Heaps

Priority Queues

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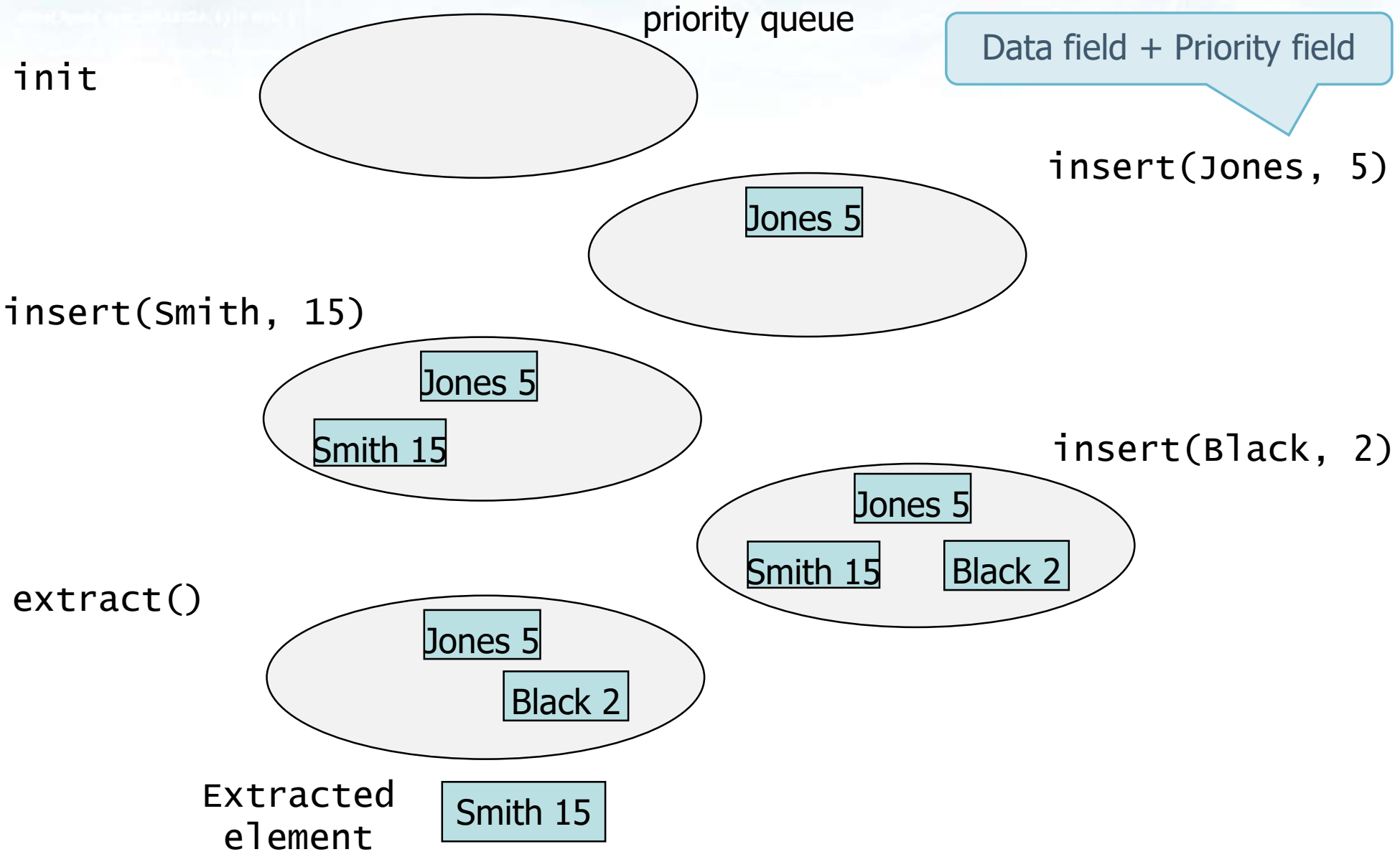
Priority Queues

- Heaps have many applications beyond heap-sort
- A priority queue is a data structure to store elements including a priority field such that all main operations are based on such a field
- Priority queues have several applications
  - Job scheduling
  - Etc.

Priority Queues

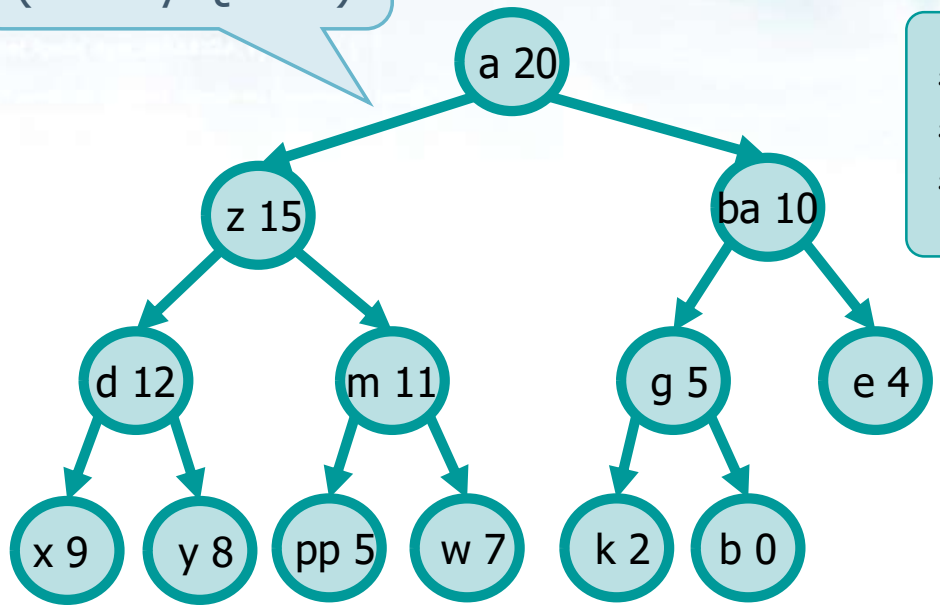


Priority Queues

- It is possible to implement
  - Min-priority queues
  - Max-priority queues
- Main operations
  - Insert, extract maximum, read maximum, change priority
- Possible alternative data structure implementations
  - Unordered array/list
  - Ordered array/list

Example

PQ
(Priority Queue)




```
#define LEFT(i)    (2*i+1)
#define RIGHT(i)   (2*i+2)
#define PARENT(i)  ((int)(i-1)/2)
```

Array
representation

pq->A

a	z	ba	d	m	g	e	x	y	pp	w	k	b		
20	15	10	12	11	5	4	9	8	5	7	2	0		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

pq->heapsize = 13

Heap ←→ PQ

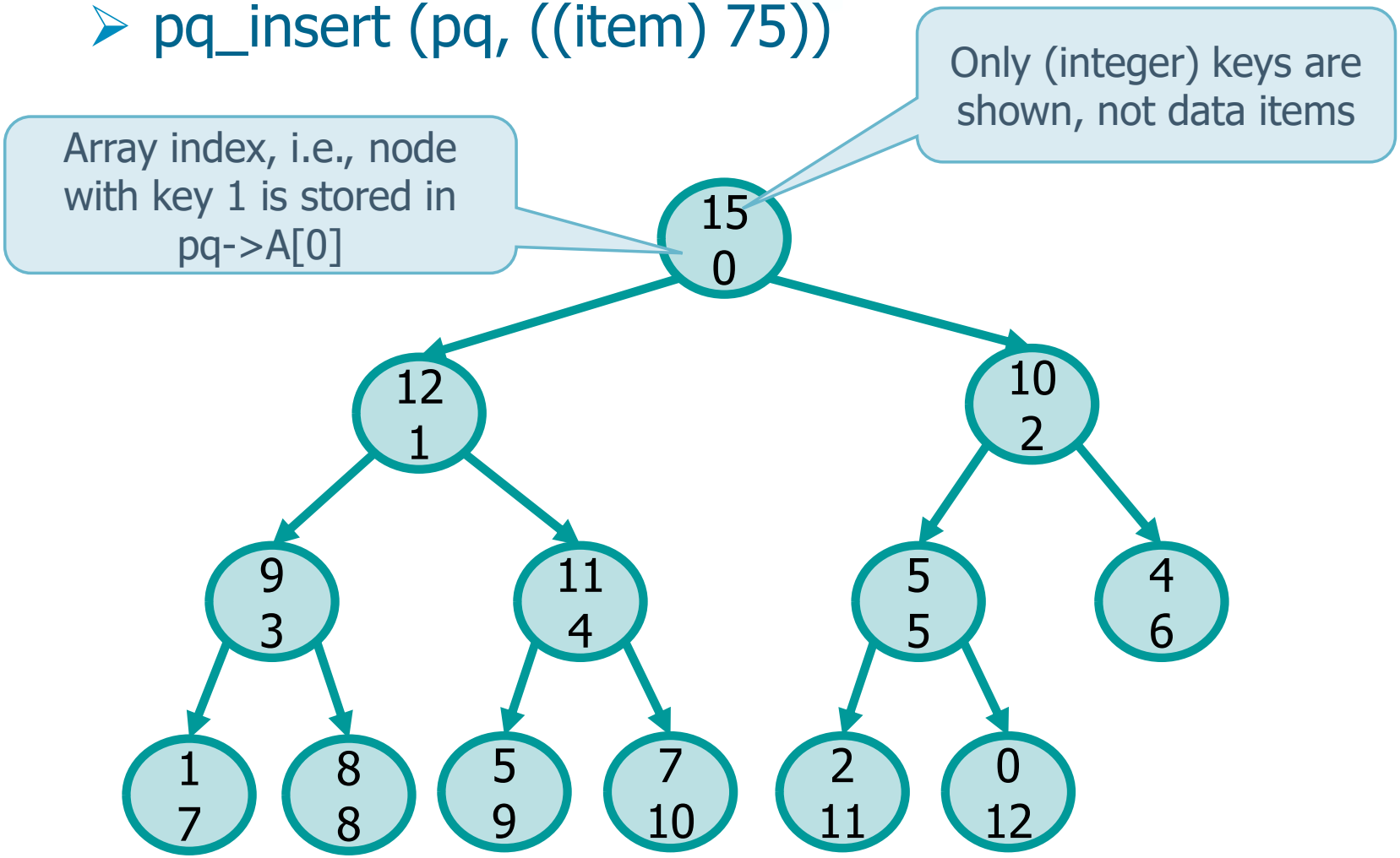
Array (maximum)
maxN = 15

Function pq_insert

- Add a leaf to the tree
  - It grows level-by-level from left to right satisfying the structural property
- From current node up (initially the newest leaf) up to the root
  - Compare the parent's key with the new node's key, moving the parent's data from the parent to the child when the key to insert is larger
  - Otherwise insert the data into the current node
- Complexity
  - $T(n) = O(\lg n)$

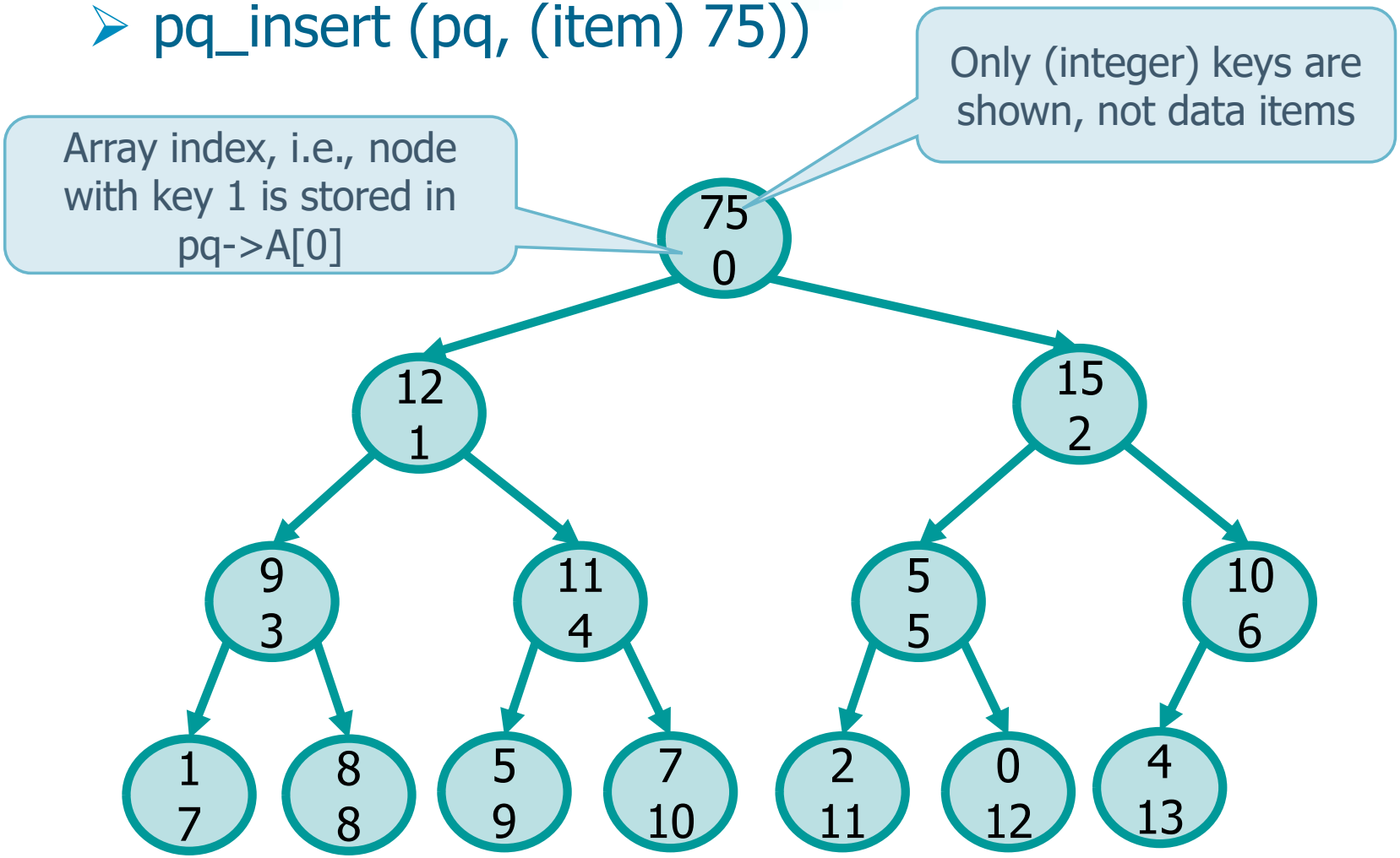
Example

- Call function
  - pq_insert (pq, ((item) 75))



Solution

- Call function
  - pq_insert (pq, (item) 75))



Implementation

Function **item_less** compares keys

```
void pq_insert (PQ pq, Item item) {
  int i;

  i  = pq->heapsize++;
  while( (i>=1) &&
         (item_less(pq->A[PARENT(i)], item)) )
    pq->A[i] = pq->A[PARENT(i)];
    i = PARENT (i);
  }
  pq->A[i] = item;

  return;
}
```

Increase the heap size

Move node down

Move up toward the root

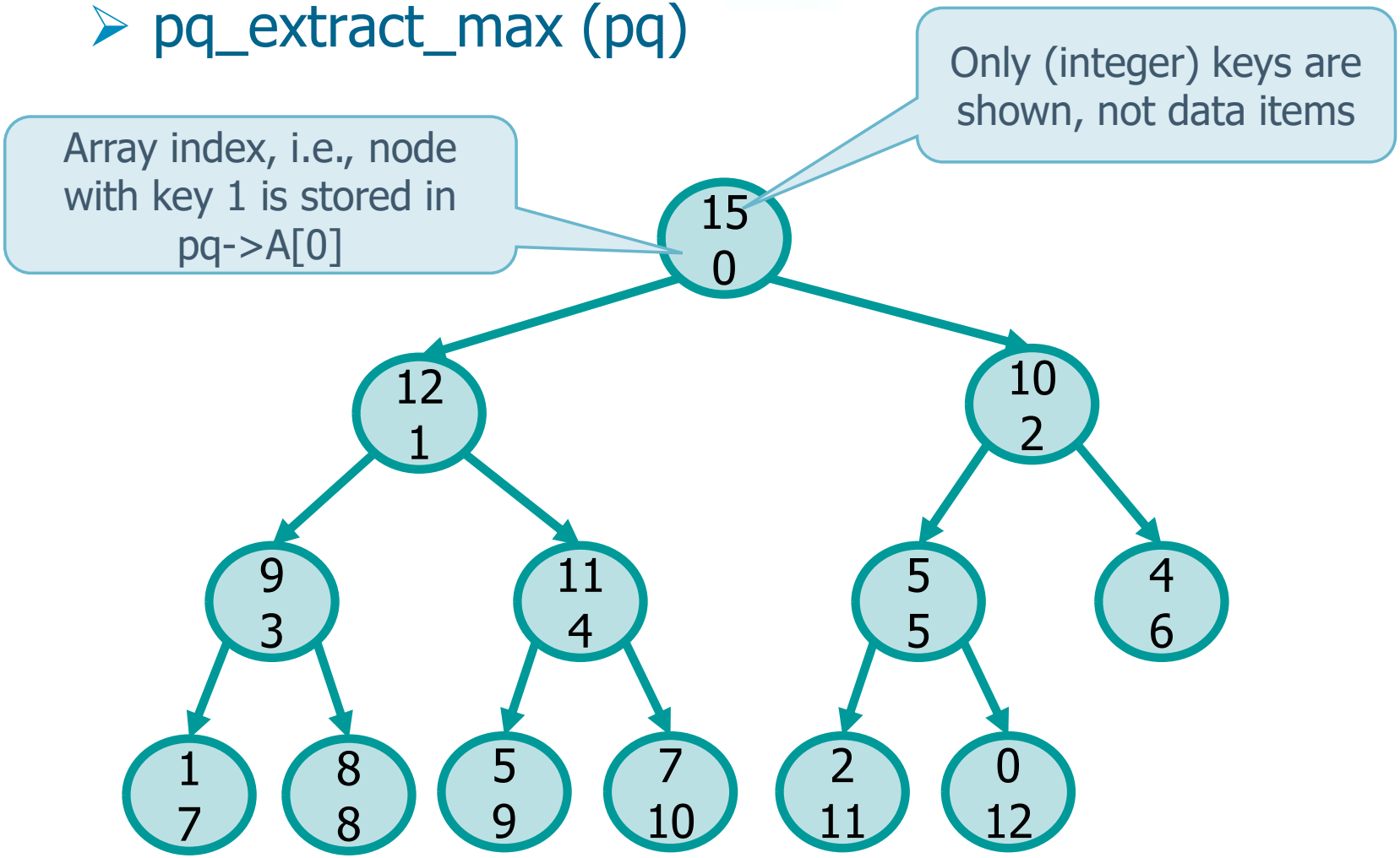
Insert new element in its final destination

Function pq_extract_max

- Modify the head, by extracting the largest value, stored into the root
  - Swap root with the last leaf (the rightmost onto the last level)
  - Reduce by 1 the heap size
  - Restore the heap property by applying heapify
- Complexity
  - $T(n) = O(\lg n)$

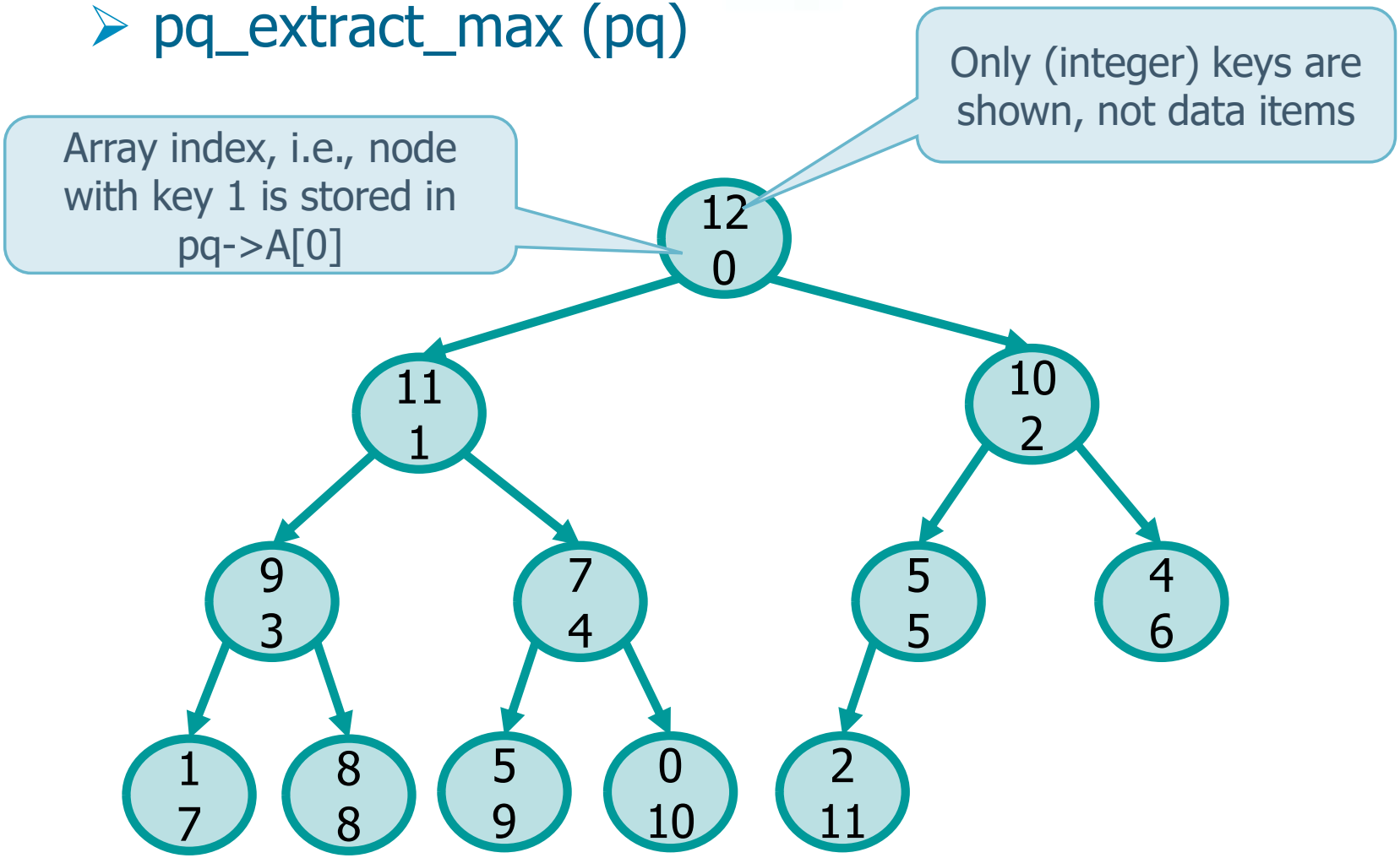
Example

- Call function
  - pq_extract_max (pq)



Solution

- Call function
  - pq_extract_max (pq)



Implementation

```
Item pq_extract_max(PQ pq) {
  Item item;

  swap (pq, 0, pq->heapsize-1);
  item = pq->A[pq->heapsize-1];
  pq->heapsize--;
  heapify (pq, 0);

  return item;
}
```

Extract max and move last element into the root node

Reduce heap size

Heapify from root

Function pq_change

- Modify the key of an element **in a given position** given its index
- Can be implemented as two separate operations
  - Decrease key
    - When a key is decreased, we may need to move it downward
    - To move a key downward, we can adopt the same process analyze in **heapify**
      - Heapify keeps moving the key from the parent to the child with the largest key until the key is inserted into the current node

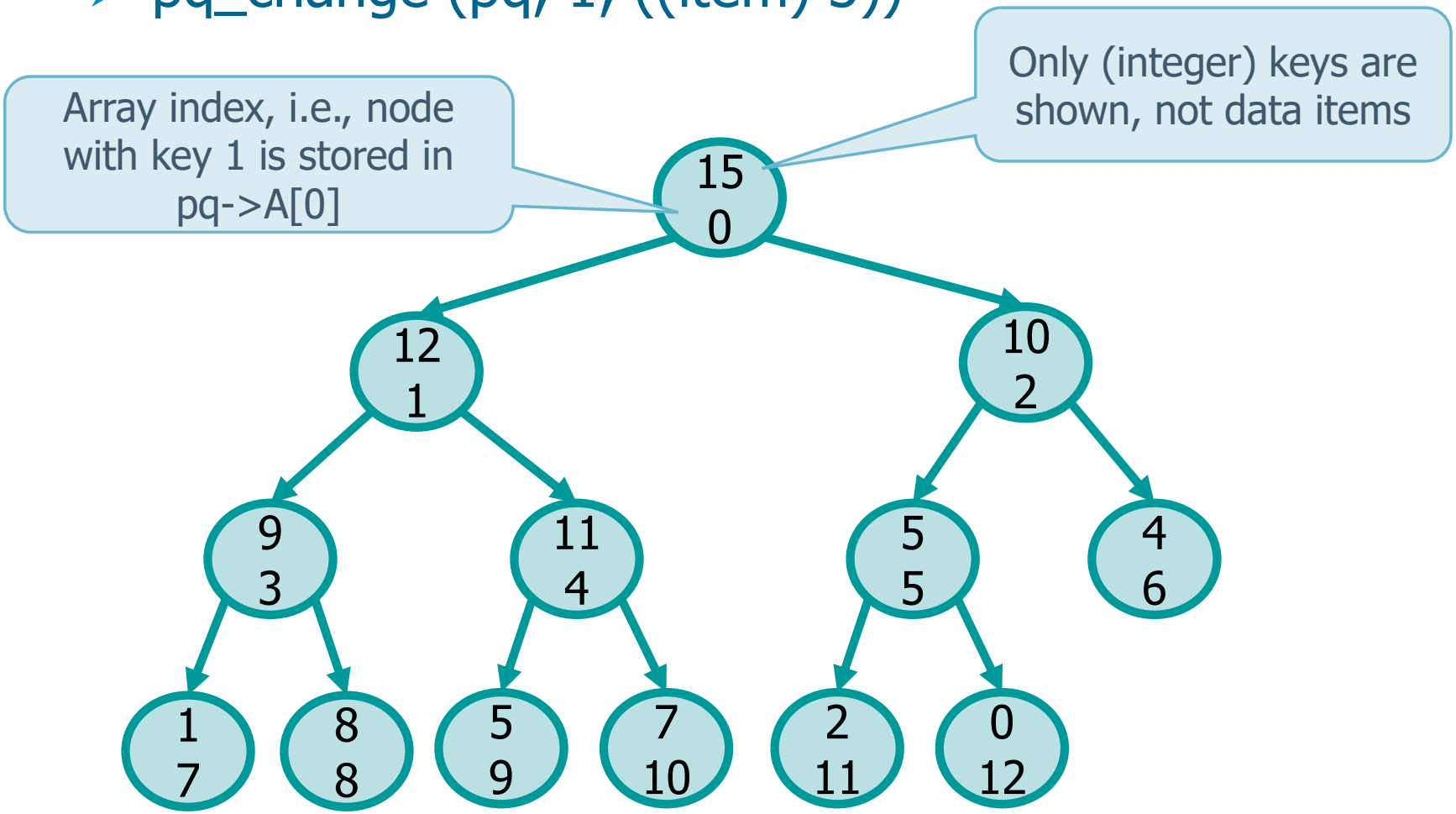
Function pq_change

- Increase key
  - When a key is increased, we may need to move it upward
  - To move a key upward, we can adopt the same process analyze in **pq_insert**
    - We move the key up into the parent until the key is inserted into the current node

- Complexity
  - Dependent on the tree height
  - T(n) = O(lg n)

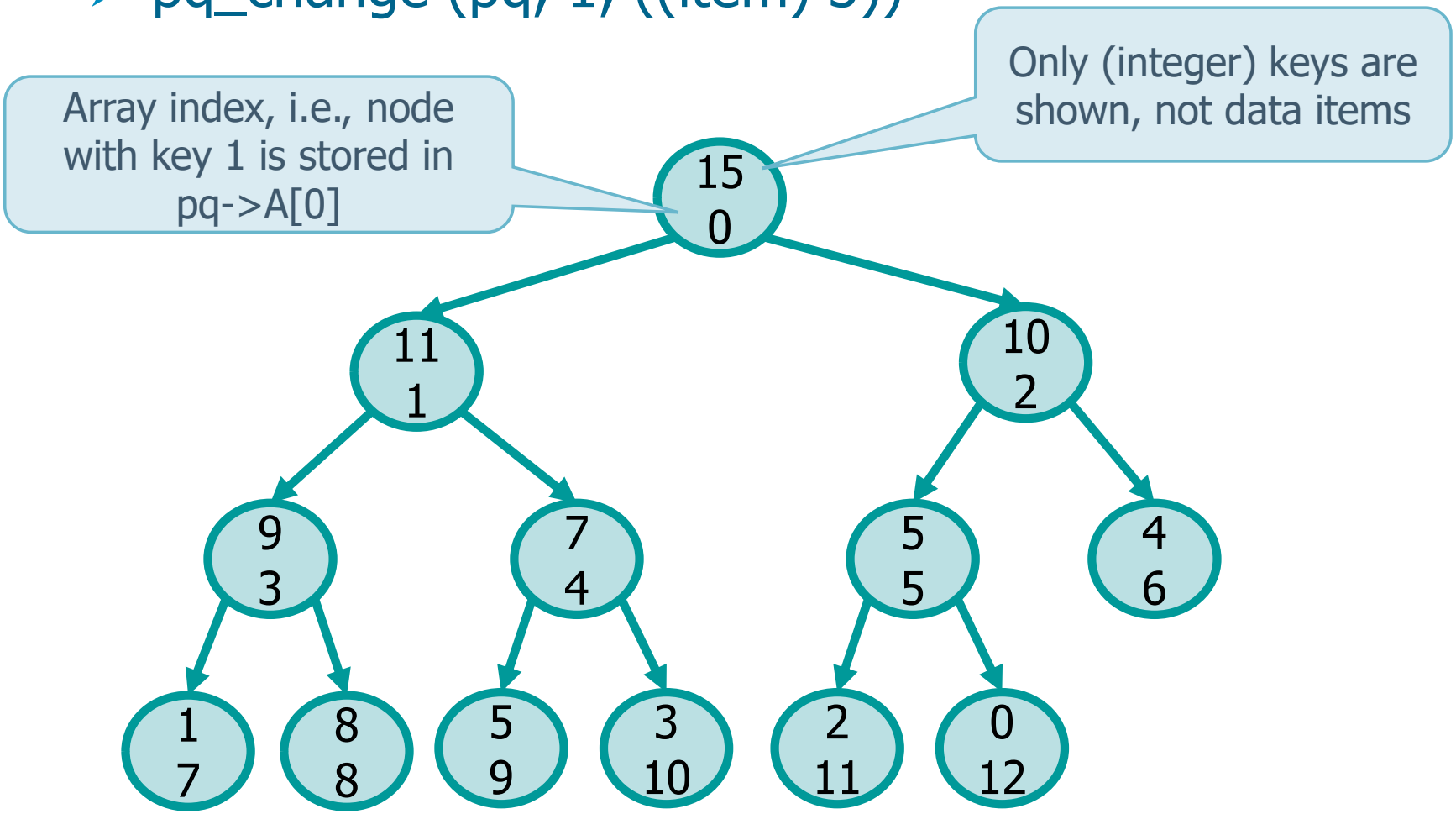
Example: Decrease key

- Call function
  - pq_change (pq, 1, ((item) 3))



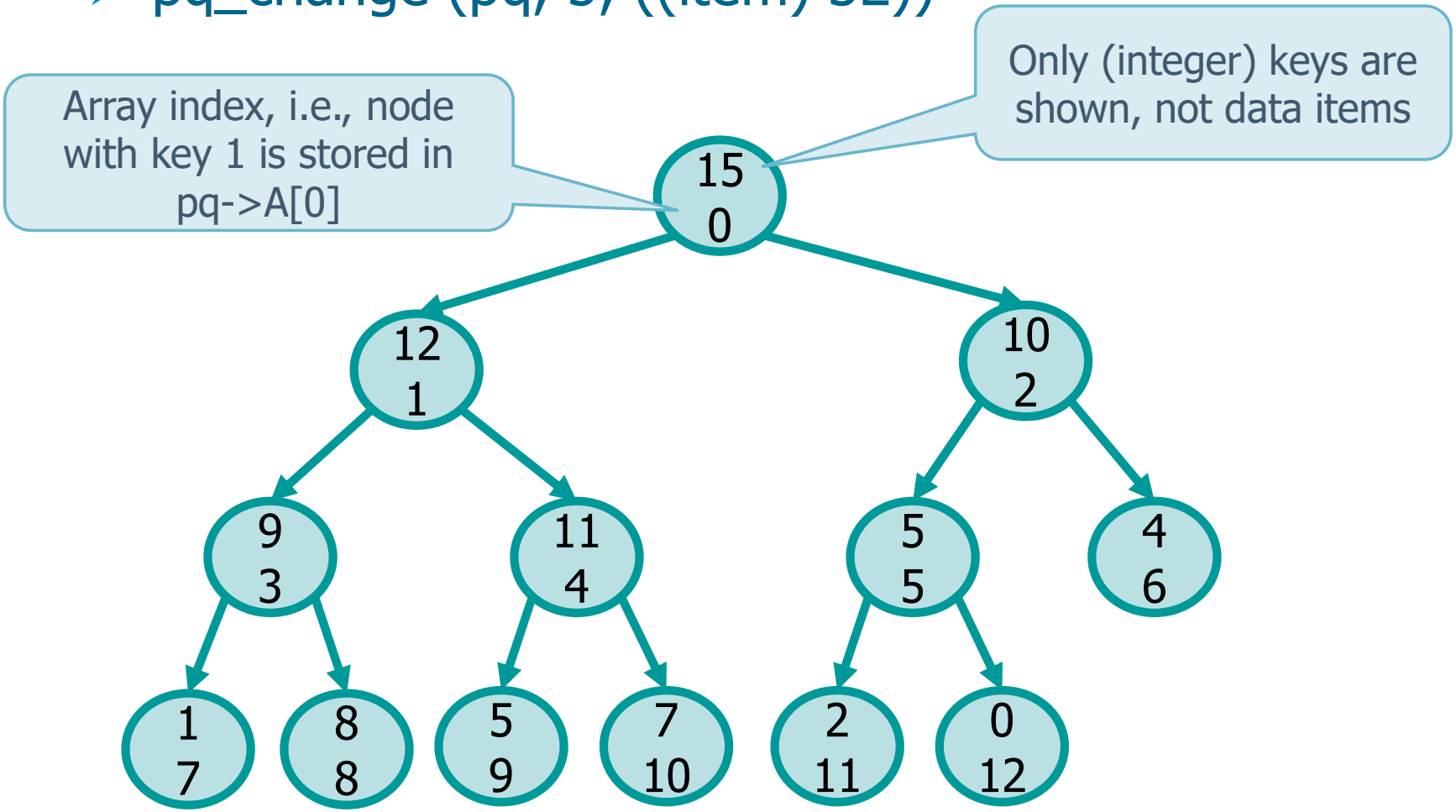
Solution

- Call function
  - pq_change (pq, 1, ((item) 3))



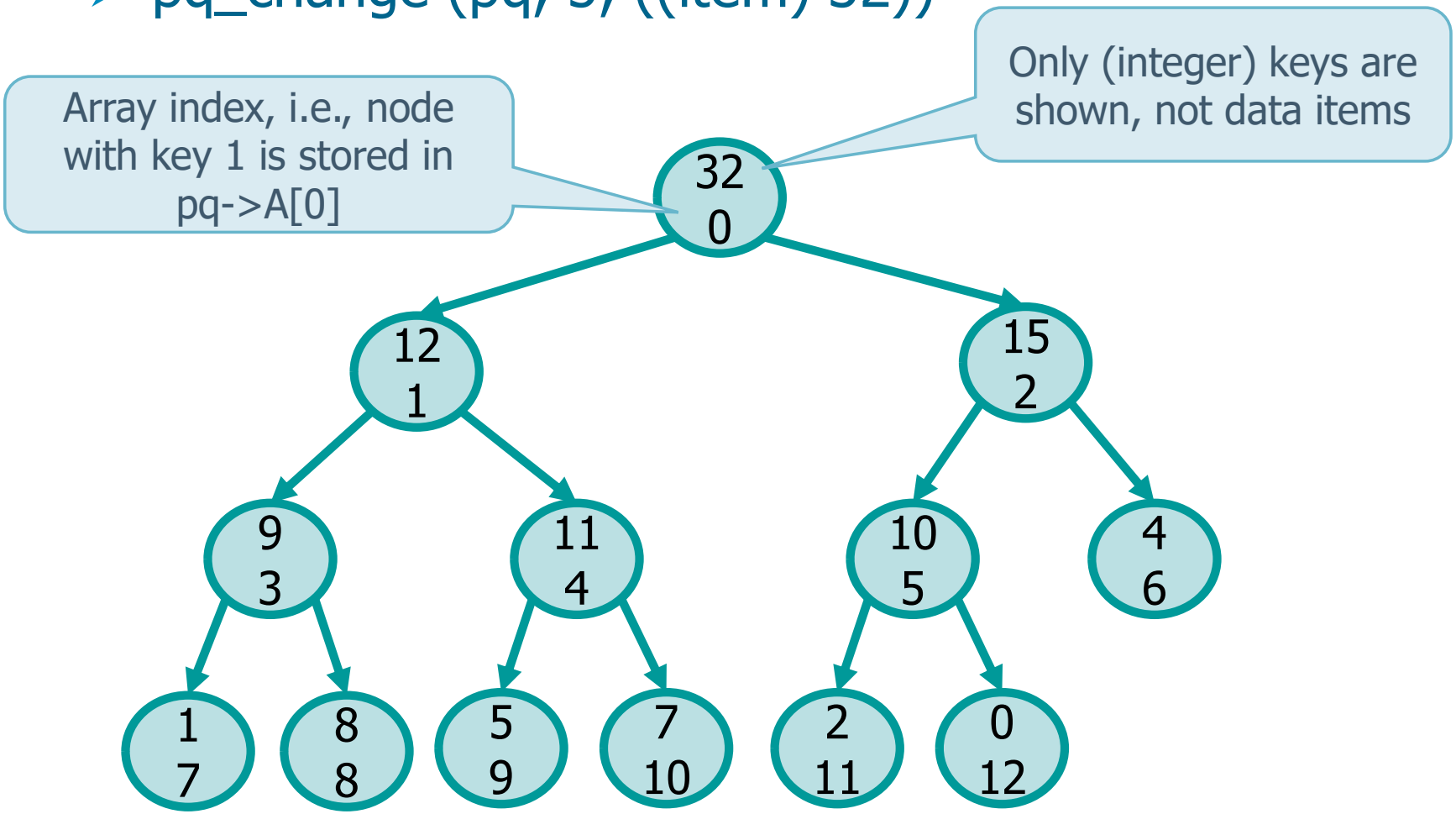
Example: Increase key

- Call function
  - pq_change (pq, 5, ((item) 32))



Solution

- Call function
  - pq_change (pq, 5, ((item) 32))



Implementation

```
void pq_change (PQ pq, int i, Item item) {
  if (item_less (item, pq->A[i]) {
    decrease_key (pq, i);
  } else {
    increase_key (pq, i, item);
  }
}
void decrease_key (PQ pq, int i) {
  pq->A[i] = item;
  heapify (pq, i);
}
void increase_key (PQ pq, int i) {
  while( (i>=1) &&
         (item_less(pq->A[PARENT(i)], item)) ) {
    pq->A[i] = pq->A[PARENT(i)];
    i = PARENT(i);
  }
  pq->A[i] = item;
}
```

Exercise

- Insert the following values into an initially empty **max-priority** queue
  - 11 31 77 34 65 1 76 48 55 24 9 98 90 5 13 88

Notice that this is not an application of **heapsort** but of **pq_insert**

Exercise

- Insert the following values into an initially empty **min-priority** queue
  - 11 31 77 34 65 1 76 48 55 24 9 98 90 5 13 88

Notice that this is not an application of **heapsort** but of **pq_insert**