Math 33B: Differential Equations

Homework 4: Homogeneous, second order linear ODEs

Due on: Fri., May 3, 2013 - 9:00 AM *Instructor: Aliki M.*

Please include your *name*, *UID* and *discussion section* on the submitted homework.

Problem 1

Find the Wronskian determinant, $W(x)$ of the following pair of functions:

(i) $f(x) = e^{2x}$, $g(x) = e^{-3x/2}$
(ii) $f(x) = e^{-2x}$, $g(x) = xe^{-2x}$
(iii) $f(x) = x$, $g(x) = xe^{x}$

[Answers: (i) $-\frac{7}{2}e^{x/2}$, (ii) e^{-4x}, (iii) x^2e^{x}]

Problem 2

Verify that $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are two solutions to the differential equation $t^2y'' - 2y = 0$, $t > 0$.

Then show that $y(t) = c_1t^2 + c_2t^{-1}$ is also a solution to the ODE for any c_1 and c_2.

Problem 3

Verify that $y_1(x) = \cos(2x)$ and $y_2(x) = \sin(2x)$ are two solutions to the differential equation $y'' + 4y = 0$.

Do they constitute a fundamental set of solutions?

Problem 4

Show that $y = x^2 \sin x$ and $y = 0$ are both solutions to

$$x^2y'' - 4xy' + (x^2 + 6)y = 0$$

and that they both satisfy $y(0) = 0$ and $y'(0) = 0$.

Explain why this doesn't contradict the hypotheses of the existence and uniqueness theorem.

Problem 5

Find the particular solution $y(x)$ of

$$y'' - 3y' + 2y = 0$$

for which $y(0) = 1$ and $y'(0) = 0$.

[Answer: $y(x) = 2e^{x} - e^{2x}$]

Problem 6

Find the general solution of each of the following equations:

(i) $y'' + 4ky' - 12k^2y = 0$

(ii) $y'' + 8y = 0$

(iii) $y'' - 2ay' + a^2y = 0$

[Answers: (i) $y(x) = c_1e^{-6kx} + c_2e^{2kx}$, (ii) $y(x) = c_1\cos(2\sqrt{2}x) + c_2\sin(2\sqrt{2}x)$, (iii) $y(x) = e^{ax}(c_1 + c_2x)$]

Problem 7

Find all the nonzero solutions, $y(t)$ of the equation,

$$y'' - 2y' + 10y = 0,$$

and determine their behavior as $t \to \infty$.

[Answer: $y(t) = e^t\big(c_1\cos(3t) + c_2\sin(3t)\big)$]