Signal and Systems Formula Sheet

Countinuous Time - Chapter 2

Dirac's delta: $\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1$, $\int_{-\infty}^{\infty} \delta(a[t-t_0])\, dt = \frac{1}{|a|} S(t_0)$
$\int_{-\infty}^{\infty} f(t)\delta(t-a)\, dt = f(a)$

Countinuous Time - Chapter 3

Time Average: $\bar{x} = \frac{1}{T2-T1}\int_{T1}^{T2} x(t)\, dt$
Energy of a signal: $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$
Spettro di potenza: $S_x(f) = |X(f)|^2$
Valore Medio: $\bar{x} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt$
Power of a periodic signal: $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$

Countinuous Time - Chapter 4

Schwarz's Inequality (Cauchy-Schwarz Inequality):
For vectors u and v in an inner product space, it states:
$|\langle u, v\rangle|^2 \leq \langle u, u\rangle \cdot \langle v, v\rangle$
Orthonormal Set:
An orthonormal set u_i is a set of vectors in an inner product space such that:
$\langle u_i, u_j\rangle = \delta_{ij}$
Orthonormal Basis:
An orthonormal basis is a set of vectors that is both orthonormal and forms a basis for the vector space. If u_i is an orthonormal basis, then for any vector v in the space:
$v = \sum_i \langle v, u_i\rangle \cdot u_i$
Gram-Schmidt:
Canonical bases:
$x(t) \approx x_0(t) = \sum_{n=-\infty}^{\infty} x(n\Delta t)\Pi_{\Delta t}(t - n\Delta t)$

Countinuous Time - Chapter 5

Fourier Basis: $e^{j\omega t}$
Where:
- e is the base of the natural logarithm (approximately 2.71828).
- j represents the imaginary unit ($j^2 = -1$).
- ω is the angular frequency (measured in radians per second).
- t is time.

Discrete Time - Chapter 1

Trasformata di Fourier Discreta (DFT): $X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}$
Parseval's Rule Discrete Time: $\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$
Base di Fourier per sequenze periodiche:
La base di Fourier per sequenze periodiche è utilizzata per rappresentare segnali discreti periodici di lunghezza N. I componenti della base includono armoniche discrete: $e_k[n] = e^{j\frac{2\pi kn}{N}}, \quad k = 0, 1, 2, \ldots, N-1$
Dove:
- e è la base dell'esponenziale complesso.
- j rappresenta l'unità immaginaria.
- k è un intero che rappresenta l'armonica.
- n è l'indice temporale discreto.

Gram-Schmidt:
1. Ortogonalizzazione: $u_i = v_i - \sum_{j=1}^{i-1} \frac{\langle v_i, u_j\rangle}{\langle u_j, u_j\rangle} u_j$
2. Normalizzazione (Ortonormalizzazione): $u_i = \frac{u_i}{\|u_i\|}$

Discrete Time - Chapter 2

Linear: $y[n] = a_1 y_1[n] + a_2 y_2[n]$
Time-invariant: $y[n] = x[n] * h[n]$

Trigonometry

Identità Trigonometriche Fondamentali:
$\sin^2(\theta) + \cos^2(\theta) = 1$
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$, $\sec(\theta) = \frac{1}{\cos(\theta)}$, $\csc(\theta) = \frac{1}{\sin(\theta)}$, $\cot(\theta) = \frac{1}{\tan(\theta)}$
Seno e Coseno di Somma e Differenza:
$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
Seno e Coseno dell'Angolo Duplo: $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
Seno e Coseno dell'Angolo Metà: $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
Formule delle Funzioni Inverse: $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$
$\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$
Formule del Prodotto: $\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
$\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
Sinc: $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}, \quad \text{per } x \neq 0$

Countinuous Time - Chapter 3

Trasformata di Fourier: $X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} \, dt$
Periodic signal: $x(t) = \sum_{n=-\infty}^{\infty} x(t + nT)$
Inner product: $\langle \mathbf{v}, \mathbf{w} \rangle = \sum_{i=1}^{n} v_i w_i$
Parseval's Rule: $\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df$
Spettro di potenza: $S_x(f) = |X(f)|^2$
Valore Medio: $\bar{x} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt$
Energy of a signal: $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$
Power of a periodic signal: $P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$
Time Average: $\bar{x} = \frac{1}{T2 - T1} \int_{T1}^{T2} x(t) \, dt$
Instantaneous Power: $P(t_0) = |x(t_0)|^2$
Scalar Product: $\langle x, y \rangle = \int x(t) \cdot y(t) \, dt$
Norm: $\|x\| = \sqrt{\int |x(t)|^2 \, dt}$
Orthogonality: $\langle x, y \rangle = 0$