ARE213 Problem Set #2A

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1 Problem #1

1.1 Part A

Here we will consider the within estimator, as suggested. This suggests that we want to find $\widehat{\beta_{FE}}$ by running the following regression:

$$\ddot{Y_{it}} = \ddot{X}'_{it}\widehat{\beta_{FE}} + \ddot{\epsilon_{it}} \tag{1}$$

Where $\ddot{Y_{it}} = Y_{it} - \bar{Y}_i$, $\ddot{X_{it}} = X_{it} - \bar{X}_i$, $\bar{Y}_i = \frac{1}{T}\sum_{t=1}^{T} Y_{it}$, and $\bar{X}_i = \frac{1}{T}\sum_{t=1}^{T} X_{it}$. Based on OLS, our fixed effects estimator is:

$$\widehat{\beta_{FE}} = (\ddot{X}'_{it}\ddot{X_{it}})^{-1}\ddot{X}'_{it}\ddot{Y_{it}} \tag{2}$$

We know that $\ddot{X_i} = \boldsymbol{A}X_i$ and $\ddot{Y_i} = \boldsymbol{A}Y_i$, so

$$\widehat{\beta_{FE}} = (X'_{it}\boldsymbol{AA}X_{it})^{-1}X_{it}\boldsymbol{AA}Y_{it} \tag{3}$$

However, since AA=A, and thus $\boldsymbol{AA} = \boldsymbol{A}$, this reduces to

$$\widehat{\beta_{FE}} = (X'_{it}\boldsymbol{A}X_{it})^{-1}X_{it}\boldsymbol{A}Y_{it} \tag{4}$$

And looking at the first differences estimator, where we get $\widehat{\beta_{FD}}$ by running the regression

$$\Delta Y_{it} = \Delta X'_{it}\widehat{\beta_{FD}} + \Delta\epsilon_{it} \tag{5}$$

where $\Delta Y_{it} = Y_{it} - Y_{it-1}$, $\Delta X_{it} = X_{it} - X_{it-1}$, and $\Delta\epsilon_{it} = \epsilon_{it} - \epsilon_{it-1}$.

Because we only have T=2, this differencing estimator can only be estimated for t=2, so we regress

$$\Delta Y_{i2} = \Delta X'_{i2}\widehat{\beta_{FD}} + \Delta\epsilon_{i2} \tag{6}$$

$$Y_{i2} - Y_{i1} = (X_{i2} - X_{i1})'\widehat{\beta_{FD}} + \epsilon_{i2} - \epsilon_{i1} \tag{7}$$

thus, our first difference estimator is

$$\widehat{\beta_{FD}} = ((X_{i2} - X_{i1})'(X_{i2} - X_{i1}))^{-1}(X_{i2} - X_{i1})'(Y_{i2} - Y_{i1}) \tag{8}$$