Math 33B-2, Fall 2012, Quiz 3 (Th)

Section __________ Name KEY

Q1 (5 pts). Find the general solution of $\mathbf{y}' = A\mathbf{y}$ for

$$A = \begin{pmatrix} 4 & 2 \\ 1 & 5 \end{pmatrix}.$$

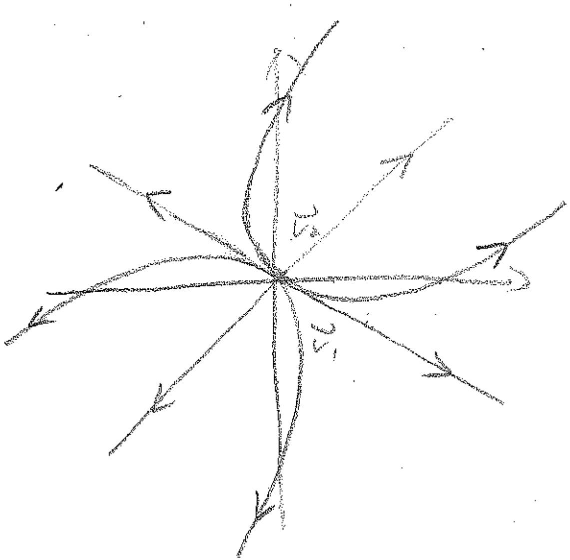
And sketch a rough approximation of a solution in each region determined by the half-line solutions. Classify the type of the equilibrium point.

$p(\lambda) = \det(A - \lambda I) = \det\begin{pmatrix} 4-\lambda & 2 \\ 1 & 5-\lambda \end{pmatrix} = \lambda^2 - 9\lambda + 18 = (\lambda-3)(\lambda-6)$

$\lambda_1 = 3.\ (A - \lambda_1 I)\vec{v}_1 = 0 \Rightarrow \begin{pmatrix} +1 & +2 \\ 1 & 2 \end{pmatrix}\vec{v}_1 = 0 \Rightarrow \vec{v}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$\lambda_2 = 6\ (A - \lambda_2 I)\vec{v}_2 = 0 \Rightarrow \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix}\vec{v}_2 = 0 \Rightarrow \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\vec{y}(t) = c_1 e^{3t}\begin{pmatrix} 2 \\ -1 \end{pmatrix} + c_2 e^{+6t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



Nodal source.

Q2 (5 pts). Find the solution of the initial value problem for system $\mathbf{y}' = A\mathbf{y}$, with

$$A = \begin{pmatrix} -2 & 1 \\ -9 & 4 \end{pmatrix}, \qquad \mathbf{y}(0) = \begin{pmatrix} 5 \\ 3 \end{pmatrix}.$$

$$p(\lambda) = \det(A - \lambda I) = \det\begin{pmatrix} -2-\lambda & 1 \\ -9 & 4-\lambda \end{pmatrix} = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2$$

$$\lambda = 1. \quad (A - \lambda I)\vec{v}_1 = 0 \Rightarrow \begin{pmatrix} -3 & 1 \\ -9 & 3 \end{pmatrix}\vec{v}_1 = 0 \Rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{Y}_1 = e^t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

The other solution is $\vec{Y}_2 = e^{\lambda t}(\vec{v}_2 + t\vec{v}_1)$

with $\begin{cases} A\vec{v}_1 = \lambda \vec{v}_1 \\ (A - \lambda I)\vec{v}_2 = \vec{v}_1 \end{cases}$

$$\Rightarrow \lambda = 1, \quad \vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

let $\vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\quad (A - \lambda I)\vec{w} = \begin{pmatrix} -3 & 1 \\ -9 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \vec{v}_1$

$$\Rightarrow \vec{v}_2 = \vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\vec{Y}_2 = e^t\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \end{pmatrix}\right) = e^t\begin{pmatrix} t \\ 1+3t \end{pmatrix}$$

$$\vec{y}(t) = C_1\vec{Y}_1 + C_2\vec{Y}_2 = e^t\left(C_1\begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2\begin{pmatrix} t \\ 1+3t \end{pmatrix}\right)$$

$$\vec{y}(0) = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 5 \\ 3 \end{pmatrix} = C_1\begin{pmatrix} 1 \\ 3 \end{pmatrix} + C_2\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} C_1 = 5 \\ C_2 = -12 \end{matrix}$$

$$\Rightarrow \vec{y}(t) = e^t\left(5\begin{pmatrix} 1 \\ 3 \end{pmatrix} - 12\begin{pmatrix} t \\ 1+3t \end{pmatrix}\right) = e^t\begin{pmatrix} 5-12t \\ 3-36t \end{pmatrix}$$