Math 226A: Problem Set 1
due Friday, October 11

1. Petersen, Chapter 1, Exercise 5 (a) on p. 18.

2. Consider the upper-half plane

$$\mathbb{R}^2_+ = \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$$

with the hyperbolic metric

$$\frac{dx^2 + dy^2}{y^2}.$$

Show that the vertical line segment between $(0, 1)$ and $(0, 2)$ is the shortest path between these points.

3. Consider $\mathbb{R}^2_+$ with the hyperbolic metric as above. Let $v_0 = (0, 1)$ be a tangent vector at the point $(0, 1)$ of $\mathbb{R}^2_+$. Let $v(t)$ be the parallel transport of v_0 along the curve $x = t, y = 1$. Show that $v(t)$ makes an angle t with the direction of the y-axis, measured in the clockwise sense.

4. Petersen, Chapter 2, Exercise 5 on p. 56.

5. (The first part of Petersen, Chapter 2, Exercise 6 on p. 56.) For any point p in a Riemannian manifold (M, g), show that there exist coordinates $x^1, \ldots, x^n$ near p such that $\partial_i = e_i$ and $\nabla \partial_i = 0$ at p.

6. Petersen, Chapter 2, Exercise 10 (e) on p. 57.

7. Petersen, Chapter 2, Exercise 11 on p. 58.

8. Petersen, Chapter 2, Exercise 13 on p. 58.