V46

Der Faraday-Effekt

Fritz Agildere
fritz.agildere@udo.edu

Amelie Strathmann
amelie.strathmann@udo.edu

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TU Dortmund – Fakultät Physik

Inhaltsverzeichnis

1 Zielsetzung

Im diesem Versuch soll die Faraday-Rotation ausgenutzt werden, um die effektive Masse der Leitungselektronen in negativ dotiertem Galliumarsenid (n-GaAs) zu bestimmen.

2 Theorie [1]

2.1 Bandstruktur

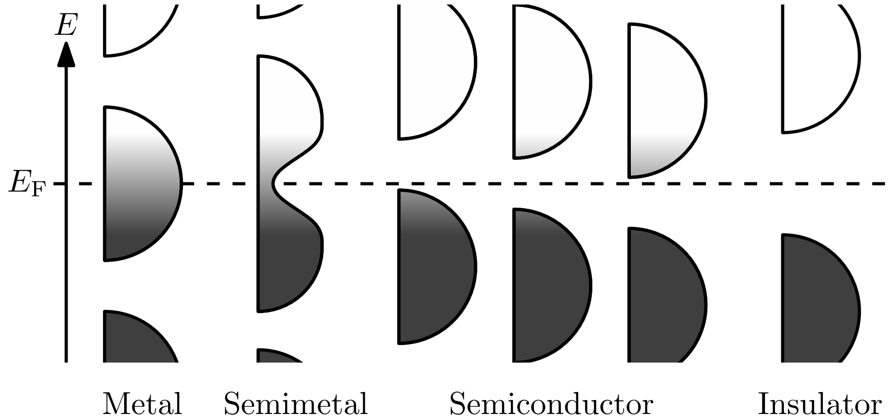

Abbildung 1: Energieschemata verschiedener Materialklassen im Vergleich. [3]

2.2 Dotierung

2.3 Faraday-Effekt

$$\boldsymbol{E}(z) = \frac{1}{2}\left(\boldsymbol{E}_R(z) + \boldsymbol{E}_L(z)\right) \tag{1}$$

$$\boldsymbol{E}_R(z) = E_0\left(\hat{\boldsymbol{x}} - i\hat{\boldsymbol{y}}\right)e^{ik_R z} \tag{2}$$
$$\boldsymbol{E}_L(z) = E_0\left(\hat{\boldsymbol{x}} + i\hat{\boldsymbol{y}}\right)e^{ik_L z} \tag{3}$$

$$\boldsymbol{E}(0) = E_0\hat{\boldsymbol{x}} \tag{4}$$

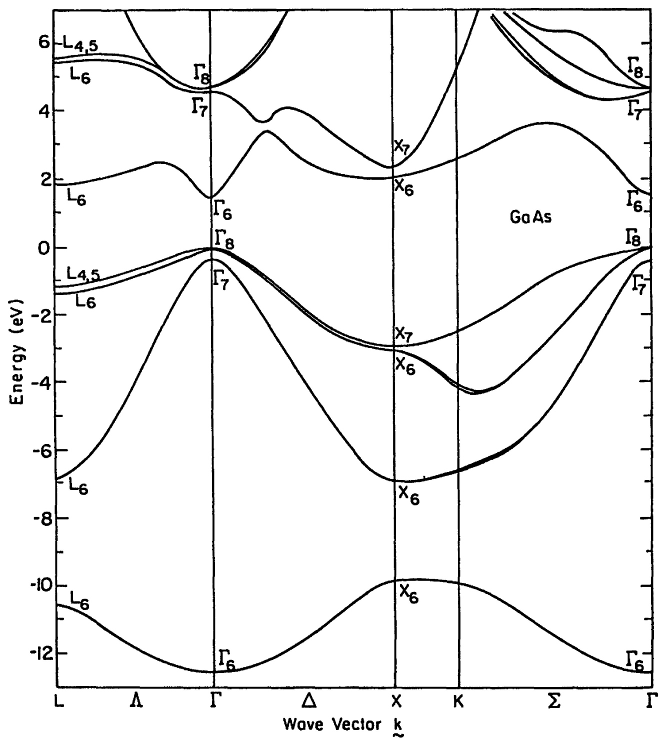

Abbildung 2: Berechnete Bandstruktur von GaAs um die Bandlücke. [2]

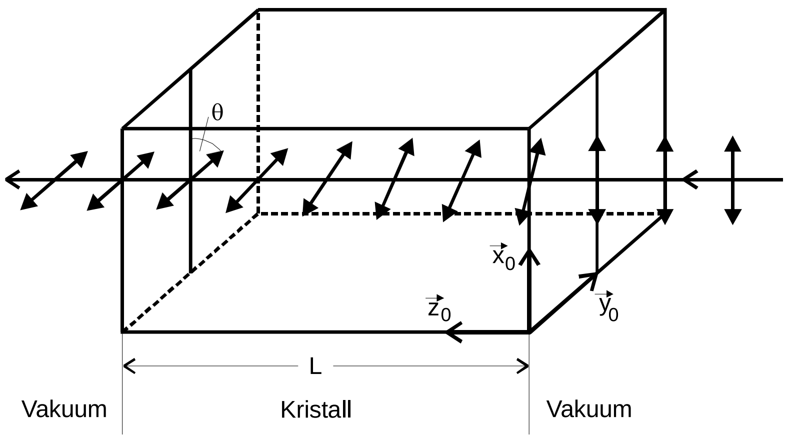

Abbildung 3: Drehung der Polarisationsebene einer Lichtwelle beim Durchgang durch einen Kristall. [1]

$$\boldsymbol{E}(L) = \frac{1}{2}E_0\left(\left(e^{ik_RL} + e^{ik_LL}\right)\hat{\boldsymbol{x}} + \left(e^{ik_RL} - e^{ik_LL}\right)\hat{\boldsymbol{y}}\right) \tag{5}$$

$$\psi \equiv \frac{L}{2}(k_R + k_L) \tag{6}$$

$$\theta \equiv \frac{L}{2}(k_R - k_L) \tag{7}$$

$$\boldsymbol{E}(L) = \frac{1}{2}E_0\left(\left(e^{i\psi}e^{i\theta} + e^{i\psi}e^{-i\theta}\right)\hat{\boldsymbol{x}} + \left(e^{i\psi}e^{i\theta} - e^{i\psi}e^{-i\theta}\right)\hat{\boldsymbol{y}}\right) \tag{8}$$

$$\boldsymbol{E}(L) = E_0e^{i\psi}(\cos\theta\,\hat{\boldsymbol{x}} + \sin\theta\,\hat{\boldsymbol{y}}) \tag{9}$$

$$v = \frac{\omega}{k} \tag{10}$$

$$n = \frac{c}{v} = \frac{ck}{\omega} \tag{11}$$

$$\theta = \frac{L\omega}{2c}(n_R - n_L) \tag{12}$$

$$\boldsymbol{\chi} = \begin{pmatrix} \chi_{xx} & 0 & 0 \\ 0 & \chi_{yy} & 0 \\ 0 & 0 & \chi_{zz} \end{pmatrix} \tag{13}$$

$$\boldsymbol{\chi} = \begin{pmatrix} \chi_{xx} & i\chi_{xy} & 0 \\ -i\chi_{xy} & \chi_{xx} & 0 \\ 0 & 0 & X_{zz} \end{pmatrix} \tag{14}$$

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} \approx \varepsilon_0(1+\chi)\boldsymbol{E} \tag{15}$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{E}) = -\boldsymbol{\nabla} \times \frac{\partial \boldsymbol{B}}{\partial t} = -\mu_0 \frac{\partial}{\partial t} \boldsymbol{\nabla} \times \boldsymbol{H} = -\mu_0 \frac{\partial}{\partial t} \left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t} \right) \tag{16}$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{E}) \approx -\mu_0 \frac{\partial^2 \boldsymbol{D}}{\partial t^2} \approx -\varepsilon_0 \mu_0 (1 + \chi) \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\frac{1}{c^2} (1 + \chi) \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \tag{17}$$

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\boldsymbol{r} - \omega t)} \tag{18}$$

$$\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{E}) = -\frac{\omega^2}{c^2} (1 + \chi) \boldsymbol{E} \tag{19}$$

$$\boldsymbol{k} = k \hat{\boldsymbol{z}} \tag{20}$$

$$\boldsymbol{E} = E_x \hat{\boldsymbol{x}} + E_y \hat{\boldsymbol{y}} + E_z \hat{\boldsymbol{z}} \tag{21}$$

$$\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{E}) = -k^2 (E_x \hat{\boldsymbol{x}} + E_y \hat{\boldsymbol{y}}) \tag{22}$$

$$\boldsymbol{\chi} \boldsymbol{E} = (\chi_{xx} E_x + i \chi_{xy} E_y) \hat{\boldsymbol{x}} + (\chi_{xx} E_y - i \chi_{xy} E_x) \hat{\boldsymbol{y}} + \chi_{zz} E_z \hat{\boldsymbol{z}} \tag{23}$$

$$\frac{\omega^2}{c^2} (1 + \chi_{zz}) E_z = 0 \tag{24}$$

$$E_z = 0 \tag{25}$$

$$\boldsymbol{C} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{\omega^2}{c^2} (1 + \chi_{xx}) - k^2 & i \frac{\omega^2}{c^2} \chi_{xy} \\ i \frac{\omega^2}{c^2} \chi_{xy} & -\frac{\omega^2}{c^2} (1 + \chi_{xx}) + k^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{26}$$

$$0 = -\det \boldsymbol{C} = \left(\frac{\omega^2}{c^2}(1+\chi_{xx}) - k^2\right)^2 + i^2\frac{\omega^4}{c^4}\chi_{xy}^2 \tag{27}$$

$$k_\pm = \frac{\omega}{c}\sqrt{1+\chi_{xx} \pm \chi_{xy}} \tag{28}$$

$$v_L^R = \frac{c}{\sqrt{1+\chi_{xx} \pm \chi_{xy}}} \tag{29}$$

$$v = \frac{c}{\sqrt{1+\chi_{xx}}} \tag{30}$$

$$E_{x_L}^{\ R} = \pm iE_y \tag{31}$$

$$\theta = \frac{L}{2}(k_+ - k_-) = \frac{L\omega}{2c}\left(\sqrt{1+\chi_{xx}+\chi_{xy}} - \sqrt{1+\chi_{xx}+\chi_{xy}}\right) \tag{32}$$

$$\theta \approx \frac{L\omega}{2c\sqrt{1+\chi_{xx}}}\chi_{xy} = \frac{L\omega v}{2c^2}\chi_{xy} = \frac{L\omega}{2cn}\chi_{xy} \tag{33}$$

$$m\frac{\partial^2 \boldsymbol{r}}{\partial t^2} + K\boldsymbol{r} = -e_0\boldsymbol{E} - e_0\frac{\partial \boldsymbol{r}}{\partial t} \times \boldsymbol{B} \tag{34}$$

$$\boldsymbol{E} \sim \boldsymbol{r} \sim e^{-i\omega t} \tag{35}$$

$$-m\omega^2\boldsymbol{r} + K\boldsymbol{r} = -e_0\boldsymbol{E} + ie_0\omega\boldsymbol{r} \times \boldsymbol{B} \tag{36}$$

$$\boldsymbol{P} = -Ne\boldsymbol{r} \tag{37}$$

$$-m\omega^2\boldsymbol{P} + K\boldsymbol{P} = Ne^2\boldsymbol{E} + ie_0\omega\boldsymbol{P}\times\boldsymbol{B} \tag{38}$$

$$\boldsymbol{B} = B\hat{\boldsymbol{z}} \tag{39}$$

$$(K - m\omega^2)P_x = Ne_0^2E_x + ie_0\omega P_yB \tag{40}$$
$$(K - m\omega^2)P_y = Ne_0^2E_y - ie_0\omega P_xB \tag{41}$$
$$(K - m\omega^2)P_z = Ne_0^2E_z \tag{42}$$

$$\varepsilon_0(K - m\omega^2)(\chi_{xx}E_x + i\chi_{xy}E_y) = Ne_0^2E_x + i\varepsilon_0e_0\omega(i\chi_{yx}E_x + \chi_{xx}E_y)B \tag{43}$$

$$\varepsilon_0(K - m\omega^2)\chi_{xx} = Ne_0^2 - \varepsilon_0e_0\omega\chi_{yx}B \tag{44}$$
$$\varepsilon_0(K - m\omega^2)\chi_{xy} = \varepsilon_0e_0\omega\chi_{xx}B \tag{45}$$
$$\tag{46}$$

$$\varepsilon_0(K - m\omega^2)(i\chi_{yx}E_x + \chi_{xx}E_y) = Ne_0^2E_y - i\varepsilon_0e_0\omega(\chi_{xx}E_x + i\chi_{xy}E_y)B \tag{47}$$

$$\varepsilon_0(K - m\omega^2)\chi_{xx} = Ne_0^2 + \varepsilon_0e_0\omega\chi_{xy}B \tag{48}$$
$$\varepsilon_0(K - m\omega^2)\chi_{yx} = -\varepsilon_0e_0\omega\chi_{xx}B \tag{49}$$

$$\chi_{xy} = -\chi_{yx} \tag{50}$$

$$\chi_{xy} = \frac{Ne_0^3\omega B}{\varepsilon_0\left((K - m\omega^2)^2 - (e_0\omega B)^2\right)} \tag{51}$$

$$\theta \approx \frac{LNe_0^3\omega^2 B}{2\varepsilon_0 cn\left((K-m\omega^2)^2-(e_0\omega B)^2\right)} = \frac{LNe_0^3\omega^2 B}{2\varepsilon_0 cm^2\left(\left(\frac{K}{m}-\omega^2\right)^2-\frac{e_0^2B^2}{m^2}\omega^2\right)} \tag{52}$$

$$\omega_0 \equiv \sqrt{\frac{K}{m}} \tag{53}$$

$$\omega_C \equiv \frac{e_0 B}{m} \tag{54}$$

$$\theta \approx \frac{LNe_0^3\omega^2 B}{2\varepsilon_0 cm^2\left((\omega_0^2-\omega^2)^2-\omega_C^2\omega^2\right)} \tag{55}$$

$$\omega_0^2-\omega^2 \gg \omega_C\omega \tag{56}$$

$$\theta \approx \frac{LNe_0^3\omega^2 B}{2\varepsilon_0 cm^2(\omega_0^2-\omega^2)^2} \tag{57}$$

$$\omega = 2\pi\nu = 2\pi\frac{c}{\lambda} \tag{58}$$

$$\omega \ll \omega_0 \tag{59}$$

$$\theta \approx \frac{LNe_0^3\omega^2 B}{2\varepsilon_0 cm^2\omega_0^4} = \frac{2\pi^2 LNe_0^3 cB}{\varepsilon_0 m^2\lambda^2\omega_0^4} \tag{60}$$

$$\omega \gg \omega_0 \tag{61}$$

$$\theta \approx \frac{LNe_0^3 B}{2\varepsilon_0 cm^2\omega^2} = \frac{LNe_0^3\lambda^2 B}{8\pi^2\varepsilon_0 c^3 m^2 n} \tag{62}$$

$$\frac{\theta}{L} \approx \frac{Ne_0^3 B}{2\varepsilon_0 cm^{*2}\omega^2} = \frac{Ne_0^3\lambda^2 B}{8\pi^2\varepsilon_0 c^3 m^{*2} n} \tag{63}$$

3 Aufbau

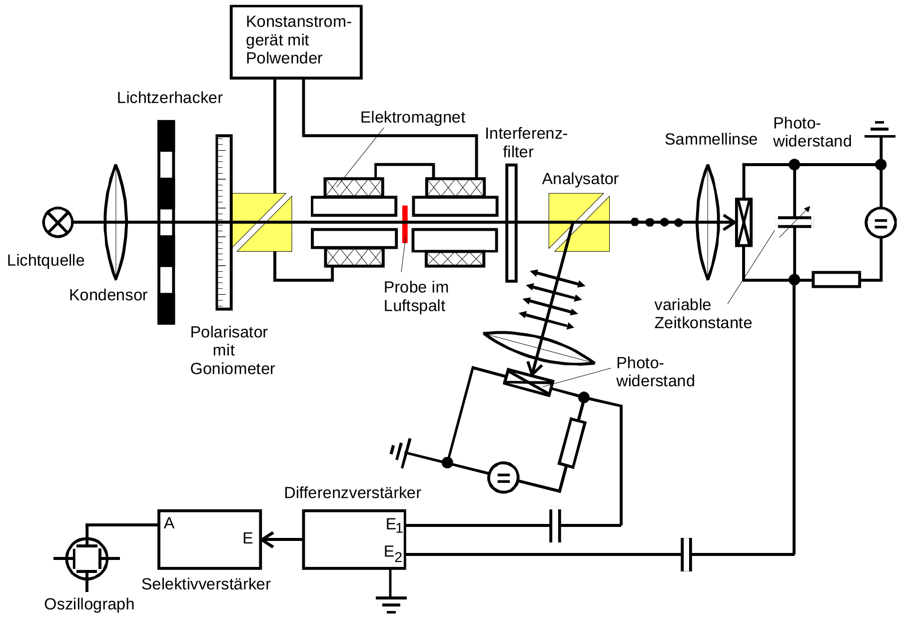

Abbildung 4: Schematische Darstellung der Messapparatur. [1]

4 Durchführung

5 Auswertung

5.1 Magnetfeld

5.2 Faraday-Rotation

5.2.1 Dotierte Proben

5.2.2 Reine Probe

5.3 Effektive Masse

6 Diskussion

Literatur

[1] *Anleitung zu Versuch 46, Der Faraday-Effekt*. TU Dortmund, Fakultät Physik. 2024.

[2] „Band Structure of Gallium Arsenide“. In: Marvin L. Cohen und James R. Chelikowsky. *Electronic Structure and Optical Properties of Semiconductors*. Springer Berlin, Heidelberg, 1988, S. 103. ISBN: 978-3-642-97080-1. DOI: `https://doi.org/10.1007/978-3-642-97080-1`.

[3] *Valence and Conduction Bands*. 2013. URL: `https://en.wikipedia.org/wiki/file:band_filling_diagram.svg`.

Anhang