1 **Suppose $P(x,y,z)$, $Q(x,y,q)$, and $R(x,y,z)$ are C^∞ functions on $\mathbb{R}^3$ which vanish identically if $|x| \geq 5$ and, $|y| \geq 5$, or $|z| \geq 5$. Prove that the volume integral**

$$\int_{-6}^{6}\int_{-6}^{6}\int_{-6}^{6} d(Pdy \wedge dz + Qdx \wedge dz + Rdx \wedge dy) = 0$$

(Do this directly, not by quoting Stokes' Theorem: this is a special case of a proof of Stokes' Theorem!)

2 **Suppose that $V = P(x,y,z)\frac{\partial}{\partial x} + Q(x,y,z)\frac{\partial}{\partial y} + R(x,y,z)\frac{\partial}{\partial z}$ is a C^∞ vector filed on $\mathbb{R}^3$ with $V \neq \vec{0}$ at the origin. Find a necessary and sufficient condition for there to exist a C^∞ function $\lambda(x,y,z)$ in some neighborhood of the origin such that λV is the gradient of a C^∞ function on the neighborhood.**

3 **Let $T_t : \mathbb{R}^3 \to \mathbb{R}^3$ be the right-hand rule rotation around the positive z-axis by t degrees and $S_s : \mathbb{R}^3 \to \mathbb{R}^3$ be the right-hand rule rotation around the positive x-axis by t degrees.**

3.1 **Find the infinitesimal generators of the flows T_t and S_t, i.e., the vector fileds X and Y, respectively, on $\mathbb{R}^3$ whose flows are $\{T_t\}$ and $\{S_t\}$.**

3.2 **Compute the commutator**

$$T_{-t} \circ S_{-t} \circ T_t \circ S_t.$$

3.3 **Compare the result of (B) (lowest order non-identically zero term) with the Lie bracket $[X,Y]$.**

4 **Take as given that a C^∞ 2-form ω on S^2 is of the form $d\theta$ for some C^∞ 1-form θ if and only if $\int_{X^2} \omega = 0$. Use this to show that every C^∞ 2-form Ω on $\mathbb{R}P^2$ has the form $d\Lambda$ for some C^∞ 1-form Λ. (Do not just quote DeRham's Theorem here.)**

5.1 **Suppose $F : S^1 \to \mathbb{R}^3$ is a C^∞ function such that dF is nowhere zero (on S^1). Prove that there is a two-dimensional subspace P of $\mathbb{R}^3$ such that $\pi_P \circ F : S^1 \to \mathbb{R}^3$ has nowhere vanishing differential, where π_P = orthogonal projection on P.**

5.2 **Show by example (a picture with explanation is all right) that there is such an F that is also 1 to 1 (injective) but is such that, for all P, $\pi_P \circ F$ fails to be injective.**

5.3 **Show that if $F : S^1 \to \mathbb{R}^4$ is C^∞ and injective then there is a three-dimensional subspace H of $\mathbb{R}^4$ such that $\pi_H \circ F$ is injective, where π_H = orthogonal projection on H.**