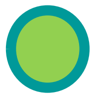
Trees

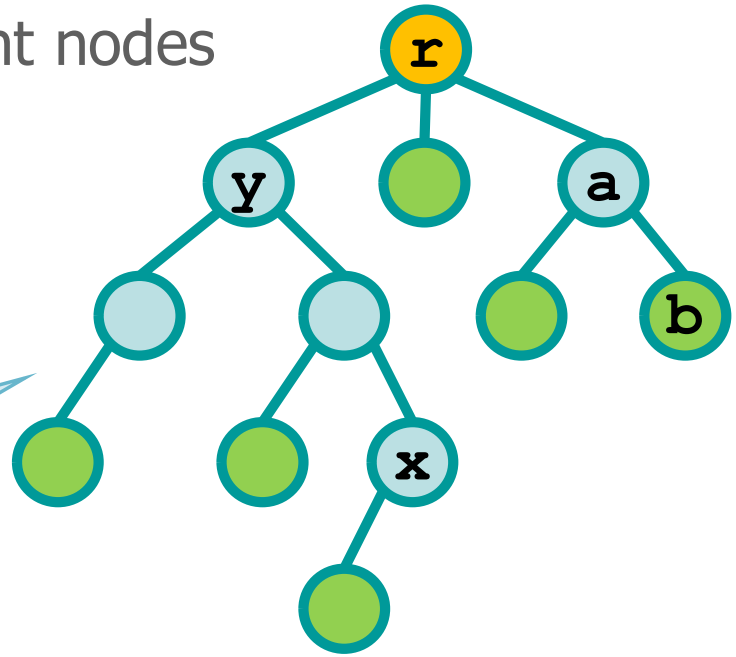
Definitions

Stefano Quer
Dipartimento di Automatica e Informatica
Politecnico di Torino

Rooted trees

- A rooted tree is a tree where there is a node r called root
  - Parent/child relationship
    - y is an ancestor of x if y belongs to the path from r to x. In this case x is a descendant of y
    - y is a proper ancestor of x iff x ≠ y
    - Parent and a child are adjacent nodes
  - The root has no parent 

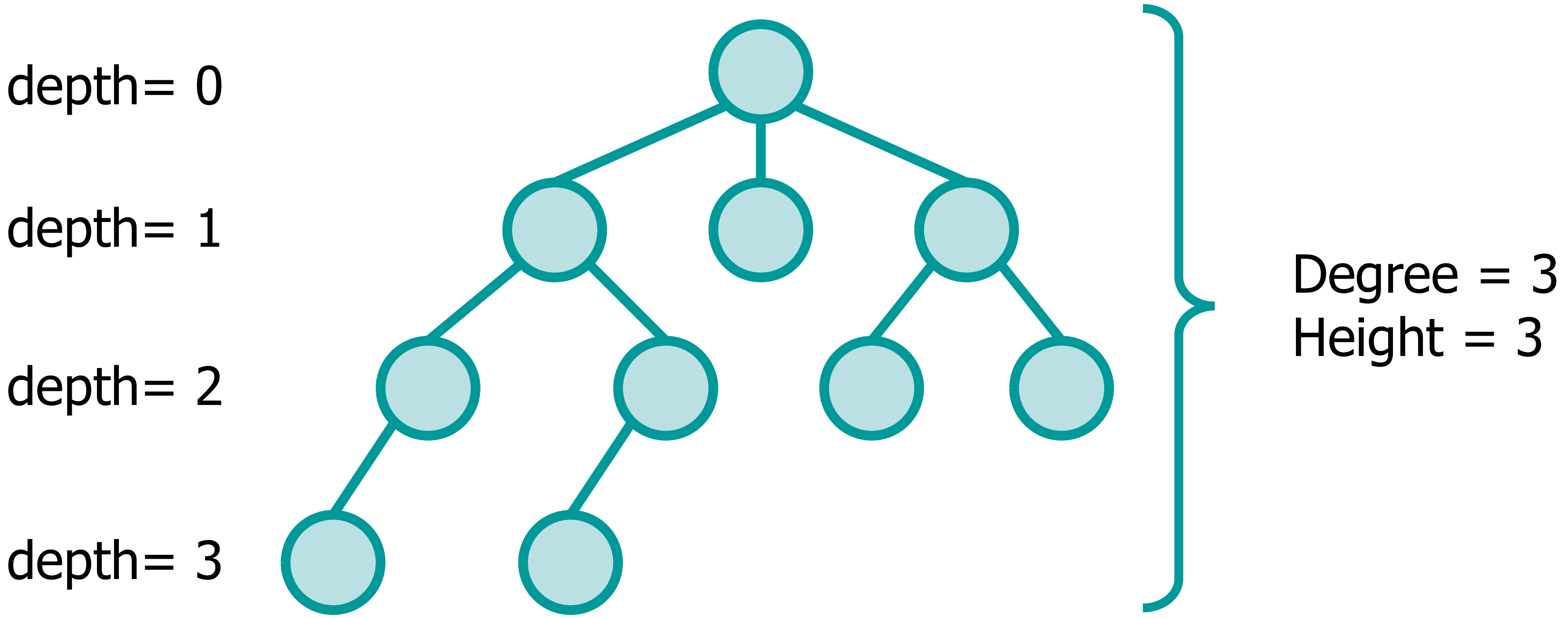
  - Leaves have no children 



y ancestor of di x
x descendant of y
a parent of b
b child of a

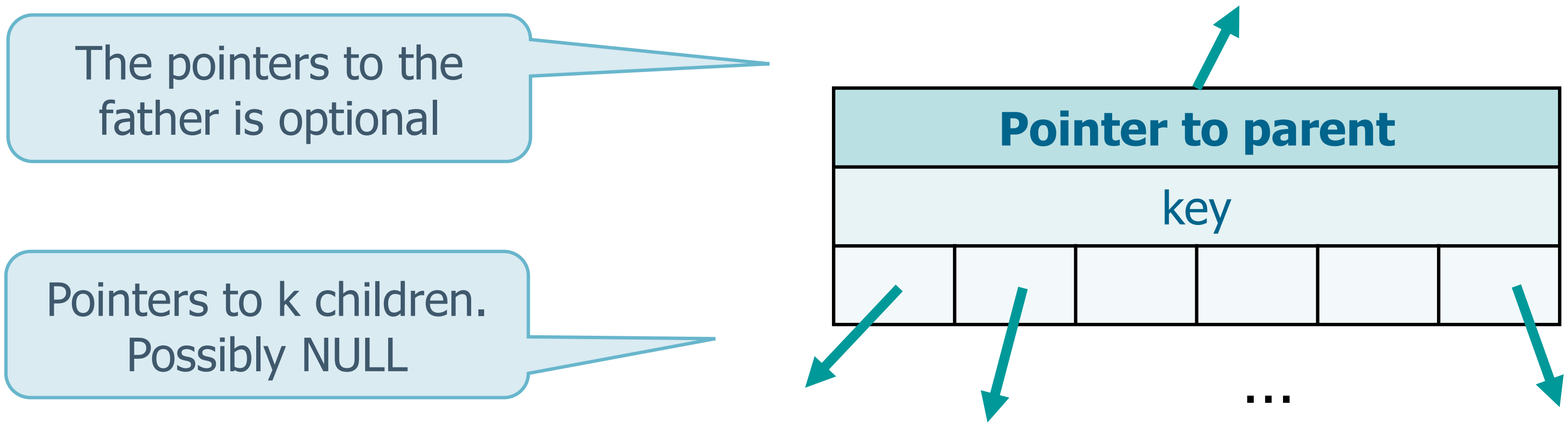
Properties of a rooted tree

- Given a rooted tree T the following are common definitions
  - Degree (T) = maximum number of children
  - Depth (x) = length of the path from the root to x
  - Height (T) = maximum depth of a node



Representation of a tree

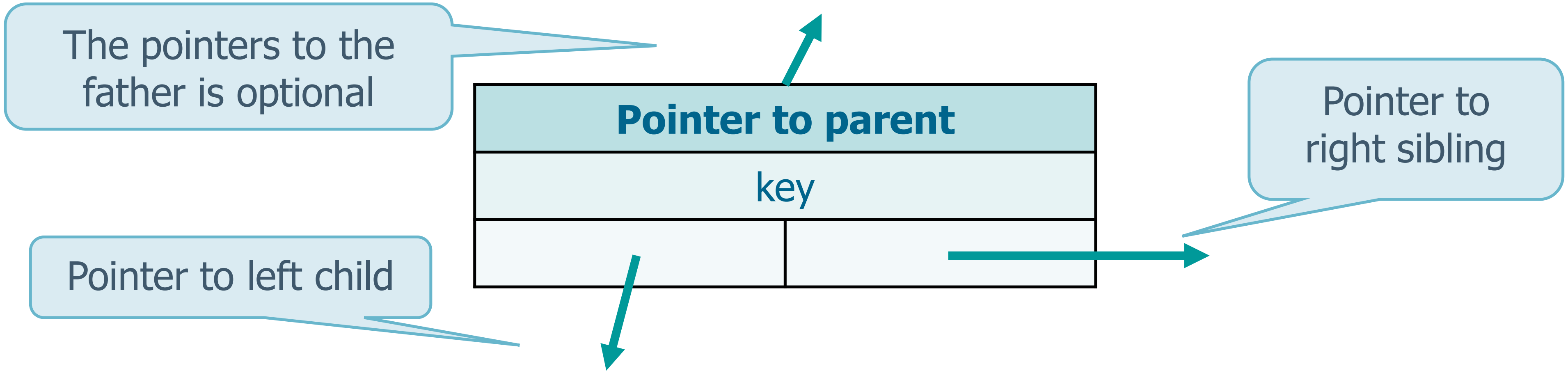
- There are at least two representations for nodes of a tree of degree k
  - Each node may store a pointer to the parent, the key, and k pointers to k children



    - Unefficient if only few nodes have indeed degree k
      - Space is allocated for all k pointers, but many are NULL)

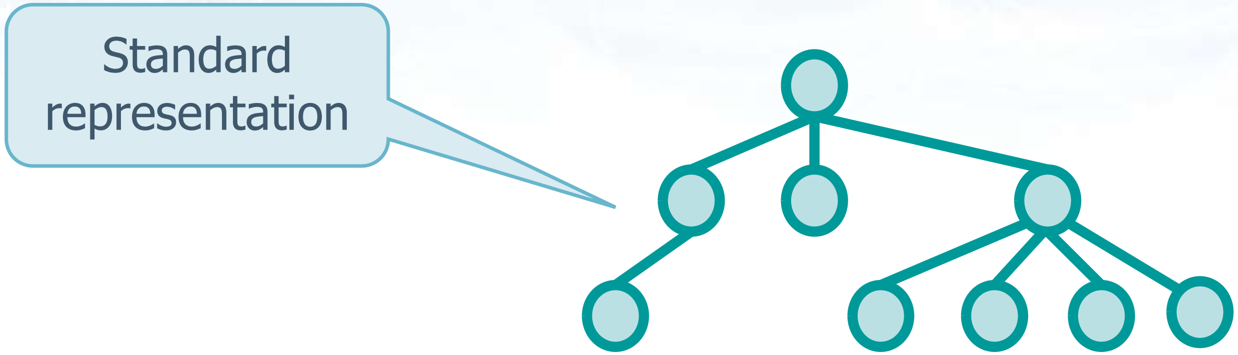
Representation of a tree

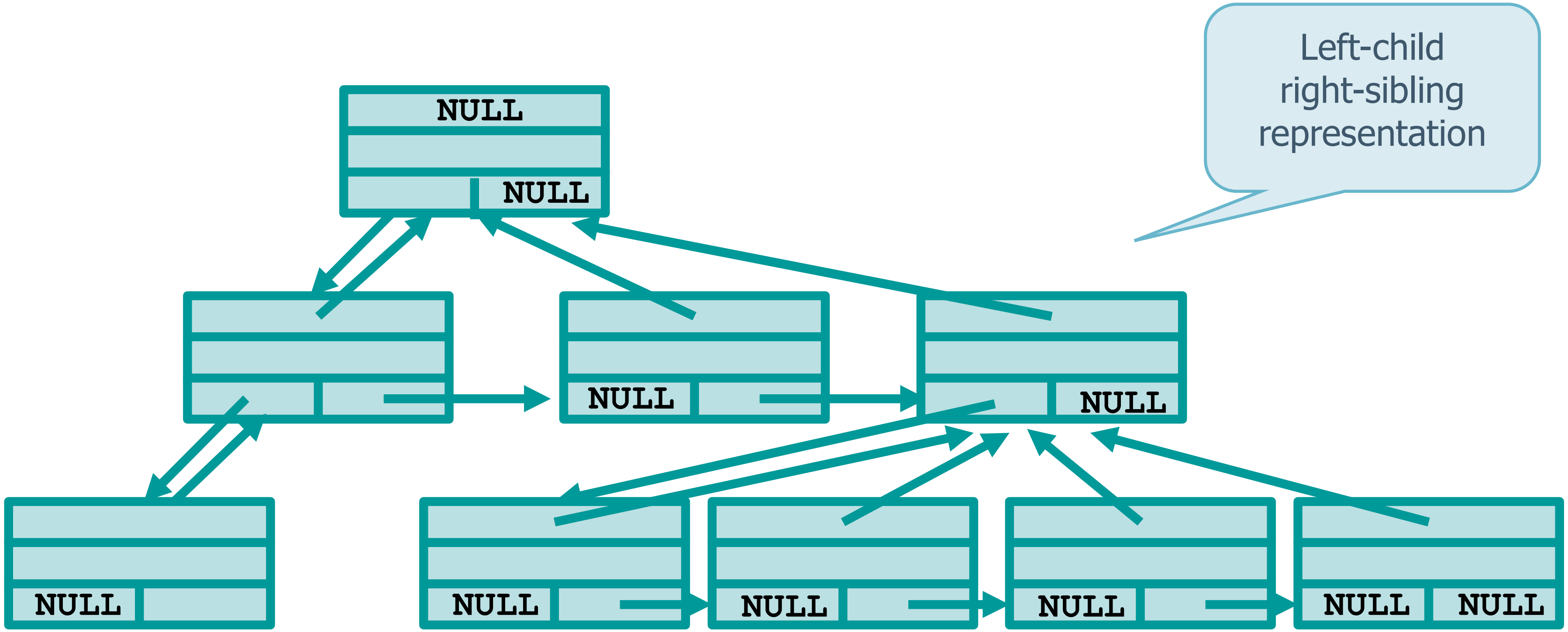
- Each node may also store a pointer to parent, the key, 1 pointer to left child, 1 pointer to right sibling



  - Efficient, as each node specifies always 2 pointers, no matter the degree of the tree

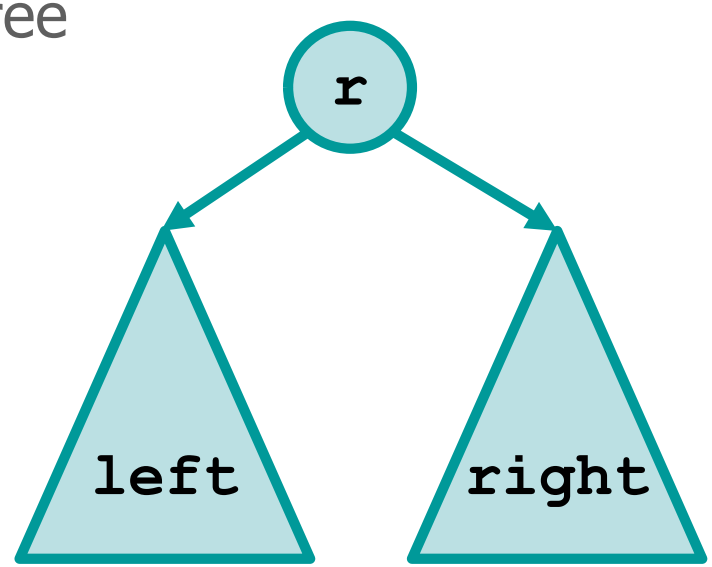
Representation of a tree





Binary trees

- Definition
  - Tree of degree 2
  - Recursively T is
    - Empty set of nodes
    - Root, left subtree, right subtree

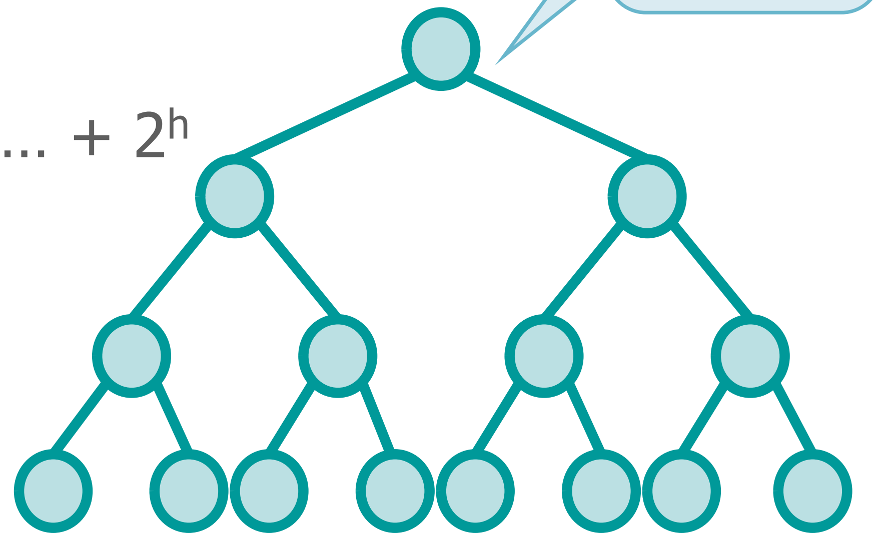


Complete Binary Trees

- A complete binary tree must satisty two conditions
  - All leaves have the same depth
  - Every node is either a leaf or it has 2 children
- In a complete binary tree of height h
  - The number of leaves is 2^h
  - The number of nodes is
    - $\Sigma_{0 \leq i \leq h} 2^i = 2^0 + 2^1 + 2^2 \ldots + 2^h$
      $= 2^{h+1} - 1$

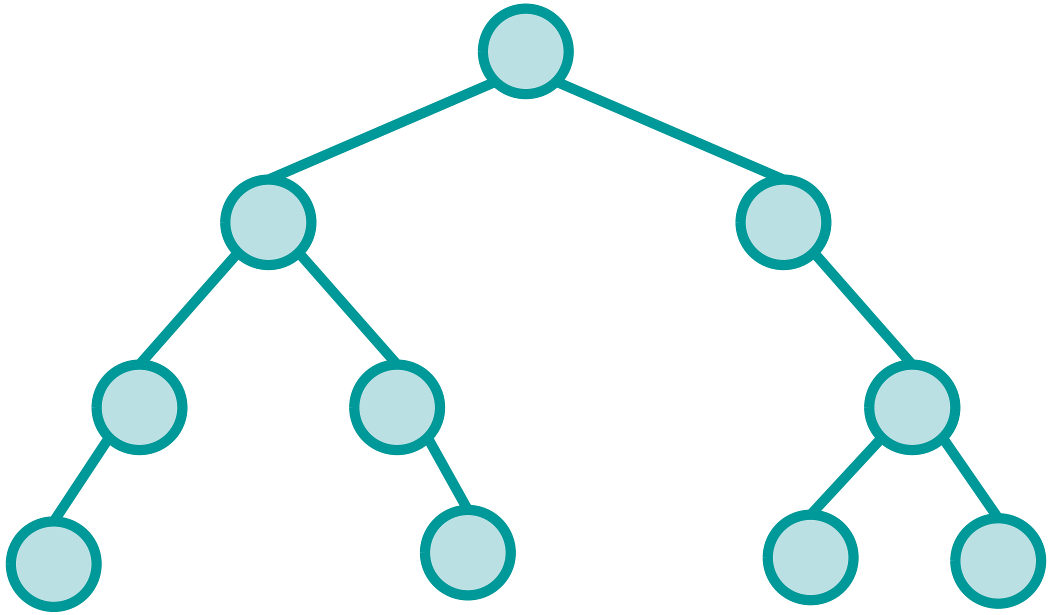
Finite geometric progression with ratio = 2

h = 3
8 leaves
15 nodes



Balanced binary trees

- In a balanced binary tree all paths root-leaves have the same length



  - If T is complete, then T is also balanced
  - The opposite is not necessarily true

Balanced binary trees

- A binary tree is said to be almost balanced if the length of all paths from root to leaves differs at most by 1