Homework 10

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1 Question 1

1.1 Question

Let $\{A_i\}_{i\in I}$ be a collection of rings. Show that the following are equivalent:

1. The natural embedding $\coprod_{i\in I} \text{Spec } A_i \to \text{Spec } A$ has dense image.
2. The inclusion $\text{nil}(A) \subset \prod_{i\in I} \text{nil}(A_i)$ is an equality.
3. There exists $n > 0$ and a cofinite subset[1] J of I such that $a^n = 0$ for all $a \in \text{nil}(A_i)$ and all $i \in J$.

1.2 Answer

2 Question 2

2.1 Question

Let $A = \prod_{n=1}^{\infty} \mathbb{Z}/p^n\mathbb{Z}$.

1. Prove that $\mathfrak{p} \in \text{Spec } A$ is a closed point (i.e. $\mathfrak{p}$ is a maximal ideal) if and only if $\kappa(\mathfrak{p})$ is of characteristic p. Furthermore, for such $\mathfrak{p}$ prove that $A/\mathfrak{p} \cong \mathbb{Z}/p\mathbb{Z}$.
2. Prove that if $\mathfrak{p} \in \text{Spec } A$, then $\kappa(\mathfrak{p})$ is of characteristic zero if and only if $\mathfrak{p}$ is not closed.

2.2 Answer

[1] i.e. a subset with finite complement