Math 33B Sample Midterm 2 Questions

These are questions from previous years' midterms. They are intended to give you an idea of the types of questions you may encounter on the exam. They should not be considered as a comprehensive study guide. Note that the exam is 50 minutes long, so an appropriate number of questions will be asked (not this many).

1. Give general solutions to the following differential equations:

   (a) $y'' + 6y' + 18y = 0$

   (b) $y'' + 4y' - 12y = 0$

   (c) $y'' + 4y' + 4y = 0$

2. Solve the initial value problem

$$y'' + 4y' - 12y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

3. Solve the initial value problem

$$y'' + 8(y')^2 = 0, \quad y(1) = 0, \quad y'(1) = e^{-1}$$

4. Find general solutions to the following differential equations

   (a) $y'' + 36y = 0$

   (b) $y'' + 36y = \sec 6x$

5. Find particular solutions to the following differential equations using the method of undetermined coefficients:

   (a) $y'' + 6y' + 18y = e^x$

   (b) $y'' + 6y' + 18x = x^2$

6. (a) Suppose that the functions $y_1(t)$ and $y_2(t)$ are both solutions to $y'' + \sin(t^3)y = 0$ with $y_1(6) = y_2(6) = 0$. If $y_2'(6) = 1$, show there is a constant C such that $y_1(t) = Cy_2(t)$ for all t. What is the constant?

   (b) Now let $y_1(t)$ and $y_2(t)$ be any two solutions to the differential equation in part (a). Show that the Wronskian of y_1 and y_2 must be constant.

7. (a) Find the solution to the initial value problem $y'' + 3y' - 4y = 0$ with $y(0) = 5$ and $y'(0) = -5$.

   (b) Find the solution to the initial value problem $y'' - 4y' + 5y = 0$ with $y(0) = 3$ and $y'(0) = 5$.

8. Find particular solutions to the following equations

   (a) $y'' + 2y' + 2y = 4t$

   (b) $y'' + 2y' + 2y = e^{3t} + 12$

   (c) $y'' + 2y' + 2y = \cos 2t$

9. (a) Find the general solution of the system $\mathbf{y}' = A\mathbf{y}$, where

$$A = \begin{pmatrix} -1 & -2 \\ 4 & 3 \end{pmatrix}$$

   (b) Find the solution to the initial value problem for the equation in (a) with

$$\mathbf{y}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

10. Consider a mass, spring, damper system with spring constant, $k = 1$, damping coefficient, $c = 3$, and mass, $m = 10$. The motion of the system is given by $my'' + cy' + ky = 0$.

    (a) Solve the system for the motion of the mass with $y(0) = 5$ and $y'(0) = -v_o$.

    (b) Is this system over-damped, under-damped, or critically damped?

    (c) Sketch what the solution would look like on a y-t plot.