

AUTOMATIC CONTROL

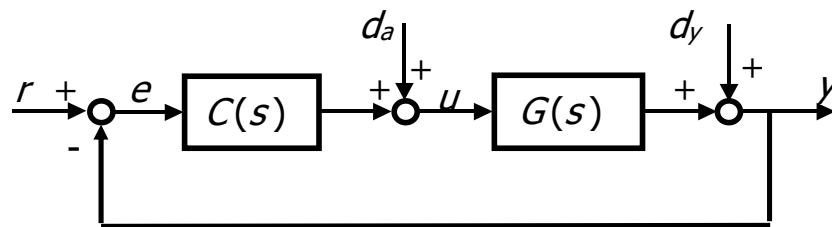
Computer Engineering and Electronic and Communications Engineering

Laboratory practice n. 4

Objectives: Response computation of feedback systems, feedback control systems transfer functions and simulation. Steady state analysis.

Problem 1 Computation of transfer functions and response of a feedback control system.

Consider the feedback control system



where $G(s) = \frac{1}{(1+s)^2}$ and $C(s) = \frac{(1+s)^2}{s(1+s/4)}$

- Compute the time response of the control input $u(t)$, when $r(t)$ is a unit step and the other inputs (i.e., d_a and d_y) are set to zero.
- Compute, if possible, the steady state response $e_{ss}(t)$ of the tracking error $e(t)$ when $d_y(t)=0.5\sin(t)\varepsilon(t)$ and the other inputs (i.e., $r(t)$ and $d_a(t)$) are set to zero.
- Compute, if possible, the steady state response $y_{ss}(t)$ of the controlled output $y(t)$ when $r(t) = 3\varepsilon(t)$, $d_y(t)= 2\varepsilon(t)$ and $d_a(t)= 0$.

Answer:

$$u(t) = (3e^{-2t} - 2te^{-2t} + 1)\varepsilon(t), \quad e_{ss}(t) = 0.4123 \sin(t-2.2531), \quad y_{ss}(t) = 3\varepsilon(t)$$

Problem 2 feedback control systems simulation

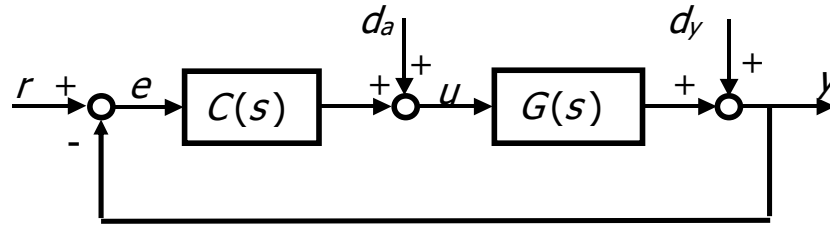
Packet information flow in a router working under TCP/IP can be modelled, in a suitable working condition, through the following transfer function (see C. V. Hollot *et al.*, "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows", IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 47, NO. 6, pp. 945-959, 2002.)

$$G(s) = \frac{q(s)}{p(s)} = \frac{\frac{c^2}{2N} e^{-sR}}{\left(s + \frac{2N}{R^2 c}\right) \left(s + \frac{1}{R}\right)}$$

where

- q = queue length (packets)
- p = probability of packet mark/drop
- c = link capacity (packets/s)
- N = load factor (number of TCP sessions)
- R = round trip time (s)

The objective of an active queue management (AQM) algorithm is to choose automatically the packet mark/drop probability p , so that the sender can tune the window size to keep the queue length at a constant level. This system can be represented by the standard feedback structure reported below.



Several AQM algorithms are available, but the one that has received special attention is the random early detection (RED) algorithm. The RED algorithm dynamics can be approximated through the linear controller:

$$C(s) = \frac{\ell}{1 + \frac{s}{\kappa}}$$

- Given the following controller and plant parameter values

$$\ell = 1.86 \cdot 10^{-4}, \kappa = 0.005, c = 3750, N = 60, R = 0.246$$

build a suitable Simulink scheme to simulate the given AQM. In particular, evaluate the transient performance in terms of maximum overshoot, 10-90% rise time and settling time 1%, the steady state tracking error and the maximum amplitude of the input magnitude $\max_t |u(t)|$ when the reference signal is a step function with amplitude 1 and the other inputs are set to 0.

- Repeat the previous points when:

$$C(s) = \frac{K_c \left(1 + \frac{s}{z}\right)}{s}, K_c = 9.64 \cdot 10^{-6}, z = 0.53$$

Answer:

$$\begin{aligned} \hat{s} = 0, t_r' &\approx 34.59 \text{ s}, t_{s,1\%} \approx 74.12 \text{ s}, |e_r^\infty| = 0.09, \max_t |u(t)| = 1.67 \cdot 10^{-5} \\ \hat{s} = 0, t_r' &\approx 2.99 \text{ s}, t_{s,1\%} \approx 6.49 \text{ s}, |e_r^\infty| = 0, \max_t |u(t)| = 2.238 \cdot 10^{-5} \end{aligned}$$