Signal and Systems Formula Sheet

Countinuous Time - Chapter 2

Dirac's delta: $\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1$, $\int_{-\infty}^{\infty} \delta(a[t-t_0])\, dt = \frac{1}{|a|} S(t_0)$,
$\int_{-\infty}^{\infty} x(t)\delta(t)\, dt = x(0)$, $\int_{-\infty}^{\infty} f(t)\delta(t-a)\, dt = f(a)$
$x(t) * \delta(t-t_0) = x(t_0)\delta(t-t_0)$, $x(t) * \delta(t) = x(t)$, $x(t) * \delta(t-\theta) = x(t-\theta)$

Countinuous Time - Chapter 3

Time Average: $\bar{x} = \frac{1}{T2-T1} \int_{T1}^{T2} x(t)\, dt$
Instantaneous Power: $p(t) = |x(t)|^2$
Average Power (Periodic Signal): $P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt = E/T$
Energy of a signal: $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$
Periodic signal: $x(t) = \sum_{n=-\infty}^{\infty} x(t+nT)$

Countinuous Time - Chapter 4

Norm: $\|x\| = \sqrt{\int |x(t)|^2\, dt}$
Distance: $\|\mathbf{v}\| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$
Inner Product: $\langle f, g \rangle = \int_a^b f(x)\overline{g(x)}\, dx$
Orthogonality: $\langle \mathbf{u}, \mathbf{v} \rangle = 0$
Signal distance: $d = \sqrt{\sum_i (x_i - y_i)^2}$
Parseval's Rule: $\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$
Schwarz's Inequality: $|\langle \mathbf{u}, \mathbf{v} \rangle| \leq \|\mathbf{u}\| \cdot \|\mathbf{v}\|$
Projection of a Vector onto an Orthonormal Set:
$\mathbf{v}_{\text{proj}} = \langle \mathbf{v}, \mathbf{u}_1 \rangle \mathbf{u}_1 + \langle \mathbf{v}, \mathbf{u}_2 \rangle \mathbf{u}_2 + \ldots + \langle \mathbf{v}, \mathbf{u}_n \rangle \mathbf{u}_n$
Approximation: $\mathbf{v}_{\text{approx}} = \sum_{i=1}^{n} \langle \mathbf{v}, \mathbf{u}_i \rangle \mathbf{u}_i$
Error: $\mathbf{v}_{\text{error}} = \mathbf{v} - \mathbf{v}_{\text{approx}}$

Countinuous Time - Chapter 5

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi n t}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Countinuous Time - Chapter 6

Continuous Fourier Transform (CFT): $X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t}\, dt$
Inverse Continuous Fourier Transform: $x(t) = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi f t}\, df$
Discrete Fourier Transform (DFT): $X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn}$
Inverse Discrete Fourier Transform (IDFT): $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j\frac{2\pi}{N}kn}$
Fourier Transform Pairs: $X(f) \rightleftharpoons x(t)$ - $X(k) \rightleftharpoons x(n)$
Frequency Shift Property: $\mathcal{F}\{x(t)e^{j2\pi f_0 t}\} = X(f - f_0)$
Time Scaling (or Compression) Property: $\mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{f}{a}\right)$
Time Shifting Property: $\mathcal{F}\{x(t-t_0)\} = X(f) \cdot e^{-j2\pi f t_0}$
Convolution Theorem: $\mathcal{F}\{x(t) * h(t)\} = X(f) \cdot H(f)$
Parseval's Theorem: $\int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(f)|^2\, df$

Countinuous Time - Chapter 7

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi n t}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Countinuous Time - Chapter 8

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi n t}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Countinuous Time - Chapter 9

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi n t}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Countinuous Time - Chapter 10

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi n t}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Countinuous Time - Chapter 11

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi n t}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Discrete Time - Chapter 1

Trasformata di Fourier Discreta (DFT): $X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}$
Parseval's Rule Discrete Time: $\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$
Base di Fourier per sequenze periodiche:
La base di Fourier per sequenze periodiche è utilizzata per rappresentare segnali discreti periodici di lunghezza N. I componenti della base includono armoniche discrete: $e_k[n] = e^{j\frac{2\pi kn}{N}}, \quad k = 0, 1, 2, \ldots, N-1$
Dove:
- e è la base dell'esponenziale complesso.
- j rappresenta l'unità immaginaria.
- k è un intero che rappresenta l'armonica.
- n è l'indice temporale discreto.

Gram-Schmidt:
1. Ortogonalizzazione: $u_i = v_i - \sum_{j=1}^{i-1} \frac{\langle v_i, u_j \rangle}{\langle u_j, u_j \rangle} u_j$
2. Normalizzazione (Ortonormalizzazione): $u_i = \frac{u_i}{\|u_i\|}$

Discrete Time - Chapter 2

Linear: $y[n] = a_1 y_1[n] + a_2 y_2[n]$
Time-invariant: $y[n] = x[n] * h[n]$

Discrete Time - Chapter 3

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi nt}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$

Discrete Time - Chapter 4

Fourier Series: $c_n = \frac{1}{T} \int_0^T f(t) e^{-i\frac{2\pi nt}{T}}\, dt$
Fourier Basis: $e^{j\omega t}$