1 Let M be a closed (compact, without boundary) manifold. Show that any smooth function

$$f : M \to \mathbb{R}$$

has a critical point.

2.1 Show that every closed 1-form on S^n, $n > 1$, is exact.

2.2 Use this to show that every closed 1-form on $\mathbb{R}P^n$, $n > 1$ is exact.

3 Let M^d be a d-dimensional manifold and $\omega_1, \ldots, \omega_p$ be pointwise linearly independent 1-forms. If $\theta_1, \ldots, \theta_p$ are 1-forms so that

$$\sum_{i=1}^{p} \omega_i \wedge \theta_i = 0$$

then there exist smooth functions f_{ij} such that

$$\theta_i = \sum_{i=1}^{p} f_{ij} \omega_i, \quad i = 1, \ldots, p$$