Math 33B: Differential Equations

Homework 5: Second order linear differential equations

Due on: Fri., May 10, 2013 - 9:00 AM *Instructor: Aliki M.*

Please include your *name*, *UID* and *discussion section* on the submitted homework.

Problem 1

Find the general solution of each of the following equations:

(i) $y'' - 3y' + 2y = 14\sin 2x - 18\cos 2x$

(ii) $y'' - 2y' + y = 6e^x$

[Answers: (i) $y(x) = c_1e^x + c_2e^{2x} + 2\sin 2x + 3\cos 2x$,
(ii) $y(x) = c_1e^x + c_2xe^x + 3x^2e^x$]

Problem 2

If k and b are positive constants, find the general solution of

$$y'' + k^2y = \sin bx$$

when:

(i) $b \neq k$

(ii) $b = k$

[Answers: $y(x) = c_1\cos(kx) + c_2\sin(kx) + \dfrac{1}{k^2 - b^2}\sin(bx)$
(ii) $y(x) = c_1\cos(kx) + c_2\sin(kx) - \dfrac{x\cos(kx)}{2k}$]

Problem 3

The equation,

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0 \tag{1}$$

is the special case of Bessel's equation,

$$x^2y'' + xy' + (x^2 - p^2)y = 0$$

when $p = \dfrac{1}{2}$.

(i) Verify that if $x > 0$, $y_1 = x^{-1/2}\sin x$ is one solution to Eq. (1).

(ii) Given that $y_1 = x^{-1/2}\sin x$ is a solution, find the second solution, y_2.

[Answer: $y_2 = -x^{-1/2}\cos x$]

Problem 4

You are given that $y_1 = t^2$ and $y_2 = t^{-1}$ are solutions to the homogeneous equation,

$$t^2 y'' - 2y = 0, t > 0.$$

(Note, you've already verified that y_1 and y_2 satisfy the homogeneous ODE in Homework 4 - Problem 2).

Now, find the general solution to the nonhomogeneous equation,

$$t^2 y'' - 2y = 3t^2 - 1, t > 0.$$

[Answer: $y(t) = c_1 t^2 + c_2 t^{-1} + t^2 \ln t + \frac{1}{2}$]

Problem 5

Verify that $y_1 = 1 + t$ and $y_2 = e^t$ are solutions to the homogeneous equation,

$$ty'' - (1+t)y' + y = 0, t > 0.$$

Then, find the general solution to the nonhomogeneous equation,

$$ty'' - (1+t)y' + y = t^2 e^{2t}, t > 0.$$

[Answer: $y(t) = c_1(1+t) + c_2 e^t + \frac{1}{2}e^{2t}(t-1)$]