

V47

## Molar Heat of Copper

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# 1 Objective

By investigating the molar heat capacity of copper, we test some of the most prominent models for their validity and determine select characteristic quantities of solids.

## 2 Theory [1]

In the description of condensed matter, tools from classical thermodynamics and statistical physics as well as quantum mechanical considerations need to be applied. Throughout the following paragraphs, we explore some of the more well known models in the subfield of thermal properties, specifically for the explanation of the heat capacity in solids.

### 2.1 Heat Capacity

The heat capacity of any material is defined via

$$C \equiv \frac{\Delta Q}{\Delta T}$$

as the amount of heat  $\Delta Q$  corresponding to a unit change  $\Delta T$  in temperature. This is clearly dependent on the amount of matter, so to allow for comparisons between different materials we can normalize with volume  $V$  or mass  $M$  to obtain a specific heat capacity.

#### 2.1.1 Molar Heat

In our case, we are interested in the heat capacity per number of particles making up a sample. This leads to

$$c \equiv \frac{\Delta Q}{n\Delta T}$$

for the molar heat capacity, where  $n$  is the number of mol contained in the substance. To determine  $n$  one can use readily available values of the molar mass  $m$  or molar volume  $v$  for the given experimental parameters.

#### 2.1.2 Measurement

During the measuring process, it is necessary to constrain certain parameters. To grasp the following relations, we examine the first and second laws of thermodynamics

$$dU = \delta Q - \delta W = TdS - PdV$$

where  $U$  is the internal energy,  $Q$  and  $W$  are heat and work,  $T$  stands for temperature,  $S$  for entropy,  $P$  for pressure and  $V$  for volume. The symbols  $d$  and  $\delta$  denote exact and inexact differentials, respectively.

From this relation, we identify  $\delta Q = TdS$  and thereby

$$C = T \frac{\partial S}{\partial T}$$

when translating the defining statement to infinitesimal notation.

### 2.1.2.1 Isochoric Case

At fixed volume, all units of heat result in temperature changes exclusively. The isochoric heat capacity can be written as

$$C_V = T \left. \frac{\partial S}{\partial T} \right|_V$$

and constitutes an interesting quantity to study the behaviour of materials due to the isolation of otherwise connected effects.

### 2.1.2.2 Isobaric Case

When the pressure is kept constant instead, some of the heat exchange also results in expansion or contraction of the sample and therefore reduces the total temperature difference. Intuition therefore tells us that  $C_P > C_V$  where we write

$$C_P = T \left. \frac{\partial S}{\partial T} \right|_P$$

for the isobaric heat capacity. This situation is more easily realizable in experiments but comes at the cost of no longer separating different mechanisms affecting the substance.

### 2.1.2.3 Connection

In order to convert between  $C_P$  and  $C_V$  we need some generally applicable relationship between the two quantities. We establish the differential form for entropy

$$dS = \left. \frac{\partial S}{\partial T} \right|_V dT + \left. \frac{\partial S}{\partial V} \right|_T dV$$

in terms of volume as an extensive and temperature as an intensive property. We obtain

$$\left. \frac{\partial S}{\partial T} \right|_P = \left. \frac{\partial S}{\partial T} \right|_V + \left. \frac{\partial S}{\partial V} \right|_T \left. \frac{\partial V}{\partial T} \right|_P$$

by differentiating, allowing us to reformulate our desired quantities

$$C_P - C_V = T \left( \left. \frac{\partial S}{\partial T} \right|_P - \left. \frac{\partial S}{\partial T} \right|_V \right) = T \left. \frac{\partial S}{\partial V} \right|_T \left. \frac{\partial V}{\partial T} \right|_P$$

as the difference between them.

A combination of Jacobian coordinate transformations and Maxwell relations yields

$$C_P - C_V = \alpha_V^2 \kappa_T TV$$

with the isothermal bulk modulus

$$\kappa_T = -V \left. \frac{\partial P}{\partial V} \right|_T$$

and the volumetric thermal expansion coefficient

$$\alpha_V = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_P$$

of the material. For isotropic materials, the last term is equivalent to the form  $\alpha_V = 3\alpha_L$  via the linear analogue

$$\alpha_L = \frac{1}{L} \left. \frac{\partial L}{\partial T} \right|_P$$

which gives

$$C_P - C_V = 9\alpha_L^2 \kappa_T TV$$

as the final expression. For typical solids with strong binding forces this difference is negligible, though it becomes more relevant for liquid or gaseous states of matter.

## 2.2 Models

All approaches discussed in this section model condensed matter as perfect crystal lattices with normal modes of participating members as purely harmonic oscillations. In the Hooke regime, the center of mass does not change with increasing or decreasing energy. Accordingly, the volume is constant under heat exchange and the condition  $C_V = C_P$  is always fulfilled. This is of course unphysical, but it should serve as a decent approximation for the materials we are interested in. To account for thermal contraction, we would need to consider anharmonic effects by expanding the assumed potential minimum to higher orders.

### 2.2.1 Classical Physics

The equipartition theorem from classical thermodynamics affords a mean energy

$$\langle u \rangle = \frac{1}{2} k_B T$$

to each available degree of freedom. For a singular uncoupled harmonic oscillator, we have the spacial components of position and velocity as free coordinates. In this view, a three dimensional system of  $N$  atoms has an internal energy

$$U = 3Nk_B T = 3nRT$$

when we impose zero equilibrium potential. Here  $R = N_A k_B$  is the universal gas constant,  $N_A$  is the Avogadro constant and  $k_B$  is the Boltzmann constant.

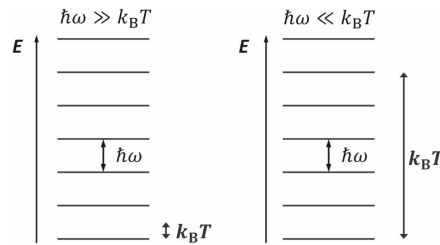
Because  $dV = 0$  and therefore  $dU = \delta Q$  we can simply write

$$C = \frac{\partial U}{\partial T} = 3Nk_B = 3nR$$

to find the heat capacity as predicted through the empirical law by Dulong and Petit.

### 2.2.2 Quantum Mechanics

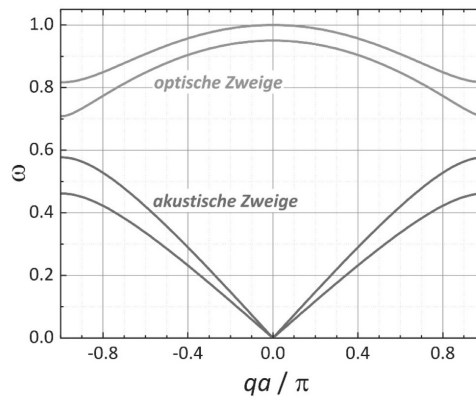
The classical result estimates the heat capacity reasonably well for high temperatures. At lower temperatures however, we expect quantum mechanical effects to play a significant role. This is explained by the quantization of energy in multiples of  $\hbar\omega$  dividing the range of allowed states into discrete levels. At temperatures where  $k_B T \gg \hbar\omega$  the energy space appears continuous to thermal excitations whereas  $k_B T \ll \hbar\omega$  forces most members to remain frozen in the ground state. To visualize this description, figure 1 presents a conceptual sketch.



**Figure 1:** Exemplary energy discretization in low and high temperature limits. [1]

#### 2.2.2.1 Phonons

As shown in figure 2 collective lattice vibrations show two types of dispersion relation.



**Figure 2:** Schematic phonon branches inside the first Brillouin zone. [1]

Optical branches result from out of phase oscillations, where neighboring atoms have opposite velocity vectors. Conversely, acoustic branches represent in phase or parallel vibrational modes. For a crystal with  $k$  basis atoms, there are 3 acoustic and  $3k - 3$  optical dispersion curves. The differences between these solutions can be more or less relevant under different circumstances.

We call such quantized sound waves phonons, which are quasi particles with similar behaviour to photons as the gauge bosons representing quantized light waves. The most important difference is the vanishing chemical potential  $\mu = 0$  indicating that phonons are not conserved.

In accordance with the previous statement, the mean equilibrium population

$$\langle n \rangle = \frac{1}{e^{\beta\hbar\omega} - 1}$$

follows statistics as described by Bose and Einstein, where

$$\beta \equiv \frac{1}{k_B T}$$

is the thermodynamic beta and the exponentials trace back to Boltzmann weighting as well as the convergence of the geometric series. From the energy levels of a quantum harmonic oscillator

$$E_n = \hbar\omega \left( \frac{1}{2} + n \right)$$

we find the internal energy

$$U = \sum_{\mathbf{q}r} \frac{1}{2} \hbar\omega_{\mathbf{q}r} + \sum_{\mathbf{q}r} \frac{\hbar\omega_{\mathbf{q}r}}{e^{\beta\hbar\omega_{\mathbf{q}r}} - 1}$$

where we sum over all allowed reciprocal vectors  $\mathbf{q}$  and polarisations  $r$  to account for multiatomic bases. The heat capacity follows as

$$C = \frac{\partial}{\partial T} \sum_{\mathbf{q}r} \frac{\hbar\omega_{\mathbf{q}r}}{e^{\beta\hbar\omega_{\mathbf{q}r}} - 1}$$

in the most general case. For high temperatures, we get

$$C = \sum_{\mathbf{q}r} k_B$$

by expanding the exponential to linear order. With  $\sum_{\mathbf{q}r} = 3N$  this reproduces the classical result as asserted by the correspondence principle.

2.2.2.2 Einstein

2.2.2.3 Debye

2.2.2.4 Electrons

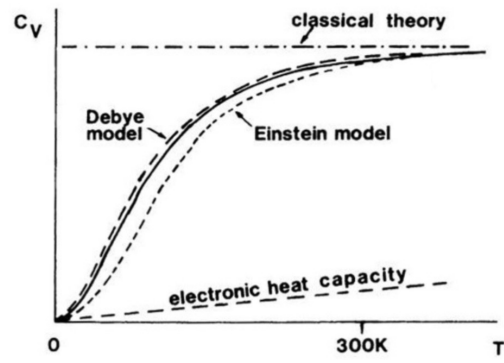


Figure 3: [3]



### 3 Setup

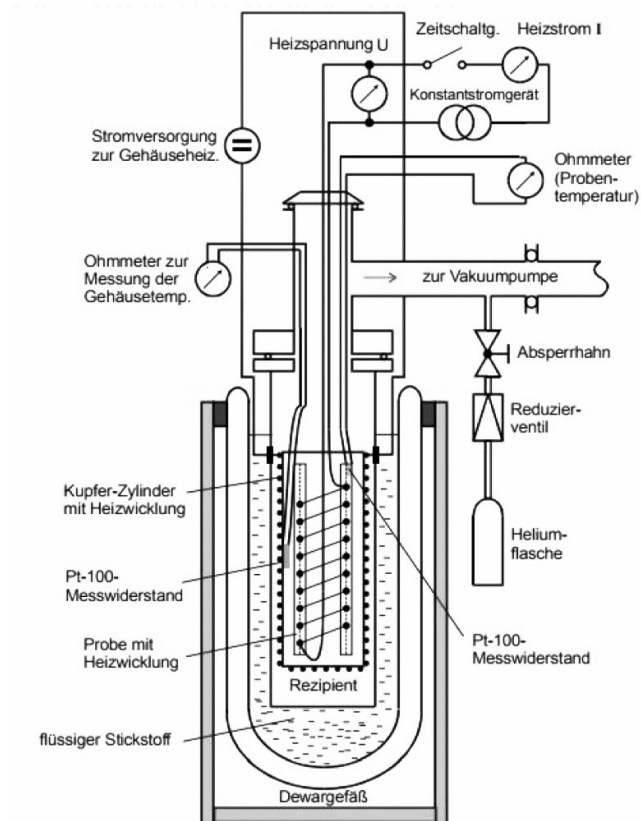


Figure 4: [2]

### 4 Procedure

### 5 Results

|       |       |       |       |        |        |        |        |       |       |
|-------|-------|-------|-------|--------|--------|--------|--------|-------|-------|
| T[°C] | -200  | -190  | -180  | -170   | -160   | -150   | -140   | -130  | -120  |
| R[Ω]  | 18,44 | 22,71 | 27,03 | 31,28  | 35,48  | 39,65  | 43,80  | 47,93 | 52,04 |
| T[°C] | -110  | -100  | -90   | -80    | -70    | -60    | -50    | -40   | -30   |
| R[Ω]  | 56,13 | 60,20 | 64,25 | 68,28  | 72,29  | 76,28  | 80,25  | 84,21 | 88,17 |
| T[°C] | -20   | -10   | 0     | +10    | +20    | +30    | +40    |       |       |
| R[Ω]  | 92,13 | 96,07 | 100   | 103,90 | 107,79 | 111,67 | 115,54 |       |       |

Figure 5: [2]

|                                    |       |       |       |       |       |       |       |       |
|------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| T [K]                              | 70    | 80    | 90    | 100   | 110   | 120   | 130   | 140   |
| $\alpha$ [ $10^{-6}$ grd $^{-1}$ ] | 7,00  | 8,50  | 9,75  | 10,70 | 11,50 | 12,10 | 12,65 | 13,15 |
| T [K]                              | 150   | 160   | 170   | 180   | 190   | 200   | 210   | 220   |
| $\alpha$ [ $10^{-6}$ grd $^{-1}$ ] | 13,60 | 13,90 | 14,25 | 14,50 | 14,75 | 14,95 | 15,20 | 15,40 |
| T [K]                              | 230   | 240   | 250   | 260   | 270   | 280   | 290   | 300   |
| $\alpha$ [ $10^{-6}$ grd $^{-1}$ ] | 15,60 | 15,75 | 15,90 | 16,10 | 16,25 | 16,35 | 16,50 | 16,65 |

Figure 6: [2]

| $\theta_0/T$ | 0       | 1       | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       |
|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0            | 24,9430 | 24,9310 | 24,8930 | 24,8310 | 24,7450 | 24,6340 | 24,5000 | 24,3430 | 24,1630 | 23,9610 |
| 1            | 23,7390 | 23,4970 | 23,2360 | 22,9560 | 22,6600 | 22,3480 | 22,0210 | 21,6800 | 21,3270 | 20,9630 |
| 2            | 20,5880 | 20,2050 | 19,8140 | 19,4160 | 19,0120 | 18,6040 | 18,1920 | 17,7780 | 17,3630 | 16,9470 |
| 3            | 16,5310 | 16,1170 | 15,7040 | 15,2940 | 14,8870 | 14,4840 | 14,0860 | 13,6930 | 13,3050 | 12,9230 |
| 4            | 12,5480 | 12,1790 | 11,8170 | 11,4620 | 11,1150 | 10,7750 | 10,4440 | 10,1190 | 9,8030  | 9,4950  |
| 5            | 9,1950  | 8,9030  | 8,6190  | 8,3420  | 8,0740  | 7,8140  | 7,5610  | 7,3160  | 7,0780  | 6,8480  |
| 6            | 6,6250  | 6,4090  | 6,2000  | 5,9980  | 5,8030  | 5,6140  | 5,4310  | 5,2550  | 5,0840  | 4,9195  |
| 7            | 4,7606  | 4,6071  | 4,4590  | 4,3160  | 4,1781  | 4,0450  | 3,9166  | 3,7927  | 3,6732  | 3,5580  |
| 8            | 3,4468  | 3,3396  | 3,2362  | 3,1365  | 3,0403  | 2,9476  | 2,8581  | 2,7718  | 2,6886  | 2,6083  |
| 9            | 2,5309  | 2,4562  | 2,3841  | 2,3146  | 2,2475  | 2,1828  | 2,1203  | 2,0599  | 2,0017  | 1,9455  |
| 10           | 1,8912  | 1,8388  | 1,7882  | 1,7393  | 1,6920  | 1,6464  | 1,6022  | 1,5596  | 1,5184  | 1,4785  |
| 11           | 1,4400  | 1,4027  | 1,3667  | 1,3318  | 1,2980  | 1,2654  | 1,2337  | 1,2031  | 1,1735  | 1,1448  |
| 12           | 1,1170  | 1,0900  | 1,0639  | 1,0386  | 1,0141  | 0,9903  | 0,9672  | 0,9449  | 0,9232  | 0,9021  |
| 13           | 0,8817  | 0,8618  | 0,8426  | 0,8239  | 0,8058  | 0,7881  | 0,7710  | 0,7544  | 0,7382  | 0,7225  |
| 14           | 0,7072  | 0,6923  | 0,6779  | 0,6638  | 0,6502  | 0,6368  | 0,6239  | 0,6113  | 0,5990  | 0,5871  |
| 15           | 0,5755  | 0,5641  | 0,5531  | 0,5424  | 0,5319  | 0,5210  | 0,5117  | 0,5020  | 0,4926  | 0,4834  |

Figure 7: [2]

## 6 Discussion

## References

- [1] Rudolf Gross and Achim Marx. *Festkörperphysik*. Berlin, Boston: De Gruyter Oldenbourg, 2022. ISBN: 9783110782394. DOI: doi : 10 . 1515 / 9783110782394. URL: <https://doi.org/10.1515/9783110782394>.
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## Appendix