AUTOMATIC CONTROL

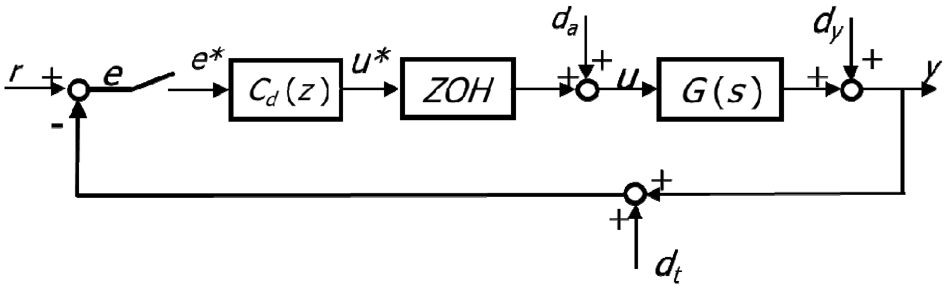
Computer Engineering

Laboratory practice n. 7

Objectives: Design of digital control systems through emulation. DT LTI systems solution and stability. Nonlinear dynamical systems equilibrium and stability.

Problem 1 digital control design through emulation

Given the digital control architecture.



where $G(s) = \dfrac{40}{s^2+4s-9.81}$

Assume a sampling time $T_s = 0.02$ s. Design a digital controller $C_d(z)$ in order to meet the following requirements.

1. $\left|e_r^{\infty}\right| \leq 0.25,\ r(t) = 2t\varepsilon(t)$
2. $\left|y_{d_t}^{\infty}\right| \leq 0.01,\ d_t(t) = \delta_t \sin(\omega_t t),\ \left|\delta_t\right| \leq 0.1,\ \omega_t \geq 90$ rad/s
3. $\hat{S} \leq 20\%$
4. $t_r \leq 0.3$s

Problem 2 **(DT LTI systems solution)**

Given the discrete time LTI system (sampling time is supposed T_s = 1 s):

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ 0.1 & -0.3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 7 & 7 \end{bmatrix} x(k)$$

Compute the state x(k) and the output y(k) responses when $x(0)=[0,0]^T$, and u(k) = ε(k).

(Answer:
$$x(k) = \begin{bmatrix} 0.8\overline{3} + 0.9524(-0.5)^k - 1.7857(0.2)^k \\ 0.8\overline{3} - 0.4761(-0.5)^k - 0.3571(0.2)^k \end{bmatrix} \varepsilon(k), y(k) = (11.\overline{6} + 3.3341(-0.5)^k - 15(0.2)^k)\varepsilon(k))$$
)

Problem 3 **(DT LTI systems solution, modal analysis and stability)**

Given the discrete time LTI system (sampling time is supposed T_s = 1 s):

$$x(k+1) = \begin{bmatrix} 0.5 & -1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 2 & 4 \end{bmatrix} x(k)$$

- Compute the output y(k) response when $x(0)=[2,1]^T$, and u(k) = $0.9^k\varepsilon(k)$.
- Compute the system natural modes and perform the modal analysis.
- Study the internal and BIBO stability properties of the given system.

(Answer: $y(k) = (-2 \cdot 0.5^k + 10 \cdot 0.9^k)\varepsilon(k)$, the natural modes are 0.5^k (geometrically convergent) and 1^k (bounded), the system is internally stable and BIBO stable))

Problem 4 (DT LTI systems internal stability)

Study the internal stability properties of a discrete time LTI system characterized by the following dynamic matrix A (sampling time is supposed T_s = 1 s).

$$A = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(Answer: the system is stable)

Problem 5 Equilibrium solution computation and stability

For the pendulum system with state equation

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\dfrac{g}{l}\sin(x_1(t)) - \dfrac{\beta}{ml^2}x_2(t) + \dfrac{1}{ml^2}u(t) \end{cases}$$

$m = 0.1\,kg$, $l = 0.5\,m$, $\beta = 0.1\,N s rad^{-1}$, $g = 9.81\,ms^{-2}$

Compute the equilibrium input $\bar{u}$ that corresponds to the equilibrium state $\bar{x} = \begin{bmatrix} \frac{\pi}{2} \\ 0 \end{bmatrix}$ and study its stability properties through the linearized model.

Answer: $\bar{u} = mgl \approx 0.49$ Nm, no conclusion can be drawn about the equilibrium stability

Problem 6 Equilibrium solution computation

Consider the nonlinear dynamical system below

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -19.62\sin(x_1(t)) - 4x_1(t) - 4x_2(t) + 40u(t) \end{cases}$$

Compute the equilibrium states $\bar{x}$ corresponding to the equilibrium input $\bar{u} = 0$ and study their stability properties through the linearized model.
(Hint: the solving equation for equilibrium computation can be trivially solved graphically through a MATLAB plot.)

Answer: $\bar{x}' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \bar{x}'' = \begin{bmatrix} -4.15 \\ 0 \end{bmatrix}, \bar{x}''' = \begin{bmatrix} 4.15 \\ 0 \end{bmatrix}, \bar{x}'''' = \begin{bmatrix} -4.85 \\ 0 \end{bmatrix}, \bar{x}''''' = \begin{bmatrix} 4.85 \\ 0 \end{bmatrix}$, all the equilibrium points are asymptotically stable