

Signal Analysis Formulae Sheet

Countinuous Time - Chapter 2-3

Trasformata di Fourier: $X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} dt$

Periodic signal: $x(t) = \sum_{n=-\infty}^{\infty} x(t + nT)$

Inner product: $\langle \mathbf{v}, \mathbf{w} \rangle = \sum_{i=1}^n v_i w_i$

Parseval's Rule: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$

Spettro di potenza: $S_x(f) = |X(f)|^2$

Valore Medio: $\bar{x} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt$

Energy of a signal: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

Power of a periodic signal: $P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

Time Average: $\bar{x} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} x(t) dt$

Instantaneous Power: $P(t_0) = |x(t_0)|^2$

Scalar Product: $\langle x, y \rangle = \int x(t) \cdot y(t) dt$

Norm: $\|x\| = \sqrt{\int |x(t)|^2 dt}$

Orthogonality: $\langle x, y \rangle = 0$

Countinuous Time - Chapter 4

Schwarz's Inequality (Cauchy-Schwarz Inequality):

For vectors u and v in an inner product space, it states:

$$|\langle u, v \rangle|^2 \leq \langle u, u \rangle \cdot \langle v, v \rangle$$

Orthonormal Set:

An orthonormal set u_i is a set of vectors in an inner product space such that:

$$\langle u_i, u_j \rangle = \delta_{ij}$$

Orthonormal Basis:

An orthonormal basis is a set of vectors that is both orthonormal and forms a basis for the vector space. If u_i is an orthonormal basis, then for any vector v in the space:

$$v = \sum_i \langle v, u_i \rangle \cdot u_i$$

Countinuous Time - Chapter 5

Fourier Basis: $e^{j\omega t}$

Where:

- e is the base of the natural logarithm (approximately 2.71828).
- j represents the imaginary unit ($j^2 = -1$).
- ω is the angular frequency (measured in radians per second).
- t is time.

Discrete Time - Chapter 1

Trasformata di Fourier Discreta (DFT): $X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}$

Parseval's Rule Discrete Time: $\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$

Base di Fourier per sequenze periodiche:

La base di Fourier per sequenze periodiche è utilizzata per rappresentare segnali discreti periodici di lunghezza N . I componenti della base includono armoniche discrete: $e_k[n] = e^{j\frac{2\pi kn}{N}}$, $k = 0, 1, 2, \dots, N-1$

Dove:

- e è la base dell'esponenziale complesso.
- j rappresenta l'unità immaginaria.
- k è un intero che rappresenta l'armonica.
- n è l'indice temporale discreto.

Gram-Schmidt:

1. Ortogonalizzazione: $u_i = v_i - \sum_{j=1}^{i-1} \frac{\langle v_i, u_j \rangle}{\langle u_j, u_j \rangle} u_j$
2. Normalizzazione (Ortonormalizzazione): $u_i = \frac{u_i}{\|u_i\|}$

Discrete Time - Chapter 2

Linear: $y[n] = a_1 y_1[n] + a_2 y_2[n]$

Time-invariant: $y[n] = x[n] * h[n]$

Trigonometry

Identità Trigonometriche Fondamentali:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)}$$

Seno e Coseno di Somma e Differenza:

$$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$$

$$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$$

$$\sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)$$

$$\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$$

Seno e Coseno dell'Angolo Duplo: $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2 \cos^2(\theta) - 1 = 1 - 2 \sin^2(\theta)$$

Seno e Coseno dell'Angolo Metà: $\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$$

Formule delle Funzioni Inverse: $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$

$$\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$$

Formule del Prodotto: $\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$

$$\cos(A) \cos(B) = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

Sinc: $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, per $x \neq 0$

