Math 33B: Differential Equations

Practice problems I

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Problem 1

$$y^3 - 4y - x^3 + 1 = 0$$

The solution is defined within $|x^3 - 1| < \dfrac{16}{3\sqrt{3}}$.

Problem 2

$$y = -\frac{\sqrt{x^2+1}}{\sqrt{2}}$$

The solution is defined for all values of x, i.e. within $-\infty < x < \infty$.

Problem 3

Critical points occur at $y = 0$ and $y = 1$. The equilibrium solutions are therefore at $y(x) = 0$ and $y(x) = 1$ for all values of x.

Problem 4

$$y(x) = \frac{2}{e^{x^2}+1}$$

Problem 5

(i)
$$\frac{dy}{dt} = 6 - \frac{y}{25}, \quad y(0) = 75$$

(ii) At $t = 25 \ln 3$ mins ($t \approx 27.47$ minutes).

(iii) 150 lbs.

Problem 6

(i)
- IVP:
$$\frac{dy}{dt} = \frac{9}{1000} - \frac{0.15y}{1800}, \quad y(0) = 0$$
- Solution:
$$y(t) = 108(1 - e^{-t/12000})$$

(ii) At or before 36 minutes after the smoking starts.

Problem 7

(i)

$$y(x) = \Big(\frac{k}{b - \lambda a}\Big)e^{-\lambda x} + ce^{-bx/a}$$

(iii) The limiting value for y is 0.

Problem 8

$a = 1$ and the general solution is,

$$e^{2xy} + x^2 = c_1.$$

Problem 9

(i) For $a \leq 0$, $y = 0$ is the only critical point and in both cases $y = 0$ is an asymptotically stable equilibrium solution.

For $a > 0$, $y = 0$ and $y = \pm\sqrt{a}$ are the critical points. $y = 0$ is asymptotically unstable and $y = \pm\sqrt{a}$ are both asymptotically stable.

Problem 10

(i) $y = 0$ and $y = 1$ are the critical points.

$y = 0$ is asymptotically unstable while $y = 1$ is asymptotically stable.

(ii) As $t \to \infty$, all solutions approach $y = 1$ meaning that the entire population will be infected by the disease (note that a starting point of $y_0 < 0$ has no physical sense).