EUROPEAN COMMISSION METHODOLOGY TO ESTIMATE OUTPUT GAP

The growth of potential product of an economy is one of the fundamental economic variables for economic analysis. In particular, the potential growth rate acquires special relevance for understanding the cyclical position of the economy or assessing the orientation of fiscal policy. Thus, in the case of Spain for example, this variable is necessary to calculate the structural component of the public deficit, in accordance with the regulations of the Stability and Growth Pact at the European level and the Law on Budgetary Stability and Financial Sustainability at the national level. In this sense, the existence of fiscal rules based on the cyclical position of the economy, instead of on nominal variables, represents an advance in the process of monitoring the, more than common, fiscal deficits of the EU countries. However, potential output is not observable, so it has to be estimated. This estimate is not without controversy and carries a high degree of uncertainty. There is a wide variety of methodologies that can lead to very different results in their calculation. A common feature of the different available methodologies is that they offer results in growth estimates with a high pro-cyclical nature. In periods of increase they offer results that could be overestimating potential output and, conversely, in periods of recession they would reduce it excessively (see, for example, Havik et al. 2014).

Output gap refers to the state of the economy in relation to the economic cycle. The International Monetary Fund defines it as the difference between real GDP and potential GDP, with potential GDP being the level of GDP that uses its inputs in the most efficient way possible, or that uses resources at full capacity. Potential GDP can also be considered that level of GDP that can be maintained without affecting inflation levels.

Thus, a positive output gap would imply that the economy is operating above its full capacity, which would mean an excess of demand that, according to economic theory, should result in an increase in inflationary pressures. A positive output gap is also often associated with excessive rigidity in the labor market, which leads to upward pressure on wages and can also turn out into higher inflation.

A negative output gap means that the level of production is lower than what would be obtained with an economy at full capacity, and its explanation would come from a weak demand for goods and services that could lead to deflation and economic recession.

In this way, a positive or negative output gap would have implications in the way fiscal and monetary policy should be used.

The problem with output gap is that it is an unobservable variable, so its estimation must resort to econometric statistical techniques. These techniques can be grouped into three large groups:

1) Extraction of cycle and trend in economic series.

2) Estimation from a production function.

3) Estimation from Dynamic Stochastic General Equilibrium Models (DSGE).

The first category can be subdivided into univariate and multivariate methods. These methods are based on the fact that an economic series (GDP) can be decomposed into a component called trend (potential GDP) and another component called cycle (output gap).

UNIVARIATE METHODS

Time Series Approach

To estimate a linear trend, we can perform a linear regression against time that includes the time trend and a constant. The adjusted values of the regression contain the information related to the trend, so that the cyclical component, the output gap, is obtained through the difference between the original values and the trend obtained.

Obviously, this methodology to decompose the series into cycle and trend, when contemplating the entire series and assuming linearity in the trend, it will present very few oscillations around the trend.

Although what has been done so far is perfectly valid to decompose a series in cycle and trend, it is highly recommended to apply natural logarithms to our economic series before proceeding to calculate potential GDP and output gap, thus defining the output gap as the difference in logarithms between real and potential GDP:

$$\hat{y}_t = LOG\left(Y_t / \bar{Y}_t \right)$$

One of the fundamental characteristics of this linear estimation method is that it considers a constant potential GDP growth throughout the series.

Hodrick-Prescott Filter

It is the most frequent univariate method for the decomposition of a series in cycle and trend. Its biggest gap lies in the bias it presents in the last observations of the sample, which makes it less useful in real-time applications. In addition, it requires a smoothing parameter whose choice is somewhat arbitrary, although there is a certain consensus in using a value of 1,600 for quarterly series and 100 for annual series. On the other hand, the hypotheses on which it is based for its calculation are not exempt from criticism:

1) Real GDP does not deviate too much from its potential GDP level.

2) Potential GDP growth is relatively smooth.

However, its simplicity and transparency play in its favor.

Kalman Filter

State-space models play a very important role in cycle and trend decompositions, considering, in a generic way, a decomposition of the observable series y_t into a set of unobservable series or states.

Using the dlm package in R we can estimate and compare different dynamic linear models by applying the Kalman filter to our original series in logarithms.

The specification of a dynamic linear model (DLM) is done by the following equations:

$y_t=F_t\theta_t+v_t$, $vt \sim N(0,Vt)$ MEASURE EQUATION

$\theta_t=G_t\theta_{t-1}+w_t$, $wt \sim N(0,Wt)$ TRANSITION EQUATION

and considering an a priori probability distribution for $\theta_0 \sim N(m_0,C_0)$

So, in our case we would have two states, p_t and c_t, which would be assimilated respectively with the trend component and with the output gap or cyclical component, so that F = (1 1) and $V_t = 0$. We would therefore have to assume some hypothesis regarding the behavior of the two unobservable components. Consider, for example, for the trend a random walk plus stochastic drift, which in turn follows a random walk, and for the cyclical component a second-order autoregressive process:

$$y_t = p_t + c_t$$

$$p_t = g_{t-1} + p_{t-1} + \upsilon_t$$

$$g_t = g_{t-1} + w_t$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-1} + e_t$$

MULTIVARIATE METHODS

Multivariate Kalman Filter

One of the main characteristics of dynamic linear models is that they allow us to introduce variables that help us to interpret our results more economically. Thus, let us consider the unemployment rate, and suppose that its cyclical component is proportional to the cyclical component of GDP. Since the unemployment rate is a countercyclical variable, this constant of proportionality must be negative, but that must be told by the model. In this way, with $y_{1,t}$ being the unemployment rate, we will introduce the following new equations in the univariate model already considered:

$$y_{1,t} = p_{1,t} + c_{1t}$$

$$p_{1,t} = p_{1,t-1} + \upsilon_{1,t}$$

$$c_{1,t} = \alpha_1 c_t + e_{1,t}$$

We can keep using *dlm* package or we may introduce a new R library, *KFAS*, whose calculation of optimal values is somewhat more efficient than in dlm by introducing a fuzzy component in the estimation. But first, we have to obtain some values for the initial points of the model, since this is a critical aspect when using the Kalman filter. We can carry out this search for the initial values from the Hodrick-Prescott decomposition of the series and the specifications that we have considered.

PRODUCTION FUNCTION METHODS

Cobb-Douglas function and HP filters approach

The production function of the model that we are going to use is the Cobb-Douglas function:

$Y_t = A_t (K_t)^{\alpha} (L_t)^{\beta}$

where Y is the GDP, L is the employed labor force, K is the physical capital, A is the level of technological development, and α and β are the respective elasticities with respect to GDP, assuming that α + β = 1.

The level of technological development, A, is also known as total factor productivity, TFP, and is usually calculated as a residual (Solow residual), being the part of GDP growth not explained by capital or labor force.

In our case, if we consider the NAIRU[1] as the trend component of the unemployment rate, obtained by applying Hodrick-Prescott filter, u_t^f, we can obtain the potential employment level as $L_t^f = (1 - u_t^f)L_t^{s,f}$, where $L_t^{s,f}$ is the level of full employment, which would be given by the product of the potential participation rate and the number of people in working age.

Estimating TFP from: $lnA_t = lnY_t - \propto lnK_t - (1-\propto)lnL_t$, taking 1-α as the sharing of Compensation of Employees in GDP, we can obtain, also through HP filter, its trend component; $\zeta_t = lnA_t^f$.

Thus, it would be very easy to estimate potential GDP; $Y_t^f = \exp(\zeta_t)(K_t)^\alpha(L_t^f)^{1-\alpha}$, and, therefore, also output gap as the difference between actual GDP and potential.

Cobb-Douglas production function and Kalman filters

We can follow previous methodology but using Kalman filters instead of HP filters when needed.

EUROPEAN COMMISION METHODOLOGY

The output gap and potential growth have become basic concepts in the fiscal surveillance process of the European Union within the framework of the Stability and Growth Pact. Potential growth is an indicator of the ability of an economy to generate sustainable non-inflationary growth, while the output gap indicates the degree of overheating or stagnation of the economy in relation to its potential growth.

The methodology used by the European Commission to estimate the output gap is also based on the Cobb-Douglas production function, $Y_t = TFP_t(K_t)^\alpha(L_t)^{1-\alpha}$.

Statistics series used to estimate output gap by European Commision can be downloaded from AMECO[2]. A remarkable aspect is that the series end two years after the last official data published, taking as valid the forecasts made by the different countries.

Another peculiarity of the methodology followed by the Commission is that the input of the labor force is defined in hours worked, considering the figure from the National Accounts and not from the Labor Force Survey.

According to the Commission's methodology, a potential participation rate is first calculated, although considering in this case the working-age population of 15 to 74 years instead of habitual 16 to 64 years. This potential participation rate is calculated using a Hodrick-Prescott filter[3].

The second step would be to calculate a potential employment level consistent with a stable non-inflationary wage unemployment level, NAWRU. This is probably the most

[1] NAIRU, non accelerating inflation rate of unemployment.
[2] https://ec.europa.eu/info/business-economy-euro/indicators-statistics/economic-databases/macro-economic-database-ameco_en
[3] A lambda of 10 has been used instead of the usual 100 for annual data according to the Commission's methodology.

complicated step in the commission methodology due to the many calculations and assumptions that must be made.

The Commission's methodology allows the use of two types of approximations to represent the Phillips Curve; the Traditional Phillips Curve (TKP), of adaptive expectations, and the New Phillips Curve of Neo-Keynesian cut (NKP). For ease reasons, the NKP has been considered due to:

• Simplicity: It uses fewer explanatory variables.

• Interpretation: Due to the use of rational expectations, more adjusted to the current theoretical literature.

• Less procyclicality of the estimates.

According to the Commission's methodology, the equations of measurement of the state-space model would be:

$LUR_t = uP_t + uG_t$

$DWS_t = Intercept + \beta_0 * uGtil_t + \beta_0 * (\phi_c - 0.99)/(\phi_c * 0.99 - 1.0) * \phi_{G1} * uGtil_{t-1} + cw_t$

LUR being the series of the unemployment rate and DWS the series relating to wage inflation, calculated in the following way:

DWS = WI / 100-LP- (PCE-1)

Where WI is the variation in the nominal wage per worker, LP is the interannual variation in labor productivity, and PCE is the variation in the private consumption deflator.

The transition equations of the model would be:

$uP_t=uP_{t-1}+uslope_{t-1}+auP_t$

$uslope_t=uslope_{t-1}+auslope_t$

$uG_t= \phi_{G1}*uG_{t-1}+\phi_{G2}*uG_{t-2}+auG_t$

$uGtil_t=\phi_C*uGtil_{t-1}+uG_t$

$cw_t= \phi_C *cw_{t-1}+aw_t$

We have been used *dlm* package in R to estimate initial values for the unknown parameters, and *KFAS* package to get smoothed Kalman filtered series for unemployment rate; the NAWRU series.

To the NAWRU series thus obtained, the Commission proposes to make two consecutive adjustments. A first adjustment that consists of forcing a convergence of the series at t + 10, (t + 8 considering the estimates of the two years immediately following as endogenous) to a structural NAWRU calculated from a series of panel data for all the countries of the Union[4]. These panel

[4] Actually two sets of panel data are used, one for the oldest Member States, and another set for the latest Member States to join the Union.

data include a series of structural indicators (taxation, unemployment protection, active employment policies, degree of union membership) and a series of cyclical indicators (TFP, real interest rate, employment in construction). From the mean values of the cyclical indicators and the last values of the series for the structural indicators, an anchor value of the NAWRU is obtained. With this value of the anchor we force the convergence of the NAWRU series[5].

The second adjustment made by the Commission's methodology consists of modifying the anchor based on the comparison with the value that would have been given if the TKP approach had been used instead of NKP.

Next, we calculate the trend value of hours worked by multiplying the potential employment level by the trend value of average hours worked.

Finally, only the TFP calculation and its potential value would remain. However, the Commission's methodology is also somewhat complex when it comes to calculating the potential TFP.

First, the TFP would be calculated using the Solow residual. Next, once the TFP is obtained, the potential TFP is calculated from a multivariate Kalman filter considering an auxiliary variable, CU, which is constructed from different surveys on utilization capacity in industry, construction and services[6]. The estimated state space model for the calculation of the potential TFP is very similar to the one considered for the calculation of the NAWRU[7].

And finally, the potential GDP and the output gap would be calculated in the same way it has been explained before.

<u>Changes for Spring 2020 forecasts</u>

Methodology as explained has been changed for the Spring 2020 forecasts aimed at keeping potential as stable as possible relative to the Autumn 2019 baseline although *"it needs to be accepted that the Autumn 2019 baseline path is now obsolete"*[8]. Three main changes have been accomplished:

- Average hours worked series has been smoothed using a Hodrick-Prescott filter.
- Capacity Utilization (CUBS); a yearly value for 2020 is not available in the harmonized Business and Consumer Surveys, so a proxy based on the available forecasts on TFP growth in 2020 from ECFIN's country desks, scaled using an adjustment factor based on the change in the CUBS in the year following the financial crisis has been used.
- NAWRU; "labour hoarding" dummy variables for 2020 (and 2021) have been added for member states where the Phillips curve relationship breaks down (namely EL, ES, FR, PT, HU, LV, PL, SK, CY and BG).

[5] "NAWRU Estimation Using Structural Labour Market Indicators", DISCUSSION PAPER 069 | OCTOBER 2017.
[6] "The Production Function Methodology for Calculating Potential Growth Rates & Output Gaps", Economic Papers 535 | November 2014.
[7] Actually, for 2020 estimates the methodology has been slightly changed and a Bayesian State Space Model has been used to estimate potential TFP instead of the traditional structural state space model.
[8] "OGWG Note TELCO on COVID shock 27042020".

Alternative methodology proposal

Obviously, European Commission Methodology hasn't followed Occam's razor principle, aka principle of parsimony, and we can identify, at least, three questionable issues in the official model:

1) The consideration of hours instead of the number of employed persons: It must be taken into account that by using the average hours per employed person we are actually incorporating an additional source of variability to the model, thus increasing its error and uncertainty about the future evolution of its variables. In fact, this series has been smoothed for Spring 2020 forecasts.
2) The anchor and subsequent adjustment in the NAWRU estimate.
3) The use of the auxiliary variables on capacity utilization to estimate the potential level of TFP.

In addition to eliminating these three steps in the estimation procedure, we could also use the traditional lambda of 100 when using the Hodrick-Prescott filter with annual data, although this point doesn't bring much improvement to the results.

In this way, in addition to applying the principle of parsimony, or Occam's razor, considerably reducing the number of parameters to estimate and the number of variables to be used, we obtain a model whose theoretical foundation is even stronger than that of the Commission itself considering to the correlations of the output gap with the unemployment rate, with the NAWRU, with the unemployment rate gap, and with inflation[9]:

Correlations Table	*UR*	*NAWRU*	*UR gap*	*Inflation*
Commission Output Gap	-0.53	-0.30	-0.59	0.39
Alternative Simplified Methodology	-0.70	-0.49	-0.84	0.53

This alternative proposal has also the advantage that can be easily reproducible or replicable with R and without the need to settle a numeric seed.

You can find the code to build this shiny app on *https://github.com/frsabido/OutputGap*

DISCLAIMER

The views and opinions expressed in this document are those of the author, Francisco Sabido Martín, and do not necessarily represent official policy or position of the Spanish Government.

[9] These results correspond to Spring 2019 data for Spain.