AUTOMATIC CONTROL

Computer Engineering, Electronic and Communications Engineering

Laboratory practice n. 1

Objectives: computation of the time response of LTI dynamical systems through the solution of the system state equations (in Laplace domain), state space and transfer function representation of electric circuits.

Problem 1

Consider the following LTI system

$$\dot{x}(t)=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}x(t)+\begin{bmatrix}5\\ 8\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}-1 & 3\end{bmatrix}x(t)+8u(t)$$

Compute the state x(t) and the output y(t) responses when $x(0)=[2,2]^T$ and the input u(t) is a step signal with amplitude 2.

Solution: $x(t)=\begin{bmatrix}3.0\overline{6}\,e^{5t}-0.\overline{6}e^{-t}-0.4\\ 6.1\overline{3}\,e^{5t}+0.\overline{6}e^{-t}-4.8\end{bmatrix}\varepsilon(t) \quad y(t)=(15.\overline{3}\,e^{5t}+2.\overline{6}e^{-t}+2)\varepsilon(t)$

Problem 2

Given the LTI system

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}4 & 0\\ 10 & 1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

Compute the output y(t) response when $x(0)=[0,0]^T$, and $u_1(t) = 0$, $u_2(t) = 2\,\delta(t)$.

Solution: $y(t)=(2.3094e^{-0.5t}\cos(0.866t-1.5708))\varepsilon(t)$)

Problem 3

Given the LTI system

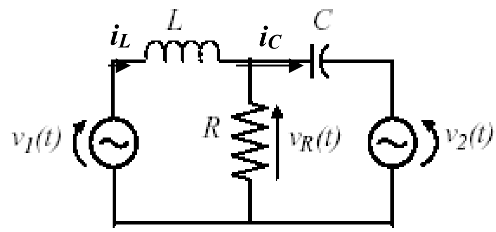
$$\dot{x}(t)=\begin{bmatrix}0 & 6\\ -1 & -5\end{bmatrix}x(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

compute the exponential matrix e^{At} of the system; (hint: recall that $e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$).

Solution: $e^{At} = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & 6e^{-2t} - 6e^{-3t} \\ -e^{-2t} + e^{-3t} & -2e^{-2t} + 3e^{-3t} \end{bmatrix}$

Problem 4

In the electric dynamical system below, generator voltages $v_1(t)$ e $v_2(t)$ are the inputs while the voltage $v_R(t)$ is the output:



Using basic circuit equations derive matrices A, B, C and D of the state space representation. (Hint. Use $i_L(t)$ e $v_C(t)$ as state variables)

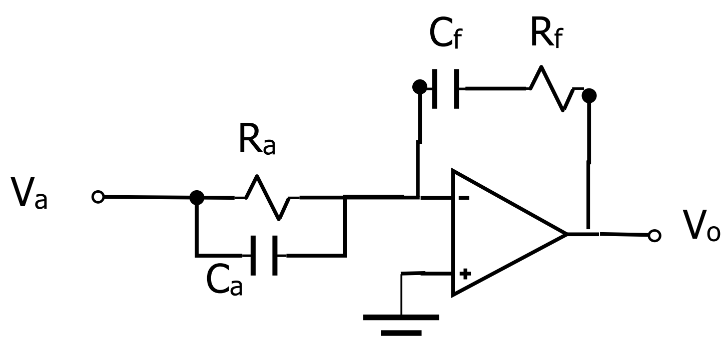
Solution:

$$u(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \quad x(t) = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad y(t) = v_R(t) = x_2(t) + u_2(t)$$

$$A = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad B = \begin{bmatrix} 1/L & -1/L \\ 0 & -1/RC \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

Problem 5

Show that the transfer function $H(s) = V_o(s)/V_a(s)$ of the circuit below is not proper.



Solution: $$H(s) = -\frac{(1+R_fC_fs)(1+R_aC_as)}{R_aC_fs}$$