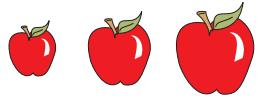


Gravitation différentielle



Résonances



Résonances

Fonction perturbatrice

$$\mathbf{u} = (a, e, i), \mathbf{v} = (\Omega, \omega, M) \quad R = R(\mathbf{u}, \mathbf{v}, \mathbf{u}', \mathbf{v}')$$

$$M = n(t - \tau)$$

... Fourier ...

$$\mathbf{k} = (k_1, k_2, k_3), \quad k_i \in \mathbb{Z}, \quad j^2 = -1$$

$$R = \Re \left\{ Gm' \sum_{\mathbf{k}, \mathbf{k}'} A_{\mathbf{k}, \mathbf{k}'} (\mathbf{u}, \mathbf{u}') e^{j(\mathbf{k} \cdot \mathbf{v} + \mathbf{k}' \cdot \mathbf{v}')} \right\}$$



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Première équation planétaire

$$\frac{da}{dt} = -\frac{2}{n^2 a} \frac{\partial R}{\partial \tau} = \frac{2}{na} \frac{\partial R}{\partial M}$$

donc

$$a^{1/2} da = 2Gm' dt \Re \left\{ \sum_{\mathbf{k}, \mathbf{k}'} j k_3 A_{\mathbf{k}, \mathbf{k}'} (\mathbf{u}, \mathbf{u}') e^{j(\mathbf{k} \cdot \mathbf{v} + \mathbf{k}' \cdot \mathbf{v}')} \right\}$$

Tout est constant sauf $a, M = M_o + n_o(t - t_o)$ et $M' = M'_o + n'_o(t - t_o)$



Résonances

Fonction perturbatrice

$$\mathbf{u} = (a, e, i), \mathbf{v} = (\Omega, \omega, M) \quad R = R(\mathbf{u}, \mathbf{v}, \mathbf{u}', \mathbf{v}')$$

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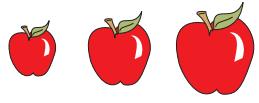
donc

$$a^{1/2} da = 2Gm' dt \Re \left\{ \sum_{\mathbf{k}, \mathbf{k}'} j k_3 A_{\mathbf{k}, \mathbf{k}'} (\mathbf{u}, \mathbf{u}') e^{j(\mathbf{k} \cdot \mathbf{v} + \mathbf{k}' \cdot \mathbf{v}')} \right\}$$

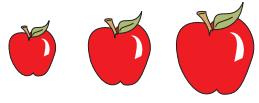
Tout est constant sauf $a, M = M_o + n_o(t - t_o)$ et $M' = M'_o + n'_o(t - t_o)$

$$a^{3/2}(t) = a_o^{3/2} + 3Gm' \Re \left\{ \sum_{\mathbf{k}, \mathbf{k}'} \frac{k_3 A_{\mathbf{k}, \mathbf{k}'} (\mathbf{u}, \mathbf{u}')}{k_3 n_o + k'_3 n'_o} e^{j(\mathbf{k} \cdot \mathbf{v} + \mathbf{k}' \cdot \mathbf{v}')} \right\}$$

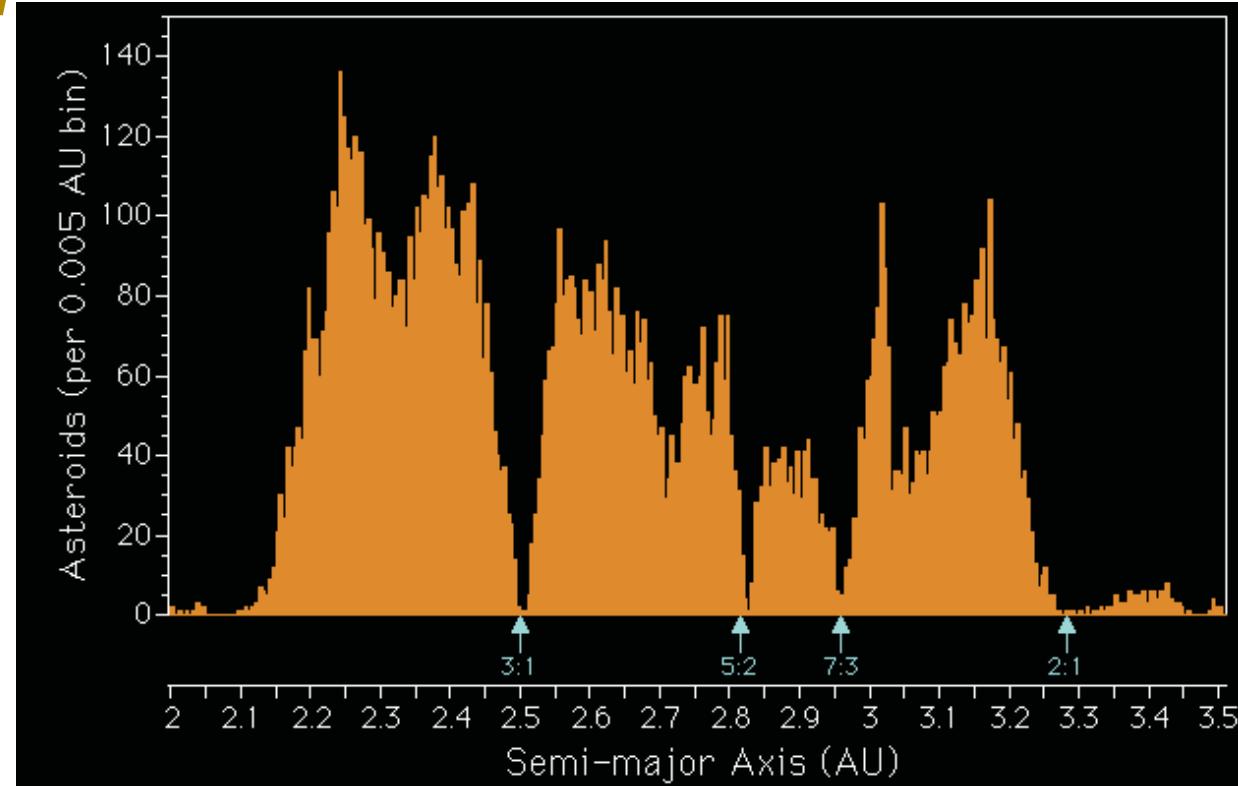
Résonance pour $k_3 n_o + k'_3 n'_o \approx 0$, i.e. $n_o/n'_o \in \mathbb{Q}$

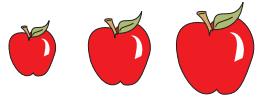


Exemples ...

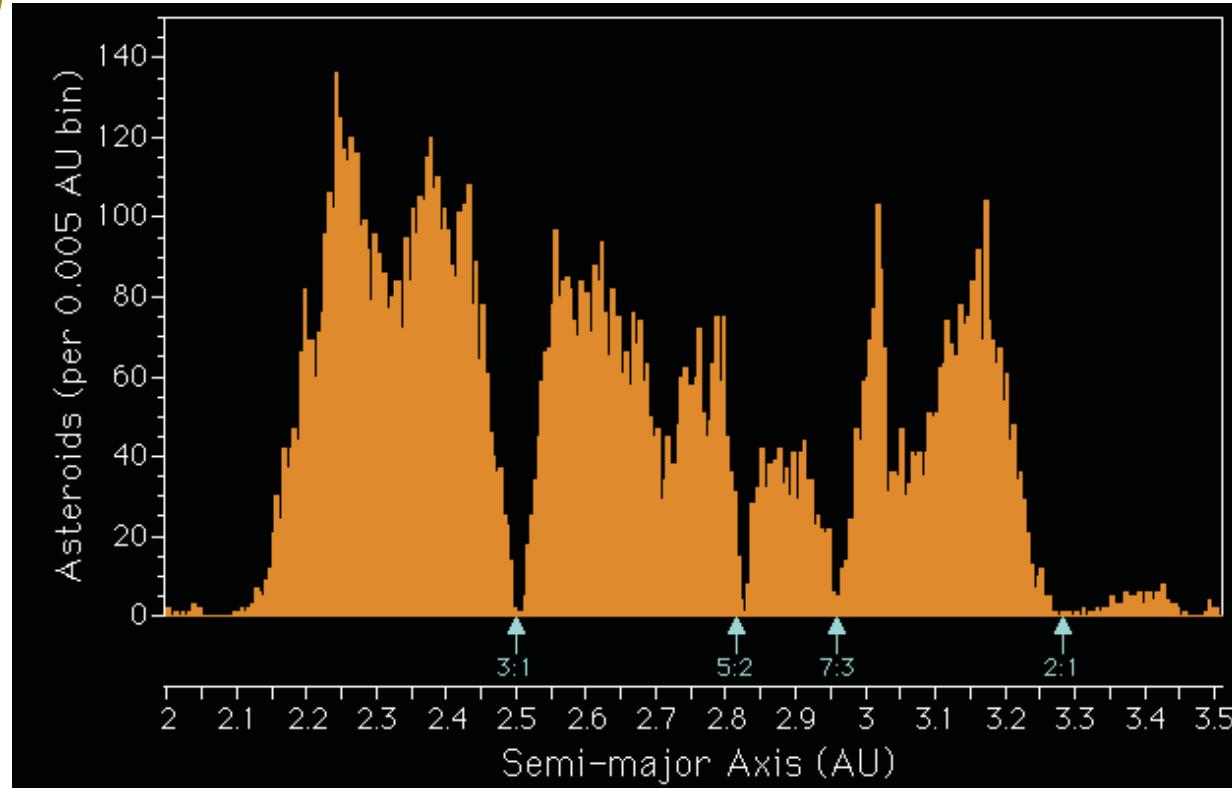


Exemples ...





Exemples ...



Division de Cassini, Mimas

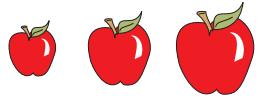
$$T_m = 0.942 \text{ j}, a_m = 3.08$$

$$\frac{T^2}{a^3} = \text{cste} = 0.03037$$

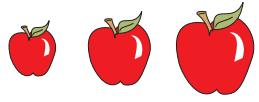
$$a_c = 1.95$$

$$T_c = \sqrt{\left(\frac{T^2}{a^3}\right)_m \times a_c^3}$$
$$= 0.474 \simeq \frac{1}{2} T_m$$





Effets de marée



Effets de marée

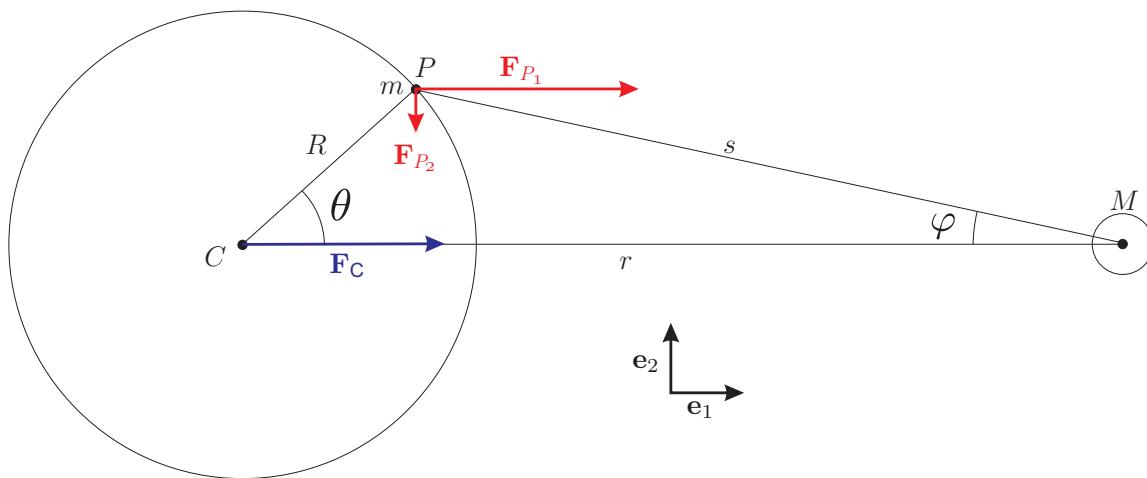
Déformation des corps



Effets de marée

Déformation des corps

Action d'un corps de masse M sur un élément de masse m à la surface d'un corps de masse totale M_{\oplus} .



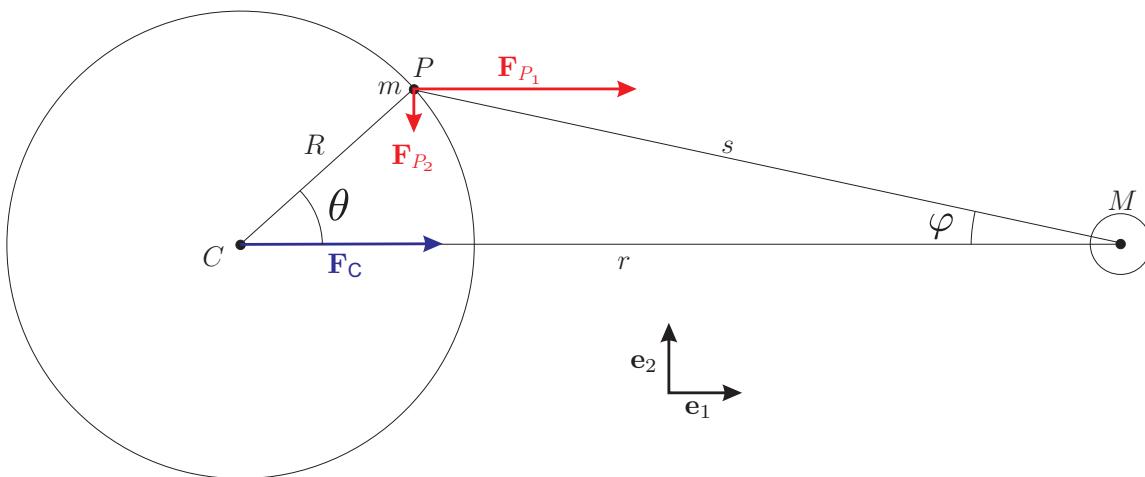
$$\mathbf{F}_C = G \frac{Mm}{r^2} \mathbf{e}_1 \quad \mathbf{F}_P = G \frac{Mm}{s^2} (\cos \varphi \mathbf{e}_1 - \sin \varphi \mathbf{e}_2)$$



Effets de marée

Déformation des corps

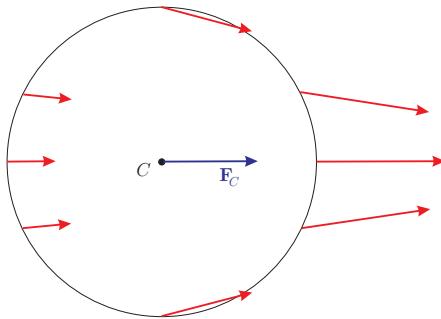
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$$s^2 = R^2 \sin^2 \theta + (r - R \cos \theta)^2 = r^2 [1 - y]$$

avec $y = \rho (2 \cos \theta + \rho)$ et $\rho = R/r$



Force différentielle de gravitation en P

$$\Delta \mathbf{F} = \mathbf{F}_P - \mathbf{F}_C = \frac{GMm}{r^2} \left[\left(\frac{\cos \varphi}{1-y} - 1 \right) \mathbf{e}_1 - \frac{\sin \varphi}{1-y} \sin \varphi \mathbf{e}_2 \right]$$

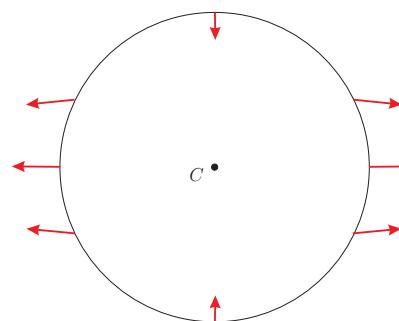
$$\cos \varphi = \frac{r - R \cos \theta}{s} = \frac{1 - \rho \cos \theta}{\sqrt{1-y}} \quad \sin \varphi = \frac{R \sin \theta}{s} = \frac{\rho}{\sqrt{1-y}}$$

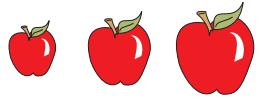
si $\rho \ll 1, \dots \Delta \mathbf{F} = \frac{GMm}{r^2} \rho [\delta_1(\rho, \theta) \mathbf{e}_1 + \delta_2(\rho, \theta) \mathbf{e}_2]$ avec

$$\delta_1(\rho, \theta) = 2 \cos \theta - \frac{3}{2} \rho (1 + 2 \cos \theta - 5 \cos^2 \theta) + \mathcal{O}(\rho)$$

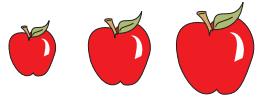
$$\delta_2(\rho, \theta) = -\sin \theta - \frac{3}{2} \rho \sin 2\theta + \mathcal{O}(\rho)$$

δ_1 : paire en θ
 δ_2 impaire en θ
 \rightarrow Elongation de M_{\oplus}





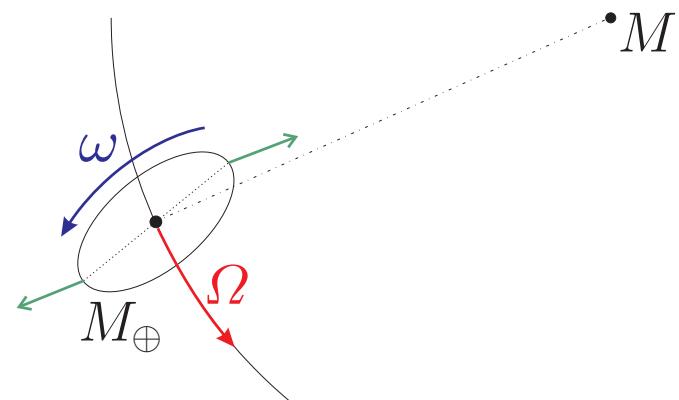
Synchronisation



Synchronisation

$\omega < \Omega$: couple accélérateur

$\omega > \Omega$: couple décélérateur



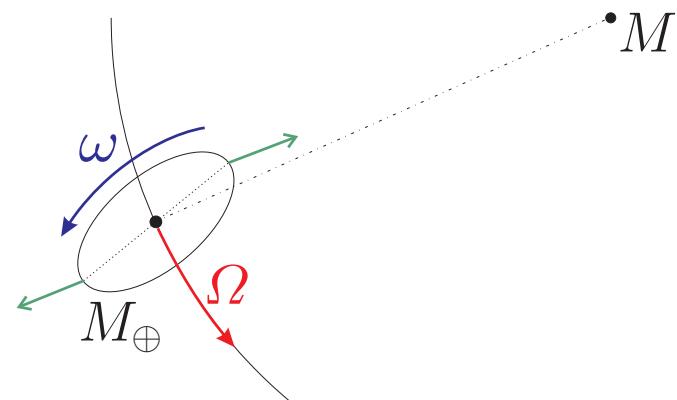
Effets de marée : En général, $\lim |\omega - \Omega| = 0$



Synchronisation

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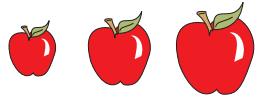


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■ Terre → Lune : Effectif

■ Lune → Terre : 1 jour ↗ 2 ms/siècle (pas constant)

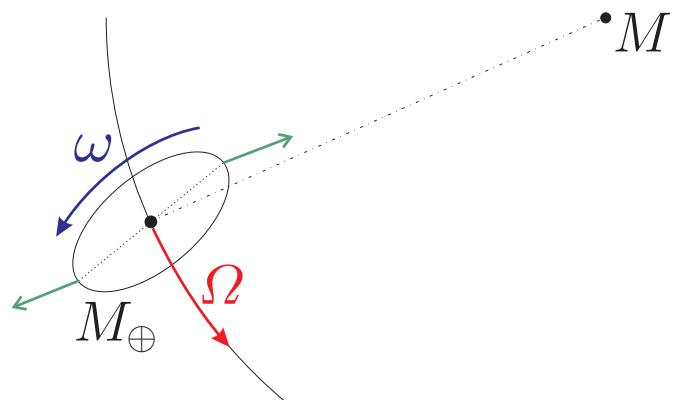
Relevés paléoclimatiques : Il y a $9 \cdot 10^8$ ans, il y avait 481 jours de 18 heures/an.



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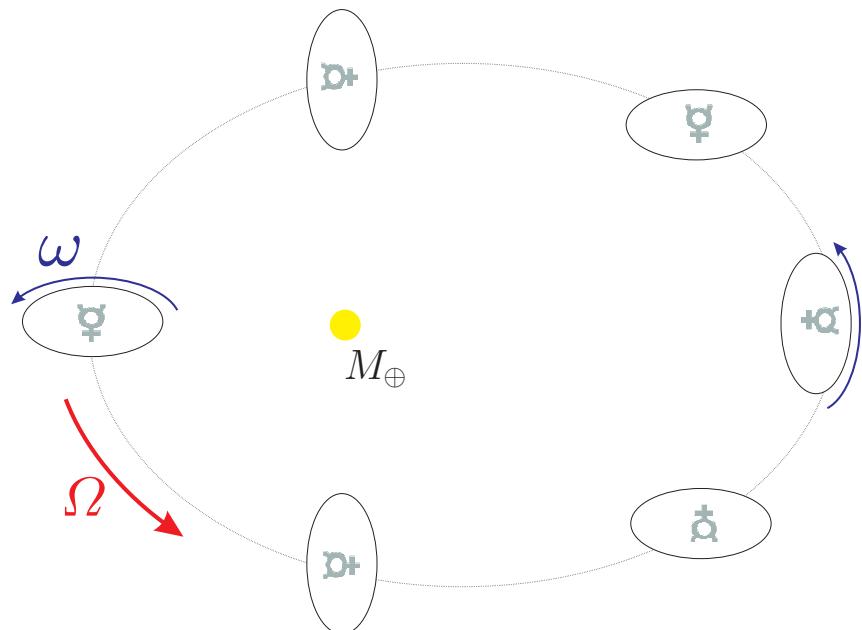
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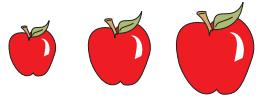
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Relevés paléoclimatiques : Il y a $9 \cdot 10^8$ ans, il y avait 481 jours de 18 heures/an.

Soleil → Mercure : Couplage Spin-Orbite (3 ans = 2 jours)



Hyp. : a et $e = \text{cste}$
Goldreich & Peale, 1968
Rambaux & Bois, 2003

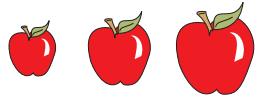


Migration et circularisation

Effets de marée \Rightarrow Synchronisation

$$\text{Moment cinétique } J = M_{\oplus}(\Omega^2 + \omega^2)r^2 = \text{cste}$$

Si $\omega \nearrow$ alors $r \searrow$ (Terre-Lune : $+4 \text{ cm/an}$)
Si $\omega \searrow$ alors $r \nearrow$



Migration et circularisation

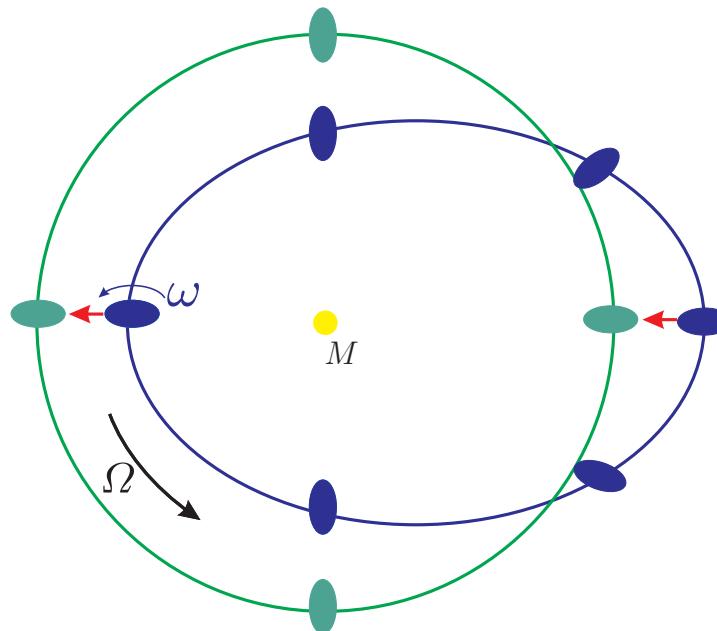
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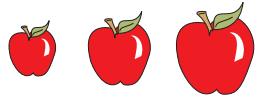
Synchronisation = rapide (Hut, 1981)
Si orbite excentrique ($e \neq 0$, $\Omega \neq \text{cste}$)

$v_p > v_a$
 $a \nearrow \quad a \searrow$
Effet : $e \rightarrow 0$



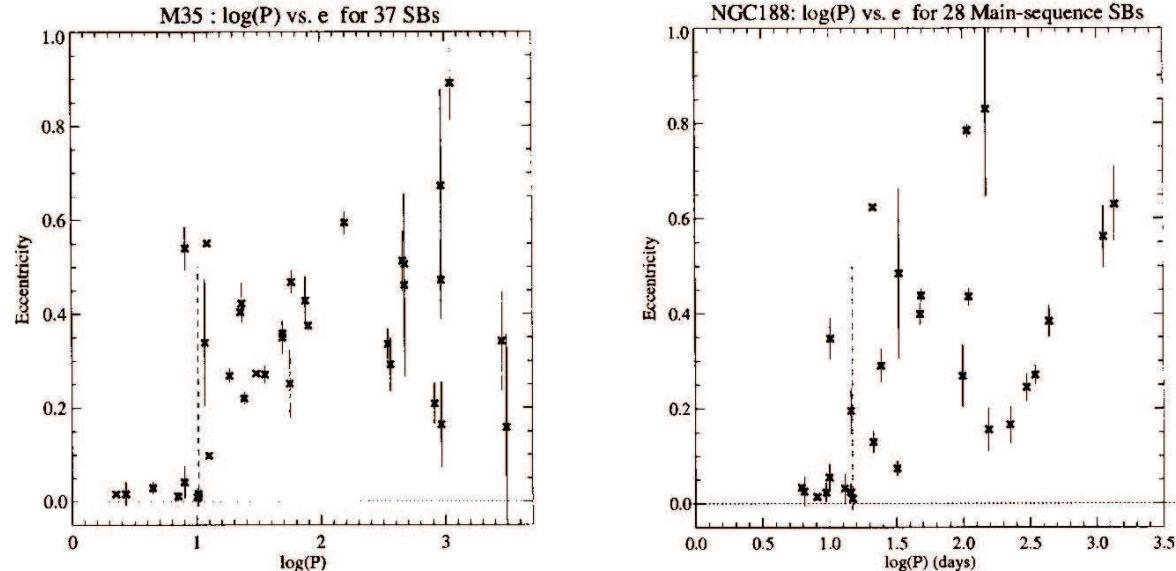
$$T_{circ} \propto (R/r)^6$$

Zahn, 1966, Lecar et al, 1976

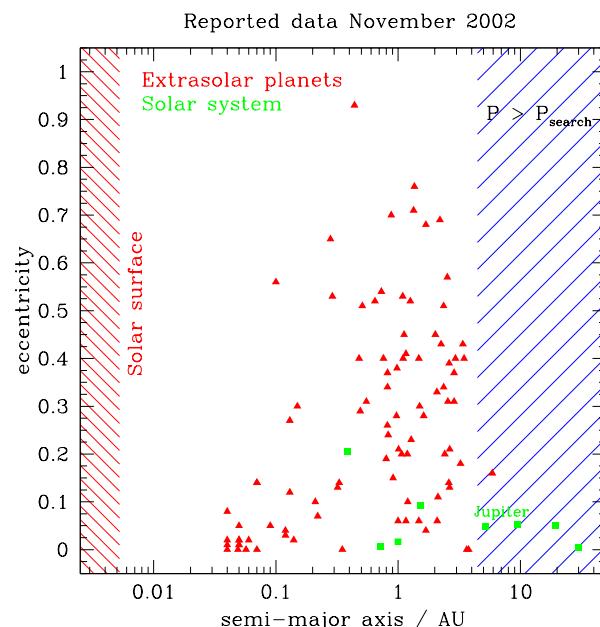


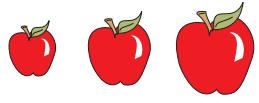
Coupe = observée ...

- Binaires dans des amas(Meibom&Mathieu.Astroph0303197)

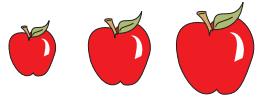


- Exoplanètes (catalogues...)





Chauffage interne



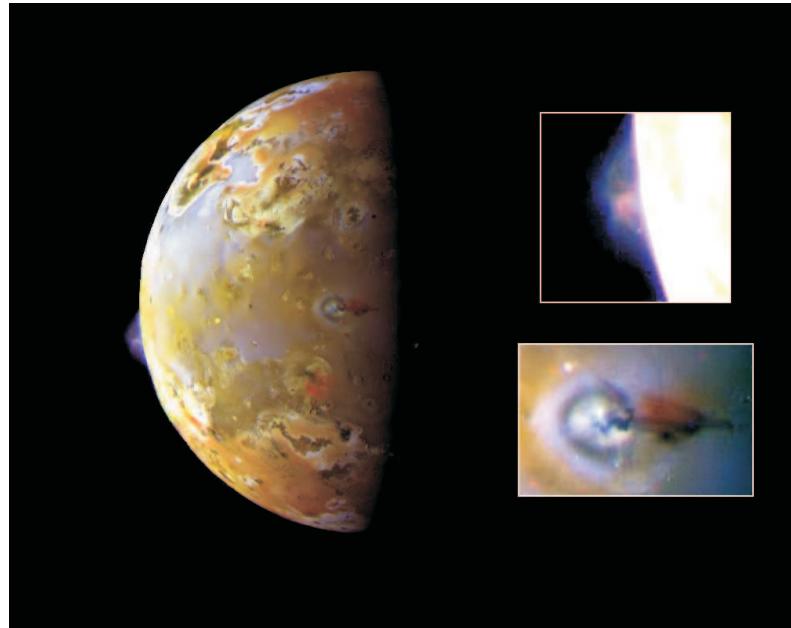
Chauffage interne

Volcanisme de Marées

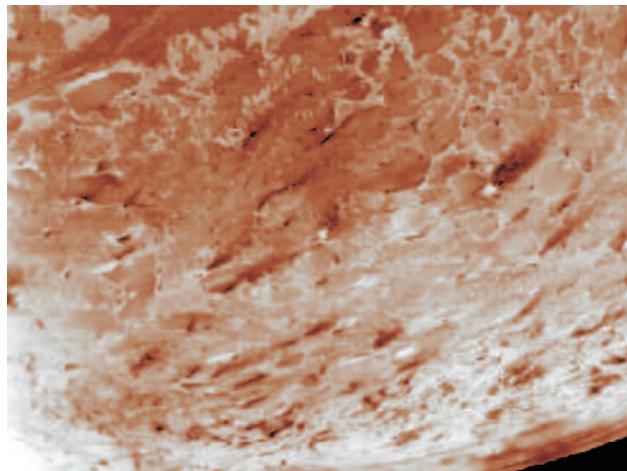
Io

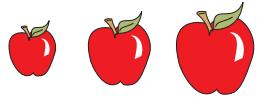
Peale & Cassen, 1979

Confirmation : Voyager II



Triton
Geyser
(azote liquide)
Voyager II





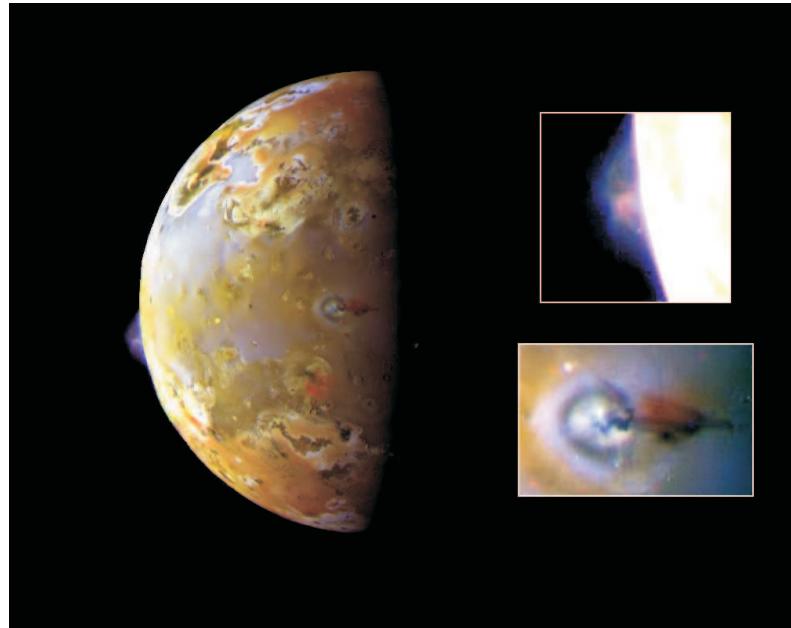
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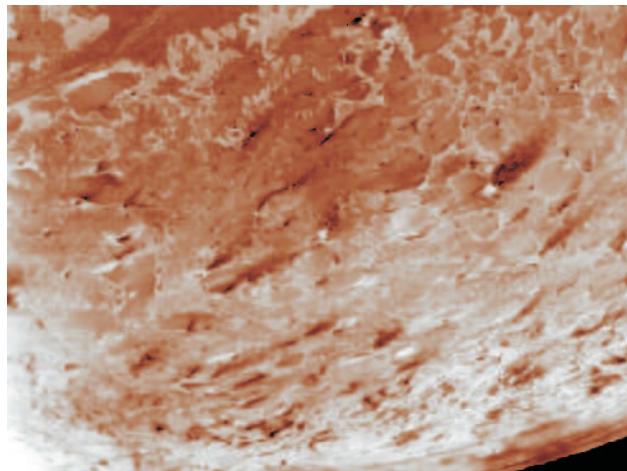
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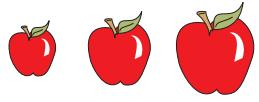


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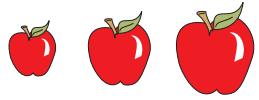


Planètes extrasolaires

Pégasides : $e \approx 0$ très chaude)
⇒ Bodenheimer et al, 2001, Guillot & Showmann, 2002



Limite de Roche



Limite de Roche

P : Corps sphérique homogène, m_p, ρ_p .

S : Satellite de P , sphérique homogène, m_s, ρ_s , orbite circulaire à la distance r , spin ω .

Forces de marée \Rightarrow Etirement \Rightarrow Brisure ?



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■ Forces d'étirement: Marée : $2Gm_p r_s r^{-3}$, Centrifuge : $\omega^2 r_s$

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Condition d'existence du satellite : Etirement < Cohésion

$$2 \frac{Gm_p r_s}{r^3} + \omega^2 r_s < \frac{Gm_s}{r_s^2}$$



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Hypothèse : rotation synchrone de la révolution (marées, Kepler)

$$\omega^2 = \frac{Gm_p}{r^3} \quad \text{Condition} \quad : \quad r > \left(3\frac{m_p}{m_s}\right)^{\frac{1}{3}} r_s = \left(3\frac{\rho_p}{\rho_s}\right)^{\frac{1}{3}} r_p$$

Cas général : Non homogène, non sphérique, orbite d'exentricité $e \dots$

$$r > \kappa \left(\frac{\rho_p}{\rho_s}\right)^{\frac{1}{3}} r_p \quad \kappa \in [1, 3]$$

... Anneaux de Saturne ...



Références

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- **Rambaux G. & Bois E.**, Astronomy and Astrophysics , astroph0306387, 2003
- **Hut P.** ,Astronomy and Astrophysics, **99**, 126, 1981
- **Zahn J.P**,Annales d'Astrophysique., **29**, 313, 1966
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- **Guillot T. & Showmann**,Astronomy and Astrophysics, **99**, 126, 2002
- **Peale&Cassen**,Science, **203**, 892, 1979