Laboratory practice n. 2

Objectives: Modal analysis, internal stability, BIBO stability of LTI systems.

Problem 1: Modal analysis of LTI systems

Given the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

perform the modal analysis.

(Answer $e^{-0.5t}\cos(0.866t+\varphi) \to$ exponentially convergent)

Problem 2: Modal analysis of LTI systems

Given the LTI system

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 5 \\ 8 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -1 & 3 \end{bmatrix} x(t) + 8u(t)$$

perform the modal analysis.

(Answer $e^{5t} \to$ exponentially divergent ; $e^{-t} \to$ exponentially convergent)

Problem 3: Modal analysis and internal stability of LTI systems

Perform the modal analysis and study the internal stability properties of an LTI system characterized by the following dynamic matrix *A*.

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(Answer: $e^{-2t} \to$ exponentially convergent $\varepsilon(t) \to$ bounded constant $t \to$ polynomially divergent ;
the system is unstable)

Problem 4: Internal and BIBO stability of LTI systems

An LTI system is described by the following state space representation:

$$\dot{x}(t) = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix} x(t)$$

Study its internal and BIBO stability properties.

(Answer: internal stability → unstable, BIBO stability → stable)

Problem 5: Internal stability of LTI systems

Given $p \in \mathbb{R}$, then discuss the internal stability properties of the LTI system characterized by the following dynamic matrix *A.*

$$A = \begin{bmatrix} p^2 - 1 & 0 & 0 \\ 0 & p - 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(Hint: notice that matrix *A* is diagonal…
Answer: $-1 < p < 1$ → asymptotically stable, $p = \pm 1$ → stable, $p > 1$, $p < -1$ → unstable)

Problem 6: BIBO stability of LTI systems

Analyse the BIBO stability properties of an LTI system having the following dynamic transfer function in the presence of variations of the real parameter *p*:

$$H(s) = \frac{4}{s^2 + (p+1)s + 4p - 2}$$

(Hint: apply Descartes' rule of sign to the denominator polynomial…
Answer: the system is BIBO stable for $p > 0.5$)