Exercise session 2

Topics: data and functions approximation

1. Find polynomials of degree $n = 5, 9, 13$ interpolating the Runge function $f(x) = 1/(1 + x^2)$ at equally spaced nodes in the interval $[-5, 5]$. Graphically represent the Runge Function, the interpolating polynomials and interpolation data. Repeat the exercise using Chebyshev nodes:

$$t_i = -\cos\left(\frac{(2i-1)\pi}{2(n+1)}\right), \quad i = 1, ..., n+1$$

defined on the interval $[-1, 1]$ properly scaled in the generic interval $[a, b]$ by means of the change of variable $x_i = \frac{b-a}{2}t_i + \frac{b+a}{2}$. Which of the two choices of nodes leads to an increasingly accurate approximation as the polynomial degree goes up?

2. Approximate with interpolating polynomials of degree $n = 5, 10, 15$ the following functions:

$$f_1(x) = \sin(\mathrm{x}), \quad \text{over } [0, \pi],$$

$$f_2(x) = \frac{1}{1+x^2}, \quad \text{over } [-2\pi, 2\pi].$$

Use both equally spaced and Chebyshev-Lobatto nodes:

$$t_i = -\cos\left(\frac{(i-1)\pi}{n)}\right), \quad i = 1, ..., n+1$$

properly scaled in the interval of interest. For each n graphically represent the interpolation absolute error at 100 equally spaced points in the considered interval and print the maximum absolute error. For each function compare the graphs reporting the error of the two sets of nodes and deduce tre most advantageous choice of nodes.

3. Graphically represent the *not-a-knot* cubic spline interpolating $f(x) = 1/(1+x^2)$ over 6, 10, 14 equally spaced nodes in the interval $[-5, 5]$. Compare the graphs with the ones of Exercise 1 and comment the results.

4. Use the Matlab function `spline` to construct cubic splines:

$S_3(x)$ fulfilling *not-a-knot* condition,

$\bar{S}_3(x)$ fulfilling $\bar{S}_3'(x_0) = f'(x_0)$ and $\bar{S}_3'(x_n) = f'(x_n)$

interpolating $f(x) = (1 - x^2)^{5/2}$ over the nodes $x_i = -1 + 2i/n$, $i = 0, 1, ..., n$, $n = 2^k$, $k = 2, 3, 4, 5$. Graphically represent interpolation errors linked to $S_3(x)$ and $\bar{S}_3(x)$ at 100 equally spaced points in the interpolation interval $[-1, 1]$. Find which of the two gives more accurate result. Print maximum absolute error for each value of k and deduce which of the two approximations is more accurate (justify your answer).

5. According to the air quality database of Regione Piemonte, on November 13, 2015 the control unit placed near Lingotto, detected the following nitrous oxides concentration in the air (micrograms/cubic meters) at hourly intervals starting from 13:00 untill 8:00 of next day. Find an approximation of nitrous oxides concentration levels at 14:30 and at 7:30 by means of linear spline.

hour	nitrous oxides	hour	nitrous oxides
1	243	11	138
2	209	12	95
3	181	13	56
4	179	14	32
5	180	15	21
6	166	16	12
7	163	17	11
8	157	18	61
9	187	19	146
10	192	20	186