Assignment 3

Group members' name:

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Question 1

(a) Pigeonholes : score (0-100) = 101

pigeon : students : ?

- Pigeonhole state that pigeon > pigeonholes which is there will always be at least two pigeons in one hole.
- if each students have each one of the score ~~which is~~ from 0 to 100 and at least two students received the same score, then the students in the class need to be at least 102.

(b) pigeons - students (n = ?).

pigeonhole - grade (m = 5)

$k = 6$

$k = \left\lceil \frac{n}{m} \right\rceil$

$6 = \left\lceil \frac{n}{5} \right\rceil$, m = 30.

$\therefore$ minimum students = 30.

Question 2.

2(a) $P(A) = 0.7$

(b) $P(B) = 0.3$

(c) $P(C|A) = 0.2$

(d) $P(A \cap C) = ?$

$P(C|A) = \frac{P(C \cap A)}{P(A)}$

$0.2 = \frac{P(C \cap A)}{0.7}$

$P(C \cap A) = 0.14$.

(e) $P(B \cap C) = ?$

$P(C|B) = 0.4$

$P(C|B) = \frac{P(C \cap B)}{P(B)}$

$0.4 = \frac{P(C \cap B)}{0.3}$

$P(C \cap B) = 0.12$.

(f) $P(C) = ?$

$P(B|C) = \frac{P(C|B)P(B)}{P(C|B)P(B) + P(C|A)P(A)}$

$= \frac{(0.4)(0.3)}{(0.4)(0.3) + (0.2)(0.7)}$

$= 0.462$

$P(B|C) = \frac{P(B \cap C)}{P(C)}$ $\therefore P(C) = 0.26$.

~~(d) P(A|C)~~

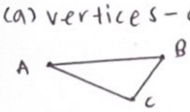
(g) $P(A|C) = \frac{P(C|A)P(A)}{P(C|A)P(A) + P(C|B)P(B)}$

$= \frac{(0.2)(0.7)}{(0.2)(0.7) + (0.4)(0.3)}$

$= 0.538$.

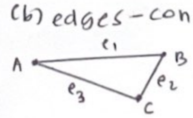
Question 3.

(a) vertices - objects that are connected together.



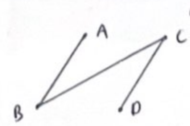
$V = \{A, B, C\}$

(b) edges - connection between the vertices.



$E = \{e_1, e_2, e_3\}$

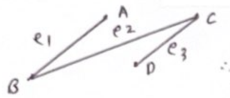
(c) adjacent vertices - two vertices that are connected by an edge are called adjacent.



$\therefore$ B and C are adjacent.

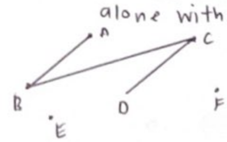
Question 3

(d) incident edge - edge that connects a particular vertex.



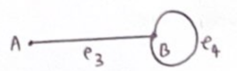
$\therefore$ e_1 and e_2 are incident edge.

(e) isolated vertex - vertex that is not connected to any other vertex by an edge. It stands alone without any direct connections.



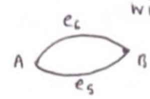
$\therefore$ E and F are isolated vertex.

(f) loop - an edge with just one endpoint.



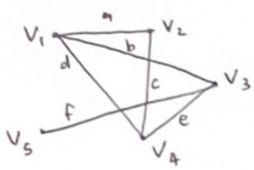
$\therefore$ e_4 is a loop.

(g) parallel edge - two or more dictinct edges with the same set of endpoints.



$\therefore$ e_5 and e_6 are parallel.

Question 4.



$\deg(V_1) = 3$

$\deg(V_2) = 2$

$\deg(V_3) = 3$

$\deg(V_4) = 3$

$\deg(V_5) = 1$.

Question 5.

i. incidence matrix

$V = \{1,2,3,4,5,6\}$ $E = \{a,b,c,d,e,f,g,h,i,k\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccccccc} a & b & c & d & e & f & g & h & i & k \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

(ii) adjancency matrix.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Question 6

* both graph have the same number of vertices and edges.
* $f: Y \to Z$, where $Y = \{A,B,C,D,E,F\}$ and $Z = \{1,2,3,4,5,6\}$;

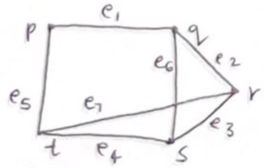
$f(A) = 6$; $f(B) = 5$; $f(C) = 4$; $f(D) = 3$; $f(E) = 2$; $f(F) = 1$.

$$A_Y = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$A_Z = \begin{array}{c} \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{array}\begin{array}{c} \begin{array}{cccccc} 6 & 5 & 4 & 3 & 2 & 1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore$ both graphs are isomorphic.

Question 7



(i) (p, e_5, t), $(p, e_1, q, e_6, s, e_4, t)$, $(p, e_1, q, e_2, r, e_7, t)$, $(p, e_1, q, e_2, r, e_3, s, e_4, t)$, $(p, e_1, q, e_6, s, e_3, r, e_7, t)$.

(ii) (p, e_5, t), $(p, e_1, q, e_6, s, e_4, t)$, $(p, e_1, q, e_2, r, e_7, t)$, $(p, e_1, q, e_2, r, e_3, s, e_4, t)$, $(p, e_1, q, e_6, s, e_3, r, e_7, t)$.

(ii) shortest path : (p, e_5, t)

longest path : $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ /

$(p, e_1, q, e_6, s, e_5, r, e_7, t)$

(iii) shortest trail : (p, e_5, t)

longest trail : $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ /

$(p, e_1, q, e_6, s, e_3, r, e_7, t)$.