

Math 226A: Problem Set 3

due Friday, November 15

1. Petersen, Chapter 5, Exercise 1 on p. 149.
2. Petersen, Chapter 5, Exercise 2 on p. 149.
3. Petersen, Chapter 5, Exercise 8(c) on p.149. (The definition of *totally geodesic* is on p. 145.)
4. Let p be a point in a Riemannian manifold (M, g) and $\sigma \subset T_p M$ a two-dimensional subspace. For small $r > 0$, let $\Sigma_\sigma \subset M$ be the (diffeomorphic) image of $B(0, r) \cap \sigma \subset T_p M$ under the exponential map \exp_p . Show that the sectional curvature $\sec(\sigma)$ at p (computed inside M) is equal to the sectional curvature (that is, Gaussian curvature) of the surface Σ_σ at p , in the induced metric.
5. Let $SO(n)$ be the Lie group of orthogonal matrices of determinant 1. Equip $SO(n)$ with a bi-invariant Riemannian metric g of volume one, as constructed in the previous homework. The tangent space $T_I SO(n)$ can be identified with the space $\mathfrak{so}(n)$ of skew-adjoint matrices. Show that the exponential map (with respect to g)

$$\exp_I : \mathfrak{so}(n) \rightarrow SO(n)$$

coincides with the usual matrix exponentiation $A \rightarrow e^A$.

Hint: Feel free to use Proposition 12 on p.79 in Petersen's book. Compare also exercise 19 in Petersen, Chapter 5, p.151.

6. Petersen, Chapter 6, Exercise 5 on p.184.
7. Petersen, Chapter 6, Exercise 8 on p.184. (See the definition of a *Killing field* on p.23.)
8. Petersen, Chapter 6, Exercise 10 on p.184.