MATH 33B: DIFFERENTIAL EQUATIONS — APRIL 2013

Example: drawing direction fields.

Plot the direction field for:

$$\frac{dy}{dx} = x^2 + y^2 \quad (1)$$

on $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ & $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

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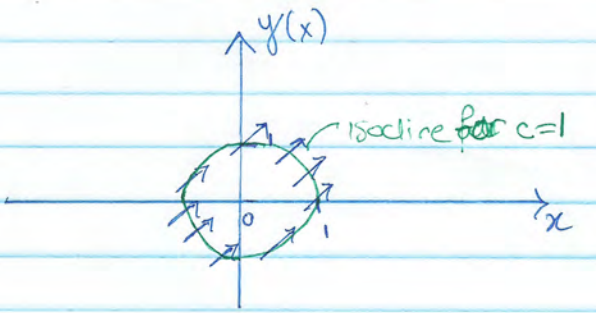
1) Using the method of isoclines, we set Eq. (1) equal to a constant, c:

$$\therefore \boxed{x^2 + y^2 = c} \quad (2)$$

Eq. (2) represents the equation of the isoclines. These are represented by a family of circles of radius $\sqrt{c}$ centered at (0,0).

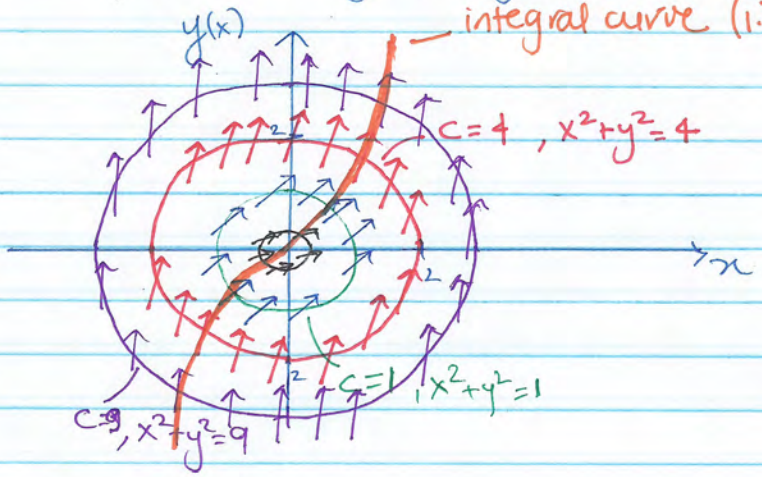
2) We pick different values of c and draw several isoclines
e.g. at $c=1$, (2) $\Rightarrow x^2 + y^2 = 1$ (this is a circle with radius 1)



3) Now, draw arrows along the isoclines that have a slope equal to c.

e.g. for c=1, the arrows drawn on the green circle above all have slope of c=1.

Repeating steps 2) and 3) for various values of c gives us the direction field for $y' = x^2 + y^2$.



Having drawn the direction field, we can visualize the behavior of the solution given that we know that this will be tangent to the arrows.

A solution satisfying (0,0) is shown in orange above.

The direction field for $y'=x^2+y^2$ and an integral curve passing through (0,0) is shown in orange
[Computed in MATLAB]