Separable equations:

$$\frac{dy}{dt} = g(t)f(y) \quad \Rightarrow \quad \int \frac{dy}{f(y)} = \int g(t)\ dt.$$

Linear equations:

$$x' = ax + f \quad \Rightarrow \quad u(t)x(t) = \int u(t)f(t)\ dt + C, \text{ for the integrating factor } u(t) = e^{-\int a(t)dt}.$$

Integrating factor for the form $P(x,y)\ dx + Q(x,y)\ dy$ is

$$\mu(x) = e^{\int h(x)dx}, \qquad \text{if } h = \frac{1}{Q}\Big(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\Big) \text{ is a function of } x \text{ only,}$$

$$\mu(x) = e^{-\int g(y)dy}, \qquad \text{if } g = \frac{1}{P}\Big(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\Big) \text{ is a function of } y \text{ only.}$$

Harmonic motion:

$my'' + \mu y' + ky = 0$, with m, μ, k being mass, damping constant, and spring constant respectively.

Simple harmonic motion (undamped case):

$$y = a\cos(\omega_0 t) + b\sin(\omega_0 t) = A\cos(\omega_0 t - \phi),$$

with amplitude $A = \sqrt{a^2+b^2}$ and phase $-\pi < \phi \le \pi$ such that $\cos(\phi) = \dfrac{a}{\sqrt{a^2+b^2}}$, $\sin(\phi) = \dfrac{b}{\sqrt{a^2+b^2}}$

Method of undetermined coefficients; searching for a particular solution y_p of $y'' + py' + qy = f$:

$$y_p(t) = \begin{cases} ae^{rt}, & f(t) = e^{rt} \\ a\cos(\omega t) + b\sin(\omega t), & f(t) = \cos(\omega t) \text{ or } \sin(\omega t) \\ p(t), & f(t) = P(t); \quad \text{polynomials } P \text{ and } p \text{ of same degree} \\ p(t)\cos(\omega t) + q(t)\sin(\omega t), & f(t) = P(t)\cos(\omega t) \text{ or } f(t) = P(t)\sin(\omega t);\ P, p, q \text{ of same degree} \\ e^{rt}(a\cos(\omega t) + b\sin(\omega t)), & f(t) = e^{rt}\cos(\omega t), \text{ or } f(t) = e^{rt}\sin(\omega t) \\ e^{rt}(p(t)\cos(\omega t) + q(t)\sin(\omega t)), & f(t) = e^{rt}P(t)\cos(\omega t), \text{ or } f(t) = e^{rt}Q(t)\sin(\omega t);\ P, p, q \text{ of same degree} \end{cases}$$

Variation of parameters for $y'' + p(t)y' + q(t)y = g(t)$:

$y_p = v_1 y_1 + v_2 y_2$, where y_1 and y_2 form fundamental solution set for $y'' + p(t)y' + q(t)y = 0$,
and v_1, v_2 satisfy $\quad v_1' y_1 + v_2' y_2 = 0,\ v_1' y_1' + v_2' y_2' = g(t)$.

Characteristic polynomial of A: $p(\lambda) = \det(\lambda I - A)$. For 2×2 matrix A, $p(\lambda) = \lambda^2 - T\lambda + D$, where T is the trace of A and D is the determinant of A.

General solution of $y' = Ay$, for an $n \times n$ matrix A:
$y = C_1 y_1 + \cdots + C_n y_n$, for solutions $y_1, \ldots, y_n$ for which $y_1(t), \ldots, y_n(t)$ are independent vectors.