Design Procedure

Steady-State Design

The steady-state requirements depend on the **system type** (the number of poles at the origin of the Loop Transfer Function $L(s) = C(s)G(s)$) and on the **gain** $K_g = \lim s \to 0 s^g L(s)$. Since the Plant Transfer function $G(s)$ is fixed, those requirements can be translated on suitable choices of the values K_C and g_c. In order to proceed systematically with the design of $C(s)$ we can express it as $C(s) = C_{SS}(s)C_T(s)$ where $C_{SS}(s) = \frac{K_C}{s^{g_C}}$ is the **steady-state controller** and $C_T(s) = \frac{N_C^{'}(s)}{D_C^{'}(s)}, \lim s \to 0 \frac{N_C^{'}(s)}{D_C^{'}(s)} = 1$ is the **transient controller**. To properly meet the requirements we need to shape C_{SS} to:

1. Add a suitable number of poles at the origin to get the required value of g (system type)
2. Tune the value of K_C in order to get the required value of K_g.

How do we choose the sign of K_C to ensure system stability?

- If the plant transfer function $G(s)$ does not include poles and/or zeros with strictly positive real part, then the sign of K_c is chosen so that $K_g = K_G \cdot K_C > 0$
- If the plant transfer function $G(s)$ includes poles and/or zeros with strictly positive real part, then the sign of K_C is chosen according to a conceptual procedure.

Conceptual procedure:

1. Draw the Nyquist diagram of the $L^{'}(s) = C_{SS}(s)G(s)$ using the sign of K_C so that $K_g = K_G \cdot K_C > 0$
2. If $L^{'}(s)$ leads to a stable feedback system, then this is the correct sign choice, provided that $C_T(s)$ will be designed to avoid significant modifications of the frequency response of $L^{'}(s)$ near the critical point
3. Else if $L^{'}(s)$ leads to an unstable feedback system, discuss whether a suitable choice of $C_T(s)$ may be able to stabilize the feedback system (e.g. through a phase lead action) around the critical point
4. If the previous steps fail, repeat the procedure changing the sign of K_C to verify that it's the correct choice

Transient Design

Transient requirements of a feedback system, are defined considering the controlled output response y(t) when the reference signal r(t) is a step function, because it introduces a sudden change in the desired behavior of the controlled output causing critical solicitations during the transient phase. In this case, the following indices are used to define the transient performance of a feedback control system for the **time response**

- Maximum overshoot $\hat{s} \to$ accuracy
- Rise time $t_r \to$ trigger off quickness.
- Settling time $t_{s,\alpha\%} \to$ extinction quickness.

These can be translated into **Frequency Response** requirements

- Resonant Peak of the complementary sensitivity function T_P
- Resonant Peak of the sensitivity function S_P
- Crossover frequency ω_c

Using a graphical procedure we can obtain the **minimum dampening coefficient** ζ from the requirement on the maximum value of the overshoot $\hat{s}$. From the dampening coefficient, using the formulas or the graphical approach we can get the desired values of T_P and S_P.

Remember that the obtained values of T_P and S_P are not in dB and need to be converted before using the functions *SGrid* and *TGrid* for the plots of the Constant Magnitude Loci

The rise time t_r and settling time $t_{s,\alpha\%}$ requirements can be translated in a crossover frequency ω_c requirement. Using the formulas or the graphical procedure, we get **two different values** for ω_c. The desired crossover frequency is the maximum between the two found values $\omega_{c,des} = \max(\omega_{c,t_r}, \omega_{c,t_{s,\alpha\%}})$

The requirements have been obtained **assuming that $T(s)$ is exactly described by second order prototype model**. However, this is hardly the case, and we will need to evaluate if the requirements have been met through simulation

Transient Controller Design

After obtaining the desired values of T_P,S_P and ω_c but before the design of the transient controller C_T we must do this preliminary steps:

1. Consider the Loop transfer function $L^{'}(s) = C_{SS}(s)G(s)$ obtained during the steady-state design. Plot the frequency response $L^{'}(j\omega)$ on the Nichols Plane
2. Mark the point corresponding to $\omega_{c,des}$
3. Plot the constant magnitude loci T_P and S_P

We need to shape $L^{'}(s)$ in order for the point corresponding to $\omega_{c,des}$ to be **tangent** to the constant magnitude loci and to be on the **x-axis** ($y = 0dB$). We have multiple ways to approach this

Lead Network

$C_D(s) = \frac{1+\frac{s}{\omega_D}}{1+\frac{s}{m_D\omega_D}}, \omega_D > 0, m_D > 1$ The lead network introduces a **desired phase lead** and a **possibly undesired magnitude increase**. The greater m_D the greater the phase lead. Basic guidelines for design:

- Quantify the amount of the phase lead $\Delta\phi$ needed at $\omega_{c,des}$ in order to shift the value of $\angle L^{'}(j\omega_{c,des})$ outside the "influence" of the constant magnitude loci
- m_D is is choosen on the basis of the required value of $\Delta\phi$
- ω_D is fixed to obtain that the phase lead $\Delta\phi$ occurs exactly at $\omega_{c,des}$

We use the **Universal Lead Network Diagrams**. It is **strongly suggested** to use multiple lead networks if the required phase lead is **greater than 60°**

Lag Network

$C_I(s) = \frac{1+\frac{s}{m_I\omega_I}}{1+\frac{s}{\omega_I}}, \omega_I > 0, m_I > 1$ The Lag Network introduces a **desired magnitude decrease** and a **possible undesired phase lag**. The greater m_I the greater the magnitude decrease. Basic guidelines for design:

- Quantify the amount of the magnitude decrease $\Delta\phi$ needed at $\omega_{c,des}$ in order to shift the value of $\angle L^{'}(j\omega_{c,des})$ outside the "influence" of the constant magnitude loci
- m_I is is choosen on the basis of the required value of $\Delta\phi$
- ω_I is fixed to obtain that the magnitude decrease $\Delta\phi$ occurs exactly at $\omega_{c,des}$

We can use two different methods to decide the values of ω_I, m_I:

1. **Empyrical method**: To limit the phase lag at $\omega_{c,des}$ the zero of the lag network $m_I\omega_I$ should be *sufficiently far* from $\omega_{c,des}$. We have $\omega_{c,des} \approx \alpha m_I \omega_I, \alpha >> 1 \rightarrow \omega_I = \frac{\omega_{c,des}}{\alpha m_I}$. As a rule of thumb, start with $\alpha = 10$ and try to go down with its values. Increasing α decreases the phase lag introduced at $\omega_{c,des}$
2. **More precise method**: using the Lead Universal Phase Diagrams we get $\omega_I = \frac{\omega_{c,des}}{\omega_{norm}}$

Real Negative Zero

$C_Z(s) = 1 + \frac{s}{\omega_Z}, \omega_Z > 0$ The maximum phase lead introduced by $C_Z(s)$ is 90°. If we need more than that multiple real negative zeros can be employed.

Issues: Since the transfer function of a real negative zero is not proper, it is not guaranteed, in general, that the final overall controller $C(s) = C_{SS}(s)C_Z(s)$ is proper. If $C(s)$ is not proper, we need to add **closure poles** that have to be placed at higher frequencies with respect to highest frequency of the controller zeros. $C(s) = \frac{K_C(1+\frac{s}{\omega_{Z_1}})(1+\frac{s}{\omega_{Z_2}})}{s(1+\frac{s}{\omega_P})}, \omega_P >> \max(\omega_{Z_1}, \omega_{Z_2})$

Sensor Noise: Steady State Response to Sinusoidal Disturbances

At steady state, in the presence of sinusoidal $d_t = \delta_t \sin(\omega_t)$ we have $y_{ss} = \delta_t|T(j\omega)|\sin(\omega t + \angle(-T(j\omega))$. The steady-state output error is $|y_{d_t}^{\infty}| = \max(|y_{ss}(t)|)) = \delta_t|T(j\omega)|$. The steady-state output error is required to be bounded by a given constant $|y_{d_t}^{\infty}| \leq \rho_t$. This constraint can be translated into a Constant Magnitude Locus $M_T^{HF} = \frac{\rho_t}{\delta_t}$, and this **introduces a constraint on the value of ω_c such that $\omega_{c,des} \leq 0.1\omega_t$**

Output Disturbance: Steady State Response to Sinusoidal Disturbances

At steady state, in the presence of sinusoidal $d_y = \delta_y \sin(\omega_y)$ we have $y_{ss} = \delta_y|S(j\omega)|\sin(\omega t + \angle S(j\omega)$. The steady-state output error is $|y_{d_y}^{\infty}| = \max(|y_{ss}(t)|)) = \delta_y|S(j\omega)|$. The steady-state output error is required to be bounded by a given constant $|y_{d_y}^{\infty}| \leq \rho_y$. This constraint can be translated into a Constant Magnitude Locus $M_S^{LF} = \frac{\rho_y}{\delta_y}$, and this **introduces a constraint on the value of ω_c such that $\omega_{c,des} \geq 10\omega_y$**

Actuator Saturation

Physical limitations actuator devices impose hard constraints on the control input u(t). The above described actuator limitations, impose to the control input u(t) a **saturation constraint** of the form $|u(t)| \leq u_M, \forall t \geq 0$. The saturation constraints can be described as a nonlinear static function of the control input. To satisfy this requirement the only approach is **"a posteriori"**. We need to simulate the behaviour of the system (usually with a step input) and check if the requirement is met. If it's not met a common procedure is to **reduce the value of the actual crossover frequency** ω_c (keep in mind that **this may cause unsatisfaction of transient requirements**)