

Final practice, Math 33B - Lec 3, Winter 2013
Instructor: Tonći Antunović

Name and student ID: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (a) (2 points) Is the differential equation

$$2xy \, dx + (x^2y - 1) \, dy = 0$$

exact?

- (b) (2 points) Find all the values of y_0 for which the initial value problem might have multiple solutions

$$y' = x^4 + \sqrt[3]{y^2}, \quad y(0) = y_0.$$

- (c) (2 points) Is there a continuous function $f: \mathbb{R} \mapsto \mathbb{R}$ such that the autonomous equation $x' = f(x)$ has exactly two equilibrium solutions both of which are stable?

- (d) (2 points) Can

$$y = C_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + C_2 e^{-t} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} + C_3 e^{-2t} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

be the general solution to the equation $y' = Ay$ for some A ?

- (e) (2 points) Classify the equilibrium point of $y' = Ay$ where

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 1 \end{pmatrix}.$$

2. (10 points) Solve the initial value problem

$$x' - x = xt + t + 1, \quad x(0) = -2.$$

3. (10 points) A tank contains 50 gallons of pure water. Salty water starts pouring into the tank at the rate of 1 gallon per minute containing 0.5 pounds of salt per gallon. At same time a whole opens at the bottom of the tank and the water starts flowing out of the tank through it at the rate of 0.1 gallon per minute. Find the amount of salt in the tank after t minutes. What is the limiting amount of salt and the limiting concentration of salt as $t \rightarrow \infty$. You can assume the tank to have an unlimited capacity.

4. (10 points) Determine all the equilibrium solutions of the equation $x' = (x^5 - x)(e^{x-1} - 1)$, and for each determine whether it's stable or unstable. Sketch the direction field.

5. (10 points) Find the general solution of the equation

$$y'' - 2y' + y = e^t(1 + t^{-2}).$$

6. (10 points) An object of weight $m = 3$ is attached to the spring of constant $k = 12$. Find the damping constant μ which would make the system critical. The object starts oscillating on the spring without any damping so that it's extended by 1 upward and is given the velocity 2 downward. Determine the displacement of the object after time t , the amplitude and the phase of the motion.

7. (10 points) Solve the initial value problem $y' = Ay$, $y(0) = y_0$ where

$$A = \begin{pmatrix} 2 & -4 \\ 5 & -6 \end{pmatrix}, \quad y_0 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

8. (10 points) Sketch the phase portrait of the equation $y' = Ay$ and determine whether $\lim_{t \rightarrow \infty} y(t)$ exists for all solutions y if

(i)

$$A = \begin{pmatrix} -1 & 2 \\ 3 & 0 \end{pmatrix},$$

(ii)

$$A = \begin{pmatrix} 2 & 6 \\ -3 & -7 \end{pmatrix},$$

9. (10 points) Find the functions x_1, x_2, x_3 which satisfy

$$x_1' = -x_1 - x_2 - x_3$$

$$x_2' = -x_1$$

$$x_3' = 2x_1 - x_2$$

and $x_1(0) = x_2(0) = x_3(0) = 1$.

10. (10 points) Find the general solution to $y' = Ay$ if

(i)

$$A = \begin{pmatrix} -1 & -2 & 2 \\ 0 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix},$$

(ii)

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$