1 Suppose $P(x,y,z)$, $Q(x,y,q)$, and $R(x,y,z)$ are C^∞ functions on $\mathbb{R}^3$ which vanish identically if $|x| \geq 5$ and, $|y| \geq 5$, or $|z| \geq 5$. Prove that the volume integral

$$\int_{-6}^{6}\int_{-6}^{6}\int_{-6}^{6} d(Pdy \wedge dz + Qdx \wedge dz + Rdx \wedge dy) = 0$$

(Do this directly, not by quoting Stokes' Theorem: this is a special case of a proof of Stokes' Theorem!)

2 Suppose that $V = P(x,y,z)\frac{\partial}{\partial x} + Q(x,y,z)\frac{\partial}{\partial y} + R(x,y,z)\frac{\partial}{\partial z}$ is a C^∞ vector filed on $\mathbb{R}^3$ with $V \neq \vec{0}$ at the origin. Find a necessary and sufficient condition for there to exist a C^∞ function $\lambda(x,y,z)$ in some neighborhood of the origin such that λV is the gradient of a C^∞ function on the neighborhood.

3 Let $T_t : \mathbb{R}^3 \to \mathbb{R}^3$ be the right-hand rule rotation around the positive z-axis by t degrees and $S_s : \mathbb{R}^3 \to \mathbb{R}^3$ be the right-hand rule rotation around the positive x-axis by t degrees.

3.1 Find the infinitesimal generators of the flows T_t and S_t, i.e., the vector fileds X and Y, respectively, on $\mathbb{R}^3$ whose flows are $\{T_t\}$ and $\{S_t\}$.

3.2 Compute the commutator

$$T_{-t} \circ S_{-t} \circ T_t \circ S_t.$$

3.3 Compare the result of (B) (lowest order non-identically zero term) with the Lie bracket $[X,Y]$.