General Physics Module 1 Quarter 2

English (Davao City National High School)

SHS

AIRs - LM in
General Physics 1
Quarter 2 – Week 1
Module 1- Torque and Moment of Inertia

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General Physics 1

Grade 11/12 Quarter 2 - Module 1- Torque and Moment of Inertia

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General Physics 1

Quarter 2 – Week 1

Module 1- Torque and Moment of Inertia

Lesson 1. Torque and Moment of Inertia

Target

Newton's First Law of motion, states that, unless a net force is acted on a body, it remains in its state of being at rest or of constant motion. If a net force is acted on a body, it changes its state (rest or motion). In order for a body remain in the state of equilibrium, balance force must act on a body, whether the body is at rest or in constant motion. However, balance forces is not a guarantee that the body is in complete equilibrium.

In your previous lesson about forces in equilibrium, you have learned how to determine the resultant force under the action of concurrent forces.

In this lesson, you will be learning how parallel non-concurrent forces can keep objects in equilibrium but has the tendency to rotate. The concepts of torque and moment of inertia will be the focus of this module.

After finishing this Learning Material, you are expected to:

1. calculate the moment of inertia about a given axis of single object and multiple-object systems. (STEM_GP12RED IIa-1)
2. calculate magnitude and direction of torque using the definition of torque as a cross product. (STEM_GP12REDIIa-3)
3. describe rotational quantities under static equilibrium using vectors. (STEM_GP12REDIIa-4 &5)

Jumpstart

In the first condition of equilibrium, stated that, the summation of the concurrent forces acting on a body must be zero. Statics deals with the study of forces acting on bodies at rest. A body at rest or a body at constant motion is said to be in equilibrium. However, there may be forces, each acting at different point on a body. Let say, a rock lying on a surface (Figure 1). Since every part of the rock has small particles with masses, every part is attracted to the center of the Earth. The Earth is so huge, all the downward forces exerted on the rock are virtually parallel.

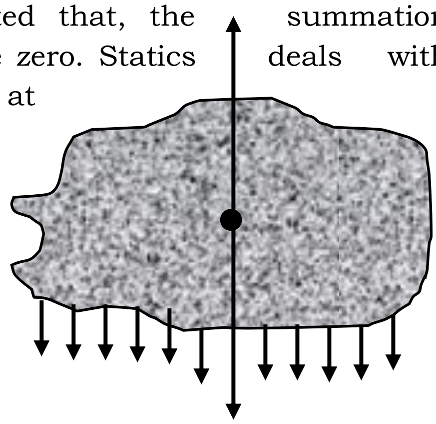


Figure. 1.

The weight of the stone can be thought of as a force vector that is the resultant of all these parallel force vectors. The single point in which the weight of the rock is concentrated is known as the "center of gravity."

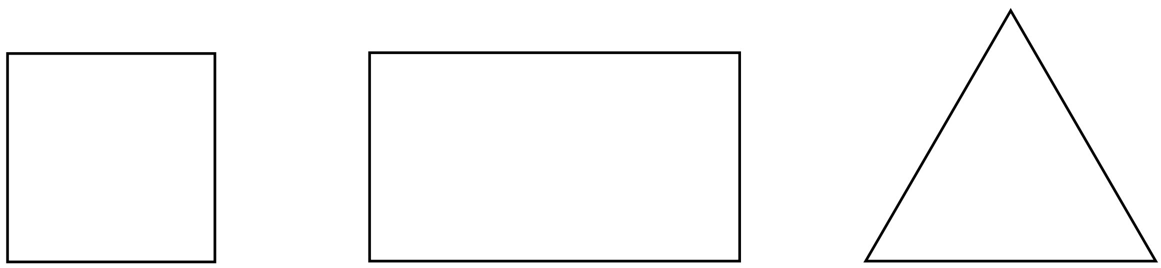
Activity 1. Where is the Center of Gravity?

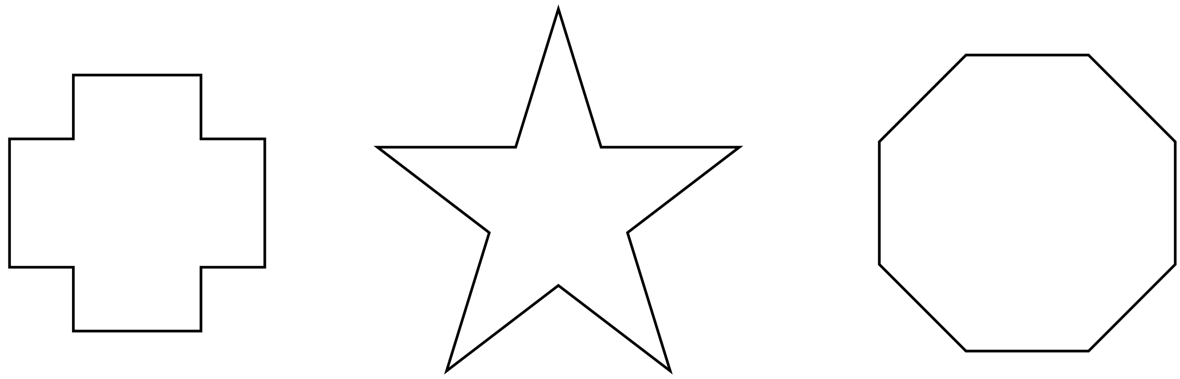
What You Need

Ruler or foot ruler, pencil or pen, figures of regularly-shaped objects

A. Regular planar shape objects

Direction: Below are regular planar shape objects. Determine the center of gravity of each figure. The center of gravity for regularly-shape object may be found at its geometric center.





Q1. How did you find the center of gravity of each shape? ______________________
____
____

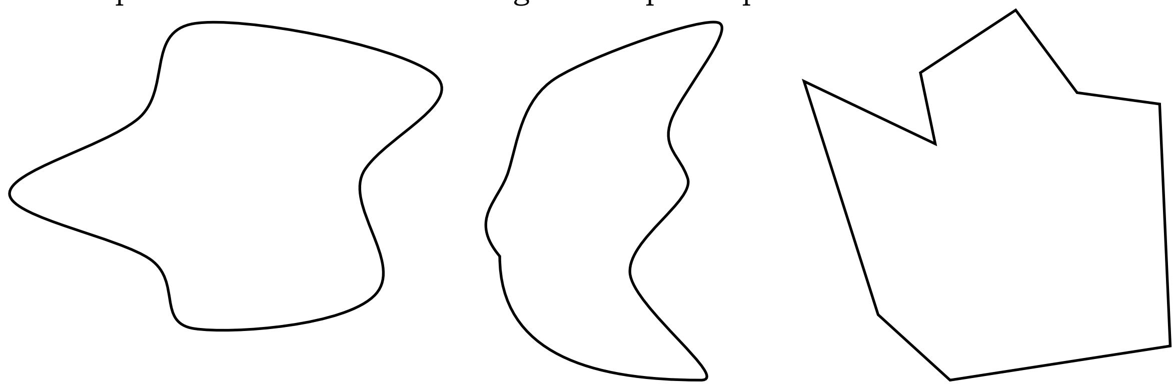
B. Irregularly-shape objects

What You Need

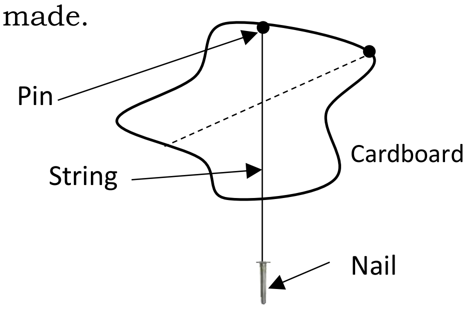
4-cardboard shapes, scissors, pin or thumbtack, 20 cm string, pen/pencil, nail

Direction:

1. Cut 3 pieces of cardboard following the shape templates below.



2. Attach a nail to the end of your string. This is your plumb line, following the direction of the gravitational force.
3. Hang the plumb line on the pin. Pierce the pin anywhere along the edge of the shape, so that your shape is free to rotate.
4. Hold the pin and wait until the shape and the plumb line have settled. Mark where the string crosses the shape by tracing the string's path with a pen.
5. Remove the pin and plumb line from the shape. Pierce them another point along the shape's edge, not too close to the last hole you made.
6. Repeat steps 1 to 5 two more times
7. Mark the spot where the lines intersect.
8. Repeat the steps for the other two shape.



Q1. Why do we have to draw several lines from different points?

____

____

Q2. What does the spot where your lines intersect represents?

____

Q3. Is the center of gravity exactly in the geometric center of the shape? _______

Discover

From the previous activity, you were able to found the center of gravity of some objects. The center of gravity is one important consideration when we deal about equilibrium of rigid bodies. Consider a wood attached to another wood by means of a bolt. The system is said to be in equilibrium because the forces are balance (Figure 2).

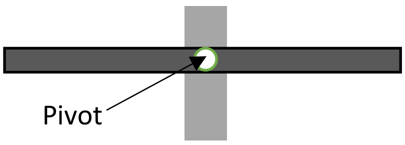


Figure 2. The horizontal wood is in equilibrium

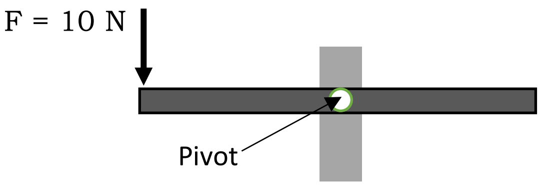


Figure 3. A downward force applied to one end of the wood

Suppose you apply a perpendicular force, say, 10 N downward on the left side of the wood (Figure 3). The tendency of the wood is to rotate about its axis called pivot or fulcrum in a clockwise rotation. Now, if another force of 10 N downward (Figure 4) is acted on the opposite side of the wood, this force can cause a clockwise rotation.

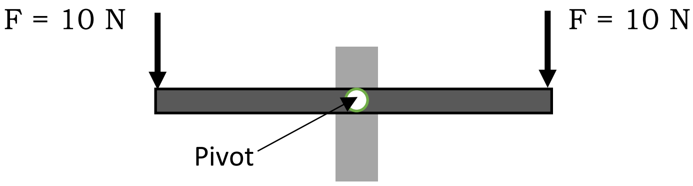


Figure 4. Two equal forces acting on the wood downward

However, the wood will stay in its condition of equilibrium, since the two forces are causing rotations oppositely. What if the force on the right side (Figure 5) is directed upward? Both forces will cause the same direction of rotation and the wood will have a continuous counterclockwise rotation.

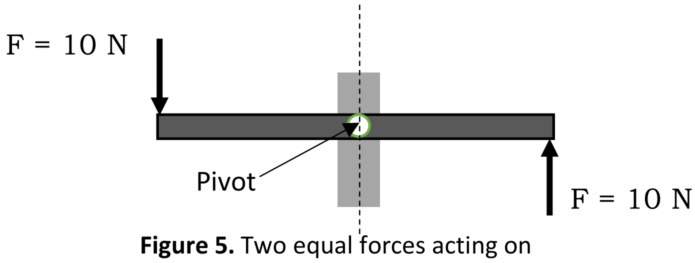


Figure 5. Two equal forces acting on the wood oppositely

TORQUES

The two forces represented in Figures 4 and 5 by the vectors F are parallel. They do not act on the same point as did the concurrent forces studied in the previous chapter. To measure the rotating effect of such parallel forces in a given plane, it is necessary to choose a stationary reference point for the measurements, we call this point as *pivot or fulcrum or axis of rotation.*

Torque is the measure of the turning effect of a rigid body. It is operationally define as the product of a perpendicular force and the length of its lever arm from the pivot or axis of rotation. Lever arm is a measure of a distance from the force applied to the pivot. In equation,

$$\tau = F.l_a$$

where:

τ – torque, expressed in N.m
F – force in newton N
l_a – lever arm in meter

If the force causing the torque makes an angle with the lever arm, then determine the component of the force perpendicular to the lever arm (Figure 6.)

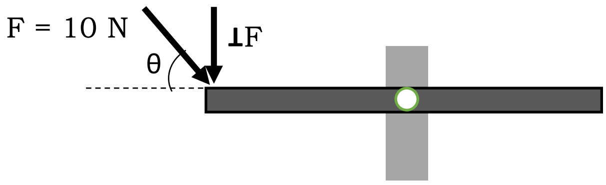


Figure 6. A force that causes rotation forms an angle

Torque is a vector quantity. The direction of the torque produced by a force is dependent on the action of the force relative to the pivot. If the force causing the rotation is ***clockwise***, torque is negative. If the force causing rotation is ***counterclockwise*** rotation, then torque is positive.

Example 1. Find the torque produced by the force of 10 N Figure 3, if the lever arm is 2 meters.

Given: F = 10 N
l_a = 2 m
τ = ?

$\tau = F.l_a$
$= (10\ N)(2\ m)$
$= 20\ N.m$

Example 2. If the force in Figure 6 makes an angle of 40^0, what is the torque produced by the force?

Given: $F = 10$ N
$l_a = 2$ m
$\theta = 40^0$
$\tau = ?$

$\tau = (F\cos\theta)\, l_a$
$= (10\text{ N})(\cos 40^0)\,(2\text{ m})$
$= 15.32$ N.m

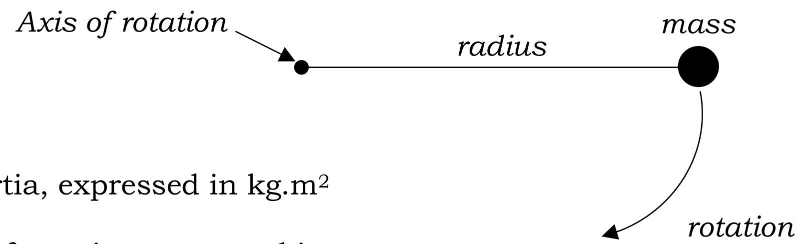
MOMENT OF INERTIA

A rigid body is a solid composed of a collection of particles. These particles remain static relative to each other and relative to the axis of rotation. Regardless of the forces acting on a rigid body, it maintain its original shape and size.

Newton's 1^{st} law of motion as applied to rotating systems states that, unless hindered by an external force, a rigid body rotating about a fixed axis will remain rotating at the same rate within the same axis.

Moment of inertia is define as the quantity that resists changes in an object's rotational state of motion. The moment of inertia of a point mass is operationally define by the equation,

$$\mathbf{I = mr^2}$$

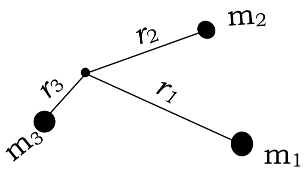


where:

I – is moment of inertia, expressed in $kg.m^2$
m – is mass in kg
r – radius from axis of rotation expressed in meter m

For a discrete number of particles, the moment of inertia I is

$$\mathbf{I = \Sigma m_i r_i^2}$$



where:

m_i – is mass of the i^{th} point particle
r_i – is the distance of the i^{th} particle from the axis of rotation

$$\mathbf{I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots\ldots\ldots}$$

Example 1. Find the moment of inertia of a mass 2 kg rotating at a radius of 0.8 m from the axis of rotation?

Given:

$m = 2$ kg

$r = 0.8$ m

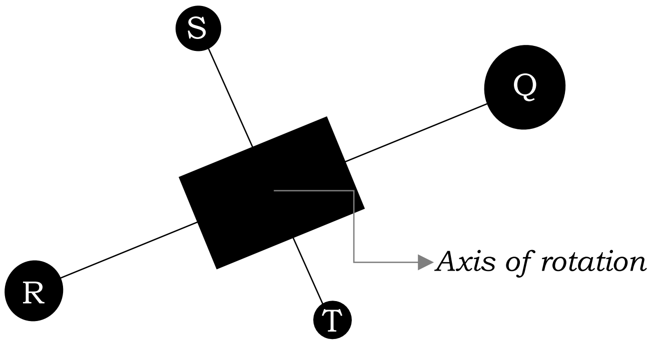
$I = ?$

$I = mr^2$

$= (2 \text{ kg})(0.8 \text{ m})^2$

$= 1.28 \text{ kg.m}^2$

Example 2. A baton is made up of particles with each end fastened by rods of negligible mass. Determine the moment of inertia of an axis where the rods intersect. The radius from the axis of rotation to particle Q of mass 120 grams and particle R whose mass is 90 grams are both 0.3 meters. Particle S with mass 60 grams and particle T of 45 grams mass are at a distance of 0.1 m and 0.08 m from the point of rotation respectively.



Given:

$m_Q = 120$ g

$r_Q = 0.3$ m

$m_R = 90$ g

$r_R = 0.3$ m

$m_S = 60$ g

$r_S = 0.15$ m

$m_T = 45$ g

$r_T = 0.1$

$I = m_Q r_Q + m_R r_R^2 + m_S r_S^2 + m_T r_T^2$

$= (0.12\text{kg})(0.3\text{m})^2 + (0.09\text{kg})(0.3\text{m})^2 + (0.06\text{kg})(0.15\text{m})^2 + (0.045\text{kg})(0.1\text{m})^2$

$= 0.0108 \text{ kg.m}^2 + 0.008 \text{ kg.m}^2 + 0.00135 \text{ kg.m}^2 + 0.00045 \text{ kg.m}^2$

$= 0.0206 \text{ kg.m}^2$

Explore

A force on a uniform bar can cause it to rotate about an axis. Each force can produce torque either clockwise or counterclockwise rotation depending on the position and direction of the force relative to the pivot or fulcrum

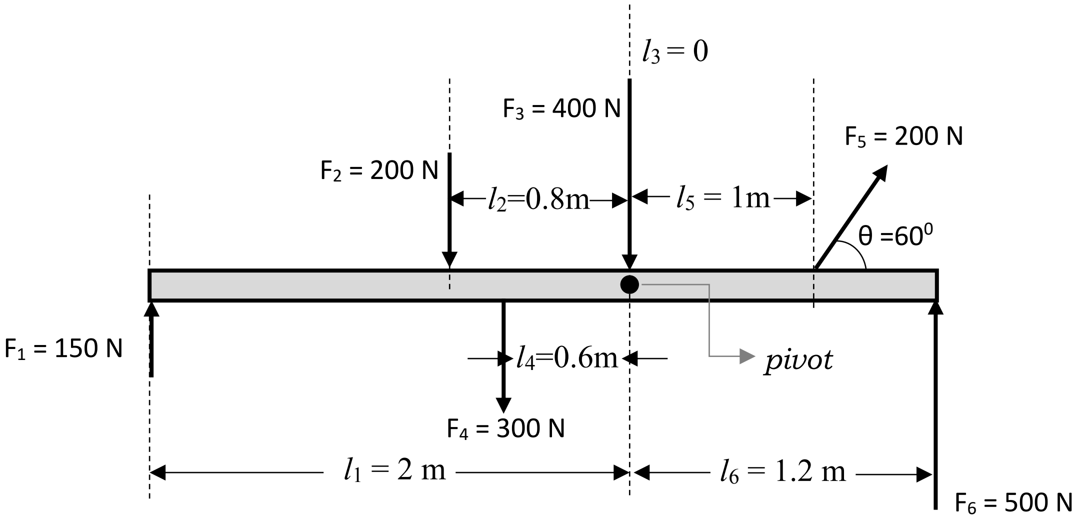
Do the following activities to enhance your skills on how the magnitude and direction of the torque produce.

Activity 1. TORQUE

Direction:

1. Determine the torque produce in a uniform bar by a force parallel force of 5 000 dynes located 30 cm from the point of rotation.

2. Refer to Figure, complete the table by calculating the torque of the uniform bar and determining the direction caused by the torque.



Table

Force	Lever arm	Torque	Direction
F_1 = 150 N	$l_1 = 2$ m		
F_2 = 200 N	$l_2 = 0.8$ m		
F_3 = 400 N	$l_3 = 0$		
F_4 = 300 N	$l_4 = 0.6$ m		
F_5 = 200 N	$l_5 = 1.0$ m		
F_6 = 500 N	$l_6 = 1.2$ m		

Activity 2. MOMENT OF INERTIA

Direction: Solve the given problems below in a separate sheet of paper.

1. A 300 kg boy is riding on a merry-go-round located 2.5 meters away from the axis of rotation. Determine the moment of inertia the experienced.

2. A 25 kg child is sitting along the rim of a merry-go-round rotating about its symmetry axis. If the mass of the merry-go-round is 100 kg and its radius is 8 m, what is the moment of inertia of the system about its axis of symmetry. Assuming that the child is treated as a point particle and the merry-go-round is modelled as a disc.

Deepen

STATIC EQUILIBRIUM

In the previous Module, you have learned the conditions necessary for the translational equilibrium of an object. The first condition states that, the sum of all concurrent forces acting on a body is zero. This condition, however, does not prevent the possibility of rotation of the object that is subject to torques.
First condition equation,

$$\Sigma F = 0$$

$$\Sigma F = \Sigma F_x + \Sigma F_y = 0$$

$$\Sigma F_x = \Sigma F_y$$

The second condition of equilibrium must be met for an object to be in complete equilibrium, states that, the sum of all the torques in a body is zero. The second condition of equilibrium equation,

$$\Sigma \tau = 0$$

$$\Sigma \tau = \Sigma \tau_{cw} + \Sigma \tau_{ccw} = 0$$

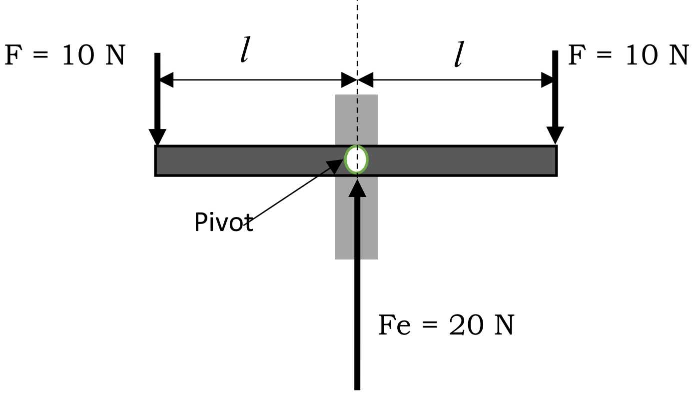
$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

Activity 1. EQUILIBRIUM OR NOT?

Direction: Refer to the diagrams to show whether a body is in the state of equilibrium or not by completing the table, based from the description below.

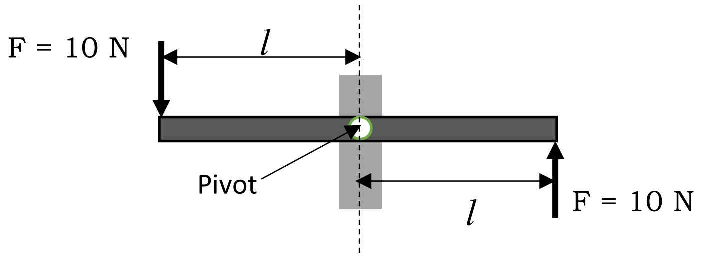
Equations:		**State**	
$\Sigma F = 0$ $\Sigma F \neq 0$	$\Sigma \tau = 0$ $\Sigma \tau \neq 0$	At rest Moves	No rotation Rotates

.



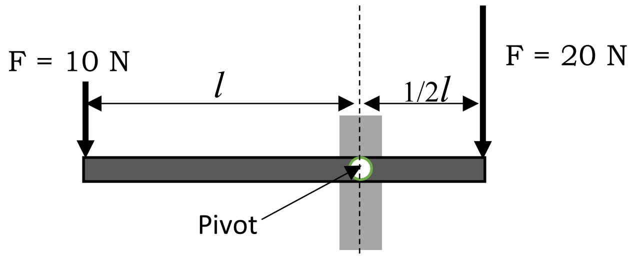
Situation A. Two equal forces acting on the wood downward balance by an equilibrant

Condition	Equation	State	Conclusion
First Condition (*Concurrent Forces*)			
Second Condition (*Non-concurrent parallel forces*)			



Situation B. Two equal forces acting on the wood oppositely

Condition	Equation	State	Conclusion
First Condition (*Concurrent Forces*)			
Second Condition (*Non-concurrent parallel forces*)			



Situation C. Two forces of different values acting on the wood downward

Condition	Equation	State	Conclusion
First Condition (*Concurrent Forces*)			
Second Condition (*Non-concurrent parallel forces*)			

Lesson 2. Rotational Kinematics

Target

In Lesson 1, you have learned that a force can produce or rotation. The First condition of equilibrium gave you an idea, that a body may be at rest if no net force is acted upon it. The second condition, has something to do with rotation of object acted upon by parallel forces along a rigid plane. If the summation of forces and torques in a system is equal to zero, then we can say that, complete equilibrium is attained.

Rotary motion occurs in a spinning bicycle wheel, the spinning crankshaft of an automobile engine, and even the rotation of Earth about its axis.

In this lesson, you will learn the applications of rotational kinematics and the relationships among the rotational quantities involving motion of a body about an internal axis.

After finishing this Learning Material, you are expected to:

1. apply the rotational kinematic relations for systems with constant angular accelerations. (STEM_GP12RED IIa-6)
2. determine angular momentum of different systems. (STEM_GP12REDIIa-9)
3. apply the torque-angular momentum relation. (STEM_GP12REDIIa-10)

Jumpstart

Note the difference between circular motion and rotary motion. What is your idea on the difference between circular motion and rotary motion?

To answer the above question, let's take a tour on this two types of motion by doing the activity below.

Activity 1. Circular or Rotational?

Direction: Check in the blank, whether the object is in circular motion or rotary motion. Use the table below for your response.

Object	Circular Motion	Rotary Motion
1. Moon's movement around the Earth		
2. Merry-go-round		
3. Spinning top		
4. Bicycle wheel		
5. Stone whirled horizontally		
6. Runner travelling around an oval		
7. Spinning skater		
8. Propeller of a chopper		
9. Ferris wheel		
10. Earth		

Questions:

Q1. Based from the table, what is circular motion?

____

____

____

Q2. What is rotational motion?

____

____

____

Discover

Rotational motion is the motion of a body about an internal axis. In the activity, spinning top, bicycle wheel and the Earth movement, are examples of rotary motion. In ***circular motion***, the axis of the motion is outside the object, we call it orbit. While ***rotary motion***, the axis of the motion is inside the moving object.

Angular Velocity

In uniform linear motion, velocity is define as the time rate of displacement. Similarly, for constant rotary motion, **angular velocity** is define as the time rate of angular displacement. **Angular displacement** is the angle about the axis of rotation through the object turns. We measure the angle in radians. One radian is defined as the angle subtended at the center of the circle by the arc whose length equals the radius of the circle, Figure 1.

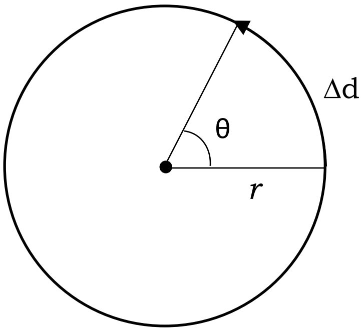


Figure 1. Measurement of radian. The arc Δd is equal to the radius *r*, angle θ is 1 radian

Relation between degrees and radians:

$$1 \text{ revolution} = 360^0 = 2\Pi \text{ radian}$$
$$\theta_{radian} = 360^0/2\Pi = 180^0/\Pi = 57.3^0$$

$$1 \text{ radian} = 57.3^0$$

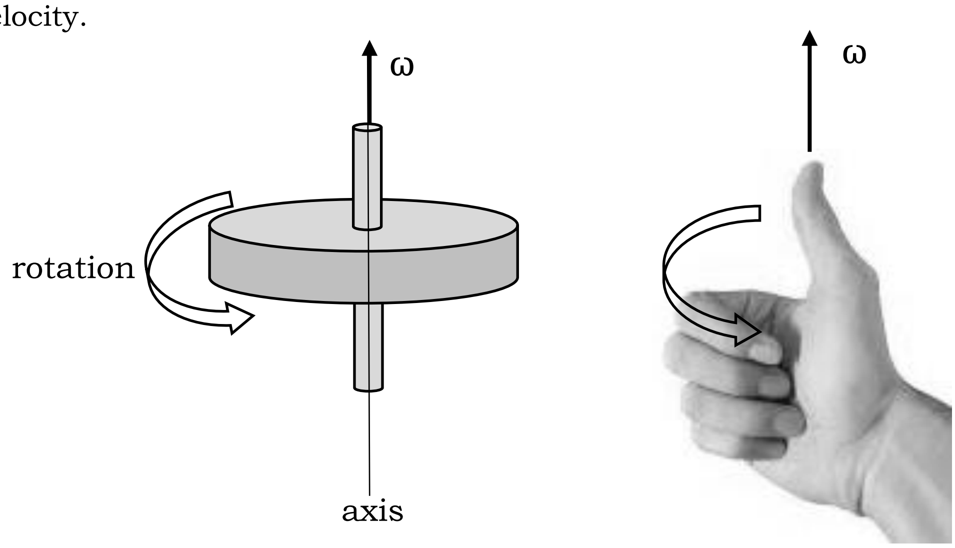
Angular velocity can be expressed using the equation,

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Where:

ω – angular velocity expressed in rad/s
Δθ – angular displacement expressed in radian
Δt – time interval in second

Angular velocity is a vector quantity, represented by a vector along the axis of rotation. We can use the right –hand rule to determine the direction of angular velocity.



https://images.app.goo.gl/Qyp5mwjwxnCEkxia7

Figure 2. The right-hand Rule of angular velocity. The angular velocity points in the direction of the thumb of the right hand. The curling of the four fingers, indicates the rotation of the object

If you curl your fingers of your right hand in the direction of rotation, your thumb points in the direction of the angular velocity, Figure 2.

Angular Acceleration

In linear motion, a change in velocity defines linear acceleration, and so with rotary motion. Changing either the rate of rotation or the direction of the axis involves a change of angular velocity. Which means, an object is in angular acceleration. The uniform rate of change in velocity is known as angular acceleration, α, in equation form,

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{\Delta t}$$

Where:

α – angular acceleration expressed in rad/s^2
Δω – change in angular velocity expressed in rad/s
Δt – time interval in second

Comparison of the Formula Between Linear Motion and Rotary Motion

Linear Motion	Rotary Motion
$v = \frac{\Delta d}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
$a = \frac{v_2 - v_1}{\Delta t}$	$\alpha = \frac{\theta_2 - \theta_1}{\Delta t}$
$v_2 = v_1 + at$	$\omega_2 = \omega_1 + \alpha t$
$d = v_1 t + \frac{1}{2} at^2$	$\theta = \omega_{1t} + \alpha t^2$
$v_2^2 = v_1^2 + 2ad$	$\omega_2^2 = \omega_1^2 + 2\alpha d$

Example 1. Find the angular velocity of a wheel in radian per second spinning at 3 000 revolution per minute?

Given:

$\omega = 3\ 000$ rev/min

$= (3\ 000\ \cancel{rev}/\cancel{min})\ (2\Pi rad)/\cancel{rev})\ (1\cancel{min}/60s) = 314.16$ rad/s

Example 2. A merry-go-round started from rest. After 0.2 second, it has covered an angular displacement of 3 radians. What is the angular velocity of the merry-go-round?

Given:

$\theta_1 = 0$

$\theta_2 = 3$ rad

$\Delta t = 0.2$ s

$$\omega = \frac{\theta_2 - \theta_1}{\Delta t} = \frac{3 \text{ rad} - 0}{0.2 \text{ s}} = 15 \text{ rad/s}$$

Example 3. A Ferris wheel accelerates uniformly from rest to an angular velocity of 10 revolutions per minute in 2 minutes. Determine the angular acceleration of the Ferris wheel in rad/s^2?

Rotational Inertia

A bicycle wheel will not spin unless a torque is applied to the wheel. Once spinning, it will continue to spin at constant angular velocity unless a torque acts on it. In both cases the wheel is in equilibrium. Newton's Law of inertia is also applies to rotary motion.

To change the angular velocity, we must apply a torque about the axis. The wheel will gain angular acceleration, which is dependent on the mass of the rotating wheel and upon the distribution of its mass with respect to the axis of rotation.

Rotational inertia is the resistance of a rotating object to changes in its angular velocity. The angular acceleration, α, is directly proportional to the torque, τ, but inversely proportional to the rotational inertia, I.

$$\alpha = \frac{\tau}{I}$$

Transposing the formula, $\tau = \alpha I$, which is analogous to Newton's second law, $F = ma$ for linear motion. To find torque, τ, we use the equation,

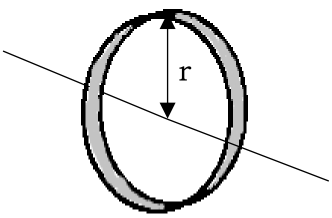
$$\tau = Fr$$

where F is the force applied tangentially at distance r from the axis of rotation.

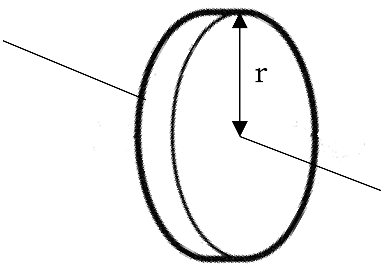
Example 1. A flywheel is acted by a torque of 50 N.m and cause to accelerate at 3 rad/s². Determine the rotational inertia of the flywheel.

$$I = \frac{\tau}{\alpha} \qquad I = \frac{50\ \text{N.m}}{3\ \text{rad/s}^2} = 16.7\ \text{kg.m}^2$$

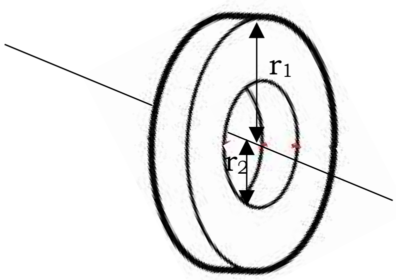
Rotational inertia takes into account both the shape and the mass of the rotating object. Equations for the rotational inertia of certain regularly shaped bodies are given in the figures.



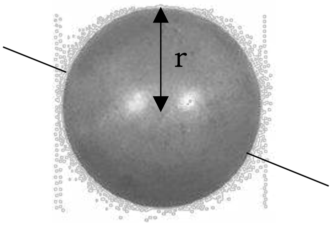
Thin ring
$I = mr^2$



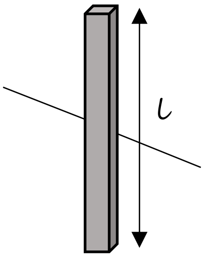
Disk
$I = \frac{1}{2} mr^2$



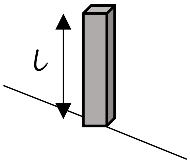
Thick ring
$I = \frac{1}{2} m(r_1^2 + r_2^2)$



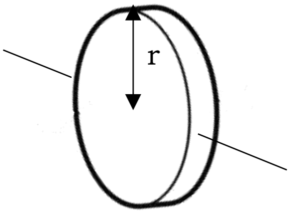
Solid Ball
$I = 2/5\ mr^2$



Thin Rod
$I = 1/12\ m\ l^2$



Thin Rod
$I = 1/3\ m\ l^2$



Disk
$I = ¼\ m\ r^2$

Example 2. A solid ball has a mass of 3 kg and a radius of 0.25 m. What is the rotational inertia of the ball?

$$
\begin{aligned}
I &= 2/5\ mr^2 \\
&= 2/5\ (3kg)\ (0.25\ m)^2 \\
&= 0.075\ kg.m^2
\end{aligned}
$$

Example 3. The length of a thin rode is 0.5 m and rotated at its axis at the center. If the mass of the thin rod is 0.15 kg, what is its rotational inertia?

$$
\begin{aligned}
I &= 1/12\ m\ l^2 \\
&= 1/12\ (0.15\ kg)\ (0.5\ m)^2 \\
&= 0.003125\ kg\ m^2
\end{aligned}
$$

Angular Momentum

In linear motion, a body moving at certain velocity has a momentum, p, equal to the product of its mass and velocity. The rotational counterpart of momentum is called angular momentum denoted by $\vec{L}$.

For particles rotating about a fixed axis, the angular momentum os a single particle can be expressed as:

$$\vec{L} = r \cdot p$$

Where r is the same position vector defined from torque-force relation, and p is the linear momentum of the particle tangent to the path taken by the particle.
In terms of the moment of inertia, the angular momentum of a rotating rigid body is:

$$\vec{L} = I\ \omega$$

The equation revealed, that, the larger the moment of inertia is, the harder is to change the state of motion of a rigid body by net torque. A net torque acts on a rigid body can change the angular momentum of the body.

The Law of Conservation of Angular Momentum states that, the total angular momentum of a system is zero, if there is no net external torque acting on a rigid body. For a non-rigid body with varying moment of inertia, conservation of angular momentum yields:

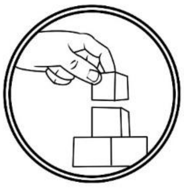
$$I_0\omega_0 = I_f\omega_f$$

Where:

I_0 – initial rotational inertia of a rigid body
I_f – final rotational inertia of a rigid body
ω_0 – initial angular velocity expressed in rad/s
ω_f – final angular velocity expressed in rad/s

In the case of rotational collision between two rigid bodies A and B, the equation for conservation of angular momentum is:

$$I_A\omega_{0A} + I_B\omega_{0B} = I_A\omega_{fA} + I_B\omega_{fB}$$



Explore

REMEMBER:

Rotary motion is the motion of a body about an internal axis. The relationships among angular displacement, velocity, acceleration, and time are similar to the corresponding relationships for linear motion.

Unless a net external torque acts on a body, its angular momentum is conserved.

Now it's your turn to apply what you have learned in this Module!

Problem To Solve!

Problem 1. Find the angular displacement in radians during the second 20 seconds interval of a wheel that accelerates from rest to 725 revolutions per minute in 1.5 minutes.

Problem 2. A disc uniformly accelerates from rest to an angular velocity of 20 revolutions per second in 10 seconds. Calculate the angular acceleration of the flywheel in rad/s^2?

Problem 3. The mass of a thick ring has a mass of 30 kg and radius of 22.5 cm. The hole in the ring is 7.5 cm in radius. What is the rotational inertia of the thick ring?

Problem 4. What is the rotational inertia of a solid ball 0.20 in diameter and has a mass of 10 kg rotating about its internal axis?

Problem 5. Show that angular momentum has the unit dimensions $kg.m^2/s$

Problem 6. A neutron star collapsed into very dense object made mostly of neutrons. The density of a neutron star is approximately 10^{14} times as great as that of ordinary solid matter. Suppose the neutron star is a uniform solid, rigid sphere, both before and after collapse. The initial radius of the star was 106 km and its final radius is 38 km. If the original star rotated once every 100 days, calculate the angular speed of the neutron star expressed in rad/s.

Deepen

You did well in the problems above. How about a more challenging exercises and problems!

Problem 1. A solid ball is rotated by applying a force of 4.7 N tangentially to it. The ball has a radius of 14 cm and a mass of 4 kg. What is the angular acceleration of the ball?

Problem 2. What is the rotational inertia of a thick ring that is rotating about an axis perpendicular to the plane of the ring and passing through its center? The ring has a mass of 1.2 kg and a diameter of 45 cm. The hole in the ring is 15 cm wide?

Problem 3. A flywheel in the shape of a thin ring has a mass of 30 kg and a diameter of 0.96 m. A torque of 13 N.m is applied tangentially to the wheel. How long will it take for the flywheel to attain an angular velocity of 10 rad/s?

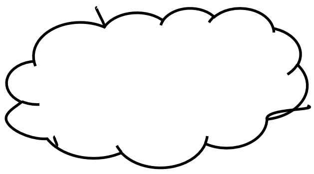
Problem 4. The mass of a disk is 5 kg and has a radius of 0.2 m rotating about an axis passing through its center. (a) What is the angular acceleration of the disc to bring it to an angular velocity of 150 rad/s in 5 seconds? (b)What constant torque is required to bring it up that angular velocity?

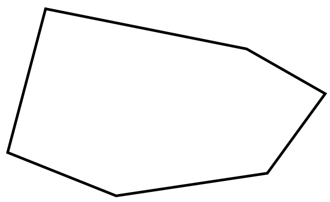
Problem 5. An engine flywheel and a clutch plate are both connected to a transmission shaft. Let the moment of inertia of the flywheel be I_1 and its angular velocity ω_1, and let the moment of inertia of the clutch plate be I_2 and its angular velocity is ω_2. The two discs have then been combined by forces which are applied at their axes of rotation so as not to cause any torque. The discs then reached a common final angular velocity after rotational collision. Find the expression for the final angular velocity.

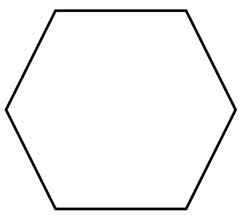
Gauge

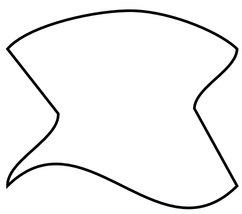
Direction. Select the ***BEST*** answer. Write the corresponding **CAPITAL LETTER** of your choice in a separate sheet of paper. Write **E** if you find no correct answer.

1. The center of gravity is a point in which the weight of an object is concentrated. Which of the following shapes below does the center of gravity is located at the geometric center?

A. 

C. 

B. 

D. 

2. In the **Figure 1** shown below, a force of 50 N is acted 300 from the horizontal uniform bar whose axis of rotation is 1/3 of its length from one end of its end. What is the equation of the torque produced by the force?

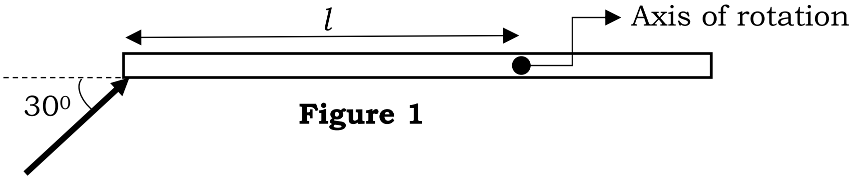


Figure 1

A. (F sin 30⁰) *l*
B. (F cos 30⁰) *l*
C. (*l* sin 30⁰) F
D. (*l* cos 30⁰) F

3. A bar is to several forces, tending to cause a rotation, as shown in the **Figure 2** below. What is the direction of the net torque?

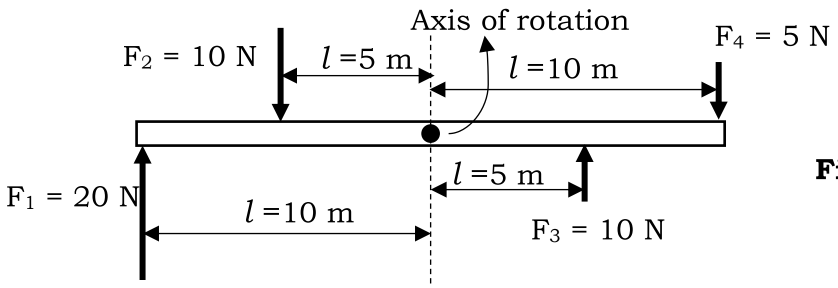


Figure 2

A. Clockwise
B. Counterclockwise
C. Upward
D. Downward

4. Refer to **Figure 2**, what is the net torque produced from the forces?
A. 15 N.m clockwise
B. 15 N.m counterclockwise
C. 100 N.m clockwise
D. 100 N.m counterclockwise

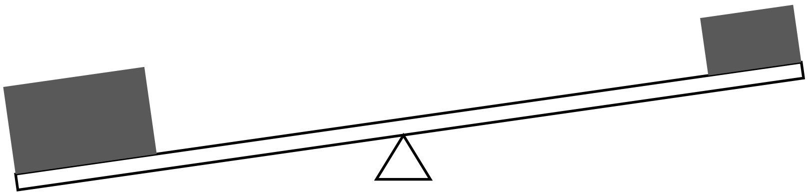
5. Applying the conditions of static equilibrium, which equation is correct to satisfy that a body is said to be in complete static equilibrium?
A. $\Sigma F = 0$ but $\Sigma\tau \neq 0$
B. $\Sigma F \neq 0$ and $\Sigma\tau \neq 0$
C. $\Sigma F = 0$ and $\Sigma\tau = 0$
D. $\Sigma\tau = 0$ but $\Sigma F \neq 0$

6. If the torque required to loosen a nut on the wheel of a car has a magnitude of 60.0 N• m, what minimum perpendicular force must be exerted by a mechanic at the end of a 45.0 cm wrench to loosen the nut?
A. 667 Nm
B. 133.3 N
C. 66.7 N
D. 13.3 Nm

7. A wheel accelerates at 5 rev/s^2. What is this in rad/s^2?
   A. 3.14
   B. 31.4
   C. 314.0
   D. None of the choices

8. A construction worker is worried about applying too much torque to a bolt using his wrench. What could he do to reduce his torque on the bolt?
   A. Increase the length of the wrench
   B. Increase the force on wrench
   C. Reduce the length of the wrench
   D. Reduce the friction between the bolt and wrench

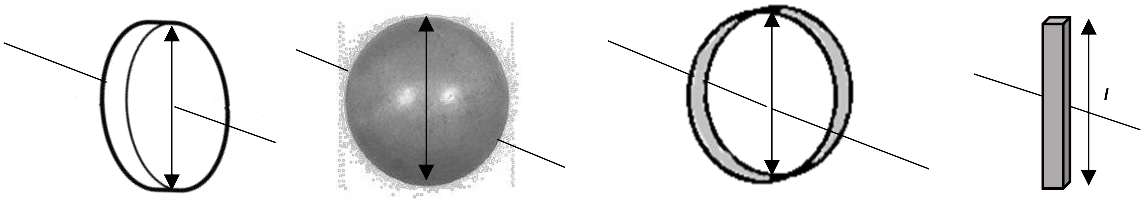
9. A seesaw is unbalanced with two different of different boxes sitting equal distances from the center. Which of the following will result in equilibrium?



   A. The larger box must be move further away from the center.
   B. The larger box must be move closer towards the center.
   C. Moving the smaller box closer to the center.
   D. Moving both boxes towards the center.

10. A Ferris wheel starts at rest and builds up to a final angular speed of 0.70 rad/s while rotating through an angular displacement of 4.9 rad. What is its average angular acceleration?
   A. 0.01 rad/s^2
   B. 1.8 rad/s^2
   C. 0.05 rad/s^2
   D. 2.6 rad/s^2

11. Four objects having masses of 2 kg each and have equal diameters. Which of the objects has the greatest rotational inertia when acted upon by a net torque?



A B C D

12. An ice skater is spinning on the surface of an ice at an angular velocity of 50 rev/min. what must he do in order to increase his angular velocity without changing his angular momentum?
   A. He must increase his rotational inertia.
   B. He must spin with arms extended.

C. He must spin with one leg and extending the other leg.
D. He must spin with his arms folded

13. The figure below shows that the body will not rotate but has the tendency to accelerate along the direction of the net force. Which of the statements is correct to satisfy the second condition of equilibrium?

A. There is a net force upward force, so body at rest will start to move upward
B. There is a net torque, so body will rotate counterclockwise.
C. The net torque about the axis is zero so body at rest has no tendency to start rotating.
D. The net force on the body is zero, so body at rest has no tendency to move upward.

14. How do you compare the initial angular momentum and the final angular momentum of an object in the law of conservation of momentum?
A. Initial angular momentum is greater than final angular momentum.
B. Initial angular momentum is less than final angular momentum.
C. Initial angular momentum is equal to final angular momentum.
D. Initial angular momentum is greater than zero, and final angular momentum is less than zero.

15. Linear momentum is closely related to angular momentum. Which of the following is a statement of the Law of Conservation of Momentum?
A. The law of conservation of angular momentum states that when external torque acts on an object, no change of angular momentum will occur.
B. The law of conservation of angular momentum states that when no external torque acts on an object, there is a little change in angular momentum
C. The law of conservation of angular momentum states that when no balanced forces act on an object, no change of angular momentum will occur.
D. The law of conservation of angular momentum states that when no external torque acts on an object, no change of angular momentum will occur.

KEY ANSWER

Lesson 1. Torque and Moment of Inertia

Jumpstart:

Activity 1. Where is the center of gravity

Q1. By drawing several lines intersecting at the midpoint of the object.

Activity 2. Irregularly shaped objects

Q1. To determine the point of intersection

Q2. The center of gravity

Q3. No, in regularly shaped object, the center of gravity is exactly at the geometric center, while irregularly shaped objects, depends on its shape where two or more lines intersect.

Explore:

Activity 1. Torque

1.) 150 000 dyne.cm

Force	Lever arm	Torque	Direction
F_1 = 150 N	$l_1 = 2$ m	300 N.m	Clockwise
F_2 = 200 N	$l_2 = 0.8$ m	16 N.m	Counterclockwise
F_3 = 400 N	$l_3 = 0$	0	
F_4 = 300 N	$l_4 = 0.6$ m	180 N.m	Counterclockwise
F_5 = 200 N	$l_5 = 1.0$ m	173.2 N.m	Counterclockwise
F_6 = 500 N	$l_6 = 1.2$ m	600 N.m	Counterclockwise

Activity 2. Moment of Inertia

1.) 1 875 $kg.m^2$

2.) 8 000 $kg.m^2$

Deepen

Situation A

Condition	**Equation**	**State**	**Conclusion**
First Condition	$\Sigma F = 0$	At rest	Body at rest has no tendency to start moving
Second Condition	$\Sigma \tau = 0$	No rotation	Body at rest has no tendency to rotate

Situation B

Condition	**Equation**	**State**	**Conclusion**
First Condition	$\Sigma F = 0$	At rest	Body at rest has no tendency to start moving
Second Condition	$\Sigma \tau \neq 0$	Rotation	There is a counterclockwise torque about its axis

Situation C

Condition	**Equation**	**State**	**Conclusion**
First Condition	$\Sigma F \neq 0$	Moves	Body at rest has the tendency to start moving
Second Condition	$\Sigma \tau = 0$	Rotation	Body at rest has no net torque. No tendency to rotate

Lesson 2. Rotational Kinematics

Jumpstart:

Activity 1. Circular or Rotational

Object	Circular Motion	Rotary Motion
1. Moon's movement around the Earth	✓	
2. Merry-go-round		✓
3. Spinning top		✓
4. Bicycle wheel		✓
5. Stone whirled horizontally	✓	
6. Runner travelling around an oval	✓	
7. Spinning skater		✓
8. Propeller of a chopper		✓
9. Ferris wheel		✓
10. Earth	✓	✓

Q1. Circular motion, the axis of the motion is outside the object, we call it orbit.

Q2. Rotary motion, the axis of the motion is inside the moving object.

Explore:

Problem 1. $\Delta\theta = 506$ rad

Problem 2. $\alpha = 1.26$ rad/s^2

Problem 3. I = 8 437.5 kg.cm^2

Problem 4. I = 0.04 kg.m^2

Problem 5. $I = I\omega$

$= (kg/m^2)\ (rev/s)$

$= kg.m^2/s$

Problem 6. Equation

$I_0\omega_0 = I_f\omega_f$

$\omega_f = 503.6$ rad/s

Deepen

Problem 1. $\alpha = 21$ rad/s^2

Problem 2. I = 0.0034 kg.m^2

Problem 3. t = 5.3 s

Problem 4. a.) $\alpha = 30$ rad/s^2

b.) $\tau = 30$ N.m

Problem 5. $L_{01} + L_{02} = L_{f1} + L_{f2}$

$I_1\omega_{01} + I_2\omega_{02} = (I_1 + I_2)\ \omega_f$

$$\omega_f = \frac{I_1\omega_{01} + I_2\omega_{02}}{I_1 + I_2}$$

Gauge

1. B
2. A
3. A
4. D
5. C
6. B
7. B
8. C
9. B
10. C
11. A
12. D
13. C
14. C
15. D

KEY ANSWER

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