Quarter 4 WEEK 7 - Physics Material

Computer Science (University of Batangas)

SHS

AIRs - LM in

General Physics 2

Quarter 4: Week 7 - Module 7

Special Relativity

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General Physics 2
Grade 12 Quarter 4: Week 7 - Module 7: Special Relativity
First Edition, 2021

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Target

It is a fact about life that some things are absolute, and some are relative. For example, the pitcher on the table is to the left of my glass. From the point of view of an observer sitting directly opposite of me, it's the other way around: My glass is to the left of the pitcher. "Left" and "right" are relative. Whether or not an object is located to the left or to the right of another depends on the observer. On the other hand, if the glass is filled to the brim with juice, all observers should agree to the fact, regardless of where they sit. That is an absolute statement, independent of who makes the observation.

Einstein's special theory of relativity (special relativity) is all about what is relative and what is absolute about time, space, and motion. Some of Einstein's conclusions are rather surprising. They are nonetheless correct, as numerous physics experiments have shown. And they have forced physicists to revise the way they think about some of their science's most basic concepts.

In your previous lesson, you have studied the properties of light that depend on its wave nature. It was also discussed how geometrical optics – ray tracing and the focusing of light by mirrors and lenses – fits together with the wave nature of light.

Now, this lesson will focus on the Special Theory of Relativity develop by Sir Albert Einstein and how this profoundly change our way of thinking about space and time. After finishing this Learning Material, you are expected to:

1. State the postulates of Special Relativity and their consequences **(STEM_GP12MPIVg-39)**
2. Apply the time dilation, length contraction and relativistic velocity addition to worded problems
3. Calculate kinetic energy, rest energy, momentum, and speed of objects moving with speeds comparable to the speed of light **(STEM_GP12MPIVg-42)**

Jumpstart

Before moving on, assess how much you know about this topic. Answer the pretest in a separate sheet of paper.

Direction: Read each statement carefully and choose the best answer that corresponds each. Write the letter of best answer.

1. Which of the following observations made by an observer in a moving inertial frame of reference is consistent with the postulates of the special theory of relativity?
   A. Moving clocks run fast.
   B. Moving sticks oriented parallel to the direction of motion become longer.
   C. Simultaneous events happening at two different places will always be simultaneous to another observer.
   D. Light always travel at the same speed in vacuum regardless of the relative motion between the source and the observer.

2. What happens to its speed If you consider a blinking light source that approaches an observer?
   A. Increases B. Decreases
   C. Remains the same D. Needs more information

3. If you consider a blinking light source that approaches an observer, what happens to its frequency?
   A. Increases B. Remains the same
   C. Decreases D. Needs more information

4. If you consider a blinking light source that approaches an observer, what happens to its wavelength?
   A. Increases B. Decreases
   C. Remains the same D. Needs more information

5. For a spaceship moving very fast with respect to the earth, the clocks on board are perceived to run slow when viewed from the _____.
   A. Earth B. space ship
   C. both places D. none of them

6. According to the special theory of relativity, all laws of nature are the same in reference frames that _____.
   A. accelerate C. move at constant speed
   B. both A and B D. none of them

7. Two meteorites are seen to strike two distant locations at the same time. As seen from a different location, the two lightning bolts _____.
   A. will also be seen at the same time
   B. will not be seen at the same time
   C. may or may not be seen at the same time
   D. cannot be determined

8. There is an upper limit on the speed of a particle. This means that there is also an upper limit on its _____.
   A. kinetic energy B. momentum
   C. both A and B D. none of them

9. Under what condition do relativity equations for length, mass, and time hold ***TRUE***?
   A. Relativistic speeds B. Everyday low speeds
   C. Both A and B D. None of them

10. While the spaceship is still at rest on earth, a woman on board finds that a wooden rod she is carrying is 1 meter long. When the spaceship is moving very fast deep into the outer space, what will the woman find out about the length of the same wooden rod she is still carrying?
   A. It will be longer. B. It will be shorter.
   C. It will be wider. D. It will still be of the same length.

11. How does the relativistic momentum of a fast-moving body compare to the momentum (mv) of the same body according to classical physics?
   A. Smaller B. Greater
   C. The same D. Cannot be determined

12. What is meant by the well-known equation, E =mc2?
   A. Mass and energy are related.
   B. Mass and energy travel at the same speed, the speed of light.
   C. When mass travels at the speed of light, it is converted to energy.
   D. When energy travels at the speed of light, it is converted to mass.

13. According to the Special Theory of Relativity, what will you notice about your own pulse rate if you travel at a very high speed?
   A. Smaller B. Just the same
   C. Greater D. Cannot be determined

14. What consequence of Special Theory of Relativity that states as clocks moving relative to an observer are measured to run more slowly as compared to clocks at rest?
   A. Length contraction B. Relativistic mass and energy
   C. Relativistic addition velocities D. Time dilation

15. What consequence of Special Theory of Relativity dictates that the length of an object moving relative to an observer is measured to be shorter along its direction of motion than when it is at rest?
    A. Length contraction
    B. Relativistic mass and energy
    C. Relativistic addition velocities
    D. Time dilation

Discover

Relativity is not new. It was already introduced around the year of 1600, Galileo explained that motion is relative. In physics, relativity arises when describing a situation from two different points of view. Wherever you happen to be, it seems like you are at a fixed point and that everything is moving with respect to you. Motion is always measured with respect to a fixed point. This is called establishing a **frame of reference.**

All frames of reference are equally valid. A passenger in a moving car is not moving with respect to the driver, but they are both moving from the point of view of a person on the sidewalk waiting for a bus. They are moving even faster as seen by a person in a car coming toward them. It is all relative.

Reference frames in which Newton's first law of motion is valid: Objects at rest remain at rest, and objects in motion remain in motion at a constant velocity in a straight line, unless acted upon by an external force; are called **Inertial Reference Frame**. The inside of a stationary house and the inside of a car moving along a road at a constant velocity are examples of Inertial reference frames.

Einstein's Special Theory of Relativity deals with how objects and events are observed and measured from inertial frames of references. Special Relativity is based on two postulates:

***First Postulate (Relativity principle)*: The laws of physics are the same in all inertial reference frames.**

***Second Postulate (Constancy of Speed of Light):* The speed of light in a vacuum is constant, independent of the motion of the light source and all observers.**

These two postulates form the foundation of Einstein's Special Theory of Relativity. It is called "special" in order to distinguish its difference from the General Theory of Relativity. Since, General Theory of Relativity deals with non-inertial reference frames while Special Theory of Relativity focuses on inertial reference frames.

The First postulate can be traced to Galilean-Newtonian Relativity, but this postulate goes further than Galileo because it is applicable to all physical laws, not only in mechanics.

The Second postulate is about the speed of light in vacuum that is always the same, $3 \times 10^8 m/s$, no matter what the speed of the observer or the source. Therefore, a person traveling toward or away from a source of light will measure the same speed for that light as someone at rest with respect to the source. This violates are everyday experience that we have to add in the velocity of the observer.

Einstein's theory requires us to give up our common sense about the notions of space and time. The following topics discuss about the strange but interesting consequences of Special theory of relativity. We will use a technique used by Einstein which is called as thought experiment.

CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY

TIME DILATION

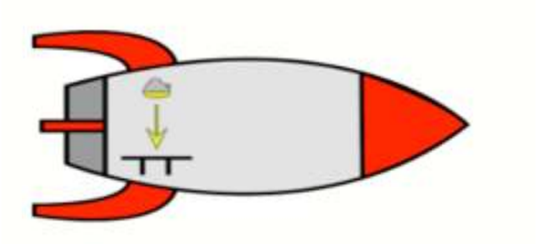


Figure 1.
Photo credits to flexbooks.ck12.org

Consider Figure 1, suppose you are in a spaceship sitting at rest on the Earth and turn on an overhead light. The light will travel downward and land on the table below. The observer in the spaceship can measure the distance traveled to the table, the time required for the light to arrive on the table and an average velocity for the light.

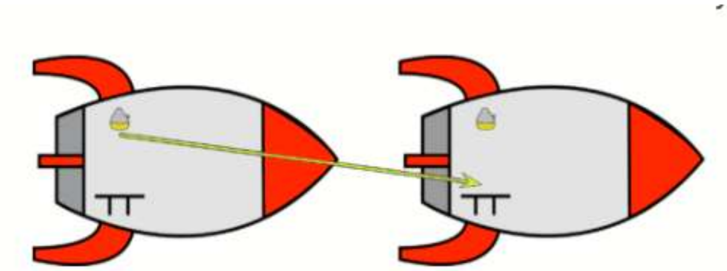


Figure 2
Photo credits to flexbooks.ck12.org

From the figure 2 above, suppose now that the spaceship is traveling past Earth at high speed and that the observer is stationary on the Earth. For the observer in the spaceship, the light falls down on the table traveling at the constant speed of light, *c*. However, for the observer on Earth, the light travels diagonally with the spaceship and also falls down on the table. Hence, the time required by the observer on Earth, will be greater than that measured by the observer on the spaceship, since the light not only went downward but also diagonally.

However, according to postulate 2, the speed of light is the same independent of the speed of the source or observer. So, it is not allowed for the light to travel farther in the same time and therefore have a greater average velocity. In other words, the speed of light as observed inside the spaceship must be $3 \times 10^8 m/s$ and the speed of light as observed by the observer on the earth must also be $3 \times 10^8 m/s$.

The time interval between two events is greater for the observer on Earth than for the observer on the spaceship. This is a result of the theory of special relativity, known as **time dilation.**

Time dilation stated as clocks moving relative to an observer are measured to run more slowly, as compared to clocks at rest.

The equation for Time Dilation is

$$\boldsymbol{\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Where,

$\boldsymbol{\Delta t}$ = dilated time/ time interval between two events in the moving reference frame;

$\boldsymbol{\Delta t_0}$ = stationary time/ time interval as measured in a stationary frame of reference;

v = relative velocity of the moving reference frame and c is the speed of light in a vacuum;

c = speed of light in a vacuum, $3 \times 10^8 m/s$

Sample problem 1:

Subatomic particles called muons are created when cosmic rays collide with atoms in the Earth's atmosphere. (a) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.80 \times 10^8 m/s$ with respect to the laboratory? A muon's mean lifetime at rest is $2.20\mu s = 2.20 \times 10^{-6} s$. (b) How far does a muon travel in the laboratory, on average, before decaying?

Given:

$\Delta t_0 = 2.20 \times 10^{-6} s$

$v = 0.60\ c = 1.80 \times 10^8 m/s$

$c = 3 \times 10^8\ m/s$

Find: Δt

Solution:

$\boldsymbol{A}$. $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$= \frac{2.20 \times 10^{-6} s}{\sqrt{1 - \frac{(0.60c)^2}{c^2}}} = \frac{2.20 \times 10^{-6} s}{\sqrt{0.64}} = \boldsymbol{2.75 \times 10^{-6} s}$$

$\boldsymbol{B}$. $d = v\Delta t$

$$= \left(1.80 \times \frac{10^8 m}{s}\right)(2.75 \times 10^{-6} s) = \boldsymbol{495\ m}$$

LENGTH CONTRACTION

Time intervals are not only things different in different reference frames. Space intervals – lengths and distances – are different as well, let us examine the given situation below.

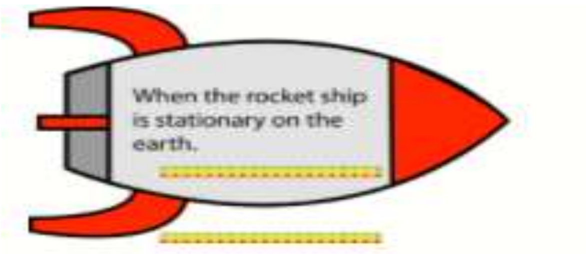
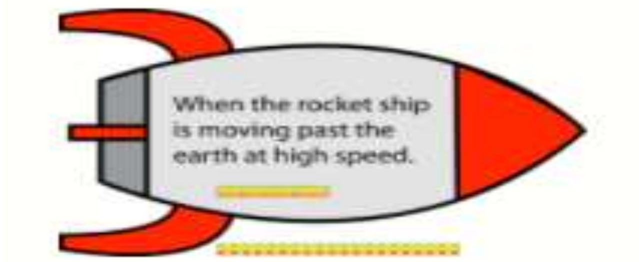


Photo credits to flexbooks.ck12.org

Based from the image above, the observer on the ship brings out his meter stick and measures the spaceship to be 20 m long while the ship sits at rest on the Earth. An observer outside the ship, standing on the Earth, also measures the ship to be 20 m with his meter stick. Suddenly, the spaceship flies past the Earth at a significant fraction of the speed of light, the observer on the ship brings out again his meter stick and measures the length of the ship and again finds it to be 20 m. The observer in Earth who is stationary measures the moving spaceship with his meter stick and was amazed to find out the moving spaceship to be less than 20 m. Just for the sake of clarity, let us say that he measures it to be 10 m.

So, how it is possible for the two observers to measure the same spaceship to be in two different lengths? When the spaceship is at rest on the Earth, the on-ship meter stick and the off-ship mater stick are the same but when the spaceship suddenly travels past the earth, the on-ship meter stick as seen by the observer one Earth has shrunk. When the observer keeps on insisting the spaceship is still 20 m long, the observer on Earth argues “No, your meter has shrunk and so has your ship. The ship now measures 10 m long using my meter stick.”



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This is a general consequence of Special Theory of Relativity and it is applicable to lengths of objects and distance between two objects. The length of an object moving relative to an observer is measured to be shorter along its direction of motion than when it is at rest, this is called as **Length Contraction**.
This can be mathematically expressed as,

$$\boldsymbol{l = l_0\sqrt{1 - \frac{v^2}{c^2}}}$$

Where,
l_0 = proper length – length measured by observes at rest or stationary
l = length measured on the moving body
v = relative speed of the reference frames
c = speed of light

It is important to note that shortening of the moving objects does not produce just a smaller object of the same shape. The object is only shortened because length contraction occurs only along the direction of motion. Therefore, the length is only shortened but its height remains the same as when it is at rest.

Sample problem 1:
A rectangular painting measures 1.00 m tall and 1.50 wide. It is hung on the side wall of a rocket ship which is moving past the Earth at a speed of 0.90c. (a) What are the dimensions of the picture according to the captain of the rocket ship? (b) What are the dimensions as seen by an observer on the Earth?
Given:
$l_0 = \ 1.50\ m$
$width = 1.00\ m$
$v = 0.90c$
$c = 3\ \times 10^8 m/s$
Find: l
Solution:

a. The captain sees a 1.00 m by 1.50 m painting because the painting is at rest (v-=0) on the rocket ship so it looks perfectly normal.
b. Only the length is shortened, so the height still the same at 1.00 m, using the equation,

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$= (1.50\ m)(\sqrt{1 - (0.90)^2} = \mathbf{0.65\ m}$$

Therefore, the painting as observed by observer on Earth has dimensions of 1.00 m by 0.65 m.

RELATIVISTIC QUANTITIES

A. RELATIVISTIC ADDITION OF VELOCITIES

Imagine a rocket ship that travels away from the Earth with speed $\boldsymbol{v}$, and assume that this rocket has fired-off another rocket ship that flies at speed u' with respect to the first. Applying the classical velocity addition and as expected that the speed u of rocket 2 with respect to Earth is $u = v + u'$. But, as the second postulate dictates us that there is no object that can travel faster than the speed of light in any reference frame. Thus, the classical addition of velocity formula is invalid. Instead, the valid formula is:

$$\boldsymbol{u} = \frac{\boldsymbol{v} + \boldsymbol{u'}}{\boldsymbol{1} + \frac{\boldsymbol{vu'}}{\boldsymbol{c^2}}}$$

Where:
$u =$ velocity of an object relative to one observer
$u' =$ velocity relative to the other observer
$v =$ relative velocity between two observers

Take note, that the relativistic velocity addition gives a result very close to classical velocity addition. For classical velocity addition is very good way of approximating the correct relativistic formula for small velocities. That is why it seems to be correct in our daily experiences.

Sample problem 1:

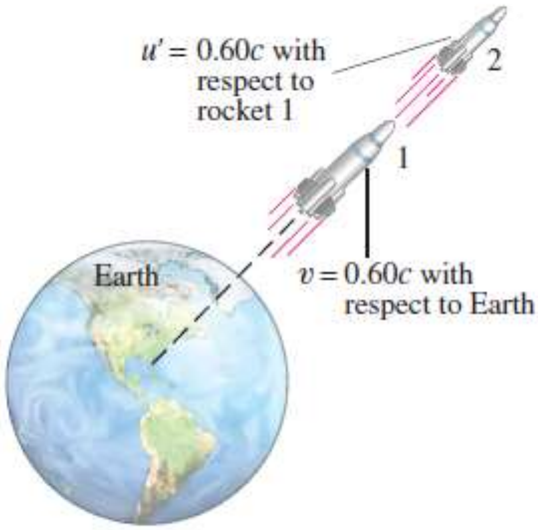

Photo credits to Giancoli Physics

Calculate the speed of rocket 2 as shown in the image with respect to Earth.
Given:
$v = 0.60\ c$
$u' = 0.60\ c$
$c = 3\ \times 10^8\ m/s$
Find: $u = ?$
Solution:

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$$= \frac{0.60c + 0.60c}{1 + \frac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = \boldsymbol{0.88c}$$

Note, that the speed of rocket 2 relative to Earth is less than c, as it must be.

B. RELATIVISTIC MOMENTUM

There is time dilation and length contraction because time intervals and length are relative, meaning their values depend on the reference frame from which they are measured. Having time intervals and length being modified, it is expected that other physical quantities such as momentum and energy are needed to be modify.
To preserve the law of conservation of momentum in relativity, the classical momentum formula is redefined as,

$$\boldsymbol{p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma mv}$$

Where:

$\gamma = \sqrt{1-\frac{v^2}{c^2}}$

$m =$ rest mass

$v =$ velocity relative to an observer

Sample problem 1:

What is the momentum of an electron traveling at a speed 0.985c? The rest mass of the electron is $9.11 \times 10^{-31} kg$.

Given:

$v = 0.985c$

$c = 3 \times 10^8 m/s$

$m = 9.11 \times 10^{-31} kg$

Find: p

Solution:

$$\boldsymbol{p = \gamma mv = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$= \frac{(9.11 \times 10^{-31} kg)(0.985c)}{\sqrt{1-\frac{(0.985c)^2}{(c)^2}}} = \mathbf{1.56 \times 10^{-21} kg \cdot m/s}$$

C. RELATIVISTIC MASS AND ENERGY

Having momentum being modified, relativistic corrections for energy and mass are also expected to be made. Einstein showed that at high speeds the formula $KE = \frac{1}{2}mv^2$ is not correct. He showed that the kinetic energy of a particle having mass m travelling at speed v is given by

$$\boldsymbol{KE = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}$$

Having $\gamma = \sqrt{1-\frac{v^2}{c^2}}$, rewriting the equation as

$$\boldsymbol{KE = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2}$$

The first term increases with the speed of the particle, while the second term $\boldsymbol{mc^2}$ is constant; it is called the **rest energy** of a particle at rest.

The equation above can be rearranged to get the *total Energy E* of the particle, as shown below:

$$\boldsymbol{E = KE + mc^2}$$

The total energy can also be written as

$$\boldsymbol{E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}}$$

For a particle at rest in a given reference frame, KE is zero, the total energy is its rest energy.

Sample problem 1:
A meson having a mass of $2.4 \times 10^{-28} kg$ travels at speed of $v = 0.80c = 2.4 \times 10^{8} m/s$. What is the Kinetic Energy? Compare it to classical calculation.
Given:
$m = 2.4 \times 10^{-28} kg$
$v = 0.80c = 2.4 \times 10^{8} m/s$
$c = 3 \times 10^{8} m/s$
Find: KE
Solution:

$$KE = mc_{21-}v_{2c2-mc2}KE = (\gamma - 1)mc^2 = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$= (2.4 \times 10^{-28} kg)(3 \times 10^{8} m/s)^2\left(\frac{1}{\sqrt{1 - 0.64}} - 1\right)$$

$$= \mathbf{1.44 \times 10^{-11} J}$$

Classically $KE = \frac{1}{2}mv^2 = \frac{1}{2}(2.4 \times 10^{-28} kg)(2.4 \times \frac{10^8 m}{s)^2} = \mathbf{6.9 \times 10^{-12} J}$

Explore

Work on the following activities to master and strengthen the basic concepts you have learned from this lesson.

Solve the following problems systematically. Use another sheet of paper for your solution.

1. A rocket ship passes you at a speed of 0.90c. A girl measured its length to be 50 m. How long would ta be when at rest?
2. An occupant on a high-speed spaceship traveling between Earth and Mars at a steady speed of 0.85c reads a brochure which takes 15.0 min according to her wristwatch. (a) How long does this take as measured by Earth-based clocks? (b) How much farther is the spaceship from Earth at the end of reading the brochure than it was at the beginning?
3. Compare the relativistic momentum of an electron to its classical momentum when it has a speed of (a) 0.88c in an accelerator used for treating cancer and (b)$4.50 \times 10^{7} m/s$ in the cathode ray tube of a TV set.
4. An accelerator at a Physics Laboratory can accelerate protons to a Kinetic energy of 10^{12} eV. The rest mass of a proton is $mc^2 = 9.38 \times 10^{8} eV$. What is the speed of that proton?

Deepen

Perform what is being asked in the problem below. Write your answer on a sheet of paper.

Based from Special Theory of Relativity, the relativistic factor $\boldsymbol{\gamma}$ which is given by $\frac{\mathbf{1}}{\sqrt{\mathbf{1}-\frac{v^2}{c^2}}}$ determines the length contraction and the time dilation. Determine the numerical values of $\boldsymbol{\gamma}$ for an object moving at speed $v = 0.01c, 0.05c, 0.10c, 0.20c, 0.30c, 0.40c$ $0.50c, 0.60c, 0.70c, 0.80c, 0.90c, 0.99c$. Make a graph of γ $versus$ v.

Gauge

I. MULTIPLE CHOICE. Read and analyze each question then select the BEST answer. Write the corresponding CAPITAL LETTER of your choice in a separate sheet of paper.

1. If you consider a blinking light source that is receding an observer, what happens to its speed?
   A. Increases B. Decreases
   C. Remains the same D. Needs more information

2. If you consider a blinking light source that is receding an observer, what happens to its frequency?
   A. Increases B. Decreases
   C. Remains the same D. Needs more information

3. If you consider a blinking light source that is receding from an observer, what happens to its wavelength?
   A. Increases B. Remains the same
   C. Decreases D. Needs more information

4. Which of the following event is ***<u>TRUE</u>*** based on the special theory of relativity?
   A. Clocks that are moving run slower than when they are at rest.
   B. Clocks that are moving run faster than when they are at rest.
   C. Clocks run at the same rate regardless of whether they are moving or not.
   D. Clock run at rates that depend on an observer's inertial frame of reference.

5. Which of the following is ***TRUE*** about the speed of light?
   A. The speed of light is slower in a moving frame of reference.
   B. The speed of light is constant in an inertial frame of reference.
   C. The speed of light has the same value for observers in all reference frames.
   D. The speed of light has a value that depends on the observer's frame of reference.

6. What situation does relativistic formula for length contraction, time dilation and relativistic mass are valid?
   A. Only for speeds less than 0.10c
   B. Only for speeds greater than 0.10c
   C. Only for speeds very close to c
   D. For all speeds

7. Suppose you are in rocket ship going faster and faster. As your speed increases and your velocity gets closer to the speed of light, which of the following can be observe in your frame of reference?
   A. Mass increases
   B. Length shortens
   C. Clock slows down
   D. All of the above

8. The spaceship which is measured to be 50 m long by the captain. When the spaceship past a space dock at 0.5c, space dock personnel measure the rocket ship to be 43.3 m long. What is the proper length?
   A. 13.3 m
   B. 43.3 m
   C. 50.0 m
   D. 93.3 m

9. As the spaceship in Number 8 passes by the space dock, the ship's captain flashes a flashlight at 1.00 s interval as measured by space-dock personnel. How often does the flashlight flash relative to the captain?
   A. Every 0.87 s
   B. Every 1.15 s
   C. Every 1.00 s
   D. cannot determine

10. What does this expression represent: $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$?
    A. Time dilation
    C. Relativistic factor
    B. Length contraction
    D. Relativistic energy

II. TRUE OR FALSE. Read each statement below carefully. Write **T** if the statement is correct and **F** if not correct. Write your answer in a separate sheet of paper.

______1. An object's length will be the same when it travels close to the speed of light.
______2. The time will noticeably slow down when travelling at speed close to c.
______3. Length and height are being changed as a consequence of Special Theory of Relativity.
______4. Relativistic mass increases with the speed of an object relative to the reference frame.
______5. Relativistic effects are significant only at high speeds or close to the speed of light.

Answer Key

Jumpstart

1. B
2. C
3. A
4. B
5. A
6. B
7. C
8. C
9. C
10. C
11. C
12. A
13. A
14. D
15. A

Explore

1. 111.71 m
2. A. 28.47 m
   B. $4.36 \times 10^{11} m$
3. A. 0.47mv = $1.13 \times 10^{-22} m$
   B. 0.99mv = $2.38 \times 10^{-22} kg \cdot m/s$
4. 0.10c

Gauge

1. C
2. B
3. A
4. A
5. C
6. D
7. B
8. C
9. A
10. C
11. F
12. T
13. F
14. T
15. T

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