MATH 33B: DIFFERENTIAL EQUATIONS April 2013

Example: Solving separable ODEs.

Solve $\frac{dy}{dx} = \cos^2 x \cos^2(2y)$ ①

Separating variables gives,

$$\frac{1}{\cos^2 2y}\frac{dy}{dx} = \cos^2 x.$$

which may be re-written as.

$$\sec^2 2y \, dy = \cos^2 x \, dx.$$

Integrating both sides gives,

$$\int \sec^2 2y \, dy = \int \cos^2 x \, dx. \quad ②$$

The LHS of Eq ② gives $\int \sec^2 2y \, dy = (\tan 2y) \cdot \frac{1}{2}$. ③

The RHS of Eq ② is $\int \cos^2 x \, dx$. ④

To integrate $\cos^2 x$ wrt x, use the double angle formula:

$$\cos 2x = \cos^2 x - \sin^2 x \quad ⑤$$

Using the identity, $\cos^2 x + \sin^2 x \equiv 1$,

$$\sin^2 x = 1 - \cos^2 x . \quad (6)$$

Sub. (6) in (5),

$$\cos 2x = 2\cos^2 x - 1$$

$$\Rightarrow \quad \cos^2 x = \frac{\cos 2x + 1}{2}. \quad (7)$$

Sub. (7) in (4),

$$\int \cos^2 x \, dx = \frac{1}{2}\int \cos 2x + 1 \quad dx$$

$$= \frac{1}{2}\left(\frac{\sin 2x}{2} + x + k\right) \quad (7).$$

Now, combining (7) & (3),

$$\frac{1}{2}\tan 2y = \frac{1}{2}\left(\frac{\sin 2x}{2} + x + k\right)$$

$$\boxed{\tan 2y = \sin x \cos x + x + k} \quad (8)$$

(note, we have used $\sin 2x = 2\sin x \cos x$)

Solving for $y(x)$;

$$\boxed{y(x) = \frac{1}{2}\tan^{-1}\left[\sin x \cos x + x + k\right]}$$

explicit general soln to the ODE given by Eq. (1).