Math 33b, Winter 2013, Tonći Antunović - Homework 3

From the textbook solve the problems:
Section 2.7: 2, 4, 6, 8, 10, 18 part(ii), 20 part(ii), 28, 30, 32.

And also the problems below:

Problem 1. Consider the initial value problem

$$x' = 2x - \tan t, \qquad x(0) = 1.$$

Without solving the equation, explain why this problem has a unique solution and determine it's interval of existence.

Problem 2. Consider the initial value problem

$$x' = e^t x^2 - 2x, \qquad x(0) = 1/2.$$

Explain why this problem has a unique solution x and show that it satisfies $0 < x(t) < e^{-t}$.

Problem 3. In the setting of questions 17-20 in the textbook solve the problem with

$$E(t) = \begin{cases} t, & 0 < t < 2, \\ -t, & t \geq 2. \end{cases}$$

Problem 4. Consider the initial value problem

$$x' = 2x + f(t), \qquad x(0) = 1,$$

where

$$f(t) = \begin{cases} 0, & t \leq 1, \\ ae^{2t}, & t > 1, \end{cases}$$

where a is a real parameter. Does there exist a non-zero value of the parameter a such that the solution to this equation is differentiable at $t = 1$?