Algorithms and Data Structures

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Contents

1 Pointers

```
sizeof(char)=1
sizeof(short)
sizeof(int)= 4 (32 bit) or 8 (32bit)
sizeof(long)
sizeof(float)
sizeof(double)
sizeof(longdouble)
```

1.0.1 Example in memory

```
struct student{
  int id;
  char a;
  int id2;
  char b;
  float percentage
}
```

In memory:

.	1 byte	1 byte	1 byte	1 byte
id	x	x	x	x
a	x			
id2	x	x	x	x
b	x			
precentage	x	x	x	x

1.1 Pointers

Pointers are varibles whose values are memory addresses

- The address operator, *
  - Returns the value of the object to which its operand (i.e., a pointer) points

  - *p = v = 5 because it's openrand p points to v
- The indirection operator, &
  - Given a variable, it takes its address
  - Given *p = v = 5 &v is the address of v

```
<type> *<pointer>;
int *pointer;
int number;

pointer = &number;
```

This means that the pointer is equal to the number address, so pointer points to the number

1.1.1 All possible cases

```
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char *argv[]) {
  int v = 5;
  int *p;
  p = &v;
```

```c
    printf("%d\n", v);
    /** printf("%d\n", *v); Error!*/
    printf("%d\n", &v);
    printf("%d\n", p);
    printf("%d\n", *p);
    printf("%d\n", &p);
    printf("%d\n", *(&v));
    /** printf("%d\n", &(*v)); Error!*/
    printf("%d\n", *(&p));
    printf("%d\n", &(*p));

    return 0;
}
5
//Error!
957891628
957891628
5
957891616
5
//Error!
957891628
957891628
```

Simbol	Meaning	Outcome
v	integer value	5
v	meaningless	Error*!
&v	Address of v	Warning*! address 957891628
p	It is the address of v that points to v	Warning! address 957891628
p	It's where p points. So it's the int value v	5
&p	p	Warning*! address 957891628
(&v)	v (integer value)	5
&(*v)	meaningless	Error!
(&p)	p	Warning*! address 957891628
&(*p)	p	Warning*! address 957891628

*The Warning is because the print is going to print an integer ("%d") but the simbol is the integer address

1.2   Final version with no Errors and no Warnings

```c
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char *argv[]) {
  int v = 5;
  int *p;
  p = &v;

  printf("%d\n", v);
  printf("%lu\n", (long unsigned int) &v);
  printf("%lu\n", (long unsigned int) &v);
  printf("%lu\n", (long unsigned int) p);
  printf("%d\n", *p);
```

```c
    printf("%lu\n", (long unsigned int) &p);
    printf("%d\n", *(&v));
    printf("%lu\n", (long unsigned int) *(&p));
    printf("%lu\n", (long unsigned int) &(*p));

    return 0;
}
```

Run:

```
5
140735223946540
140735223946540
140735223946540
5
140735223946528
5
140735223946540
140735223946540
```

Simbol	Meaning	Outcome
v	integer value	5
&v	Address of v	Address 140735223946540
p	It is the address of v that points to v	Address 140735223946540
p	It's where p points. So it's the int value v	5
&p	p	Address 140735223946540
(&v)	v (integer value)	5
(&p)	p	Address 140735223946540
&(*p)	p	Address 140735223946540

p is the pointer that points to v = 5 (*p means pointed by p) whose address is p = &v = 140735223946540

1.2.1 Example

```c
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char *argv[]) {
    float *ptr;
    float f = 7.5;

    int *ppp;
    int a = 3;

    ptr = &f;
    ppp = &a;

    printf("%.2f\n%.2f\n", f, *ptr);
    printf("%d\n%d\n%d\n", &a, &(*ppp), ppp);

    return 0;
}
```

Run:

```
7.5
```

```
7.5
32324325525
32324325525
32324325525
```

1.2.1.1 void pointer

```
int i, j;
void *pv;
 ...
pv = &i;
 ...
j = *pv;

j = i;

int i, j;
void *pv;
 ...
pv = (void *) &i;
 ...
j = (int) *pv;

j = i;
```

1.3 Pointer Arithmetic

```
int i = 10;
int *p1;
int *p2;

p1 = &i;
p1++;
```

p++ makes the pointer points to next $<$ type $>$, basically if int p++ means next byte, if char, next bit...

```
int i = 10;
int j = 20;
int *p1;
int *p2;
p1 = &i;
p2 = &j;
if (*p1 == *p2) { *1... }
if (p1 == p2) { *2... }
if (p1 > p2) { *3... }
```

- *1 Check whether the referenced values are the same even if they are placed in different position within the system memory

- *2 Check whether the two pointers refer to the same object, i.e., they store the same memory address

- *3 Check whether the address p1 comes after the address p2 into the system memory; this is often meaningless If p1==p2 also p1==*p2*

1.4 By reference

1.4.1 Wrong

```
int i; j;

swap (i, j);
```

Here you change them locally only

```c
void swap (int x, int y) {
  int tmp;

  tmp = x;
  x = y;
  y = tmp;

  return;
}
```

1.4.2   Correct

you pass the pointer &i which is then pointed by x

```c
int i; j;

swap (&i, &j);
```

Here you pass the address

```c
void swap (int *x, int *y) {
  int tmp;

  tmp = *x;
  *x = *y;
  *y = tmp;

  return;
}
```

1.4.3   Pointers and arrays

```c
int v[N];

v = &v[0]
v+i = &v[i]

*v = v[0]
*(v+i) = v[i]

#define L ...

int v[L];
int *p;

p = v;
p = &v[0];

//1
for (i=0; i<L; i++) {
  scanf ("%d", &v[i]);
  printf ("%d", v[i]);
}

//2
for (i=0, p=v; i<L; i++, p++) {
  scanf ("%d", p);
  printf ("%d", *p);
```

```
}

//3
p = &v[0];
for (i=0; i<L; i++) {
  scanf ("%d", (p+i));
  printf ("%d", *(p+i));
}
```

1.4.4 Pointers and strings

The last element is '\0'

```
int strlen (char str[]) {
  int cnt;
  cnt =0;
  while (str[cnt] != '\0')
    cnt++;
  return cnt;
}

int strlen (char str[]) {
  int cnt;
  char *p;
  cnt =0;
  p = &s[0];
  while (*p != '\0') {
    cnt++;
    p++;
  }
  return cnt;
}

int strlen (char *str) {
  int cnt;
  cnt = 0;
  while (*str != '\0') {
    cnt++;
    str++;
  }
  return cnt;
}

int strlen (char *str) {
  char *p;
  p = str;
  while (*p != '\0') {
    p++;
  }
  return (p - str);
}
```

1.4.5 Pointers and structures

```
pointer_to_structure->member_name
(*pointer_to_structure).member_name
```

```
struct student {
  char s1[L], s2[L];
  int i;
```

```
    float f;
};

struct student v;
 ...
read (&v);
 ...

void read (struct student *v) {
  char s1[DIM], s2[DIM];
  int i; float f;

  fprintf (stdout, "...: ");
  scanf ("%s%s%d%d", s1, s2, &i, &f);

  strcpy (v->s1, s1);
  strcpy (v->s2, s2);

  v->i = i;
  v->f = f;

  return;
}

struct student {
  char s1[L], s2[L];
  int i;
  float f;
};

struct student v;
 ...
read (&v);
 ...

void read (struct student *v) {
  char s1[DIM], s2[DIM];
  int i; float f;

  fprintf (stdout, "...: ");
  scanf ("%s%s%d%d", v->s1, v->s2, &v->i, &v->f);

  return;
}
```

2 Dinamic Memory Allocation

there was a cool way to go there if only i would have like to

2.1 2D array

Two-dimensional arrays can be allocated in two different ways - As a single 1D array including all elements - Easy syntax for allocation and manipulation - Difficult manipulation logic - As an array of pointers to 1D arrays of elements - Difficult syntax for allocation and manipulation - Standard manipulation logic

you start with one array allocation

```c
int **mat;

mat = (int **) malloc(r * sizeof(int *));
  if (mat==NULL)
  {
    fprintf(stderr, "memory allocation error.\n");
    exit(1);
  }

  printf("number of columns: ");
  scanf("%d", &c);
  //allocate memory for columns
  for (i=0; i<r; i++)
  {
    mat[i] = /*(int *)*/ malloc(c * sizeof(int)); //as always (int **) can be avoided.
    if (mat==NULL)
    {
      fprintf(stderr, "memory allocation error.\n");
      exit(2);
    }
  }
```

It has 2 ** beacause

```c
mat[i] = (int **) malloc (r * sizeof (int*));
```

Allocation of a new node

```
list_t *new_element ( ) {
   list_t *e_ptr;
   e_ptr = (list_t *) malloc (sizeof (list_t));
   if (e_ptr==NULL) {
      fprintf (stderr, "Memory allocation error.\n");
      exit (FAILURE);
   }
   return (e_ptr);
}
```

Function call

```
list_t *head, *new;
...
head = NULL;
...
new = new_element();
```

Initially the list is empty, thus, **head** must be initially set to NULL

{width=50%}

- mat[i][j] or (mat[i])[j]
    - Indicates a single element
    - It is a value

- mat[i]
    - Indicates an entire row
    - It is a pointer to an array of values

- mat
    - Indicates the entire matrix
    - It is a pointer to an array of pointers

head

data fields

NULL

```
list_t *p;
...
p = head;
while (p != NULL) {
   ...
   p = p->next;
}
```

We check up-front, thus the algorithm works even for empty lists

Visit the element p, i.e., p->key and all data fields

Move to the next element
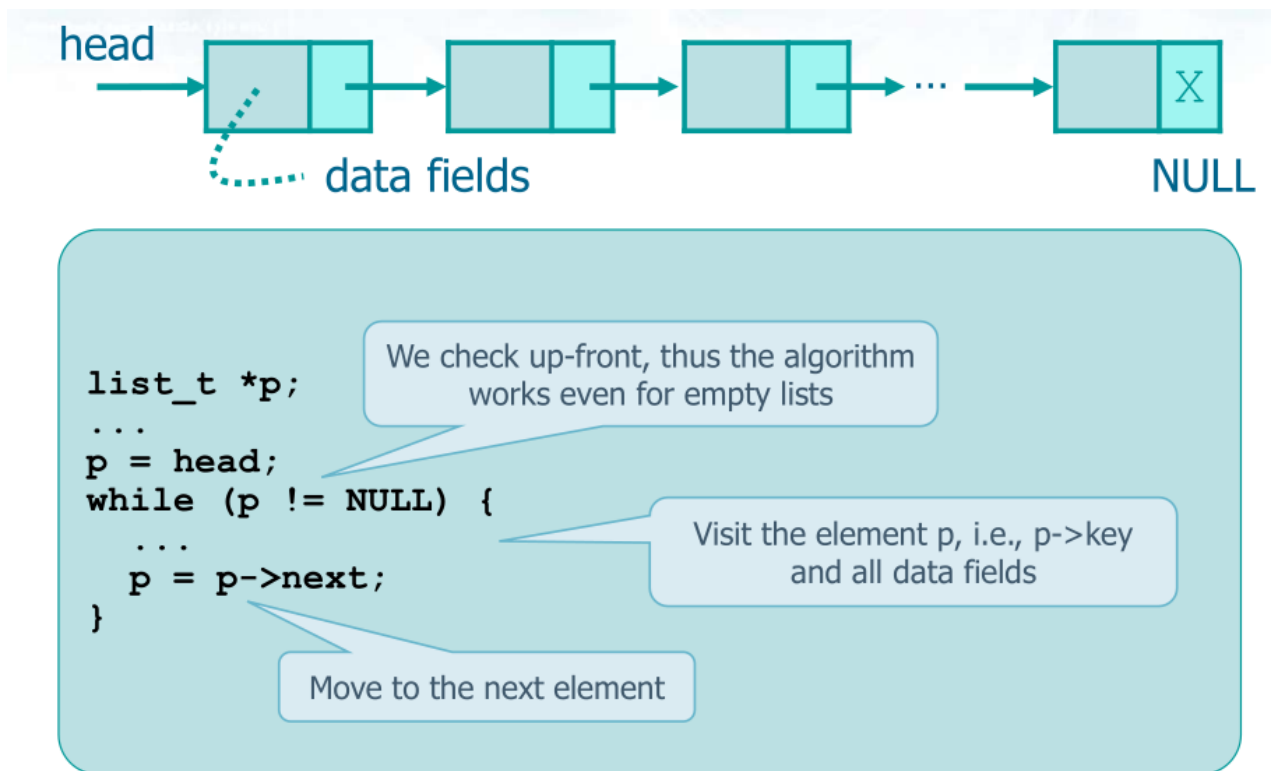
{width=50%}

mat = &mat[0]; //same notaion as p = &v;

2.1.0.1   matrix scanf   cool way to do it with strlen

```
int r, c, i;
char **mat;

printf ("Number of rows: ");
scanf ("%d", &r);

mat = (int **) malloc (r * sizeof (int *));
if (mat == NULL) {
  fprintf (stderr, "Memory allocation error.\n");
  exit (1);
}
for (i=0; i<r; i++) {
  scanf ("%s", str);

  mat[i] = malloc ((strlen(str)+1) * sizeof (char));
  if (mat[i] == NULL) {
    fprintf (stderr, "Memory allocation error.\n");
    exit (1);
  }
}
```

- As for 1D arrays, also 2D arrays may be made visible outside the environment in which they have been allocated
- As for 1D arrays, it is possible to
    - Use global variables to contain the matrix pointer
    - Adopt the return statement to return it
    - Pass the pointer to the matrix by reference
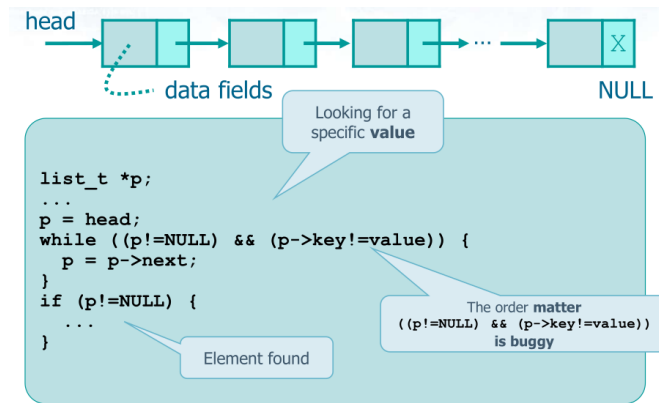
Figure 1: image

* Unfortunately, the pointer to the matrix is already a 2-star object (indirect reference)
* To pass it by reference, we have to use a 3-star object (a reference to a reference of a reference)

2.1.1 Mtrix allocation in a function

mat dies when function dies, but the content return

```
char **mat;
  ...
mat = malloc2d (nr, nc);

char **malloc2d (int r, int c) {
  int i;
  char **mat;

  mat = (char **) malloc (r * sizeof(char *));
  if (mat == NULL) { ... }
  for (i=0; i<r; i++) {
    mat[i] = (char *) malloc(c * sizeof (char));
    if (mat[i]==NULL) { ... }
  }
  return (mat);
}
```

second way
here you pass the address and the content remains there

```
char **mat;
  ...
malloc2d (&mat, nr, nc);
```

We use a 3-* object with a temporary 2* object as a support

```
void malloc2d (char ***m, int r, int c) {
  int i;
  char **mat;

  mat = (char **) malloc (r * sizeof(char *));
  if (mat == NULL) { ... }
  for (i=0; i<r; i++) {
  mat[i] = (char *) malloc(c * sizeof (char));
  if (mat[i]==NULL) { ... }
  }
```

```c
  *m = mat;
  return;
}
```

another way

```c
char **mat;
  ...
malloc2d (&mat, nr, nc);

void malloc2d (char ***m, int r, int c) {
  int i;

  (*m) = (char **) malloc (r * sizeof(char *));
  if (m == NULL) { ... }
  for (i=0; i<r; i++) {
    (*m)[i] = (char *) malloc(c * sizeof (char));
    if ((*m)[i]==NULL) { ... }
    }
  return;
}
```

2.1.2 Free

```c
void free2d (char **m, int r) {
  int i;
  for (i=0; i<r; i++) {
    free (m[i]);
  }
  free (m);
  return;
}

void free2d (char ***m, int r) {
}
  int **mat, i;
  mat = *m;
  for (i=0; i<r; i++) {
    free (mat[i]);
  }
  free (mat);
  m = NULL;
  return;
}
```

3 List

3.1 Intro

Easy to delete and add in ahead, in the middle, in the tail
- FIFO or Stack
- LIFO
- Ordered List
- Single-Linked List
- Double-Linked List
- List of List

3.2 Prototypes (Atomic operation)

3.2.1 Struct of a List

```c
#include <stdio.h>
#include <stdlib.h>
#include <ctype.h>

/* structure declaration */
typedef struct node_s {
  int val;
  struct node_s *next;
} node_t;
```

3.3 Function required to operate with list

```c
/* function prototypes */
int read (void);
node_t *insert (node_t *head, int val);
void search (node_t *head, int val);
node_t *delete_first (node_t *head);
node_t *delete_last (node_t *head);
node_t *delete (node_t *head, int val);
void display (node_t *head);
```

3.4 Main

```c
/*
 *  main program
 */
int main(void) {
  node_t *head=NULL;
  int val, stop=0;
  char choice;

  while (stop == 0) {
    fprintf(stdout, "\nAvailable commands:\n");
    fprintf(stdout, "  i: insert a value (sorted)\n");
    fprintf(stdout, "  s: search a value\n");
    fprintf(stdout, "  f: delete the first value\n");
    fprintf(stdout, "  l: delete the last value\n");
    fprintf(stdout, "  d: delete a specified value\n");
    fprintf(stdout, "  c: display the list contents\n");
    fprintf(stdout, "  e: end program\n");
    fprintf(stdout, "Make your choice: ");
    scanf("%c%*c", &choice);
```

```c
      switch (choice) {
        case 'i': val = read();
                  head = insert(head, val);
                  break;
        case 's': val = read();
                  search(head, val);
                  break;
        case 'f': head = delete_first(head);
                  break;
        case 'l': head = delete_last(head);
                  break;
        case 'd': val = read();
                  head = delete(head, val);
                  break;
        case 'c': display(head);
                  break;
        case 'e': fprintf(stdout, "End of session.\n");
                  stop = 1;
                  break;
        default : fprintf(stdout, "Wrong choice!\n");
                  break;
      }
    }

    return EXIT_SUCCESS;
}

/*
 *  read in a value
 */
int read (void) {
  int val;

  fprintf(stdout, "Value: ");
  scanf("%d%*c", &val);

  return val;
}
```

3.5   List insertion

Assigning a new value `val` to the `p` element in order to insert that element in the list
Malloc the struct

3.5.1   List visit

```c
/*
 *  insert a value in the list (sorted)
 */
node_t *insert (node_t *head, int val) {
  node_t *p, *q=head;

  p = (node_t *)malloc(sizeof(node_t));
  p->val = val;
  p->next = NULL;
```

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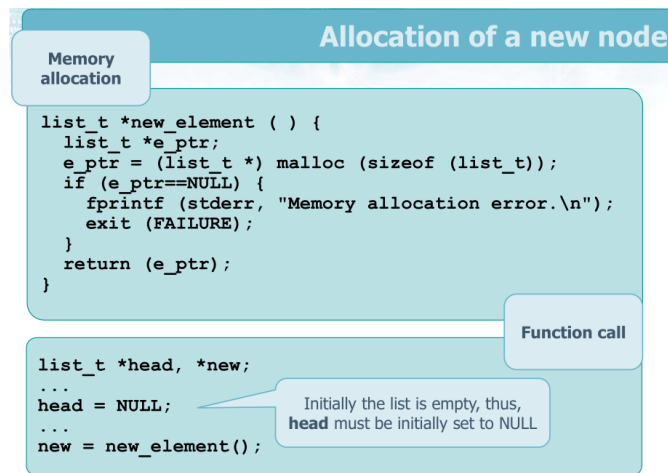
Figure 2: image
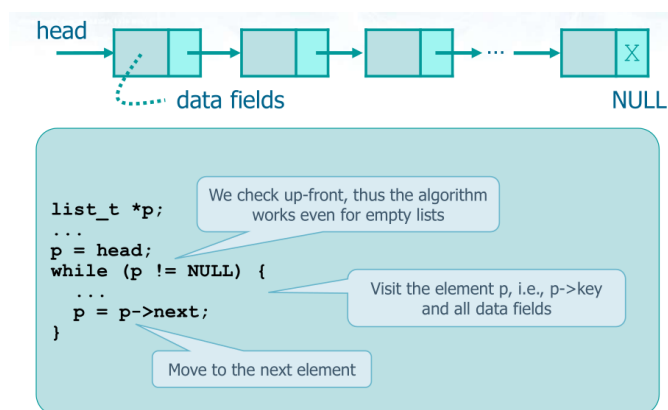
Figure 3: image

3.5.2 Head insertion

p insertion `ahead`
`O(n)` cost
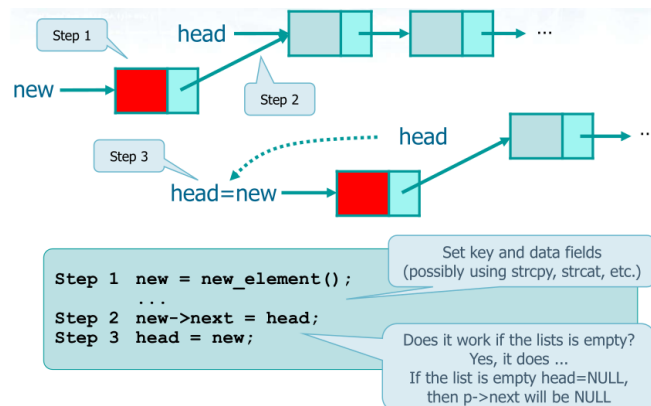
Figure 4: image

```
/* insert ahead */
if (head==NULL || val<head->val) {
  p->next = head;
  //head = p;
  return p;
}
```

3.5.3 In-order insertion
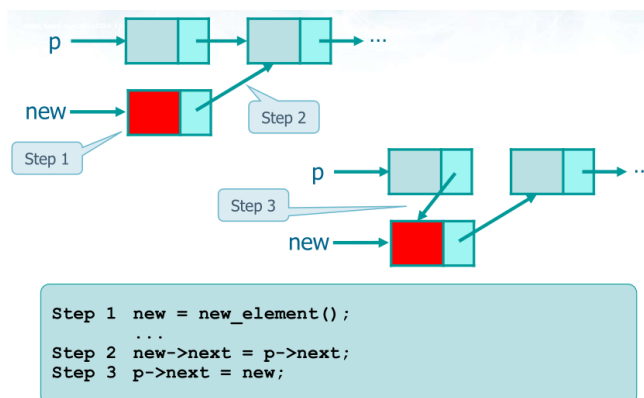
Figure 5: image

```
new->next = p->next
p->next = new
```

3.5.4 Middle Tail insertion

```
p->next = val1->next
val1->next = p
```

```
/*
 *  scan the list with the q pointer in order to find
 *  the correct position where to perform the insertion
```

```
  */
  while (q->next!=NULL && q->next->val<val) {
    q = q->next;
  }
  p->next = q->next;
  q->next = p;
  return head;
}
```

3.5.4.1 Tail insertion
Tail insertion is basically `middle insertion`
```
p->next = val1->next
val1->next = p
```

```
p->next = val1->next val1->next = p
```
or
```
p->next = NULL val1->next = p
```
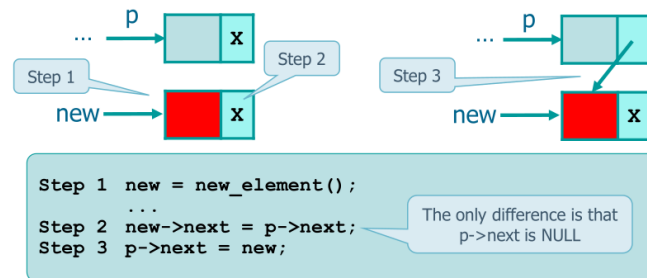

Figure 6: image

3.6 List searching

if `p->key == value` you exit and unless `p == NULL` you're good in your element of the list
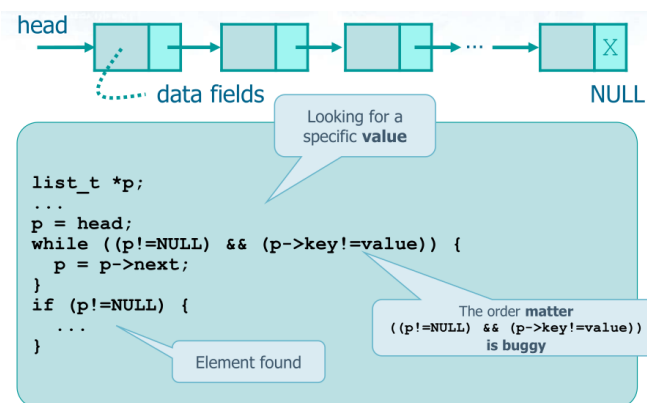
Figure 7: image

```
/*
 *  search a value in the list
 */
void search (node_t *head, int val) {
  node_t *p;
  int i;

  for (p=head, i=0; p!=NULL && p->val<val; p=p->next, i++) ;
```

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```
  if (p!=NULL && p->val==val) {
    fprintf(stderr, "Element found (index = %d)\n", i);
  } else {
    fprintf(stderr, "Element NOT found.\n");
  }
}
```

3.7   List extraction

than you free p
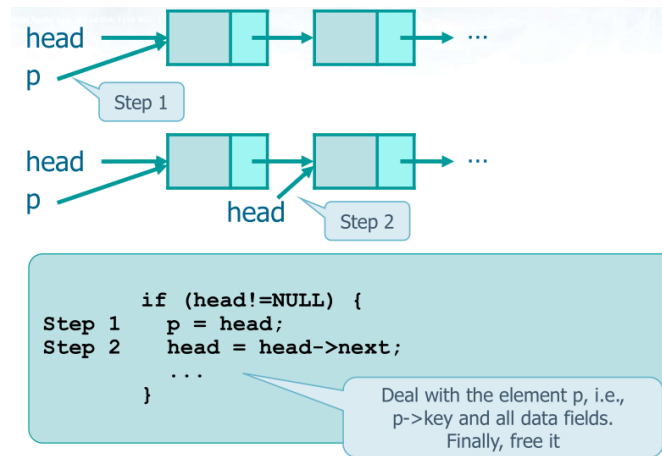
Figure 8: image

```
p = head
head = head->next
```
if there's only one element
```
p = head
head = NULL
```

3.7.1   In-order Extraction

```
p = q->next
q->next = p->next
```
or
```
q->next = q->next->next
```

p is only not to lose forever that element (red one), without step 1 I lose reference to elements and at the end ov the program I have many leaks. If I lose the reference I can not free it.

3.8   List deletion (Free)

3.8.1   Free the entire list

```
while (head !=0){
  p = head;
  head = head->next;
  free (p);
}
```

3.8.2   Deleting first element

```
/*
 *  delete the first element of the list
```
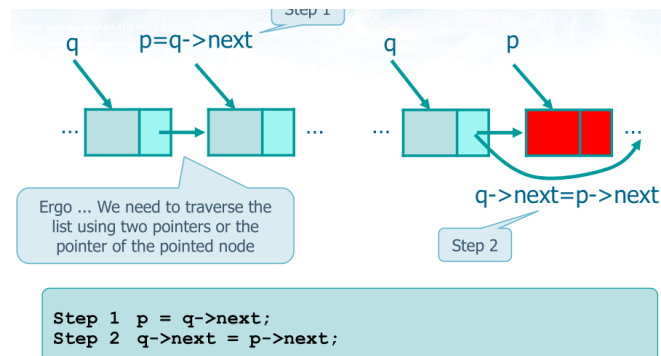
Figure 9: image
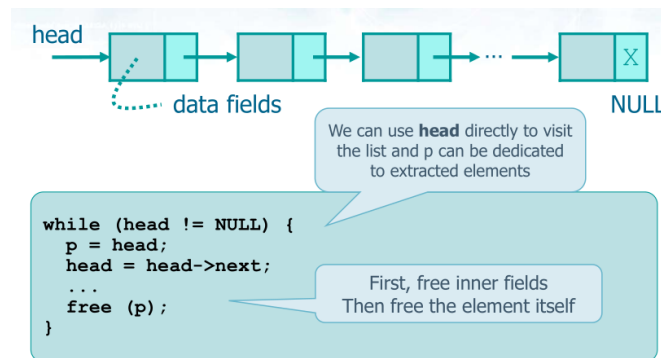
Figure 10: image

```c
 */
node_t *delete_first (node_t *head) {
  node_t *p;

  /* empty list */
  if (head != NULL) {
    p = head->next;
    free(head);
    return p;
  }

  return head;
}
```

3.8.3   Deleting a particular element

```c
/*
 *  delete a list element, keeping it sorted
 */
node_t *delete_last (node_t *head) {
  node_t *p, *q=head;

  /* empty list */
  if (head == NULL) {
    fprintf(stderr, "Error: empty list\n");
    return NULL;
  }
```

```
  /* delete ahead */
  if (head->next == NULL) {
    free(head);
    return NULL;
  }

  /* scan the list with the q pointer */
  while (q->next->next!=NULL) {
    q = q->next;
  }
  p = q->next;
  q->next = NULL;
  free(p);

  return head;
}


/*
 *  delete a list element, keeping it sorted
 */
node_t *delete (node_t *head, int val) {
  node_t *p, *q=head;

  /* empty list */
  if (head == NULL) {
    fprintf(stderr, "Error: empty list\n");
    return NULL;
  }
```

3.8.4 Deleting ahead

```
  /* delete ahead */
  if (val == head->val) {
    p = head->next;
    free(head);
    return p;
  }
```

3.9 Scan to find an elemet to delete it

```
  /*
   *  scan the list with the q pointer in order to find
   *  the element to remove from the list
   */
  while (q->next!=NULL && q->next->val<val) {
    q = q->next;
  }
  if (q->next!=NULL && q->next->val==val) {
    p = q->next;
    q->next = p->next;
    free(p);
  } else {
    fprintf(stderr, "Element NOT found.\n");
  }

  return head;
}
```

3.10 Display list

```c
/*
 *  display the list contents
 */
void display (node_t *head) {
  int i=0;

  while (head != NULL) {
    fprintf(stderr, "Element %d = %d\n", i++, head->val);
    head = head->next;
  }
}
```

3.11 Sentinel

You don't check for `null`
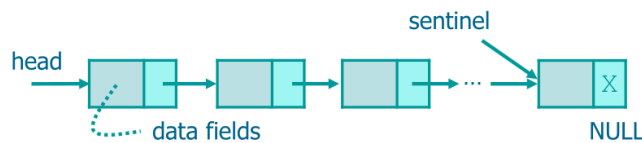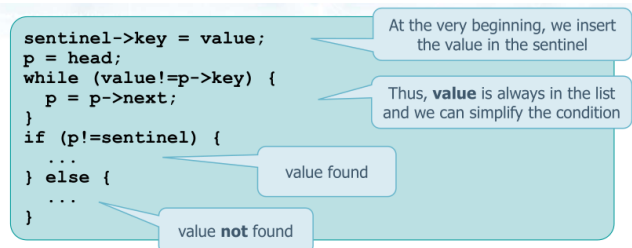
Figure 11: image

Figure 12: image

```c
sentinel->key = value;
p = head;
while (value!=p->key) {
  p = p->next;
}
if (p!=sentinel) {
  ...
value found
} else {
  ...
}
```

3.12 OT Circular buffer

in a cicle
(i++)/%
so you reuse the array instead of use a matrix

3.13 FIFO

3.13.1 Enqueue
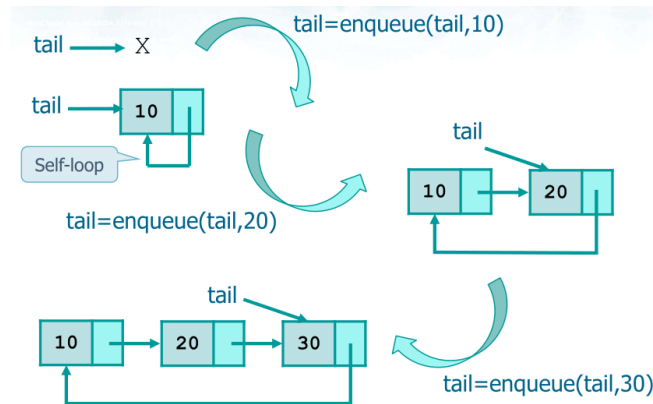
Figure 13: image
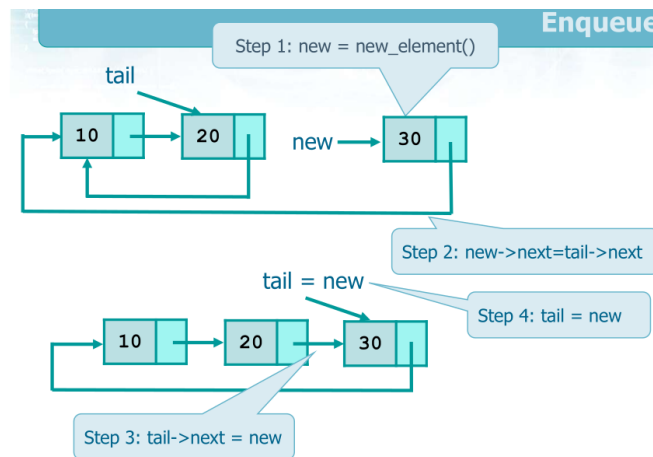
Figure 14: image

3.13.2 Dequeue

```
list_t *dequeue (list_t *tail, int *val, int *status) {
list_t *old;
if (tail != NULL) {
  if (tail == tail->next) {
    *val = tail->key;
    free (tail);
    tail
    10
    tail = NULL;
  } else {
    old = tail->next;
    *val = old->key;
```
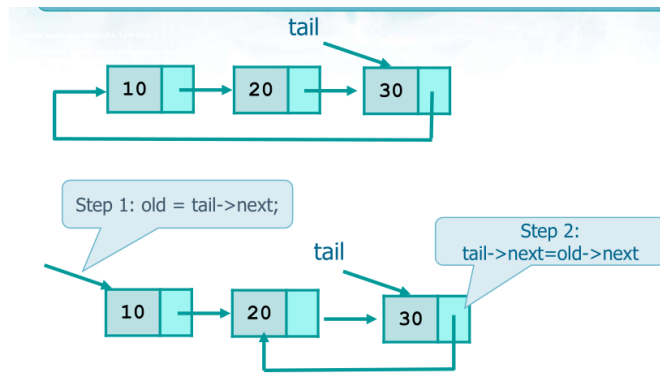
Figure 15: image

```
    tail
    X
    tail->next = old->next;
    free (old);
    }
  } else {
  *status = FAILURE;
  }
return (tail);
}
```

3.14 Ordered Linked Lists

```
do {
  ...
head = insert (head, val);
  ...
search (head, val);
  ...
head = extract (head, val);
} while ( ... );

list_t *dequeue (list_t *tail, int *val, int *status) {
  list_t *old;
  if (tail != NULL) {
    *status = SUCCESS;
    the self-loop) must be
      if (tail == tail->next) {
        implemented aside
          *val = tail->key;
        free (tail);
        tail
          10
          tail = NULL;
      } else {
        old = tail->next;
        *val = old->key;
        tail
          X
          tail->next = old->next;
        free (old);
```

```
    }
  } else {
    *status = FAILURE;
  }
  return (tail);
}
```
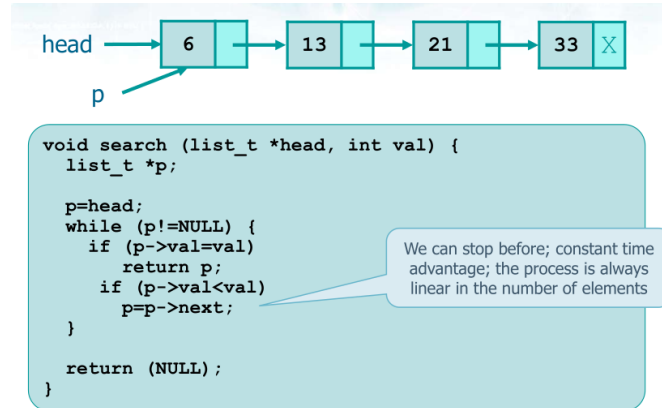

Figure 16: image

3.14.0.1   search   linear cosst, not logaritmic, so, don't use at the examination

A search can terminate - Successfully, when we find the key - Unsuccessfully when a record with a key larger (or smaller) than the search key is found - In any case - We can stop the search as soon as the key we are looking for become larger than the current node's key - This make the search more efficient - Nevertheless, the search still has a linear cost (O(n)) in the number of elements stored into the list

```
void search (list_t *head, int val) {
  list_t *p;
  p=head;
  while (p!=NULL) {
    if (p->val=val)
      return p;
      if (p->val<val)
      p=p->next;
    }
  }
  return (NULL);
```

The are several approaches to solve the problem - Use two pointers to individuate two consecutive elements - Move them along the list is a synchronized way - Use the rightmost to compare and the leftmost to insert - Use a the pointer of the pointed element to make the comparison - Reach the element referenced by the pointed element to compare, use the direct pointer to insert

3.14.0.2   insert

```
list_t *insert (list_t *head, int val) {
  list_t *p, *q=head;

  //Create a new element
  p = new_element ();
  p->val = val;
  p->next = NULL;

  Head insertion
```

```
list_t *insert (list_t *head, int val) {
  list_t *p, *q=head;

  p = new_element ();
  p->val = val;
  p->next = NULL;

  if (head==NULL || val<head->val) {
    p->next = head;
    return p;
  }
  while (q->next!=NULL && q->next->val<val) {
    q = q->next;
  }
  p->next = q->next;
  q->next = p;
  return head;
}
```

Create a new element

Head insertion

q is used to insert
q->next is used to compare

Figure 17: image
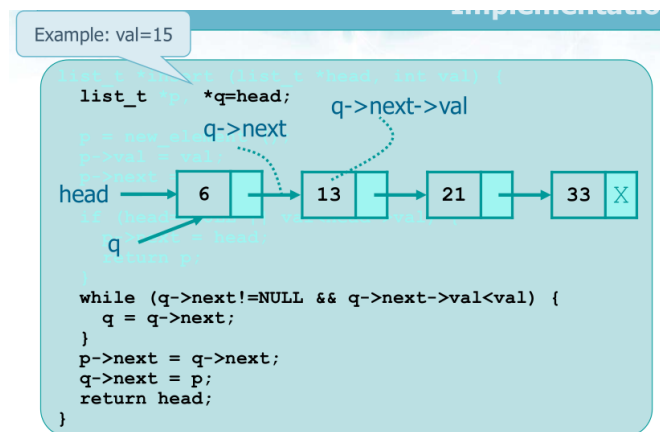
Figure 18: image
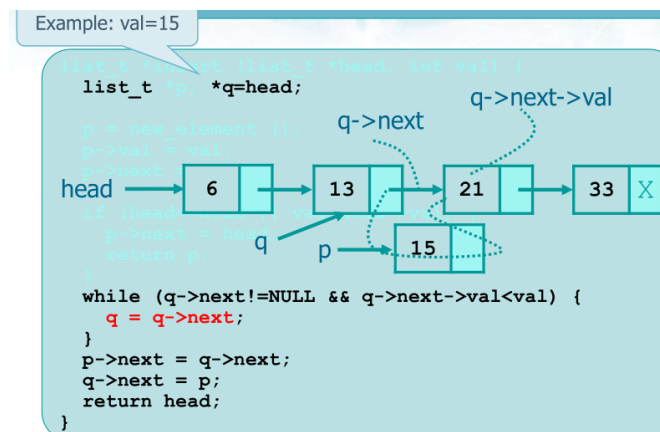
Figure 19: image

```
  if (head==NULL || val<head->val) {
    p->next = head;
    return p;
  }
  while (q->next!=NULL && q->next->val<val) {
    q = q->next;
  }
  return head;
}
```

3.14.0.3   Extraction

```
list_t *extract (list_t *head, int val) {
  list_t *p, *q=head;
  if (head == NULL) {
    fprintf(stderr, "Error: empty list\n");
    return NULL;
  }
  if (val == head->val) {
    p = head->next;
    free(head);
    Head extraction
    return p;
  }
  while (q->next!=NULL && q->next->val<val) {
    q = q->next;
  }
}
```
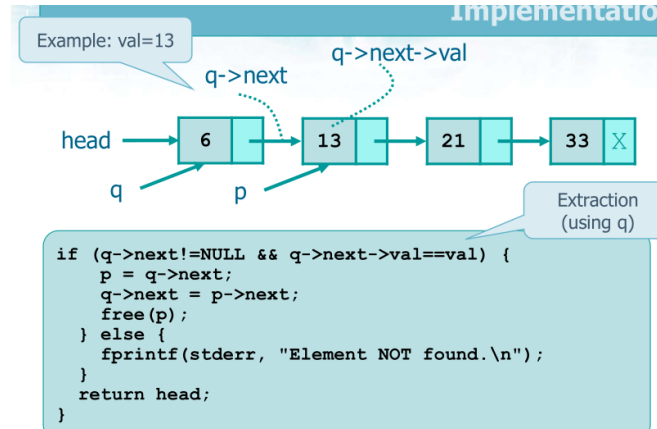

Figure 20: image

3.14.0.4   Double linked list
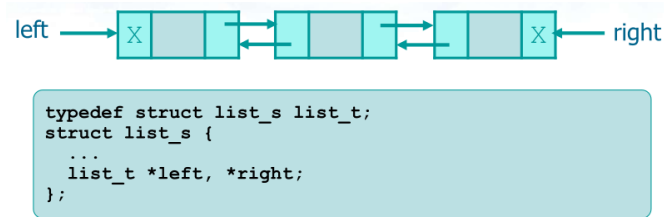
Figure 21: image
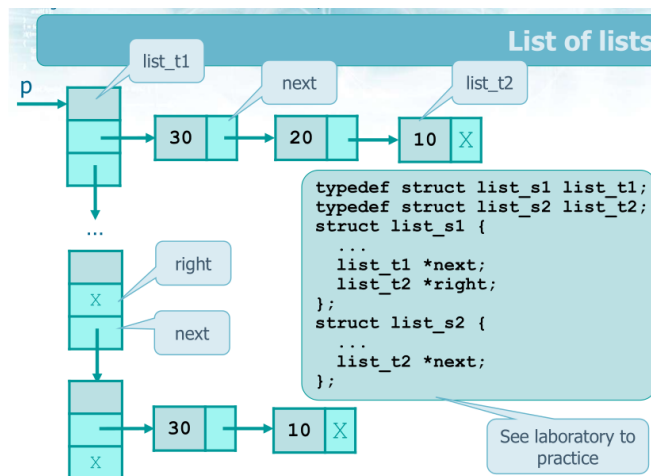
Figure 22: image

3.14.0.5 List of List

3.14.0.6 Example you can extract from head1 and you insert in head2 lifo? with a head insertion

solution

```
p = head1;
while (p != NULL){
  //p->val (p->key)
  head2 = push (head2, p->val);
  p = p->next;
}
```

Head extrction and in order insertion

❖ Given a simple linked list, invert all elements of such a list such that
  ➢ The first element becomes the last one
  ➢ The last element becomes the first one
  ➢ Input list
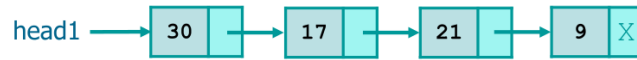
  ➢ Output list
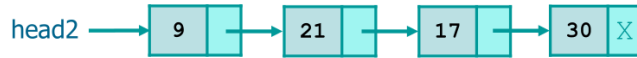
Figure 23: image

```
typedef struct list_s list_t;
struct list_s {
   int key;
   ...
   list_t *next;
};
```
Type definition,

```
list_t *list_reverse (list_t *head1) {
   list_t *tmp1, head2;
   head2 = NULL;
   while (head1 != NULL) {
      tmp1 = head1->next;
      head1 = head1->next;
      tmp1->next = head2;
      head2 = tmp1;
   }
   return head2;
}
```

Extract from list 1

Insert into list 2

Figure 24: image

❖ Given a simple linked list, sort all elements in ascending order
  ➢ Input list
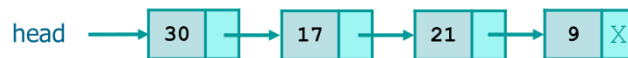
  ➢ Output list
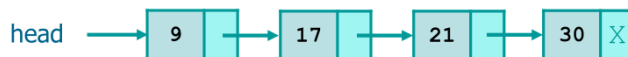
Figure 25: image

4 Sorting Algorithms

4.1 Insertion Sort

O(n^2)

Vector divided in left sorted and right unsorted, when there is a number higher in the left compared to another on the right they are switched

0	1	2	3	4	5
4	2	6	3	1	5
4	(2)	6	3	1	5
2	4	6	3	1	5
2	4	(6)	3	1	5
2	4	6	3	1	5
2	4	6	(3)	1	5
2	3	4	6	1	5
2	3	4	6	(1)	5
1	2	3	4	6	(5)
1	2	3	4	5	6

```c
//2 subarray
//left is sorted, right not sorted    i=1 means
//that the only sorted is in position 0.

#include <stdio.h>

void InsertionSort (int *A, int n);

int main(int argc, char *argv[]) {
  int n = 5;
  int arr[5] = {3, 8, 1, 7, 4};

  InsertionSort(arr, n);
  for(int i=0; i<n; i++){
    printf("%d ", arr[i]);
  }
  return 0;
}

void InsertionSort (int *A, int n) {
    int i, j, x;

    for (i=1; i<n; i++) {
        x = A[i];//first unsorted number
        j = i - 1;//J=0: only sorted number

        while (j>=0 && x<A[j]) {
            A[j+1] = A[j];//A[j] is not the smallest so it
            j--;//j=-1  // has to let the other go to left ex A[j+1=1]
        }

        A[j+1] = x;//A[j+1=0]
    }
}
```

4.2 Exchange Sort

O(n^2)

Vector divided in left unsorted and right sorted, swap of any numbers with a smaller one on its right

0	1	2	3	4	5
4	2	6	3	1	5
2	4	3	1	5	[6]
2	4	3	1	[5	6]
2	1	3	4	[5	6]
2	1	3	[4	5	6]
2	1	[3	4	5	6]
[1	2	3	4	5	6]

```
//2 subarray.
//left is unssorted, right is empty      i<n-1 means that you assume
void BubbleSort (int A[], int n) {  //   at the beginning the last is
  int i, j, temp;                   //   the greater.

  for (i=0; i<n-1; i++) {
    for (j=0; j<n-i-1; j++){        //j<n-1-i because the more you
      if (A[j] > A[j+1]) {          // go on, less numbers you have left
        temp = A[j];
        A[j] = A[j+1];
        A[j+1] = temp;
      }//in this for you find the greater unsorted
    }  //number and swapping it, you put it on the right
  }

  return;
}
```

4.3 Selection Sort

O(n^2)

Vector divided in left sorted and right unsorted, the algorithm looks for the smallest number in the right unsorted array and swaps it wiht the first element of the unsorted array

0	1	2	3	4	5
](4)	2	6	3	(1)	5
[1]	(2)	6	3	4	5
[1	2]	(6)	(3)	4	5
[1	2	3]	(6)	(4)	5
[1	2	3	4]	(6)	(5)
[1	2	3	4	5	6]

```
void SelectionSort (int A[], int n) {
    int i, j, min, temp;

    for (i=0; i<n-1; i++) {
        min = i;//first # is min

        for (j=i+1; j<n; j++) {    //it finds the smallest on the line
            if (A[j] < A[min]) {    //if the first on the right (A[j]) of the min num (A[min]) is less
                min = j;                //in that position there's the new smallest one
            }
        }                           //after this for you know which number is
                                //the smallest of the line
                                    //end now you swap it with the one you
        temp = A[i];                //assume to be the smallest (min=i   line 7)
        A[i] = A[min];
        A[min] = temp;
    }

    return;
}
```

4.4 Shell Sort

O(n^2)
swap numbers at same index in different arrays with insertion sort

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	h
7	6	8	9	8	6	2	1	8	7	0	4	5	3	0	1	0	4	9	
[3	0	1	0	4	6	2	1	5	7	3	4	5]	[7	6	8	9	8	9]	
[I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII]	[I	II	III	IV	V	VI]	13
[3	0	0	0]	[4	6	1	1]	[5	7	2	4]	[8	7	6	8]	[9	8	9]	
[I	II	III	IV]	[I	II	III	IV]	[I	II	III	IV]	[I	II	III	IV]	[I	II	III]	4
0	0	0	1	4	6	1	1	5	7	2	4	8	7	6	8	9	8	9	1

```c
void ShellSort (int A[], int n) {
    int i, j, x, h;
    h=1;

    while (h < n/3)
        h = 3*h+1;

    while (h >= 1) {
        for (i=h; i<n; i++) {
            x = A[i];
            j = i - h;

            while (j>=0 && x<A[j]) {
                A[j+h] = A[j];
                j -= h;
            }

            A[j+h] = x;
        }

        h = h/3;
    }
}
```

4.5 Counting Sort

O(n^2)

There are multiple arrays, the given one, another with every single value that is in the previous array with in each cell has the number of times that number exist in the previous array. A third array with the cumulative number of element at each index. Another array with the previous array numbers shifted by 1 index to the right. At this point number 0 at index 0 is between position 0 and 1, 1 occurrence, number 1 at index 1 is between position 1 and 4, 3 occurrences in the last vectors, number 2 at index 2 is between position 4 and 4, 0 occurrences in the last vectors, and last number, 3 at index 3 is between position 4 and 6, 2 occurrences in the last array.
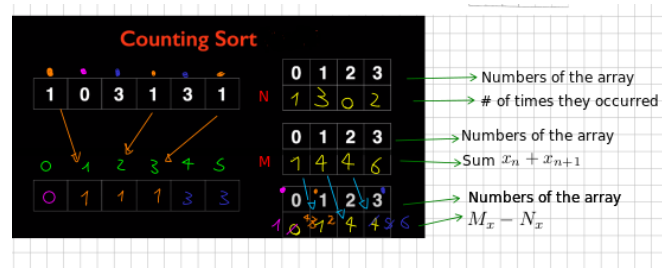
Figure 26: Counting sort

```
#define MAX 100

void CountingSort(int A[], int n, int k) {
    int i, C[MAX], B[MAX];

    for (i=0; i<k; i++)
        C[i] = 0;

    for (i=0; i<n; i++)
        C[A[i]]++;

    for (i=1; i<k; i++)
        C[i] += C[i-1];

    for (i=n-1; i>=0; i--)
    {
        B[C[A[i]]-1] = A[i];
        C[A[i]]--;
    }

    for (i=0; i<n; i++)
        A[i] = B[i];
}
```

4.6 Merge Sort

O(log_n(n))

It divides the array and then, when it merges back the numbers, it does it ordering them
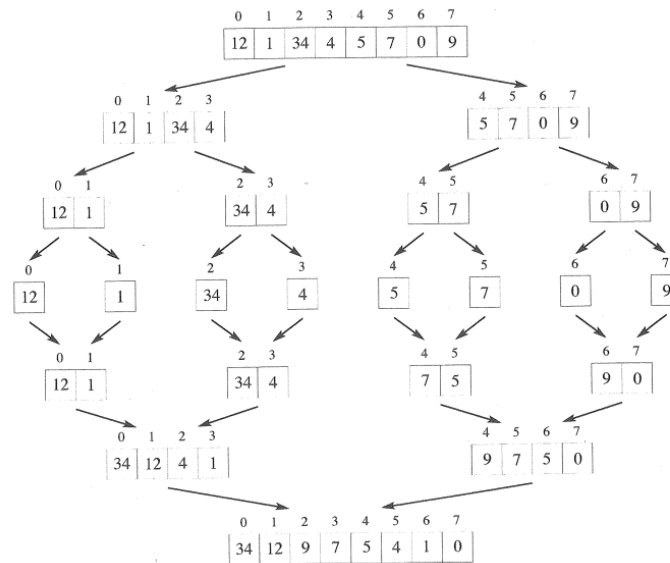
Figure 27: Merge sort

```c
#include <stdlib.h>
#include <stdio.h>

#define max 100

int insert_array(int V[]) {
  int n, i;
  printf("How many elements?: ");
  scanf("%d", &n);

  for (i=0; i<n; i++) {
    printf("Element %d: ", i);
      scanf("%d", &V[i]);
  }
  return(n);
}

void print_array(int V[], int n) {
  int i;
  for (i=0; i<n; i++) {
    printf("%d ", V[i]);
  }
  printf("\n");
  return;
}

void merge(int a[], int p, int q, int r) {
  int i, j, k=0, b[max];
  i = p;
  j = q+1;
```

```c
    while (i<=q && j<=r) {
      if (a[i]<a[j]) {
        b[k] = a[i];
        i++;
      } else {
        b[k] = a[j];
        j++;
      }
      k++;
    }
    while (i <= q) {
      b[k] = a[i];
      i++;
      k++;
    }
    while (j <= r) {
      b[k] = a[j];
      j++;
      k++;
    }
    for (k=p; k<=r; k++)
      a[k] = b[k-p];
    return;
}

void mergeSort(int a[], int p, int r) {
  int q;
  if (p < r) {
    q = (p+r)/2;
    mergeSort(a, p, q);
    mergeSort(a, q+1, r);
    merge(a, p, q, r);
  }
  return;
}

int main(void) {
  int n, V[max];
  n = insert_array(V);
  mergeSort(V, 0, n-1);
  print_array(V, n);
  return(0);
}
```

4.7   Quik Sort

O(log_n(n))

It divides the array and then, when it merges back the numbers, it does it ordering them
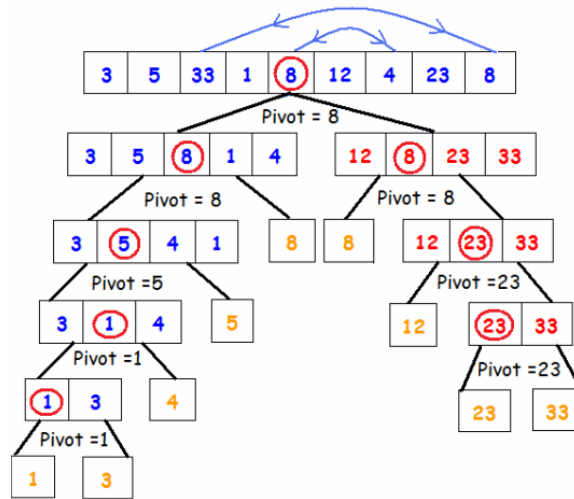
Figure 28: Quick sort

```c
#include<stdio.h>

void quicksort(int number[25],int first,int last){
  int i, j, pivot, temp;
  if(first<last){
    pivot=first;
    i=first;
    j=last;
    while(i<j){
      while(number[i]<=number[pivot]&&i<last)
        i++;
      while(number[j]>number[pivot])
        j--;
      if(i<j){
        temp=number[i];
        number[i]=number[j];
        number[j]=temp;
      }
    }
    temp=number[pivot];
    number[pivot]=number[j];
    number[j]=temp;
    quicksort(number,first,j-1);
    quicksort(number,j+1,last);
  }
}
int main(){
  int i, count, number[25];
  printf("Enter some elements (Max. - 25): ");
  scanf("%d",&count);
  printf("Enter %d elements: ", count);
  for(i=0;i<count;i++)
    scanf("%d",&number[i]);
```

```
  quicksort(number,0,count-1);
  printf("The Sorted Order is: ");
  for(i=0;i<count;i++)
    printf(" %d",number[i]);
  return 0;
}
```

4.7.1   C func

```c
#include <assert.h>
#include <stdio.h>
#include <stdlib.h>
#include <string.h>

int cmpfunc(const void *a, const void *b) { return (*(int *)a - *(int *)b); }

static int cmpstringp(const void *p1, const void *p2) {
  return strcmp(*(const char **)p1, *(const char **)p2);
}

int main(int argc, char *argv[]) {
  assert(argc > 2);

  int values[] = {88, 56, 100, 2, 25};
  char *words[] = {"ciao", "dasih", "fklgj", "jfhsg", "qhfls"};

  qsort(&argv[1], argc - 1, sizeof(char *), cmpstringp);
  qsort(values, 5, sizeof(int), cmpfunc);
  qsort(words, 5, sizeof(char *), cmpstringp);

  for (int j = 0; j < argc; j++)
    puts(argv[j]);

  for (int n = 0; n < 5; n++) {
    printf("%d ", values[n]);
    puts(words[n]);
  }
}
```

5   Complexity equation

6 Recursion

6.1 Complexity

❖ A **recursion equation** expresses the time asymptotic cost $T(n)$ in terms of

$D(n)$	Cost of dividing the problem
$T(\hat{n})$	Cost of the execution time for smaller inputs (recursion phase)
$C(n)$	Cost of recombining the partial solutions
Cost of the terminal cases	We often assume unit cost for solving the elementary problems $\Theta(1)$

$$T(n) = D(n) + \sum T(\hat{n}) + C(n)$$

$$T(n) = D(n) + \sum T(\hat{n}) + C(n)$$

We suppose that the number of subproblems of size $\hat{n}$ is a.
If $a = 1$, we have linear recursion.
If $a > 1$, we have multi-way recursion.

Complexity Analysis

Case 1

$$T(n) = D(n) + \sum T(\hat{n}) + C(n)$$

The size of the original problem n and the generated ones $\hat{n}$ are related by a **constant factor** b in general the same for all subproblems $b = {}^n/_{\hat{n}}$ and $\hat{n} = {}^n/_b$

We suppose that the number of subproblems of size $\hat{n}$ is a.
If $a = 1$ we have linear recursion.
If $a > 1$ we have multi-way recursion.

Divide | Recur $T(\hat{n})$ | Combine

$$T(n) = D(n) + a \cdot T\left({}^n/_b\right) + C(n)$$
$$T(n) = \Theta(1)$$

$n > const$
$n \leq const$

Conquer | If $b > 1$ we have divide-and-conquer approach
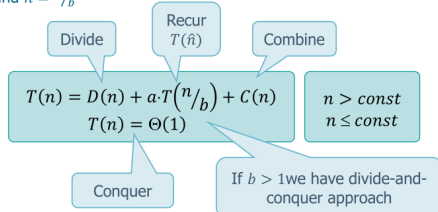
Complexity Analysis

Case 2

$$T(n) = D(n) + \sum T(\hat{n}) + C(n)$$

The size of the original problem n and the generated ones $\hat{n}$ are related by a **constant value** k_i not always the same for all subproblems $\hat{n} = n - k_i$

We suppose that the number of subproblems of size $\hat{n}$ is a.
If $a = 1$ we have linear recursion.
If $a > 1$ we have multi-way recursion.

Divide | Recur $T(\hat{n})$ | Combine

$$T(n) = D(n) + \sum_{i=0}^{a-1} T(n - k_i) + C(n)$$
$$T(n) = \Theta(1)$$

$n > const$
$n \leq const$

Conquer | If $b = 1$ we have decrease-and-conquer approach
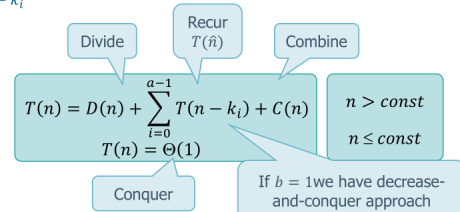
Example 1: Complexity Analysis

Divide and conquer problem	
Number of subproblems	$a = 2$
Reduction factor	$b = {}^n/_{\hat{n}} = 2$
Division cost	$D(n) = \Theta(1)$
Recombination cost	$C(n) = \Theta(1)$

$$T(n) = D(n) + a \cdot T\left({}^n/_b\right) + C(n)$$
$$T(n) = \Theta(1)$$
$n > 1$
$n \leq 1$

$n > 1$
$n \leq 1$
$$T(n) = 2 \cdot T\left({}^n/_2\right) + 1$$
$$T(1) = 1$$

$$T(n) = D(n) + a \cdot T\left({}^n/_b\right) + C(n)$$
$$T(n) = \Theta(1)$$
$n > 1$
$n \leq 1$

No cost for the combination phase

$$T(n) = 1 + 2 \cdot T\left({}^n/_2\right)$$

$$T\left({}^n/_2\right) = 1 + 2 \cdot T\left({}^n/_4\right)$$

$$T\left({}^n/_4\right) = 1 + 2 \cdot T\left({}^n/_8\right)$$

$$T\left({}^n/_8\right) = 1 + 2 \cdot T\left({}^n/_{16}\right)$$

$$\dots$$
$$T(1) = 1$$

Termination condition
$\frac{n}{2^i} = 1$
$n = 2^i$
$i = log_2 n$

```
void show (
    int v[], int l, int r
) {
    int i, c;
    if (l >= r) {
        return;
    }
    c = (r+l)/2;
    show (v, l, c);
    show (v, c+1, r);
    return;
}
```

$$T(n) = 1 + 2 \cdot T\left(\frac{n}{2}\right)$$

$$T\left(\frac{n}{2}\right) = 1 + 2 \cdot T\left(\frac{n}{4}\right)$$

$$T\left(\frac{n}{4}\right) = 1 + 2 \cdot T\left(\frac{n}{8}\right)$$

$$T\left(\frac{n}{8}\right) = 1 + 2 \cdot T\left(\frac{n}{16}\right)$$

...

$$T(1) = 1$$

$i = log_2 n$ steps

$$T(n) = 1 + 2 \cdot T\left(\frac{n}{2}\right)$$

$$T(n) = 1 + 2 + 4 \cdot T\left(\frac{n}{4}\right)$$

$$T(n) = 1 + 2 + 4 + 8 \cdot T\left(\frac{n}{8}\right)$$

$$T(n) = 1 + 2 + 4 + 8 + 16 \cdot T\left(\frac{n}{16}\right)$$

...

$$T(n) = \sum_{i=0}^{\log n} 2^i = \frac{(2^{\log n +} - 1)}{2 - 1} =$$
$$= 2 \cdot 2^{\log n} - 1 =$$
$$= 2n - 1 =$$
$$= O(n)$$

$$\sum_{i=0}^{k} x^i = \frac{(x^{k+1} - 1)}{(x - 1)}$$

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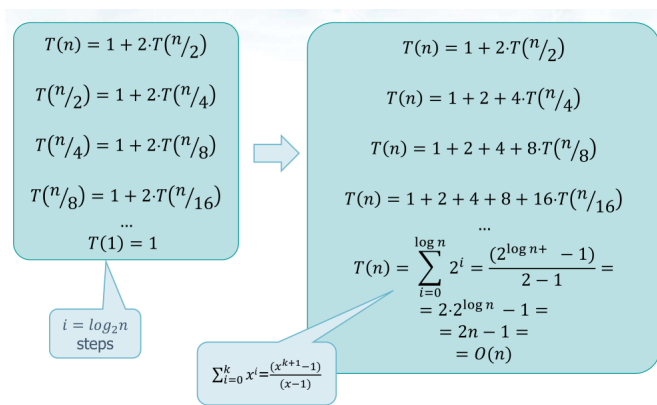
7 Heap sort

Data and Key

```
struct heap_s {
  Item *A;
  int heapsize;
} heap_t;

#define LEFT(i) (2*i+1)
#define RIGHT(i) (2*i+2)
#define PARENT(i) ((int)(i-1)/2)

#define LEFT(i) (i<<1+1)
#define RIGHT(i) (i<<1+2)
#define PARENT(i) ((i-1)>>1)

heap->A[LEFT(i)]     //is its left child
heap->A[RIGHT(i)]    //is its right child
heap->A[PARENT(i)]   //is its parentd

heap->A[0]
```
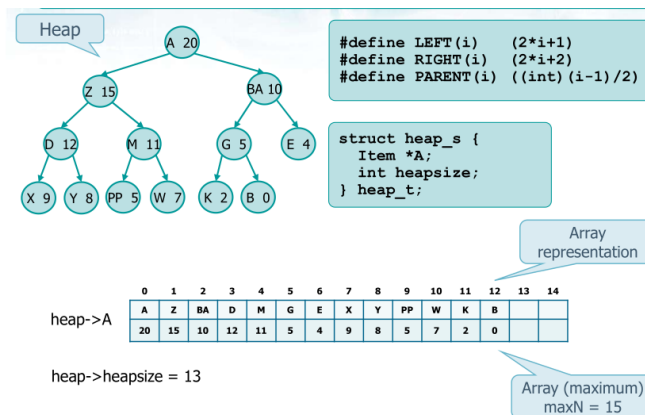

Figure 29: image

the tree is filled in the array order

largest on top, then has to be removed

7.1 Implementation

```
void heapbuild (heap_t heap) {
  int i;
  for (i=(heap->heapsize)/2-1; i >= 0; i--) {
    heapify (heap, i);
  }
  return;
}

void heapify (heap_t heap, int i) {
  int l, r, largest;
  l = LEFT(i);
  r = RIGHT(i);
  if ((l<heap->heapsize) &&
      (item_greater (heap->A[l], heap->A[i])))
    largest = l;
```

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```c
    else
      largest = i;
    if ((r<heap->heapsize)&&
        (item_greater (heap->A[r], heap->A[largest])))
      largest = r;
    if (largest != i) {
      swap (heap, i, largest);
      heapify (heap, largest);
    }
    return;
}

void heapsort (heap_t heap) {
  int i, tmp;
  heapbuild (heap);
  tmp = heap->heapsize;
  for (i=heap->heapsize-1; i>0; i--) {
    swap (heap, 0, i);
    heap->heapsize--;
    heapify (heap,0);
  }
  heap->heapsize = tmp;
  return;
}
```

8 Priority queue

$$T(n) = O(log_2 n)$$

8.1 Implementation

```
void pq_insert (PQ pq, Item item) {
  int i;
  i = pq->heapsize++;
  while( (i>=1) &&
      (item_less(pq->A[PARENT(i)], item)) )
    pq->A[i] = pq->A[PARENT(i)];
  i = PARENT (i);
}
pq->A[i] = item;
return;
}

Item pq_extract_max(PQ pq) {
  Item item;
  Extract max and move
    last element into the
    root node
    swap (pq, 0, pq->heapsize-1);
  item = pq->A[pq->heapsize-1];
  pq->heapsize--;
  heapify (pq, 0);
  Reduce heap size
}
return item;

void pq_change (PQ pq, int i, Item item) {
  if (item_less (item, pq->A[i]) {
    decrease_key (pq, i);
  } else {
    increase_key (pq, i, item);
  }
}

void decrease_key (PQ pq, int i) {
  pq->A[i] = item;
  heapify (pq, i);
}

void increase_key (PQ pq, int i) {
  while( (i>=1) &&
      (item_less(pq->A[PARENT(i)], item)) ) {
    pq->A[i] = pq->A[PARENT(i)];
    i = PARENT(i);
  }
  pq->A[i] = item;
}
```

9 Binary Search Tree

InOrder(root) visits nodes in the following order:
4, 10, 12, 15, 18, 22, 24, 25, 31, 35, 44, 50, 66, 70, 90

A Pre-order traversal visits nodes in the following order:
25, 15, 10, 4, 12, 22, 18, 24, 50, 35, 31, 44, 70, 66, 90

A Post-order traversal visits nodes in the following order:
4, 12, 10, 18, 24, 22, 15, 31, 44, 35, 66, 90, 70, 50, 25
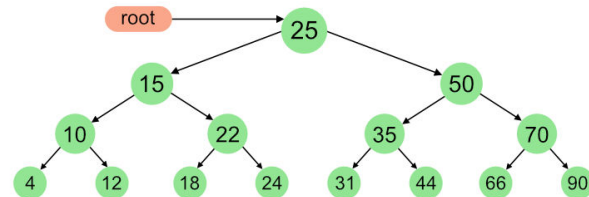
Figure 30: image

9.1 Implementation

```
void writeTree(FILE *fp, node_t *rp, int modo) {
  if (rp == NULL) {
    return;
  }

  if (modo == PREORDER) {
    writeData(fp, rp->data);
  }

  writeTree(fp, rp->left, modo);

  if (modo == INORDER) {
    writeData(fp, rp->data);
  }

  writeTree(fp, rp->right, modo);

  if (modo == POSTORDER) {
    writeData(fp, rp->data);
  }

  return;
}
```

10 Hash tables

10.1 Linear chaining

h'(k) = k % M

10.2 Open address

N « M -> Load Factor = alpha = N / M

10.3 Linear Probing

h'(k) = (h(k)+i) % M

10.4 quadratic Probing

h'(k) = (h(k)+ c_1 * i + c_2 * i^2) % M

10.5 double hasing

h'(k) = (h_1(k)+ i * h_2(k)) % M

10.6 linear probing

10.6.1 example

```
A S E R C H I N G X M P

a b c d e f g h i j  k  l  m  n  o  p  q  r  s  t  u  v  w  x  y  z
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
1 2 3 4 5 6 7 8 9 10 11 12 0  1  2  3  4  5  6  7  8  9  10 11 12 0


h'(k) = k % M = k % 13
h'(k) = (k % 13 + i) % 13

0 1 2 3 4 5 6 7 8 9 10 11 12
n a   c   e s r h i
```

10.7 quadratic probing

10.7.1 example

```
A S E R C H I N G X M P


a b c d e f g h i j k  l  m  n  o  p  q  r  s  t  u  v  w  x  y  z
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
1 2 3 4 5 6 7 8 9 10 11 12 0  1  2  3  4  5  6  7  8  9  10 11 12 0



    h'(k) = (h_1(k)+ i * h_2(k) ) % M
->  h'(k) = (h(k)+ c_1 * i + c_2 * i^2) % M


0 1 2 3 4 5 6 7 8 9 10 11 12
  a   c   e s   r
```

11   Greedy

Finding every solution is expensive

At each step you look for the best

It doesn't backtrack

11.1   Cost function

- selected a priorit
    - start from empty solution
    - sort it
    - choice
- modifyiable during the process
    - choice stored in priority queue
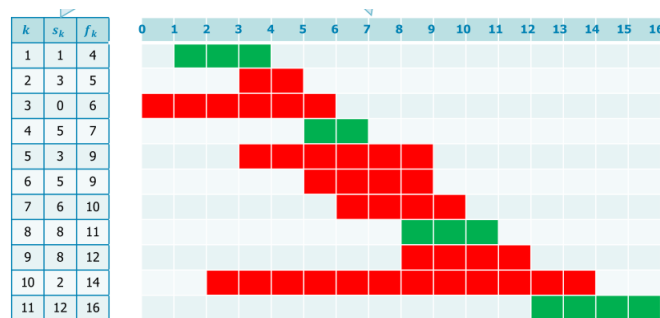
Figure 31: image

```c
typedef struct activity {
  char name[MAX];
  int start, stop;
  int selected;
} activity_t;
  ...
int cmp (const void *p1, const void *p2);
  ...
acts = load(argv[1], &n);
qsort ((void *)acts, n, sizeof(activity_t), cmp);
choose (acts, n);
```

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```
display (acts, n);
  ...

  int cmp (const void *p1, const
      activity_t *a1 = (activity_t
        activity_t *a2 = (activity_t
          return a1->stop - a2->stop;
          }
          void choose (activity_t *acts,
            int i, stop;
            }
            void *p2) {
          *)p1;
        *)p2;
      int n) {
    acts[0].selected = 1;
    stop = acts[0].stop;
    for (i=1; i<n; i++) {
      if (acts[i].start >= stop) {
        acts[i].selected = 1;
        stop = acts[i].stop;
      }
    }
  }
```

images tree...

```
{
  PQ *pq;
  pq = pq_init (maxN, item_compare);
  for (i=0; i<maxN; i++) {
    printf ("Enter letter & frequency: ");
    scanf ("%s %d", &letter, &freq);
    tmp = item_new (letter, freq);
    pq_insert (pq, tmp);
  }

  while (pq_size(pq) > 1) {
    l = pq_extract_max (pq);
    r = pq_extract_max (pq);
    tmp = item_new ('!', l->freq + r->freq);
    tmp->left = l;
    tmp->right = r;
    pq_insert (pq, tmp);
  }
  root = pq_extract_max (pq);
  pq_display (root, code, 0);
}
```

Complexity

$$T(n) = O(n * log_2 n)$$

12 Graphs

Definition - G = (V, E) - V = Finite and non empty set of vertices (simple or complex data) - E = Finite set of edges, that define a binary relation on V

- Directed/Undirected graphs
- Directed
  - Edge = sorted pair of vertices (u, v) E and u, v V
- Undirected
  - Edge = unsorted pair of vertices (u, v) E and u, v V
- Weighted
  - each edge has a weight

12.0.1 Loops

- A loop is defined as a path where
  - v 0 =v k , the starting and arrival vertices do coincide
- Self-loop
  - Loops whose length is 1
- A graphs without loops is called
- acyclic

13 Graphs visit

13.1 Breadth-First Search (BFS)

```
BFS (G, s)
  for each vertex v in V
    v.color = WHITE
    v.dtime = inf
    v.pred = NULL

  queue_init (Q)
  s.color = GRAY
  s.dtime = 0
  s.pred = NULL
  queue_enqueue (Q, s)

  while (!queue_empty (Q))
    u = queue_dequeue (Q)
    for each v in Adj(u)
      if (v.color == WHITE)
        vertex
        v.color = GRAY
        v.dtime = u.dtime + 1
        v.pred = u
        queue_enqueue (Q, v)

    u.color = BLACK
```


Figure 32: image

Figure 33: image

```c
g = graph_load(argv[1]);
printf("Initial vertex? ");
scanf("%d", &i);
src = graph_find(g, i);

graph_attribute_init (g);
graph_bfs (g, src);

n = g->g;
printf ("List of vertices:\n");
while (n != NULL) {
  if (n->color != WHITE) {
    printf("%2d: %d (%d)\n",
        n->id, n->dist, n->pred ? n->pred->id : -1);
  }
  n = n->next;
}
graph_dispose(g);

void graph_bfs (graph_t *g, vertex_t *n) {
  queue_t *qp;
  vertex_t *d;
  edge_t *e;
  qp = queue_init (g->nv);
  n->color = GREY;
  n->dist = 0;
  n->pred = NULL;
  queue_put (qp, (void *)n);

  while (!queue_empty_m(qp)) {
    queue_get(qp, (void **)&n);
    e = n->head;
    while (e != NULL) {
      d = e->dst;
      if (d->color == WHITE) {
        d->color = GREY;
        d->dist = n->dist + 1;
        d->pred = n;
        queue_put (qp, (void *)d);
```

```
      }
      e = e->next;
    }
    n->color = BLACK;
  }
  queue_dispose (qp, NULL);
}
```

13.2 Depth-First Search (DFS)

```
DFS (G)
for each vertex v in V
v.color = WHITE
v.dtime = v.endtime = inf
v.pred = NULL
time = 0
for each vertex v in V
if (v.color = WHITE)
DFS_r (G, v)
DFS_r (G, u)
time++
u.dtime = time
u.color = GRAY
for each v in Adj(u)
if (v.color == WHITE)
v.pred = u
DFS_r (G, v)
u.color = BLACK
time++
u.endtime = time
```
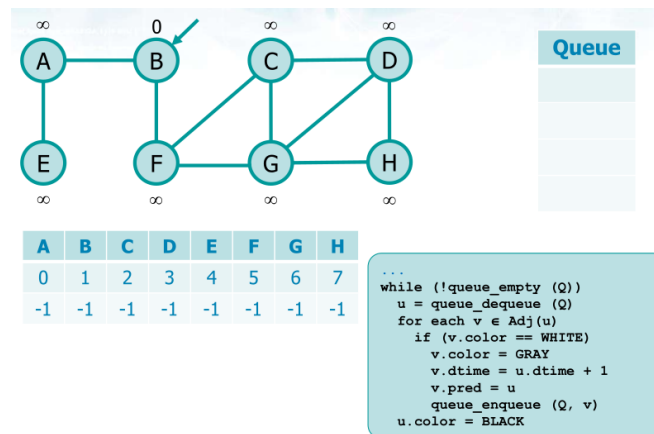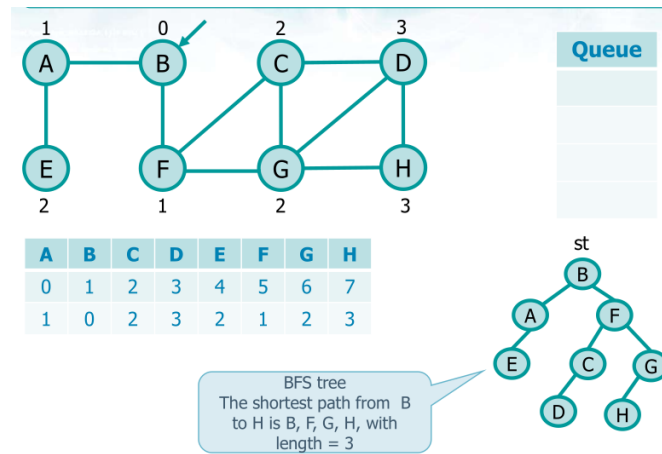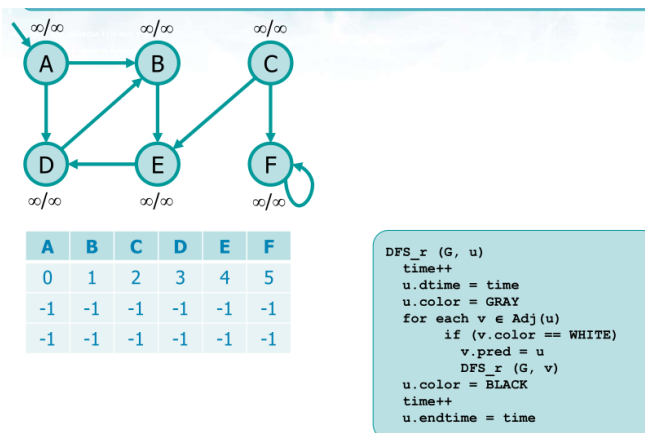

Figure 34: image
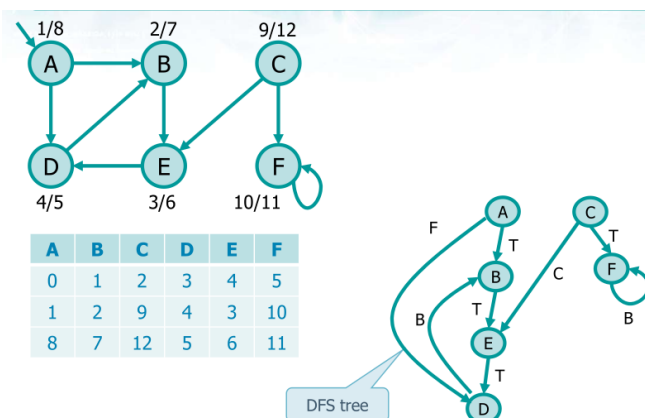
Figure 35: image

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```c
g = graph_load (argv[1]);
printf ("Initial vertex? ");
scanf ("%d", &i);
src = graph_find (g, i);
graph_attribute_init (g);
graph_dfs (g, src);
graph_dispose (g);

void graph_dfs (graph_t *g, vertex_t *n) {
  int currTime=0;
  vertex_t *tmp, *tmp2;
  printf("List of edges:\n");
  currTime = graph_dfs_r (g, n, currTime);
  for (tmp=g->g; tmp!=NULL; tmp=tmp->next) {
    if (tmp->color == WHITE) {
      currTime = graph_dfs_r (g, tmp, currTime);
    }
  }
  printf("List of vertices:\n");
  for (tmp=g->g; tmp!=NULL; tmp=tmp->next) {
    tmp2 = tmp->pred;
    printf("%2d: %2d/%2d (%d)\n",
        tmp->id, tmp->disc_time, tmp->endp_time,
        (tmp2!=NULL) ? tmp->pred->id : -1);
  }
}

int graph_dfs_r(graph_t *g, vertex_t *n, int currTime) {
  edge_t *e;
  vertex_t *t;
  n->color = GREY;
  n->disc_time = ++currTime;
  e = n->head;
  while (e != NULL) {
    t = e->dst;
    switch (tmp->color) {
      case WHITE: printf("%d -> %d : T\n", n->id, t->id);
                  break;
      case GREY : printf("%d -> %d : B\n", n->id, t->id);
                  break;
      case BLACK:
                  if (n->disc_time < t->disc_time) {
                    printf("%d -> %d : F\n",n->disc_time,t->disc_time);
                  } else {
                    printf("%d -> %d : C\n", n->id, t->id);
                  }
    }
    if (tmp->color == WHITE) {
      tmp->pred = n;
      currTime = graph_dfs_r (g, tmp, currTime);
    }
    e = e->next;
  }
  n->color = BLACK;
  n->endp_time = ++currTime;
  return currTime;
```

}

14 Graphs Applications

	A	B	C	D	E
A	0	1	0	0	0
B	0	0	1	1	0
C	0	0	1	0	0
D	0	0	0	0	1
E	1	1	0	0	0

G G^T

	A	B	C	D	E
A	0	0	0	0	1
B	1	0	0	0	1
C	0	1	1	0	0
D	0	1	0	0	0
E	0	0	0	1	0
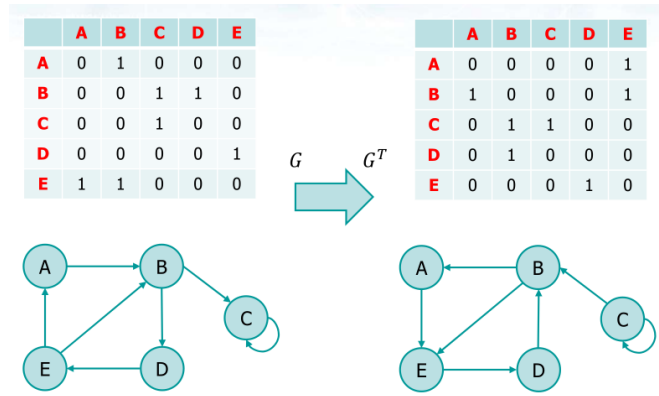
Figure 36: image

```c
graph_t *graph_transpose (graph_t *g) {
  graph_t *h;
  int i, j;
  h = (graph_t *) util_calloc (1, sizeof (graph_t));
  h->nv = g->nv;
  h->g = (vertex_t *) util_calloc (g->nv, sizeof(vertex_t));
  for (i=0; i<h->nv; i++) {
    h->g[i] = g->g[i];
    h->g[i].rowAdj = (int *) util_calloc (h->nv, sizeof(int));
    for (j=0; j<h->nv; j++) {
      h->g[i].rowAdj[j] = g->g[j].rowAdj[i];
    }
  }
  return h;
}
graph_t *graph_transpose (graph_t *g) {
  graph_t *h = NULL;
  vertex_t *tmp;
  edge_t *e;
  int i;
  h = (graph_t *) util_calloc (1, sizeof(graph_t));
  h->nv = g->nv;
  for (i=h->nv-1; i>=0; i--)
    h->g = new_node (h->g, i);
  tmp = g->g;
  while (tmp != NULL) {
    e = tmp->head;
    while (e != NULL) {
      new_edge (h, e->dst->id, tmp->id, e->weight);
      e = e->next;
    }
    tmp = tmp->next;
  }
  return h;
}

void graph_dag (graph_t *g){
  int i, *post, loop=0, timer=0;
  post = (int *)util_malloc(g->nv*sizeof(int));
```
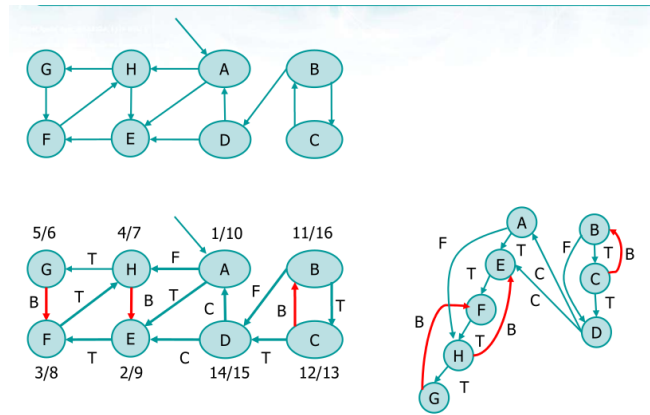
Figure 37: image
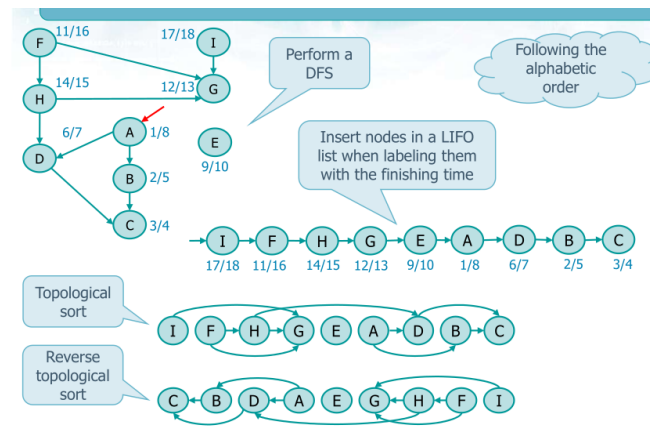
Figure 38: image

Figure 39: image

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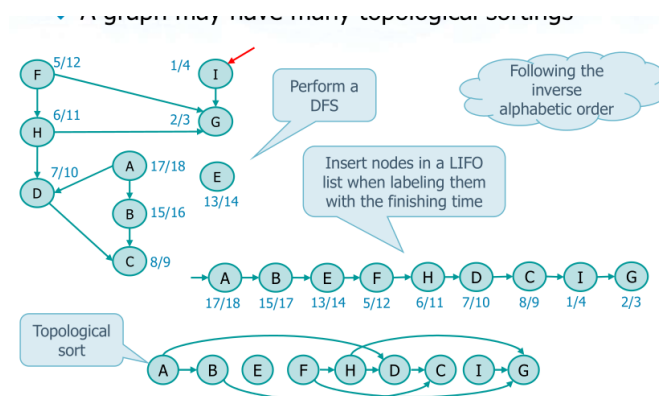
```c
  for (i=0; i<g->nv; i++) {
    if (g->g[i].color == WHITE) {
      timer = graph_dag_r (g, i, post, timer, &loop);
    }
  }
  if (loop != 0) {
    fprintf (stdout, "Loop detected!\n");
  } else {
    fprintf (stdout, "Topological sort (direct):");
    for (i=g->nv-1; i>=0; i--) {
      fprintf(stdout, " %d", post[i]);
    }
    fprintf (stdout, "\n");
  }
  free (post);
}
int graph_dag_r(graph_t *g, int i, int *post, int t,
    int *loop) {
  int j;
  g->g[i].color = GREY;
  for (j=0; j<g->nv; j++) {
    if (g->g[i].rowAdj[j] != 0) {
      if (g->g[j].color == GREY) {
        *loop = 1;
      }
      if (g->g[j].color == WHITE) {
        t = graph_dag_r(g, j, post, t, loop);
      }
    }
  }
  g->g[i].color = BLACK;
  post[t++] = i;
  return t;
}
```

14.1 Connectivity
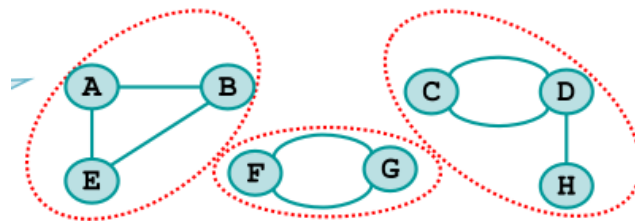
Figure 40: image

14.1.1 Bridges

14.1.2 Connected components

14.1.3 Kosaraju

compute the reverse graph (Transpose graph)

```c
g = graph_load (argv[1]);
sccs = graph_scc (g);
```
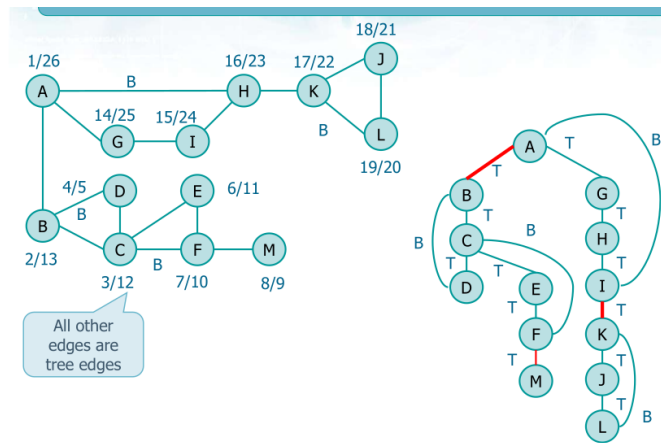
Figure 41: image
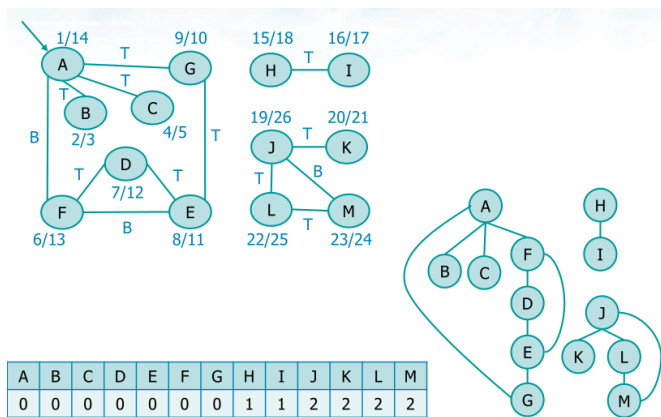
Figure 42: image

Figure 43: image

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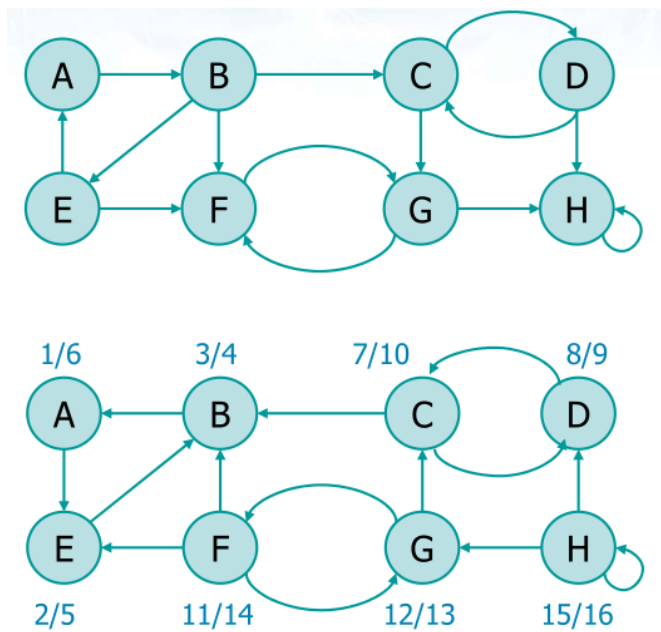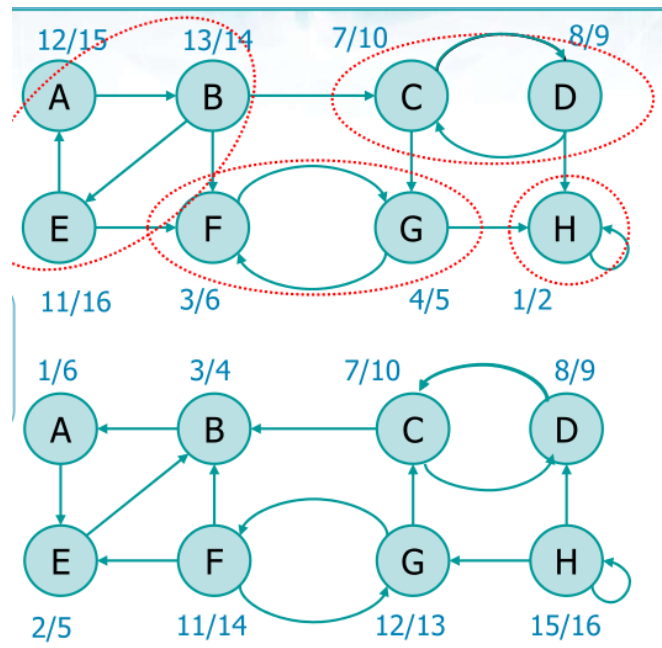
Figure 44: image

```
fprintf (stdout, "Number of SCCs: %d\n", sccs);
for (j=0; j<sccs; j++) {
  fprintf (stdout, "SCC%d:", j);
  for (i=0; i<g->nv; i++) {
    if (g->g[i].scc == j) {
      fprintf (stdout, " %d", i);
    }
  }
  fprintf (stdout, "\n");
}

graph_dispose (g);
int graph_scc (graph_t *g) {
  graph_t *h;
  int i, id=0, timer=0;
  int *post, *tmp;
  h = graph_transpose (g);
  post = (int *) util_malloc (g->nv*sizeof(int));
  for (i=0; i<g->nv; i++) {
    if (h->g[i].color == WHITE) {
      timer = graph_scc_r (h, i, post, id, timer);
    }
  }
  graph_dispose (h);
  id = timer = 0;
  tmp = (int *) util_malloc (g->nv * sizeof(int));
  for (i=g->nv-1; i>=0; i--) {
    if (g->g[post[i]].color == WHITE) {
      timer=graph_scc_r(g, post[i], tmp, id, timer);
      id++;
    }
  }
}
```

```c
    free (post);
    free (tmp);
    return id;
}

int graph_scc_r(
    graph_t *g, int i, int *post, int id, int t
    ) {
  int j;
  g->g[i].color = GREY;
  g->g[i].scc = id;
  for (j=0; j<g->nv; j++) {
    if (g->g[i].rowAdj[j]!=0 &&
        g->g[j].color==WHITE) {
      t = graph_scc_r (g, j, post, id, t);
    }
  }
  g->g[i].color = BLACK;
  post[t++] = i;
  return t;
}
```

15   Libraries

15.1   List library

15.1.1   Stack

```c
#ifndef _STACK_PUBLIC
#define _STACK_PUBLIC

#include <stdio.h>

/* macro definition */
#define stack_empty_m(sp) (stack_count(sp) == 0)

/* type declarations */
typedef struct stack stack_t;

/* extern function prototypes */
extern stack_t *stack_init(int size);
extern int stack_count(stack_t *sp);
extern int stack_push(stack_t *sp, void *data);
extern int stack_pop(stack_t *sp, void **data_ptr);
extern void stack_print(FILE *fp, stack_t *sp, void (*print)(FILE *, void *));
extern void stack_dispose(stack_t *sp, void (*quit)(void *));

#endif

#ifndef _STACK_PRIVATE
#define _STACK_PRIVATE

#include <stdio.h>
#include "stackPublic.h"
#include "util.h"

/* structure declarations */
```

```c
struct stack {
  void **array;
  int index;
  int size;
};

#endi

#include "stackPrivate.h"

/*
 *  create a new empty stack
 */
stack_t *stack_init(int size) {
  stack_t *sp;

  sp = (stack_t *)util_malloc(sizeof(stack_t));
  sp->size = size;
  sp->index = 0;
  sp->array = (void **)util_malloc(size * sizeof(void *));
  return sp;
}

/*
 *  return the number of elements stored in the stack
 */
int stack_count(stack_t *sp) { return (sp != NULL) ? sp->index : 0; }

/*
 *  store a new value in the stack (LIFO policy)
 */
int stack_push(stack_t *sp, void *data) {
  if (sp == NULL || sp->index >= sp->size) {
    return 0;
  }

  sp->array[sp->index++] = data;
  return 1;
}

/*
 *  extract a value from the stack (LIFO policy)
 */
int stack_pop(stack_t *sp, void **data_ptr) {
  if (sp == NULL || sp->index <= 0) {
    return 0;
  }

  *data_ptr = sp->array[--sp->index];
  return 1;
}

/*
 *  print all the stack elements (LIFO policy)
 */
void stack_print(FILE *fp, stack_t *sp, void (*print)(FILE *, void *)) {
  int i;
```

```c
  if (sp != NULL) {
    for (i = sp->index - 1; i >= 0; i--) {
      print(fp, sp->array[i]);
      fprintf(fp, "\n");
    }
  }
}


/*
 *  deallocate all the memory associated to the stack
 */
void stack_dispose(stack_t *sp, void (*quit)(void *)) {
  int i;

  if (sp != NULL) {
    if (quit != NULL) {
      for (i = 0; i < sp->index; i++) {
        quit(sp->array[i]);
      }
    }
    free(sp->array);
    free(sp);
  }
}
```

15.1.2 BST Library

```c
#ifndef _TREE_PUBLIC_INCLUDED
#define _TREE_PUBLIC_INCLUDED

#include "data.h"

#define PREORDER  -1
#define INORDER    0
#define POSTORDER  1

typedef struct node node_t;

data_t getData (node_t *);
node_t *createEmptyTree ();
node_t *readTree(FILE *);
node_t *searchI (node_t *, data_t);
node_t *searchR (node_t *, data_t);
node_t *treeMinI (node_t *);
node_t *treeMinR (node_t *);
node_t *treeMaxI (node_t *);
node_t *treeMaxR (node_t *);
node_t *insert(node_t *, data_t);
node_t *delete(node_t *, data_t);
void writeTree(FILE *, node_t *, int);
void freeTree(node_t *);

#endif

#ifndef _TREE_PRIVATE_INCLUDED
#define _TREE_PRIVATE_INCLUDED

#include "treePublic.h"
// #include "treeAddition.h"

struct node {
  data_t val;
  struct node *left;
  struct node *right;
};


#endif

#include "treePrivate.h"

#define FIND 0

static node_t *myAlloc(void);
#if FIND
static data_t findDeleteMax1(node_t **);
#endif
#if !FIND
static node_t *findDeleteMax2(data_t *, node_t *);
#endif

data_t getData(node_t *node) { return (node->val); }
```

```c
node_t *createEmptyTree(void) { return (NULL); }

node_t *treeMinI(node_t *rp) {
  if (rp == NULL)
    return (rp);

  while (rp->left != NULL) {
    rp = rp->left;
  }

  return (rp);
}

node_t *treeMinR(node_t *rp) {
  if (rp == NULL || rp->left == NULL)
    return (rp);

  return (treeMinR(rp->left));
}

node_t *treeMaxI(node_t *rp) {
  if (rp == NULL)
    return (rp);

  while (rp->right != NULL) {
    rp = rp->right;
  }

  return (rp);
}

node_t *treeMaxR(node_t *rp) {
  if (rp == NULL || rp->right == NULL)
    return (rp);

  return (treeMaxR(rp->right));
}

node_t *searchI(node_t *rp, data_t data) {
  while (rp != NULL) {
    if (compare(rp->val, data) == 0)
      return (rp);

    if (compare(data, rp->val) < 0)
      rp = rp->left;
    else
      rp = rp->right;
  }

  return (NULL);
}

node_t *searchR(node_t *rp, data_t data) {
  if (rp == NULL || compare(rp->val, data) == 0)
    return (rp);
```

```c
  if (compare(data, rp->val) < 0)
    return (searchR(rp->left, data));
  else
    return (searchR(rp->right, data));
}

node_t *insert(node_t *rp, data_t data) {
  node_t *p;

  /* Empty Tree: Found Position */
  if (rp == NULL) {
    p = myAlloc();
    p->val = data;
    p->left = p->right = NULL;
    return (p);
  }

  /* Duplicated Element */
  if (compare(data, rp->val) == 0) {
    return (rp);
  }

  if (compare(data, rp->val) < 0) {
    /* Insert on the left */
    rp->left = insert(rp->left, data);
  } else {
    /* Insert on the right */
    rp->right = insert(rp->right, data);
  }

  return (rp);
}

node_t *readTree(FILE *fp) {
  node_t *rp;
  data_t d;

  rp = createEmptyTree();

  while (readData(fp, &d) != EOF) {
    rp = insert(rp, d);
  }

  return (rp);
}

void freeTree(node_t *rp) {
  if (rp == NULL) {
    return;
  }

  freeTree(rp->left);
  freeTree(rp->right);
  free(rp);

  return;
```

```c
}

void writeTree(FILE *fp, node_t *rp, int modo) {
  if (rp == NULL) {
    return;
  }

  if (modo == PREORDER) {
    writeData(fp, rp->val);
  }

  writeTree(fp, rp->left, modo);

  if (modo == INORDER) {
    writeData(fp, rp->val);
  }

  writeTree(fp, rp->right, modo);

  if (modo == POSTORDER) {
    writeData(fp, rp->val);
  }

  return;
}

node_t *delete (node_t *rp, data_t data) {
  node_t *p;

  /* Empty Tree */
  if (rp == NULL) {
    printf("Error: Unknown Data\n");
    return (rp);
  }

  if (compare(data, rp->val) < 0) {
    /* Delete on the left sub-treee Recursively */
    rp->left = delete (rp->left, data);
    return (rp);
  }

  if (compare(data, rp->val) > 0) {
    /* Delete on the rigth sub-treee Recursively */
    rp->right = delete (rp->right, data);
    return (rp);
  }

  /* Delete Current Note rp */
  p = rp;
  if (rp->right == NULL) {
    /* Empty Right Sub-Tree: Return Left Sub-Tree */
    rp = rp->left;
    free(p);
    return (rp);
  }
```

```c
  if (rp->left == NULL) {
    /* Empty Left Sub-Tree: Return Right Sub-Tree */
    rp = rp->right;
    free(p);
    return rp;
  }

  /* Find Predecessor and Substitute */
#if FIND
  rp->val = findDeleteMax1(&(rp->left));
#endif
#if !FIND
  {
    data_t val;

    rp->left = findDeleteMax2(&val, rp->left);
    rp->val = val;
  }
#endif

  return (rp);
}

static node_t *myAlloc(void) {
  node_t *p;

  p = (node_t *)malloc(sizeof(node_t));
  if (p == NULL) {
    printf("Allocation Error.\n");
    exit(1);
  }

  return (p);
}

#if FIND
static data_t findDeleteMax1(node_t **rpp) {
  node_t *p;
  data_t d;

  /* Find The Rigth-Most Node (max value) */
  while ((*rpp)->right != NULL)
    rpp = &((*rpp)->right);

  p = *rpp;
  d = p->val;
  *rpp = (*rpp)->left;
  free(p);

  return (d);
}
#endif

#if !FIND
static node_t *findDeleteMax2(data_t *d, node_t *rp) {
  node_t *tmp;
```

```c
    if (rp->right == NULL) {
      *d = rp->val;
      tmp = rp->left;
      free(rp);
      return (tmp);
    }

    rp->right = findDeleteMax2(d, rp->right);
    return (rp);
}
#endif
```

15.2 Item library

15.2.1 Item with Stack

```c
#ifndef _ITEM
#define _ITEM

#include <stdio.h>
#include "util.h"

/* type declarations */
typedef int *item_t;

/* extern function prototypes */
extern int item_read(FILE *fp, void **ptr);
extern void item_print(FILE *fp, void *ptr);
extern int item_compare(void *data1, void *data2);
extern void item_dispose(void *ptr);

#endif

#include "item.h"

#define MAX 100

/*
 *  read an item from file
 */
int item_read (FILE *fp, void **data_ptr) {
  int *p;

  p = (int *)util_malloc(sizeof(int));
  if (fscanf(fp, "%d", p) == EOF) {
    return EOF;
  }
  *data_ptr = p;

  return 1;
}

/*
 *  print an item on file
 */
void item_print (FILE *fp, void *ptr) {
  item_t data = (item_t)ptr;
  fprintf(fp, "%d ", *data);
}

/*
 *  compare two items
 */
int item_compare (void *ptr1, void *ptr2) {
  item_t data1 = (item_t)ptr1;
  item_t data2 = (item_t)ptr2;

  return (*data1)-(*data2);
}
```

```c
/*
 *  free an item
 */
void item_dispose (void *ptr) {
  item_t data = (item_t)ptr;
  free(data);
  return;
}
```

15.2.2   Item with BST

15.3 Util library

15.3.1 Util with Stack

```c
#ifndef _UTIL
#define _UTIL

#include <stdio.h>
#include <stdlib.h>
#include <string.h>

/* macro definition */
#define util_check_m(expr, msg) \
  if ( !(expr) ) { \
    fprintf(stderr, "Error: %s\n", msg); \
    exit(EXIT_FAILURE); \
  }

/* extern function prototypes */
extern FILE *util_fopen(char *name, char *mode);
extern void *util_malloc(unsigned int size);
extern void *util_calloc(unsigned int num, unsigned int size);
extern char *util_strdup(char *src);
extern void util_array_dispose(void **ptr, unsigned int n, void (*quit)(void *));
extern void **util_matrix_alloc(unsigned int n, unsigned int m, unsigned int size);
extern void util_matrix_dispose(void ***ptr, unsigned int n, unsigned int m,
                                void (*quit)(void *));

#endif

#include "util.h"

/*
 *  fopen (with check) utility function
 */
FILE *util_fopen(char *name, char *mode) {
  FILE *fp = fopen(name, mode);
  util_check_m(fp != NULL, "could not open file!");
  return fp;
}

/*
 *  malloc (with check) utility function
 */
void *util_malloc(unsigned int size) {
  void *ptr = malloc(size);
  util_check_m(ptr != NULL, "memory allocation failed!");
  return ptr;
}

/*
 *  calloc (with check) utility function
 */
void *util_calloc(unsigned int num, unsigned int size) {
  void *ptr = calloc(num, size);
  util_check_m(ptr != NULL, "memory allocation failed!");
  return ptr;
}
```

```c
/*
 *  strdup (with check) utility function
 */
char *util_strdup(char *src) {
  char *dst = strdup(src);
  util_check_m(dst != NULL, "memory allocation failed");
  return dst;
}


/*
 *  array de-allocation utility function
 */
void util_array_dispose(void **ptr, unsigned int n, void (*quit)(void *)) {
  int i;

  if (quit != NULL) {
    for (i = 0; i < n; i++) {
      quit(ptr[i]);
    }
  }
  free(ptr);
}


/*
 *  matrix allocation utility function
 */
void **util_matrix_alloc(unsigned int n, unsigned int m, unsigned int size) {
  void **ptr;
  int i;

  ptr = (void **)util_malloc(n * sizeof(void *));
  for (i = 0; i < n; i++) {
    ptr[i] = util_calloc(m, size);
  }
  return ptr;
}


/*
 *  matrix de-allocation utility function
 */
void util_matrix_dispose(void ***ptr, unsigned int n, unsigned int m,
                         void (*quit)(void *)) {
  int i, j;

  for (i = 0; i < n; i++) {
    if (quit != NULL) {
      for (j = 0; j < m; j++) {
        quit(ptr[i][j]);
      }
    }
    free(ptr[i]);
  }
  free(ptr);
}
```

15.3.2   Util with BST

15.4   Data Library

15.4.1   Data with BST

```c
#ifndef _DATA_INCLUDED
#define _DATA_INCLUDED

#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <ctype.h>

#define MAXC 20

typedef int data_t;

int readData (FILE *, data_t *);
void writeData (FILE *, data_t);
int compare (data_t, data_t);

#endif
```

```c
#include "data.h"

int readData(FILE *fp, data_t *data) {
  int retValue;

  retValue = fscanf(fp, "%d", data);

  return (retValue);
}

void writeData(FILE *fp, data_t data) {
  fprintf(fp, "%d\n", data);

  return;
}

int compare(data_t d1, data_t d2) {
  if (d1 < d2) {
    return (-1);
  } else {
    if (d1 == d2) {
      return (0);
    } else {
      return (1);
    }
  }
}
```

15.5 Symbol table

```c
#ifndef ST_H_DEFINED
#define ST_H_DEFINED

#include "item.h"

typedef struct symboltable *ST;

ST      STinit(int) ;
void    STinsert(ST, Item) ;
Item    STsearch(ST, Key) ;
void    STdelete(ST, Key) ;
void    STdisplay(ST) ;

#endif


#include <stdio.h>
#include <stdlib.h>
#include <string.h>

#include "item.h"
#include "st.h"

typedef struct STnode *link;

struct STnode {
  Item item;
  link next;
};

struct symboltable {
  link *heads;
  int M;
  link z;
};

link NEW(Item item, link next) {
  link x = malloc(sizeof *x);
  x->item = item;
  x->next = next;
  return x;
}

ST STinit(int maxN) {
  int i;
  ST st = malloc(sizeof *st);

  st->M = maxN;
  st->heads = malloc(st->M * sizeof(link));
  st->z = NEW(ITEMsetvoid(), NULL);

  for (i = 0; i < st->M; i++)
    st->heads[i] = st->z;

  return st;
```

```c
}

int hash(Key v, int M) {
  int h = 0, base = 127;

  for (; *v != '\0'; v++)
    h = (base * h + *v) % M;

  return h;
}

int hashU(Key v, int M) {
  int h, a = 31415, b = 27183;

  for (h = 0; *v != '\0'; v++, a = a * b % (M - 1))
    h = (a * h + *v) % M;

  return h;
}

void STinsert(ST st, Item item) {
  int i;

  i = hash(KEYget(item), st->M);

  fprintf(stdout, "  hash index = %d\n", i);

  st->heads[i] = NEW(item, st->heads[i]);

  return;
}

Item searchST(link t, Key k, link z) {
  if (t == z)
    return ITEMsetvoid();

  if ((KEYcompare(KEYget(t->item), k)) == 0)
    return t->item;

  return (searchST(t->next, k, z));
}

Item STsearch(ST st, Key k) {
  return searchST(st->heads[hash(k, st->M)], k, st->z);
}

link deleteR(link x, Key k) {
  if (x == NULL)
    return NULL;

  if ((KEYcompare(KEYget(x->item), k)) == 0) {
    link t = x->next;
    free(x);
    return t;
  }
```

```c
    x->next = deleteR(x->next, k);

    return x;
}

void STdelete(ST st, Key k) {
    int i = hash(k, st->M);
    st->heads[i] = deleteR(st->heads[i], k);

    return;
}

void visitR(link h, link z) {
    if (h == z)
        return;

    ITEMshow(h->item);
    visitR(h->next, z);

    return;
}

void STdisplay(ST st) {
    int i;

    for (i = 0; i < st->M; i++) {
        fprintf(stdout, "st->heads[%d]: ", i);
        visitR(st->heads[i], st->z);
        fprintf(stdout, "\n");
    }

    return;
}
```

15.5.1 Item with Symbol tables

```c
#ifndef _DATO_INCLUDED
#define _DATO_INCLUDED

#define MAXST 10

typedef struct item* Item;
typedef char *Key;

Item ITEMscan();
void ITEMshow(Item data);
int ITEMcheckvoid(Item data);
Item ITEMsetvoid();
Key KEYscan();
int KEYcompare(Key k1, Key k2);
Key KEYget(Item data);
#endif

#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
```

```c
#include "item.h"

struct item {
  char *name;
  int value;
};

Item ITEMscan() {
  char name[MAXST];
  int value;

  scanf("%s%d", name, &value);

  Item tmp = (Item)malloc(sizeof(struct item));
  if (tmp == NULL) {
    return ITEMsetvoid();
  } else {
    tmp->name = strdup(name);
    tmp->value = value;
  }

  return tmp;
}

void ITEMshow(Item data) {
  fprintf(stdout, "   name = %s value = %d  ", data->name, data->value);
}

int ITEMcheckvoid(Item data) {
  Key k1, k2 = "";

  k1 = KEYget(data);
  if (KEYcompare(k1, k2) == 0)
    return 1;
  else
    return 0;
}

Item ITEMsetvoid() {
  char name[MAXST] = "";

  Item tmp = (Item)malloc(sizeof(struct item));
  if (tmp != NULL) {
    tmp->name = strdup(name);
    tmp->value = -1;
  }
  return tmp;
}
```

15.6   Graphs

16   Prototypes Library

16.1   Stack

```c
/* type declarations */
typedef struct stack stack_t;

/* structure declarations */
struct stack {
  void **array;
  int index;
  int size;
};

/* extern function prototypes */
extern stack_t *stack_init(int size);
extern int stack_count(stack_t *sp);
extern int stack_push(stack_t *sp, void *data);
extern int stack_pop(stack_t *sp, void **data_ptr);
extern void stack_print(FILE *fp, stack_t *sp, void (*print)(FILE *, void *));
extern void stack_dispose(stack_t *sp, void (*quit)(void *));
```

16.1.1   Item

```c
/* type declarations */
typedef struct item *item_t;

/* extern function prototypes */
extern int item_read(FILE *fp, void **ptr);
extern void item_print(FILE *fp, void *ptr);
extern int item_compare(void *data1, void *data2);
extern void item_dispose(void *ptr);
```

16.1.2   Util

```c
/* macro definition */
#define util_check_m(expr, msg) \
  if ( !(expr) ) { \
    fprintf(stderr, "Error: %s\n", msg); \
    exit(EXIT_FAILURE); \
  }

/* extern function prototypes */
extern FILE *util_fopen(char *name, char *mode);
extern void *util_malloc(unsigned int size);
extern void *util_calloc(unsigned int num, unsigned int size);
extern char *util_strdup(char *src);
extern void util_array_dispose(void **ptr, unsigned int n, void (*quit)(void *));
extern void **util_matrix_alloc(unsigned int n, unsigned int m, unsigned int size);
extern void util_matrix_dispose(void ***ptr, unsigned int n, unsigned int m,
    void (*quit)(void *));
```

16.2   BST

```c
#define PREORDER  -1
#define INORDER    0
```

```c
#define POSTORDER  1

typedef struct node node_t;

struct node {
  data_t val;
  struct node *left;
  struct node *right;
};

data_t getData (node_t *);
node_t *createEmptyTree ();
node_t *readTree(FILE *);
node_t *searchI (node_t *, data_t);
node_t *searchR (node_t *, data_t);
node_t *treeMinI (node_t *);
node_t *treeMinR (node_t *);
node_t *treeMaxI (node_t *);
node_t *treeMaxR (node_t *);
node_t *insert(node_t *, data_t);
node_t *delete(node_t *, data_t);
void writeTree(FILE *, node_t *, int);
void freeTree(node_t *);

void countNode (node_t *root, int *array);
void countLevel (node_t * root, int *array, int l);
void countPath (node_t * root, int *np, int *length);
int visit (node_t *root, int key1, int key2);
void visit_r (node_t *root, int key, int *d);
```

16.2.1   Data

```c
typedef int data_t;

int readData (FILE *, data_t *);
void writeData (FILE *, data_t);
int compare (data_t, data_t);
```

16.2.2   Util

```c
void check_args(int argc, char **argv);
FILE *open_file(char *filename, char *mode);
void *malloc_ck(int size);
int file_num_of_line_completed(char *filename, char *mode);
```

16.3   ST

```c
typedef struct symboltable *ST;

ST      STinit(int) ;
void    STinsert(ST, Item) ;
Item    STsearch(ST, Key) ;
void    STdelete(ST, Key) ;
void    STdisplay(ST) ;
```

16.3.1 Item

```c
typedef struct item* Item;
typedef char *Key;

Item ITEMscan();
void ITEMshow(Item data);
int ITEMcheckvoid(Item data);
Item ITEMsetvoid();
Key KEYscan();
int KEYcompare(Key k1, Key k2);
Key KEYget(Item data);
```