

ARE213 Problem Set #2A

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1 Problem #1

1.1 Part A

Here we will consider the within estimator, as suggested. This suggests that we want to find $\widehat{\beta_{FE}}$ by running the following regression:

$$\ddot{Y}_{it} = \ddot{X}'_{it} \widehat{\beta_{FE}} + \ddot{\epsilon}_{it} \quad (1)$$

Where $\ddot{Y}_{it} = Y_{it} - \bar{Y}_i$, $\ddot{X}_{it} = X_{it} - \bar{X}_i$, $\bar{Y}_i = \frac{1}{T} \sum_{t=1}^T Y_{it}$, and $\bar{X}_i = \frac{1}{T} \sum_{t=1}^T X_{it}$. Our fixed effects estimator is therefore

$$\widehat{\beta_{FE}} = (\ddot{X}'_{it} \ddot{X}_{it})^{-1} \ddot{X}'_{it} \ddot{Y}_{it} \quad (2)$$

Because we have T=2, this can be rewritten as

$$\widehat{\beta_{FE}} = ((X_{it} - \frac{1}{2}X_{i1} - \frac{1}{2}X_{i2})'(X_{it} - \frac{1}{2}X_{i1} - \frac{1}{2}X_{i2}))^{-1} (X_{it} - \frac{1}{2}X_{i1} - \frac{1}{2}X_{i2})'(Y_{it} - \frac{1}{2}Y_{i1} - \frac{1}{2}Y_{i2}) \quad (3)$$

And looking at the first differences estimator, where we get $\widehat{\beta_{FD}}$ by running the regression

$$\Delta Y_{it} = \Delta X'_{it} \widehat{\beta_{FD}} + \Delta \epsilon_{it} \quad (4)$$

where $\Delta Y_{it} = Y_{it} - Y_{it-1}$, $\Delta X_{it} = X_{it} - X_{it-1}$, and $\Delta \epsilon_{it} = \epsilon_{it} - \epsilon_{it-1}$.

Because we only have T=2, this differencing estimator can only be estimated for t=2, so we regress

$$\Delta Y_{i2} = \Delta X'_{i2} \widehat{\beta_{FD}} + \Delta \epsilon_{i2} \quad (5)$$

$$Y_{i2} - Y_{i1} = (X_{i2} - X_{i1})' \widehat{\beta_{FD}} + \epsilon_{i2} - \epsilon_{i1} \quad (6)$$

thus, our first difference estimator is

$$\widehat{\beta}_{FD} = ((X_{i2} - X_{i1})'(X_{i2} - X_{i1}))^{-1}(X_{i2} - X_{i1})'(Y_{i2} - Y_{i1}) \quad (7)$$

2 Problem #3

2.1 Part A

Running pooled bivariate OLS, adding a quadratic time trend, and adding the covariates that we expect to belong produces the models shown in Table

.

Does this typeset correctly?

Table 1: Pooled Models of Fatalities Per Capita

| | <i>Dependent variable:</i> | | |
|-------------------------|----------------------------|------------------------------|----------------------------|
| | bivariate | logfatalpc quadratic time | covariates |
| | (1) | (2) | (3) |
| primary | −0.144*** (0.026) | −0.082*** (0.026) | −0.090*** (0.025) |
| sqnetyears | | −0.001*** (0.0001) | |
| secondary | | | −0.093*** (0.020) |
| college | | | −3.263*** (0.181) |
| unemploy | | | 0.004 (0.004) |
| beer | | | 0.299*** (0.032) |
| totalvmt | | | −0.00000*** (0.00000) |
| precip | | | −0.028*** (0.006) |
| snow32 | | | −0.316*** (0.018) |
| rural_speed | | | 0.003** (0.002) |
| urban_speed | | | 0.006*** (0.002) |
| Constant | −1.703*** (0.011) | −1.626*** (0.014) | −1.669*** (0.128) |
| Observations | 1,127 | 1,127 | 1,127 |
| R ² | 0.027 | 0.079 | 0.566 |
| Adjusted R ² | 0.027 | 0.079 | 0.560 |
| F Statistic | 31.007*** (df = 1; 1125) | 348.494*** (df = 2; 1124) | 145.468*** (df = 10; 1116) |

Note:

*p<0.1; **p<0.05; ***p<0.01