AUTOMATIC CONTROL

Computer Engineering, Electronic and Communications Engineering

Laboratory practice n. 3

Objectives: Steady state behaviour, step response of prototype models, graphical representation of the frequency response function.

Problem 1: Steady state properties of LTI systems

Given the LTI system described by the transfer function

$$H(s) = \frac{1}{s^3 + 2s^2 + 5.25s + 4.25}$$

1. Compute, if possible, the steady state output response $y_{ss}(t)$ in the presence of the input

$$u(t) = \left(3\sin(0.1t) + 2\right)\varepsilon(t)$$

2. Compute, if possible, the maximum amplitude of a sinusoidal input of the form

$$u(t) = A_u \sin(3t)\varepsilon(t)$$

so that, at steady state, the maximum output amplitude satisfies $|y_{ss}(t)|<1$.
(Answer: 1. $y_{ss}(t) = (0.7038 \sin(0.1t - 0.1232) + 0.4706)\, \varepsilon(t)$ 2. $|A_u| \le 17.7658$)

Problem 2: Step response of 2nd order systems

Consider the following 2nd order system transfer functions

$$H(s) = \frac{10}{s^2 + 1.6s + 4}$$

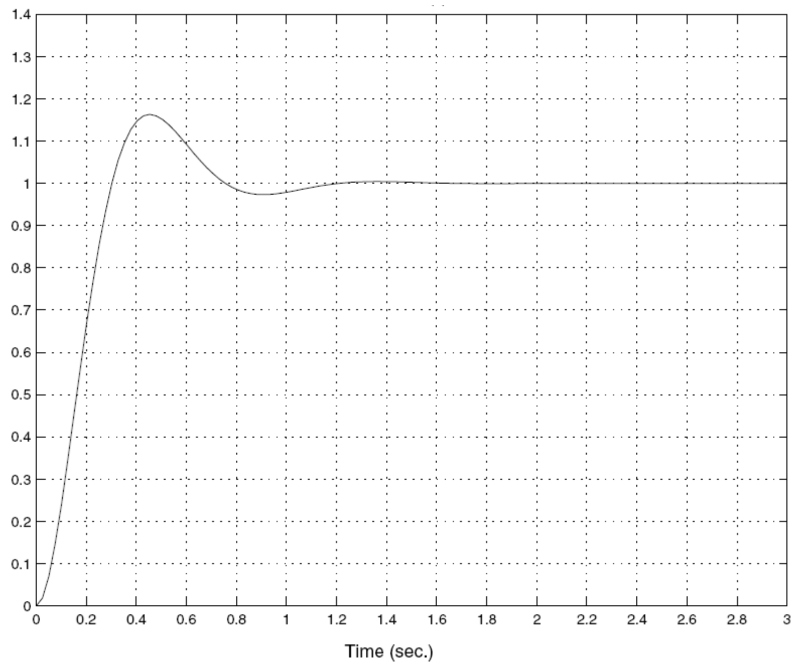
- Evaluate the natural frequency ω_n, the damping coefficient ζ and the time constant τ of the poles.
- Define H(s) in MATLAB and use the statement `step(H)` (see the online help for more details) to plot the unit step output response and, on the basis of the obtained plot, evaluate

1. Steady state value y_∞;
2. Maximum overshoot $\hat{s}$ and peak time $\hat{t}$;
3. Rise time t_r ;
4. 5% settling time $t_{s,5\%}$.

(Answer: $\omega_n = 2, \zeta = 0.4, \tau = 1.25$ s, $y_\infty = 2.5, \hat{s} = 25.38\%, \hat{t} \simeq 1.715\,s, t_r \simeq 1.08\,s, t_{s,5\%} \simeq 3.8\,s$)

Problem 3: Step response of 2nd order systems

The (zero state) output response in the presence of a step of amplitude 5 of an LTI system is reported in the picture below.



Compute the values of the parameters K, ω_n and ζ of a second order transfer function $H(s)$ of the form:

$$H(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

whose step response matches the given time course.

(Answer: $K = 0.2, \omega_n = 8, \zeta = 0.5 \rightarrow H(s) = \dfrac{12.8}{s^2 + 8s + 64}$)

Problem 4: Graphical representation of the frequency response function

Consider the following transfer functions

1. $L(s) = \dfrac{5}{s^3}$
2. $L(s) = \dfrac{0.25}{s^2(1-0.5s)^2}$
3. $L(s) = \dfrac{s-1}{s(s^2-9)}$
4. $L(s) = \dfrac{1+0.5s}{(1+s)(1-s)^2}$
5. $L(s) = \dfrac{s^2+1}{(s^2-4)(s+4)}$.

For each transfer function,

1. plot the Bode diagrams using MATLAB (function `bode)` and check the correctness of each plot based on the zeros and poles properties. To this end, it is helpful to draw the asymptotic diagrams by hand. Take into account that the `bode` statement may add +/- 360° in the phase values.
2. Draw polar and Nyquist plots by hand and check the results with MATLAB.
3. Plot the Nichols diagram with MATLAB.