Homework

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1 Chapter 8 Section 2

1.1 Problem 3

1.1.1 Question

Take the triangulation for the Möbius strip shown in Fig. 6.2, orient one of the triangles, then go round the strip orienting each triangle in a manner compatible with the one preceding it. (Of course, when you get back to where you started the orientations do not match up.) What is the boundary of the two-dimensional chain formed by taking the sum of these oriented triangles?

1.1.2 Answer

The boundary is consists of every edge which is just adjacent to one 2-simplex as well as one more edge, counted twice.

1.2 Problem 7

1.2.1 Question

Show that any cycle of K is a bounding cycle of the cone on K.

1.2.2 Answer

Proof. Let $(a_1, a_2), (a_2, a_3) \dots, (a_n, a_1)$ be a cycle in K. If x is the vertex of CK then it is easy to verify that $(a_1, a_2, x), (a_2, a_3, x) \dots, (a_n, a_1, x)$ is a collection of oriented triangle which have the given cycle as their boundary. $\square$

1.3 Problem 8

1.3.1 Question

Triangulate S^n so that the antipodal map is simplicial and induces a triangulation of P^n. If n is odd, find an n-cycle in this triangulation of P^n. What difficulties arise when n is even?

1.3.2 Answer

We triangulate by first fixing a triangulation of S^1 and then generating a triangulation of S^{n+1} by taking the double cone of our triangulation of S^n. An n-cycle in the triangulation of P^n which arises this way is just an n-cycle in the original S^n. If the space has even dimension though, this may not work since

1.4 Problem 10

1.4.1 Question

Suppose $|K|$ is homeomorphic to the torus with the interiors of three disjoint discs removed. Orient each boundary circle of $|K|$ and let z_1, z_2, z_3 be the resulting elementary 1-cycles of K. Show that $[z_3] = \lambda[z_1]+\mu[z_2]$ where $\lambda = \pm 1$, $\mu = \pm 1$. Do we have the same result if we replace the torus by the Klein bottle?

1.4.2 Answer

In the illustration we have

$$z_1 = (f, a, b, l, h, g) \quad z_2 = (b, c, d, m, i, l) \quad z_3 = (d, e, f, k, j, m)$$

so we note that $z_1 + z_2 = (f, a, c, d, m, h, g) = z_3$

This result does not hold with the Klein bottle: the fact that it is not orientable means that the sides are not aligned properly. In particular (f, i) is oriented differently in z_1, z_3 if we were to associate the edges as in a klein bottle.