2 A function $y = f(x)$ is defined implicity by the equation $(x+y^2)^3-(xy)^3-7=0$. Show that the point $(1,1)$ lies on the graph of $f(x)$ and then find the equation of the *normal* line to the graph of $f(x)$ at the point $(1,1)$.

First plug in $x = 1, y = 1$ to verify that the point lies on the graph:

$$(1+1^2)^3 - 1^3 - 7 = 0.$$

To find the equation of the normal, first find the slope of the tangent by implicit differentiation:

$$(x+y^2)^3 - (xy)^3 - 7 = 0 \Rightarrow 3(x+y^2)^2(dx+2ydy) = 3(xy)^2(xdy+ydx)$$

so at $(1,1)$ we have

$$3(1+1^2)^2(dx+2dy) = 3(1\cdot 1)^2(dy+dx) \Rightarrow 12(dx+2dy) = 3(dy+dx) \Rightarrow \frac{dy}{dx} = -3/7$$

Therefore, the slope of the *normal* is $-1/(-3/7) = 7/3$. The equation is therefore given by

$$y - 1 = 7/3(x-1) \iff y = \frac{7}{3}x - \frac{4}{3}$$