Midterm 2, Math 33b, Winter 2013
Instructor: Tonći Antunović

Printed name and student ID: ____________________

Signed name: ____________________

Section number, time and TA name: ____________________

Instructions:

- Read problems very carefully. Please raise your hand if you have questions at any time.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can. This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely. However, try to save enough time to seriously attempt to solve all the problems.
- If it's obvious that you final answers can be simplified, please simplify them. Otherwise, your final answers need to be simplified only if this is required in the statement of the problem.
- You are allowed to use only the items necessary for writing. Notes, books, sheets, calculators, phones are not allowed, please put them away. If you need more paper please raise your hand (you can write on the back pages).

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (a) (2 points) Let $f\colon \mathbb{R} \to \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} -1, & \text{for } x \leq 1, \\ 2, & \text{for } x > 1. \end{cases}$$

Determine all possible values of x_0 for which the initial value problem

$$y' = |y+1| - f(x), \qquad y(x_0) = 0$$

might have no solutions.

(b) (2 points) If y is the solution of the initial value problem

$$y' = \cos y - \sin y, \qquad y(0) = 0,$$

show that $y(t) < \pi/4$ for all $t \geq 0$.

(c) (2 points) Let y_1 be the solution of the initial value problem

$$y' = \cos y - \sin y, \qquad y(0) = 0,$$

and y_2 the solution of the initial value problem

$$y' = \cos y - \sin y, \qquad y(0) = 2\pi.$$

Show that $y_2(t) = y_1(t) + 2\pi$ for all $t \geq 0$.

(d) (2 points) Use the substitution $z = y'$ to find the general solution of the equation $y''' - y' = 0$.

(e) (2 points) If y_1 and y_2 are two particular solutions of the differential equation $y'' - ty' + e^t y = 2\sin t$ write down a differential equation whose particular solution is $y_p = y_1 - 3y_2$.

2. (10 points) Consider the differential equation $y' = (y^2 - 1)(y + 2)^2$. Find all equilibrium solutions and for each of them determine whether it's stable or unstable. Consider the initial value problem

$$y' = (y^2 - 1)(y + 2)^2, \quad y(0) = y_0.$$

Find all values of y_0 for which the solution of this initial value problem satisfies $\lim_{t\to\infty} y(t) = -2$. Find all values y_0 for which the solution satisfies $\lim_{t\to\infty} y(t) = -1$.

3. (10 points) Find the general solution of the differential equation

$$y'' + 6y' + 9y = \frac{e^{-3t}}{t^3}.$$

4. (10 points) Find the solution of the initial value problem

$$y'' + y = t^2 - \sin t, \quad y(0) = 1, \quad y'(0) = 1.$$

5. (10 points) An object of mass $m = 1$kg is attached to a spring of constant k and immersed into a liquid so that the system oscillates with the damping constant μ. The position of the object at time t is $y(t) = e^{-t} + e^{-2t}$. Find the values of k and μ as well as the initial position and velocity (at time $t = 0$). Now the system is removed from the liquid so that the damping constant is now equal to zero. The object is pulled downward from equilibrium by 1 meter and released from rest. Find the position of the object at time t and determine the amplitude and the phase of oscillations.