Homework 11

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1 Question 1

1.1 Question

1. If S is an irreducible subset of a topological space X, prove that $\overline{S}$ (the closure of S) is again irreducible.
2. If $f : X \to Y$ is a continuous map of topological space, prove that $f(S)$ is irreducible for every irreducible subset S of X.
3. If $f : X \to Y$ is a continuous map of topological spaces and $S \subset X$ is an irreducible closed subset admitting a generic point x (i.e. $S = \overline{\{x\}}$), then $\overline{f(S)}$ is irreducible closed and admits $f(x)$ as a generic point (i.e. $\overline{f(S)} = \overline{\{f(x)\}}$).

1.2 Answer

2 Question 2

2.1 Question

If A is a non-trivial zero-dimensional ring, prove that $A[x]$ is one-dimensional.

2.2 Answer

3 Question 3

3.1 Question

1. Use the correspondence between irreducible subsets of Spec A and their generic points to show that for any ring A and any $\mathfrak{p} \in$ Spec A, the closed subset $V(\mathfrak{p})$ is an irreducible component of Spec A if and

only if $\mathfrak{p}$ is a *minimal* prime ideal of $\mathfrak{p}$ (i.e. does not properly contain any other prime ideals of A).

2. Use the general existence result for irreducible components to conclude that *any* prime ideal $\mathfrak{p}$ of Spec A contains a minimal prime ideal.

3.2 Answer

4 Question 4

4.1 Question

Prove that $A[x]$ is 2-dimensional if A is a PID (not a field).

4.2 Answer