rule is $B \rightarrow A$. The arrow ' \rightarrow ' symbolises the replacement, the rewriting, of the object on the left with the object on the right of the arrow. The L-system has as axiom the value 'A' and will be expanded with these rules, creating the results in figure 1.

The first step is to use rule one, which replaces the nonterminal A with AB, resulting in the first generation. The result of the first generation ('AB') will be used to generate the second

²<https://github.com/frozenshooter/LSystems>

³Zitiert aus abop Preface - Abschnitt Modeling of Plants

⁴Zitat abop Vgl. Seite 6 Chapter 1.3

⁵Zitate The Nature of... chapter 6 page 34

⁶Zitiert von abop Chapter 1 - 1.1 Rewriting systems

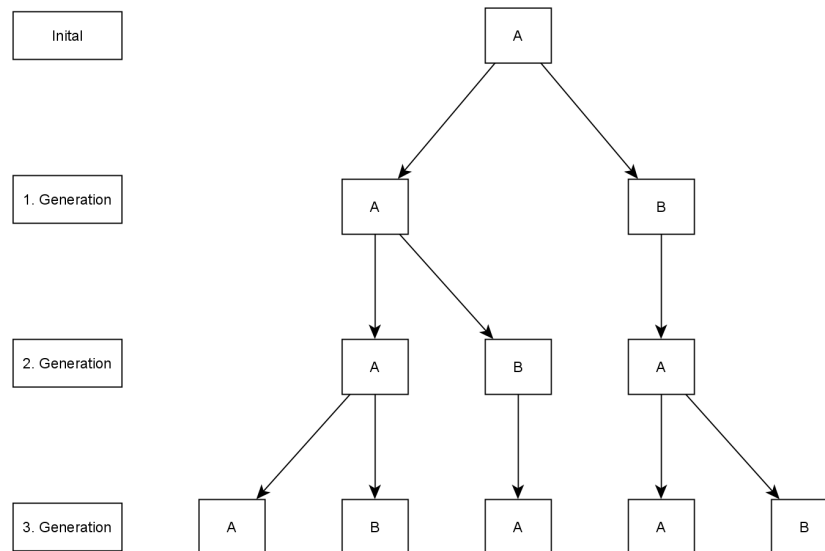


Figure 1: Simple L-system

generation. For all nonterminals the productions will be parallel applied. In this case the nonterminals A and B will be replaced, because both have existing production rules. This results in the second generation with 'ABA' as result.

This process can be successively repeated recursive for an arbitrary amount of generations and create a fractal or plant with self-similar pieces. The more generations are calculated, the more detailed is the resulting state.

2.3 Grammar

The definition of an L-system can be done similar to a Chomsky grammar, but there are some general differences. In Chomsky grammars the productions are applied sequentially, whereas in L-systems they can be applied parallel. This has some consequences for the formal properties of an L-system, for example a context-free L-system can produce a language which cannot be produced by a context-free Chomsky grammar.⁷

This paper will only focus on a class of L-systems, the DOL-systems, which can be used for string based rewriting systems. This class is deterministic (D) and context-free (O) and can be formally described by a tuple:

$$G = (V, \omega, P)$$

V: set of symbols as alphabet of the l-system - consisting of terminals and non terminals

ω : axiom - nonempty word of the alphabet, which should contain at least one nonterminal

P: set of productions

A production consists of a predecessor, a nonterminal symbol of the alphabet, and a successor, the replacement of the nonterminal in the next generation. To guarantee that the l-system is deterministic, there only can be one production for each nonterminal of the alphabet. The identity production is implicit a part of the set of productions.⁸

⁷Zitat abop vgl Seite 3 Abschnitt L-Systems

⁸Zitat vgl Seite 4 abop

If in the process of rewriting a terminal symbol is found, there will be no explicit production applied, but rather the identity production is applied. Terminals will therefore remain in future generations and won't extend an L-system with a production.

As mentioned in section 2.1 there are other extensions of a basic L-systems. The extensions introduce more possibilities for the generation process, like a non-deterministic behaviour. These extensions result in new grammars which can represent much complexer plants or fractals:

- Stochastic grammars: the system isn't deterministic anymore, because there can be multiple production rules for the same nonterminal with a probability which results in a randomisation of the generation
- Context sensitive grammars: production not only look for the nonterminal, they rather look for the symbols before and after the current symbol to process, the context.
- Parametric grammars: it is possible to set additional parameters for a symbol to influence the generation or the evaluation of the data

2.4 Interpretation as turtle commands

The L-systems introduced to this point are capable of creating a string based on a grammar. In order to create a graphic of a state of a L-system, the resulting state can be interpreted as commands of a turtle.

A turtle is a concept introduced by the language Logo as a tool for computer graphics.⁹

A state of a turtle can be described as a tuple¹⁰:

$$S = (x, y, \alpha)$$

x and y are Cartesian coordinates

α is the direction in which a turtle is facing, called heading

A turtle can move in the Cartesian coordinate system by altering the current state. You can think of it as a real turtle with a pen attached to it, walking on a paper. While moving in the coordinate system, or on the paper, the turtle can draw lines. Given this concept the turtle can receive different commands. The commands let the turtle walk on the paper or the coordinate system by altering the current state and drawing a line. For now I'll restrict this to a two dimensional coordinate system, but it is also possible to enhance it for a three dimensional coordinate system.

The walking path of the turtle is controlled by commands. Therefore a defined step size d and an angle θ is needed to calculate the next state of the turtle.

- Draw: moves one step in the current facing direction drawing a line
- Move: moves one step in the current facing direction without drawing a line
- Right-turn: turns to the right by the angle θ
- Left-turn: turns to the left by the angle θ

⁹Zitat VGL. Seite 179 Chapter 10 Logo book

¹⁰Zitat vgl. abop Seite 6 ff

Additional to the state of the turtle the state there is a state for the pen. The pen state consist of the color and its width, which will result in more colorfull or different pictures.

With the given turtle concept is it possible to interpret the result of an L-system. Therefore a mapping between the symbols in the alphabet and the commands which should be called is needed. This could be done by an arbitrary mapping between a nonterminal or a terminal and a command. For this paper the following mapping will be used, but the concept for the architecture includes the possibility to use other mappings in the future.

For the mapping alphabet V is used, which is a slightly extended version of the basic version of a L-system:

$$V = (F, f, +, -, [,])$$

This alphabet will be mapped with extended turtle commands:

Symbol	Turtle interpretation
F	Draw a line in the facing direction
f	Move in the facing direction
+	Turn right
-	Turn left
[save the current state in a stack
]	pop the last state form the stack and set it to the current state

This interpretation enables to draw the result of the L-system by iterating over every terminal and nonterminal of the result state and calling the mapped command. If no command is mapped the symbol will be skipped.

STACK ENTFERENEN UND NUR SIMPLEN WEG NUTZEN

2.5 Examples

This section will present some examples for L-system grammars which create fractals. They use the introduced mapping of section 2.4.

2.5.1 Koch curve

This fractal will be generated with the simple axiom 'F' and just one production: $F \rightarrow F+F-F+F$. The turtle will be initalized with an angle of 60° and an arbitrary length d for a step.

¹²

2.5.2 Sierpinski triangle

This fractal will be created with 'F' as axiom and two productions:

- $X \rightarrow YF+XF+Y$

¹¹Zitat vlg cad book seitev 2 unteres drittel

¹²ADD A PICTURE and perhaps a source: <http://mathforum.org/advanced/robertd/lsys2d.html>

- $Y \rightarrow XF-YF-X$

The turtle will be initialized with an angle of 60° and an arbitrary length d for a step.

¹³

¹³ADD A PICTURE and SOURCE

3 Architecture

The primary goal of this paper is creating an architecture and an implementation of a L-system as described in section 2. The focus of the architecture will be on flexibility and expandability and therefore is the following section splitted into several parts with discussions about different aspects of the final architecture.

¹⁴

3.1 Requirements overview

The architecture for a L-system, as introduced in section 2, has several requirements . This section will introduce the needed core requirements to get a flexible L-system and turtle.

The core of the concept is the L-system itself, which has to guarantee a flexible usage. The L-system holds relevant data like the grammar, consisting of production rules and an axiom. The L-system should offer a way to configure a grammar and guarantee the usage in different architectures without hardcoded grammars. The L-system should also offer to successively generate the next states of the configured grammar and access this generated result.

The turtle is another key component, which will be used to interpret the result of a L-system. The turtle has to offer the typical turtle commands as introduced in section 2.4. In order to provide a turtle that is as extensible as possible, it should be possible to implement it flexibly. Therefore an interface should be offered, that enables a fast exchange of implementations. Depending on the implementation of a turtle, it should be possible to configure the turtle and change properties, like the color or line width.

A mapping between the turtle commands and the L-system alphabet is an important part, so it is possible to call the correct turtle commands. Additional a function which is calling the mapped commands, after interpreting the L-system result, is needed to create an image of a plant or a fractal development state.

3.2 L-system

This section provides a disussion of possible implementations of the L-system and a final proposal.

A simple and rather naive idea would be an object which holds the L-system as a string and addtional the grammar, consisting of productions and an axiom. This L-system could offer a function that calculates the next generation by iterating over every char in the string. If there is a production for the current char, the char will be replaced according to the production with the successor. This function could be called for n generations to create the n-th generation of the L-system, as show in figure 2.

This idea has several design flaws resulting in a bad performance and unflexible architecture. The first flaws exist because of the lack of seperation between datastructure and processing of the data. It is not possible to simply exchange the datastructure or the function without the seperation, which affects flexibility of the L-system negative. This can be solved by extracting the functionality into a seperate function and the datastructure remains as "dumb" object. The seperated function can be a used to manipulate the current state in the datastrutcture itself and

¹⁴HIER SOLLTE MAN EVT NOCH IRGENDWO DIE SEMANTSCIHE SCHNITTSTELLE ERWÄHNEN



Figure 2: Naive L-system

also creates the opportunity to exchange the underlying datastructure, as long as the datastructure fullfills some formal properties, like the support of access to the string and grammar.

For a more flexible design additional changes could be done, because the current idea bases on chars as alphabet of the grammar. The L-system could be enhanced by allowing arbitrary objects as symbols of the alphabet, as long as they provide certain functions, like a operator to compare them. Instead of a string, the objects could for example be saved as a list.

A further flaw is the bad performance, because of the use of a string to save the current state/-generation. There are two reasons why the performance is bad, especially for larger generations. The first reason is the size of the string itself, which will grow very fast because of the nature of a L-system. Every nonterminal will be often replaced by severall symbols, which enormously increases the total size of the string for each generation. Additional to the large amount of space needed for the string, the replace in a string needs to be done for each nonterminal, which can result in a big overhead. Because of this problems, another more efficient design is needed.

This implementation only works for simple L-systems where only a few generations are needed.

In order to solve this problems and to create a more efficient and flexible architecture, let's recap the nature of L-systems. The generation of a L-system is based on a rewriting process. This process iterates over every object in the current state and calcualtes the resulting generation by applying the productions. Because of the restriction to DOL-systems, the rewriting is not context-sensitive and can be done independent from other objects of the current state. If a object gets replaced by the successor of the production, the result of this replacment can be handled independent. Consequently, the calculation of the n-th generation can be done for each object on their own and can simply be done in a recursive manner.

There is only a limited number of productions in a grammar of a L-system. Each of these productions is choosen deterministic when comparing the current nonterminal with the predecessor. The next generation is created by rewriting the current state, when a nonterminal is often in the current state, the same production can be used. If the same production is used muliple times, the resulting generation has a lot of similar objects or more specific the same string muliple times. The repeating data and the recursive calculation can be used to improve the naive idea.

The simple example in figure 3 shows the possible reduction of data using the following grammar:

Axiom: A

Production: $A \rightarrow AAAAAAAAAA$

This rather ideal example shows how it would be sufficient to save just parts of the resulting string to represent the whole generation of a L-system. For this example it could be possible to save "AAAAAAAAAA" and how often it is needed. Because of the repetitions, this is partly even for more complex grammars possible.



Figure 3: Memory reduction

With this knowledge multiple improvements can be discussed. The first possible improvement is storing the data not in a simple list, like a string, but instead in a more sophisticated datastructure. For this case a graph as datastructure is better suited. Such a graph enables a better performance for replacements and reduction of memory space.

The graph has a special sturture to achive the improvements for a L-system. The following grammar is used to demonstrate the possible improvements:

Axiom: A

1. Production: $A \rightarrow BBB$

2. Production: $B \rightarrow AAA$

A natural improvement of a graph is the replace function. This could be done by simply creating new childnodes with the replaced data. In figure 4 is a example with the given grammar. Each generation has its own nodes with data of the current state. A level of this graph represents a whole generation and can be accessed with the depth of the nodes.

This graph still has the problem of the redundant data, because every note contains its own data. For example in the second generation the part "AAA" is stored three times. This can be improved with another indirection of the data. This concept results in a datastructure containing a graph to store the dependencies and a container for the data itself. In figure 5 is an example for this indirection. Each node saves a pointer or id of the data it represents. In this graph a replace could also be done with the creation of a childnodes.

Both of the described graphs hold every calculated generation, which allows the access to an arbitrary generation with the restriction to the deepest calculated generation. To reduce the amount of storage needed even more, it is possible to neglect if all generations are needed or only the current generation. If only the last calculated generation is needed, the graph can be simplified even more. To discuss the simplification, it is necessary to look further in the different ways to do the replacement. It is possible to iterate over the node data and replace for each symbol of teh data at a time. For example with the current grammar the data "AAA", could be replaced by replacing each "A" at a time. Because of the focus of DOL-Systems, another possible method is to replace a node with all childnodes at once.

There seems to be no difference, but in combination with the proposed next step of the graph, it is relevant. If not all generations have to be stored, it is possible to remove a completely replaced

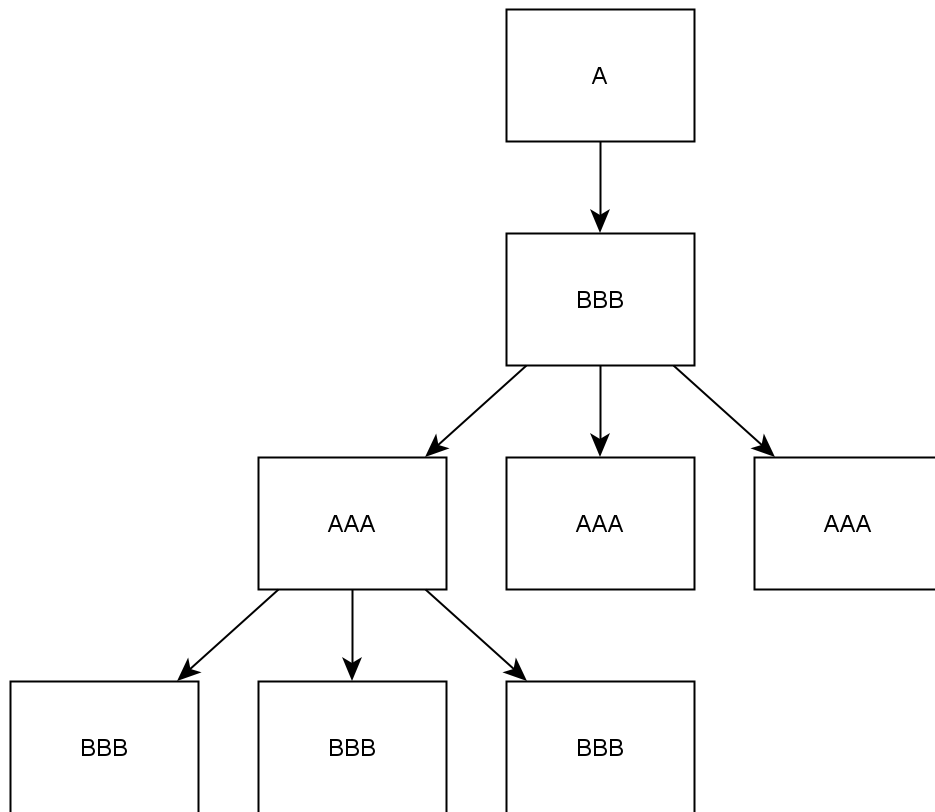


Figure 4: L-system as graph

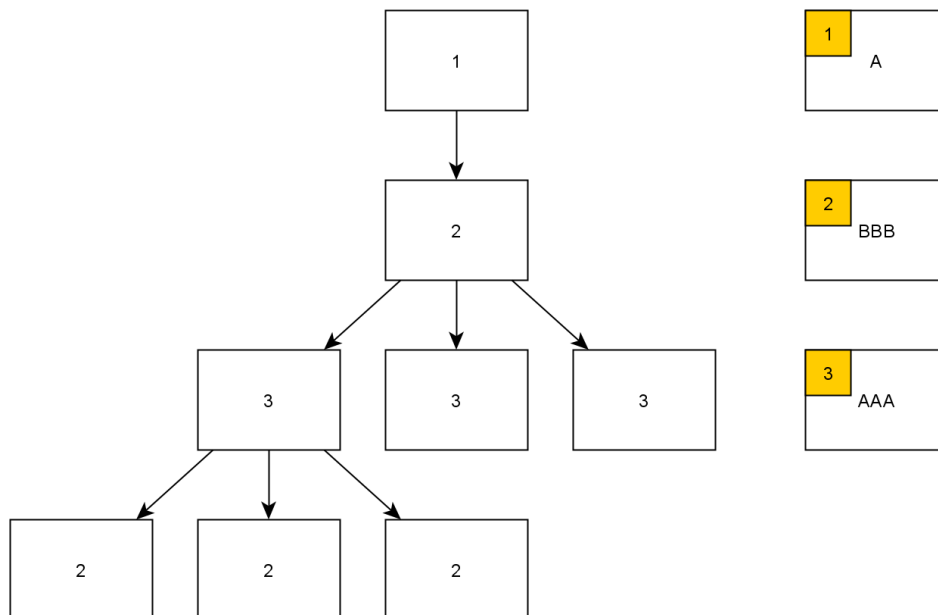


Figure 5: L-system as graph with improved memory consumption

parentnode. For example in figure 6, it is not needed to save the crossed out nodes, because they

are completely replaced. In this simple example is it already possible save only four instead of six nodes and reduce the data. This results in a flat graph with only a few nodes depth, but only the current state of a L-system. For this graph, it is important how the replacement is done. If the replacment is done with the all at once method, the parent node can be deleted directly. If the step by step replacement is done, it is nessecary to alter the parentnode data for each replacement. Only when the complete data and the parentnode is replaced, this node can be deleted. The complete replacement is simpler to implement, because the altering of parentnode data is not needed.

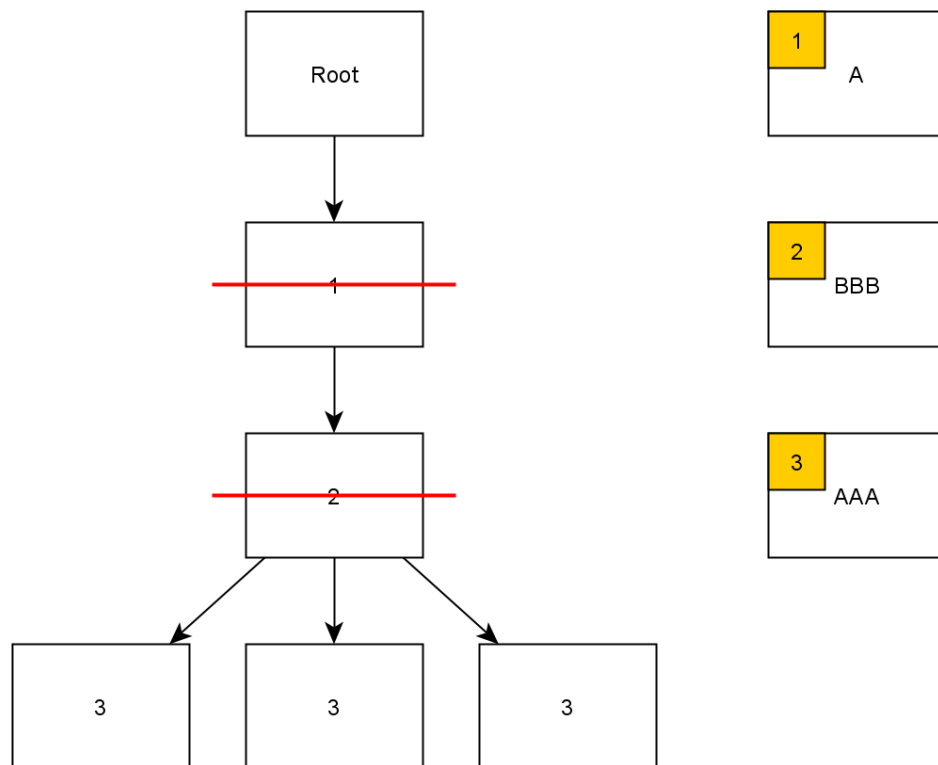


Figure 6: L-system as graph with one generation

A graph is as consequence a faster and memory saving approach in comparison to the naive L-system.

Another possible simplification is to neglect storing the generations of a L-system at all. Instead of saving the generation of a L-system in a datastructure, the result can be calculated on demand and never stored. This calculation can be done with a recursive function, that will return the generation. Clearly an advantage is the reduction of needed storage space, but this reduction is only achived at the cost of recalculation for each generation. Whenever a generation is needed, it is nessecary to calculate again and it is not possible to use a previous result. If a generation is needed multiple time, a solution could be to store the result in another object. This also enables a flexible use, because you can either only calculate the generation once and use the result directly, in this context for example to call the turtle commands, or save it for later usage.

There are the two possible ways to handle the L-system. The storage as a datastructure, based on a graph, and the recursive calculation. For this paper it is completely sufficient to calculate

the generation on demand, because it is only necessary to call the turtle command once for each object of the L-system generation. As consequence no implementation of the graph is needed for this paper. Other use cases in the future might need to store the data. This would even be possible with the recursive function, when the result is stored in another object, for example a graph. The proposal for the L-system is shown in figure 7.

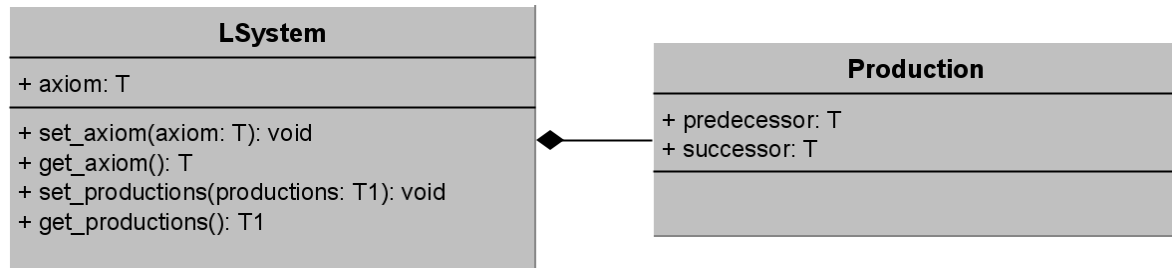


Figure 7: L-system proposal

The proposed L-system will just hold the grammar for a L-system, which will be given to the recursive function. This L-system can be used in a flexible way, because it can be configured without a restriction of the grammar. Additional, there is only a general type restriction for the axiom and the productions. The self-similarity and ergo the rewriting need to be represented by the types. The type of the axiom and the successor should have the same type, which is splittable in smaller parts. These parts must be comparable with the predecessor, which should have for that reason the same type.

Details zum aufrufen Semantische schnittstelle bei der funktion
 -> genauere beschreibung im kapitel mit der implementierung
 -> woher kommt die konfiguration

3.3 Turtle

Abstract interface with minimal set of needed functions

3.4 Turtle command mapping

3.5 Configuration data

convenient way of define a grammar - not hard coded - load from file

4 Implementation

4.1 Build System

The first important point

- Cmake as buildsystem
- reasons why cmake
- problems ?

4.2 TurtleGraphic

4.2.1 TestTurtle

4.2.2 CairoTurtle

4.2.3 Further implementations

SVG implementation

5 Tests

6 Examples

7 Problems and Restrictions

8 Outlook

15

¹⁵P. Prusinkiewicz and A. Lindenmayer, *The Algorithmic Beauty of Plants*. 2004. [Online]. Available: <http://algorithmicbotany.org/papers/abop/abop.pdf> (visited on 07/16/2020)