1 Petersen, Chapter 5, Exercise 1 on p. 149.

2 Petersen, Chapter 5, Exercise 2 on p. 149.

3 Petersen, Chapter 5, Exercise 8(c) on p. 149. (The definition of *totally geodesic* is on p. 145)

4 Let p be a point in a Riemannian manifold (M, g) and $\sigma \subset T_pM$ a two-dimensional subspace. For small $r > 0$, let $\Sigma_\sigma \subset M$ be the (diffeomorphic) image of $B(0, r) \cap \sigma \subset T_pM$ under the exponential map $\exp_p$. Show that the sectional curvature $\sec(\sigma)$ at p (computed inside M) is equal to the sectional curvature (that is, Gaussian curvature) of the surface Σ_σ at p, in the induced metric.

5 Let $SO(n)$ be the Lie group of orthogonal matrices of determinant 1. Equip $SO(n)$ with a bi-invariant Riemannian metric g of volume one, as constructed in the previous homework. The tangent space $T_ISO(n)$ can be identified with the space $\mathfrak{so}(n)$ of skew-adjoint matrices. Show that the exponential map (with respect to g)

$$\exp_I : \mathfrak{so}(n) \to SO(n)$$

coincides with the usual matrix exponentiation $A \to e^A$.

***Hint:* Feel free to use Proposition 12 on p.79 in Petersen's book. Compare also exercise 19 in Petersen, Chapter 5, p.151.**

6 Petersen, Chapter 6, Exercise 5 on p. 184.

7 Petersen, Chapter 6, Exercise 8 on p. 184. (See the definition of a *Killing field* on p.23.)

8 Petersen, Chapter 6, Exercise 10 on p. 184.