**Midterm 1 practice, Math 33b, Winter 2013**
**Instructor: Tonći Antunović**

Name and student ID: _____

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total: | 50 | |

1. (a) (2 points) Verify that $x = \frac{1}{t-1}$ is a particular solution of the equation

$$tx' + x^2 + x = 0.$$

(b) (2 points) General solution of the equation $y' = y \cos t$ is given by

$$y = Ce^{\sin t}.$$

Find the solution of the initial value problem $y' = y \cos t$, $y(\pi/2) = 2$.

(c) (2 points) The graph of the function $y(x)$, $x \geq 0$ passes through the origin $(0, 0)$. The slope of the tangent line at the point $(x, y(x))$ is equal to the area of the rectangle whose opposite vertices are $(0, 0)$ and $(x, y(x))$. Write the initial value problem that $y(x)$ satisfies (you don't have to solve it).

(d) (2 points) Does there exist a solution of the equation $y' = e^x y^2 + e^{y^5}$ such that $y'(0) = -1$? No need to solve to equation, but explain your answer.

(e) (2 points) Is the following differential equation exact

$$(x + x \sin y)\ dx + \cos y\ dy = 0.$$

2. (10 points) Find the solution of the initial value problem

$$y^2y' - e^{y^3} = te^{y^3}, \quad y(0) = 0.$$

3. (10 points) Find the general solution of the equation

$$y' + y \sin t = e^{\cos t} \sin t.$$

4. (10 points) A 10 gallon tank contains a mixture of water and a pound of salt. A pure water is entering the tank at the rate of 1 gallon per second and the mixture is leaving the tank at the same rate. Find the amount of salt in the tank after time $t$.

5. (10 points) Show that the following differential equation is exact and find the general solution.

$$(x + y \sin x)\ dx - \cos x\ dy = 0.$$