Math 33B: Differential Equations

Homework 2: Modeling with 1st order ODEs & Exact ODEs

Due on: Fri., Apr. 19, 2013 - 9:00 AM *Instructor: Aliki M.*

Please include your *name*, *UID* and *discussion section* on the submitted homework.

Problem 1

A tank initially holds 100 gallons of solution containing 30 lbs of dissolved salt. Fresh water flows into the tank at the rate of 2 gal/min while the solution (water+salt) flows out of the tank at the rate of 3 gal/min.

(i) Formulate the IVP and determine the amount of salt, $y(t)$ in the tank at any time t;

(ii) Determine the salt content of the solution after 10 minutes.

Problem 2

A tank initially contains 100 gallons of fresh water. A mixture containing 0.5 lbs/gal of salt flows into the tank at a rate of 2 gal/min and the mixture is allowed to flow out of the tank at the same rate. After 10 minutes the process is stopped and fresh water is added to the tank at the rate of 2 gal/min, with the mixture again leaving at the same rate.

Find the amount of salt in the tank after an additional 10 minutes.

Problem 3

Consider a tank initially containing 1,000 gallons of pure water. An alcohol-water mixture is added at a rate of 3 gal/min. The alcohol-water mixture being added is 75% alcohol. A valve is open at the bottom of the tank allowing the mixture to flow out at 3 gal/min.

Let $y(t)$ denote the number of *gallons* of pure alcohol in the tank.

(i) Formulate the IVP for $y(t)$;

(ii) Solve the differential equation for $y(t)$;

(iii) When will the mixture in the tank be 50% alcohol?

Problem 4

Check whether the following ODEs are exact and find the general solutions of the ones that are:

1. $(2x + y) + (x + 2y)\dfrac{dy}{dx} = 0$;
2. $(x + 2y) + (2x + y)\dfrac{dy}{dx} = 0$;
3. $[2xy - x\sin(xy)] + [y^2 - y\sin(xy)]\dfrac{dy}{dx} = 0$;
4. $[y^2 - y\sin(xy)] + [2xy - x\sin(xy)]\dfrac{dy}{dx} = 0$.

Problem 5

In the following differential equation, determine the constant a such that the equation is *exact* and solve the resulting exact equation,

$$\left(\frac{1}{x^2} + \frac{1}{y^2}\right) + \left(\frac{ax + 1}{y^3}\right)\frac{dy}{dx} = 0.$$