**Math 33b, Winter 2013, Tonći Antunović - Homework 4 solutions**

From the textbook solve the problems:
Section 2.9: 8, 10, 14, 18, 20
Section 4.1: 2, 6, 8, 16, 18
Section 4.3: 2, 4, 6, 8, 26, 30, 32

And also the problems below:

**Problem 1.** Find the general solution of the differential equation

$$y'' + 2y' - 3y = 0.$$

**Solution:** The characteristic equation is $\lambda^2 + 2\lambda - 3 = 0$ whose roots are $\lambda_1 = -3$ and $\lambda_2 = 1$ and the general solution is

$$y = C_1 e^{-3t} + C_2 e^t.$$

**Problem 2.** Find the solution of the initial value problem

$$y'' + y' = 20y, \quad y(0) = 1, \quad y'(0) = 1.$$

**Solution:** The characteristic equation of the equation $y'' + y' - 20y = 0$ is $\lambda^2 + \lambda - 20 = 0$ and the roots are $\lambda_1 = -5$ and $\lambda_2 = 4$. The general solution of this equation is

$$y = C_1 e^{-5t} + C_2 e^{4t}.$$

Now plug this into initial conditions

$$y(0) = C_1 + C_2 = 1, \quad y'(0) = -5C_1 + 4C_2 = 1.$$

This gives $C_1 = 1/3$ and $C_2 = 2/3$ so

$$y = \frac{1}{3} e^{-5t} + \frac{2}{3} e^{4t}.$$

**Problem 3.** Write down the linear homogeneous second order differential equation whose general solution is given by

$$y(t) = C_1 e^{6t} + C_2 e^{-2t}.$$

**Solution:** The equation should correspond to a characteristic equation whose roots are $\lambda_1 = 6$ and $\lambda_2 = -2$. One such equation is $(\lambda-6)(\lambda+2) = 0$ or $\lambda^2 - 4\lambda - 12 = 0$, which corresponds to the equation

$$y'' - 4y' - 12y = 0.$$

**Problem 4.** Use the substitution $z = y^2$ to find the general solution of the differential equation

$$yy'' + (y')^2 - yy' - 3y^2 = 0$$

**Solution:** Since $z' = 2yy'$ and $z'' = 2yy'' + 2(y')^2$ the above equation can be rewritten as

$$z'' - z' - 6z = 0.$$

The characteristic equation is $\lambda^2 - \lambda - 6 = 0$ whose roots are $\lambda_1 = -2$ and $\lambda_2 = 3$. Therefore the general solution is

$$z = C_1 e^{-2t} + C_2 e^{3t},$$

and

$$y = \sqrt{C_1 e^{-2t} + C_2 e^{3t}}, \qquad y = -\sqrt{C_1 e^{-2t} + C_2 e^{3t}}.$$