Recall that at the end of our last problem we had a 50 gallon container completely full of salt water. There were 21 lbs of salt in the container, and water was continuing to pour in at a rate of 4 gal/min, .5 lbs salt / gal.

$$\frac{dx}{dt} = (\text{rate in}) - (\text{rate out})$$

$$\frac{dx}{dt} = \left(4\frac{\text{gal}}{\text{min}}\right) \cdot \left(\frac{1}{2}\frac{\text{lbs}}{\text{gal}}\right) - \left(4\frac{\text{gal}}{\text{min}}\right)\frac{x}{50}$$

$$\frac{dx}{dt} = 2 - \frac{2}{25}x$$

We can solve by separation of variables

$$\frac{dx}{dt} = 2 - \frac{2}{25}x$$

$$\frac{1}{2 - \frac{2}{25}x}dx = dt$$

$$-\frac{25}{2}\left(\frac{1}{x-25}\right)dx = dt$$

$$-\frac{25}{2}\int \frac{1}{x-25}dx = \int dt$$

$$-\frac{25}{2}\log(x-25) = t + C$$

$$(x-25)^{-\frac{25}{2}} = Ce^{t}$$

$$x - 25 = Ce^{-\frac{2}{25}t}$$

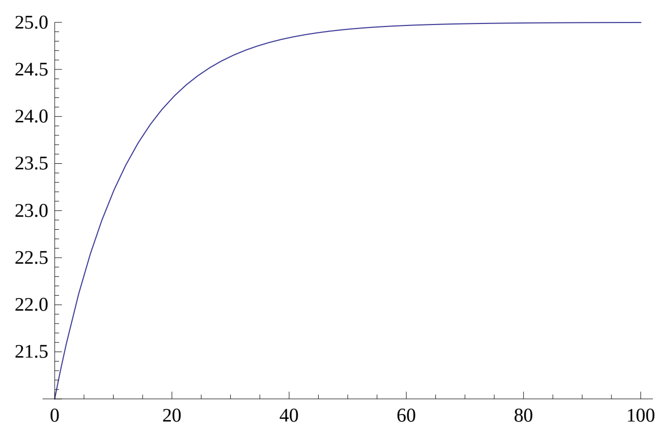
$$x = 25 + Ce^{-\frac{2}{25}t}$$

Since we started out with 21 lbs of salt in the tank, $x(0) = 18$ and we can find C.

$$x(0) = 21 = 25 + Ce^{-\frac{2}{25}\cdot 0} \Rightarrow 21 = 25 + C \Rightarrow C = -4$$

and the final solution is

$$x(t) = 25 - 4e^{-\frac{2}{25}t}$$



A plot of salt content as a function of time for the first 100 minutes.
Recall that in the previous problem, the salt content was given by

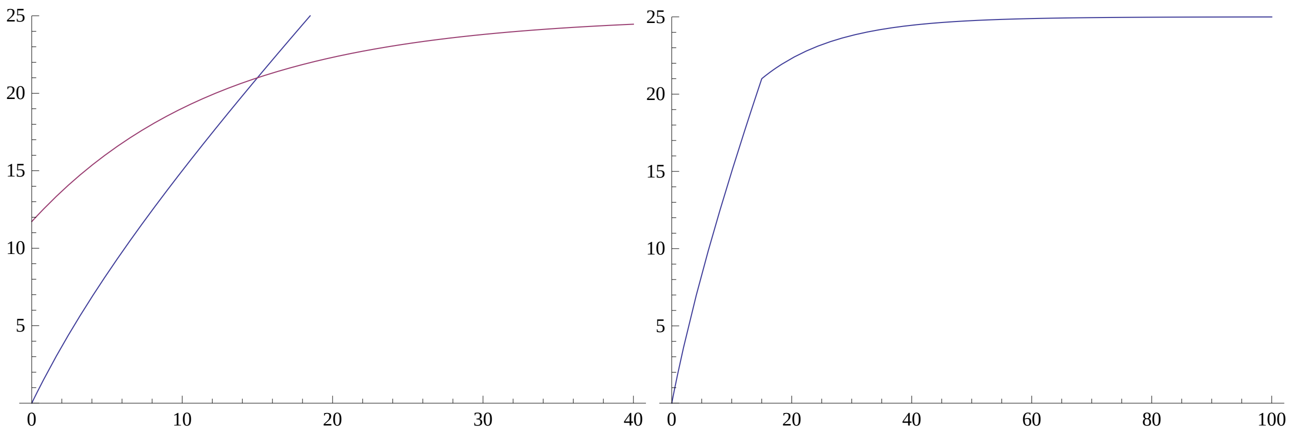
$$x_1(t) = \frac{t(t+20)}{t+10}$$

until $t = 15$ when it became full. We can shift our new solution over by 15 minutes as

$$x_2(t) = x(t-15) = 25 - 4e^{-\frac{2}{25}(t-15)}$$

Now we have a complete description of the amount of salt

$$\begin{cases} x_1 & 0 \leq x \leq 15 \\ x_2 & x > 15 \end{cases}$$



Plots of x_1, x_2, and the amount of salt respectively