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Phase Plane Portraits

1 message

Monica Visan <visan.m@gmail.com> Sat, Mar 2, 2013 at 7:49 PM
To: Jason Murphy <jason.carl.murphy@gmail.com>, Sungjin Kim <707107@gmail.com>, Sungjin Kim <i707107@math.ucla.edu>, Frederick Robinson <frobinson@math.ucla.edu>

Dear all,

Please don't say (as in the book) that solutions (or separatrices) are stable or unstable. *Stability is a property of equilibrium points, not of solutions.* I would like everybody to get this right, so that I don't have to give unnecessary zeros on the final when testing this material.

In a language that these students understand, an equilibrium point is stable if solutions starting close to it remain close to it as t \to \infty. More precisely, for any \eps>0 there exists a \delta>0 so that if a solution starts within the \delta ball centered at the equilibrium point, then it stays within the \eps ball for all t>0.

If an equilibrium point is not stable, we call it unstable.

An equilibrium point is asymptotically stable if it is stable and all solutions starting close to it converge to it as t \to \infty. More precisely, there is a \delta>0 so that if a solution starts within the \delta ball centered at the equilibrium point, then it converges to the equilibrium point as t \to \infty.

Also, I will cover in class the cases when \lamba is an eigenvalue with multiplicity 2 (both the diagonalizable and the Jordan block cases), so be prepared for questions about that. In the subcase when the eigenspace is the whole of \R^2, if \lambda<0 we call the equilibrium point a star sink, while if \lambda>0 we call the equilibrium point a star source.

I will also cover the case when one of the eigenvalues is zero. In this case we get a whole line of equilibrium points, and I will let you have fun determining if they are stable or not.

Let me know if you have any questions.

Best regards,
Monica Visan