Midterm 2 practice, Math 33b, Winter 2013
Instructor: Tonći Antunović

Name and student ID: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (a) (2 points) Specify all values t_0 for which the initial value problem

$$y' = \frac{y^2 - xy}{x^2 - x}, \quad y(x_0) = 3,$$

may fail to have any solutions.

(b) (2 points) Write down an autonomous equation $x' = f(x)$ for which $x = 1$ is an unstable equilibrium solution and such that for any $x_0 < 1$ the solution of the initial value problem

$$x' = f(x), \quad x(0) = x_0,$$

satisfies $\lim_{t\to\infty} x(t) = 1$.

(c) (2 points) Write down the solution of any second order homogeneous linear differential equation with constant coefficients which satisfies $y(0) = 0$ and $y'(0) = 0$.

(d) (2 points) Check that $y = 1/t$ and $y = t^3$ are solutions of

$$t^2 y'' - ty' - 3y = 0,$$

and write down the general solution of this equation.

(e) (2 points) The solution to the initial value problem

$$y'' - qy = 0, \quad y(0) = 1, \quad y'(0) = y_1,$$

satisfies $\lim_{t\to\infty} y(t) = 0$. If $q > 0$ is a positive constant find y_1.

2. (10 points) Find all equilibrium solutions of the autonomous equation $y' = (e^y - 1)(y-1)^2(y-2)$ and for each of them determine whether it's stable or unstable. Also determine all the possible values of the limit $\lim_{t\to\infty} y(t)$.

3. (10 points) Find the solution of the initial value problem

$$y'' - 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

4. (10 points) Find the solution of the initial value problem

$$y'' + 4y' + 4y = 2e^{-2t} + t, \quad y(0) = 0, \quad y'(0) = 1.$$

5. (10 points) An object of mass m is attached to a spring of constant k. It is determined that to make the system critically damped one would need to set the damping constant to $\mu = 4$. When there is no damping the system oscillates with period 4π. Find the values of m and k. In the case when there is no damping the object is removed from the equilibrium is to $y(0) = -\sqrt{2}$ and is given an initial velocity $y'(0) = -1/\sqrt{2}$. Find the amplitude and the phase of oscillations.