

ARE213 Problem Set #2A

Peter Alstone & Frank Proulx

November 2, 2013

1 Problem #1

1.1 Part A

Here we will consider the within estimator, as suggested. This suggests that we want to find $\widehat{\beta_{FE}}$ by running the following regression:

$$\ddot{Y}_{it} = \ddot{X}'_{it} \widehat{\beta_{FE}} + \ddot{\epsilon}_{it} \quad (1)$$

Where $\ddot{Y}_{it} = Y_{it} - \bar{Y}_i$, $\ddot{X}_{it} = X_{it} - \bar{X}_i$, $\bar{Y}_i = \frac{1}{T} \sum_{t=1}^T Y_{it}$, and $\bar{X}_i = \frac{1}{T} \sum_{t=1}^T X_{it}$. Our fixed effects estimator is therefore

$$\widehat{\beta_{FE}} = (\ddot{X}'_{it} \ddot{X}_{it})^{-1} \ddot{X}'_{it} \ddot{Y}_{it} \quad (2)$$

Because we have T=2, this can be rewritten as

$$\widehat{\beta_{FE}} = ((X_{it} - \frac{1}{2}X_{i1} - \frac{1}{2}X_{i2})'(X_{it} - \frac{1}{2}X_{i1} - \frac{1}{2}X_{i2}))^{-1} (X_{it} - \frac{1}{2}X_{i1} - \frac{1}{2}X_{i2})'(Y_{it} - \frac{1}{2}Y_{i1} - \frac{1}{2}Y_{i2}) \quad (3)$$

And looking at the first differences estimator, where we get $\widehat{\beta_{FD}}$ by running the regression

$$\Delta Y_{it} = \Delta X'_{it} \widehat{\beta_{FD}} + \Delta \epsilon_{it} \quad (4)$$

where $\Delta Y_{it} = Y_{it} - Y_{it-1}$, $\Delta X_{it} = X_{it} - X_{it-1}$, and $\Delta \epsilon_{it} = \epsilon_{it} - \epsilon_{it-1}$.

Because we only have T=2, this differencing estimator can only be estimated for t=2, so we regress

$$\Delta Y_{i2} = \Delta X'_{i2} \widehat{\beta_{FD}} + \Delta \epsilon_{i2} \quad (5)$$

$$Y_{i2} - Y_{i1} = (X_{i2} - X_{i1})' \widehat{\beta_{FD}} + \epsilon_{i2} - \epsilon_{i1} \quad (6)$$

thus, our first difference estimator is

$$\widehat{\beta}_{FD} = ((X_{i2} - X_{i1})'(X_{i2} - X_{i1}))^{-1}(X_{i2} - X_{i1})'(Y_{i2} - Y_{i1}) \quad (7)$$