GENERAL PHYSICS I

12

FIRST SEMESTER

MODULE 8

MOMENTUM, IMPULSE AND COLLISIONS

Most Essential Learning Competency:

- *Relate the momentum, impulse, force, and time of contact in a system (STEM_GP12MMICIh-58)*
- *Compare and contrast elastic and inelastic collisions. (STEM_GP12MMICIi-60)*
- *Apply the concept of restitution coefficient in collisions (STEM_GP12MMICIi-61)*
- *Solve problems involving center of mass, impulse, and momentum in contexts such as, but not limited to, rocket motion, vehicle collisions, and ping-pong. (STEM_GP12MMICIi-63)*

To the Learners

How to use this module?

Before anything else, I want you to set aside other tasks that will disturb you while learning the lessons. Read the instructions below to achieve the objectives of this kit.

1. Follow carefully all the contents and instructions indicated in every page of this module.
2. Take down important notes about the lessons. Writing enhances learning and improves your retention.
3. Perform all the provided activities in this module.
4. Let your facilitator/guardian assess your answers using the facilitator's guide.
5. Analyze conceptually the posttest and apply what you have learned.
6. Enjoy learning!

Expectations

At the end of this module, you will be able to:

- explain the meaning of the momentum of a particle, and the impulse of the new force acting on a particle causes its momentum to change;
- compare and contrast elastic and inelastic collisions;
- solve problems in which two bodies collide with each other; and
- solve problems involving momentum, impulse and collisions.

Pre-Test

Directions: Read each question below carefully. Write the letter of your best answer on the space provided before the number.

_____ 1. What is the magnitude of the momentum of a 28-g sparrow flying with a speed of 8.4 m/s?

A. 0.20 kg · m/s
B. 0.24 kg · m/s
C. 8.37 kg · m/s
D. 300 kg · m/s

_____ 2. Why is there a greater momentum in a heavy truck than a passenger car moving at the same speed? Because the truck ___________.

A. has greater mass
B. has greater speed
C. is not streamlined
D. has a large wheelbase

_____ 3. What type of collision occurs when interacting objects stick together after the collision, becoming a single compound object?

A. Elastic collision
B. Inelastic collision
C. Super elastic collision
D. Perfectly inelastic collision

_____ 4. What quantity compares the relative velocities of the interacting objects before and after the collision?

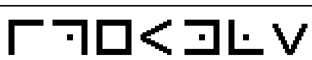
A. Coefficient of collision
B. Coefficient of friction
C. Coefficient of restitution
D. Coefficient of variation

_____ 5. A 10 kg toy truck moves at 5 m/s East. It collides head-on with a 5 kg toy car moving 10 m/s moving west. What is the total momentum of the system?

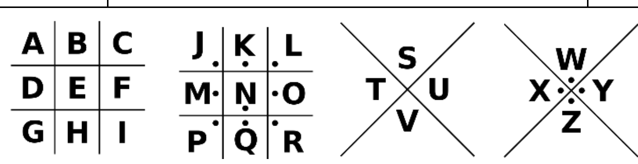
A. 0 kg · m/s
B. 10 kg · m/s
C. 30 kg · m/s
D. 50 kg · m/s

Looking Back

Directions: As a recall of your previous lessons, use the Pigpen cipher below to unjumble the words listed in the table. Write your answer in alphabet.

	Jumbled Word	Definition	Unjumbled Word
1		The quantity of motion of a moving body, measured as a product of its mass and velocity.	
2		It is the integral of a force over the time interval for which it acts.	
3		It is an instance of one moving object striking against another.	
4		It is a type of (#3) in which there is no net loss in kinetic energy in the system.	
5		It is a type of (#3) in which there is a loss of kinetic energy.	

Pigpen Cipher:



Brief Introduction

There are many questions involving forces that cannot be answered by directly applying Newton's second law, $\sum \vec{F} = m\vec{a}$. For example, when a moving truck collides head-on with a jeepney, what determines which way the wreckage moves after the collision? In playing billiards, how do you decide how to aim the cue ball to knock the nine ball into the pocket? And when a meteorite collides with the earth, how much of the meteorite's kinetic energy is released in the impact?

To answer such questions this module will introduce the concept of momentum and impulse, and the conservation of momentum. The law of conservation of momentum is valid even in situations in which Newton's laws are inadequate, such as bodies moving at very high speeds (near the speed of light) or objects on a very small scale (such as the constituents of atoms). Within the domain of Newtonian mechanics, conservation of momentum enables us to analyze many situations that would be very difficult if we tried to use Newton's laws directly. Among these are collision problems, in which two bodies collide and can exert very large forces on each other for a short time.

Momentum

The linear momentum (or "momentum" for short) of an object is defined as the product of its mass and its velocity. Momentum (plural is momenta) is represented by the symbol $\vec{\boldsymbol{p}}$. If we let $\boldsymbol{m}$ represent the mass of an object and $\vec{\boldsymbol{v}}$ represent its velocity, them its momentum $\vec{\boldsymbol{p}}$ is defined as:

$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}} \text{ (Equation 1)}$$

Velocity is a vector, so momentum too is a vector. The direction of the momentum is the direction of the velocity, and the magnitude of the momentum is $p = mv$. Because velocity depends on the reference frame, so does momentum; thus, the reference frame must be specified. The unit of momentum is that of mass × velocity, which in SI units is $\mathbf{kg \cdot m/s}$. There is no special name for this unit.

A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it, or to change its direction. Newton originally stated his second law in terms of momentum. Newton's statement of the **second law of motion**, translated into modern language, is as follows:

The rate of change of momentum of an object is equal to the net force applied to it.

We can write this as an equation,

$$\boldsymbol{\Sigma}\vec{\boldsymbol{F}} = \frac{\Delta\vec{\boldsymbol{p}}}{\Delta \boldsymbol{t}} \text{ (Equation 2)}$$

where $\boldsymbol{\Sigma}\vec{\boldsymbol{F}}$ is the net force applied to the object and $\Delta\vec{\boldsymbol{p}}$ is the resulting momentum change that occurs during the time internal $\Delta \boldsymbol{t}$**.**

We can readily derive the familiar form of the second law $\boldsymbol{\Sigma}\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}}$, from Eq. 2 for the case of constant mass. If $\overrightarrow{v_1}$ is the initial velocity of an object and $\overrightarrow{v_2}$ is the velocity after a time interval $\Delta \boldsymbol{t}$ has elapsed, then

$$\boldsymbol{\Sigma}\vec{\boldsymbol{F}} = \frac{\Delta\vec{\boldsymbol{p}}}{\Delta \boldsymbol{t}} = \frac{\boldsymbol{m}\overrightarrow{\boldsymbol{v_2}} - \boldsymbol{m}\overrightarrow{\boldsymbol{v_1}}}{\Delta \boldsymbol{t}} = \frac{\boldsymbol{m}(\overrightarrow{\boldsymbol{v_2}} - \overrightarrow{\boldsymbol{v_1}})}{\Delta \boldsymbol{t}} = \boldsymbol{m}\,\frac{\Delta\vec{\boldsymbol{v}}}{\Delta \boldsymbol{t}}$$

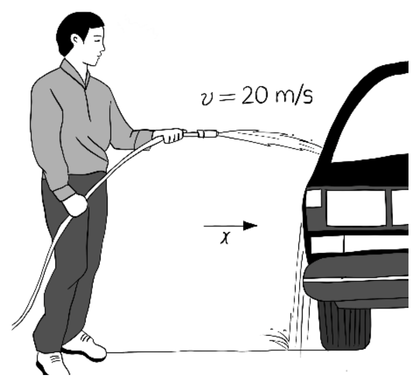
By definition, $\vec{\boldsymbol{a}} = \Delta\vec{\boldsymbol{v}}/\Delta \boldsymbol{t}$, so $\boldsymbol{\Sigma}\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}}$. Newton's statement is more general than the more familiar version because it includes the situation in which the mass may change. A change in mass occurs in certain circumstances, such as for rockets which lose mass as they burn fuel, and in the theory of relativity.

Activity 1: Momentum Change and Force

Directions: Read the problem carefully. Show your complete solution and box your final answer in the space provided below.

1. For a top player, tennis ball may leave the racket on the serve with a speed of 55 m/s (about 120 mi/h). If the shuttlecock has a mass of 0.060 kg and is in contact with the racket for 4 ms (4×10^{-3} s), estimate the average force on the ball. Would this force be large enough to lift a 60-kg person?

2. Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it (refer to Figure 1). Ignoring any splashing back, what is the force exerted by the water on the car?



**Original work*
Figure 1. A man washing a car

Impulse

From Newton's second law, the net force on one object is equal to the rate of change of its momentum:

$$\Sigma\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

If we multiply both sides of this equation by the time interval Δt, we obtain

$$\Sigma\vec{F}\Delta t = \Delta\vec{p}$$

The quantity on the left, the product of the force $\vec{F}$ times the time Δt over which the force acts, is called the impulse, denoted by $\vec{J}$, which is defined to be the product of the net force and the time interval.

$$\vec{J} = \Sigma\vec{F}\Delta t \quad \text{(assuming constant net force)}$$

Impulse is a vector quantity; its direction is the same as the net force $\Sigma\vec{F}$. Its magnitude is the product of the magnitude of the net force and the length of time that the net force acts. The SI unit of impulse is the newton-second (N·s). Because 1 N = 1 kg · m/s^2, an alternative set of units for impulse is kg · m/s, the same as the units of momentum. Because of that, impulse could also be defined as the total change in the momentum and its concept is used mainly when dealing with forces that act during a short time interval.

Activity 2: Imparted Impulse

Directions: Read the problem carefully. Show your complete solution and box your final answer in the space provided below.

1. A golf ball of mass 0.045 kg is hit off the tree at a speed of 45 m/s. The golf club was in contact with the ball for 3.5×10^{-3} s. Find (a) the impulse imparted to the golf ball, and (b) the average force exerted on the ball by the golf club.

2. A 12-kg hammer strikes a nail at a velocity of 8.5 m/s and comes to rest in a time interval at 8.0×10^{-3} s. (a) What is the impulse given to the nail? (b) What is the average force acting on the nail?

Conservation of Linear Momentum and Collisions

Just like energy, momentum is another conserved quantity. In the absence of a net external force acting on an isolated system, the total momentum is constant, hence it is conserved. Simply stated, the total momentum before is equal to the total momentum after.

The Conservation of linear momentum is best observed during collisions. A **collision** is the sudden, forceful coming together in direct contact of two or more bodies or objects. Collisions can be classified as either elastic or inelastic. In an **Elastic Collision**, the total linear momentum and Kinetic Energy are both conserved. This means that the total linear momentum and total kinetic energy of the objects interacting will be the same before and after the collision.

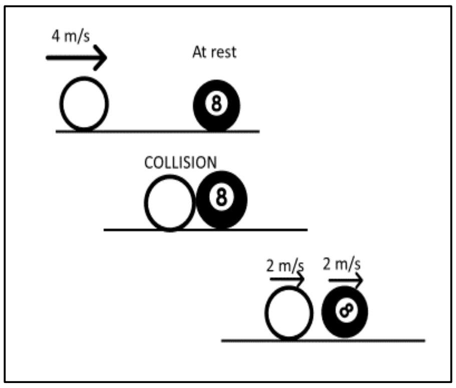


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Figure 2. Collision of Billiard Balls (A)

Figure 2 on the left shows one configuration of an elastic collision. Assume that the surface is frictionless, and the billiard balls have identical masses of 0.5 kg. The white cue ball approaches the stationary black ball with a velocity of 4 m/s (movement to the right is assumed to be positive). We can see that the initial total linear momentum is:

$$P_{Total} = P_{White} + P_{Black} = (0.5\ kg)(4\ m/s) + (0.5\ kg)(0)$$

$$P_{Total} = \mathbf{2\ kgm/s}$$

While the initial total Kinetic energy of the system is:

$$KE_{Total} = KE_{White} + KE_{Black} = \frac{1}{2}(0.5kg)(4\ m/s)^2 + \frac{1}{2}\ (0.5\ kg)(0)$$

$$KE_{Total} = \mathbf{4\ J}$$

Activity 3: Collisions

Directions: Read the problem carefully. Show your complete solution and box your final answer in the space provided below.

Calculate the Final Total Linear Momentum (P'_{Total}) and Final Total Kinetic Energy (KE'_{Total}) using the same procedure as shown above. Write your solution in the space provided below. How will you compare the Final values of the total linear momentum and Kinetic energy with the initial values?

Now, consider the situation in Figure 3 on the left. This time, after collision, the white cue ball stops, while the black ball moves with the same initial velocity as the white cue ball. Are the total linear momentum and Total kinetic energy conserved? Show your solution in the space provided.

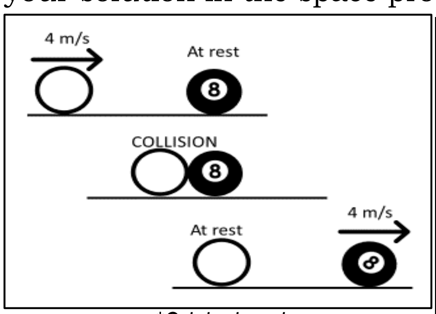

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Figure 3. Collision of Billiard Balls (B)

Another configuration of an elastic collision is shown in Figure 4 on the right. The initial values of the total linear momentum and Kinetic energy can be obtained as:

$$P_{Total} = P_{White} + P_{Black} = (0.5\ kg)(4\ m/s) + (0.5\ kg)(-3\ m/s) = \mathbf{0.5\ kgm/s}$$

$$KE_{Total} = KE_{White} + KE_{Black} = \left[\frac{1}{2}(0.5kg)(4\ m/s)^2\right] + \left[\frac{1}{2}\ (0.5\ kg)(-3\ m/s)^2\right]$$
$$= \mathbf{6.25\ J}$$

Take note that velocity directed to the left is negative.

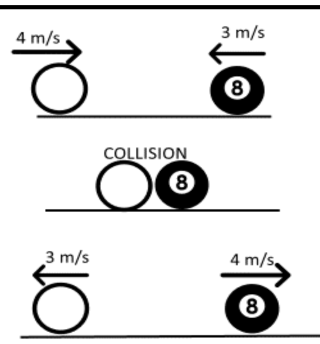

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Figure 4. Collision of Billiard Balls (C)

After the collision, the final total linear momentum and Kinetic energy can be calculated as:

$$P'_{Total} = P'_{White} + P'_{Black} = (0.5\ kg)(-3\ m/s) + (0.5\ kg)(4\ m/s) = \mathbf{0.5\ kgm/s}$$

$$KE'_{Total} = KE'_{White} + KE'_{Black} = \left[\frac{1}{2}(0.5kg)(-3\ m/s)^2\right] + \left[\frac{1}{2}\ (0.5\ kg)(4\ m/s)^2\right] = \mathbf{6.25\ J}$$

Both the total linear momentum and kinetic energy values were the same before and after the collision.

In an **Inelastic Collision**, only the Total Linear Momentum of the system is conserved. The Total Kinetic energy is not conserved. This means that the final total kinetic energy of the system is not the same as the initial value. This difference can be attributed to energy being transformed into other forms such as thermal energy.

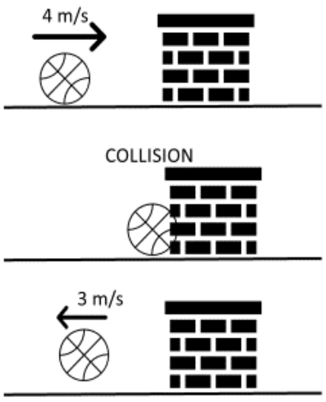

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Figure 5. Rubber ball bouncing against a wall

When a rubber ball bounces off from a wall, it gets deformed momentarily. This momentary deformation decreases the Kinetic energy of the ball as it bounces because some of the initial kinetic energy is converted into other forms of energy such as thermal energy when the ball gets deformed. The total linear momentum, however, remains the same. If the ball is 2.0 kg and the wall is 500 kg, if the total linear momentum is conserved, it means that after collision, the ball should also move! But by how much?

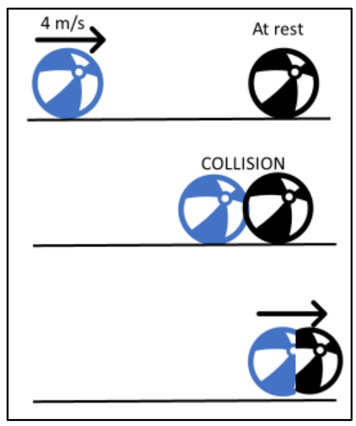
$$P_{Total} = P_{ball} + P_{wall} = \left[(2\ kg)\left(4\frac{m}{s}\right)\right] + \left[(500\ kg)\left(0\frac{m}{s}\right)\right]$$

$$P'_{Total} = P'_{ball} + P'_{wall} = \left[(2\ kg)\left(-3\frac{m}{s}\right)\right] + [(500\ kg)(X\ )]$$

Since $P_{Total} = P'_{Total}$ therefore,

$$\left[(2\ kg)\left(4\frac{m}{s}\right)\right] + \left[(500\ kg)\left(0\frac{m}{s}\right)\right] = (2\ kg)(-3\ m/s) + (500\ kg)(X\ )$$

$X = \frac{\left(8\frac{kgm}{s}\right)-\left(6\frac{kgm}{s}\right)}{500\ kg} = \mathbf{0.004\ m/s}$, which is quite negligible. This becomes even smaller if we set the wall to be part of the entire earth! Thus, instead of only considering the mass of the wall, then you will have to consider the mass of the entire earth, thus making the velocity of the wall after collision more negligible.



*Original work
Figure 6. Collision of two inflatable balls (A)

In a **perfectly inelastic collision**, the interacting objects will stick together after the collision, becoming a single compound object. This can be observed when two cars collide with each other and the cars become stuck together as they move intertwined. In Figure 6 on the right, the two inflatable balls stick together after their collision. If both inflatable balls are 1 kg in mass, we can calculate the velocity of the balls that were stuck together.

$$P_{Total} = P'_{Total} = \left[(1kg)\left(4\frac{m}{s}\right) + 0\right] = (2\ kg)(X) \qquad \mathbf{X = 2\ m/s}$$

For any collision, we can say that an elastic collision is on one extreme while a perfectly inelastic collision is on the other end of the extreme. A quantity called the **Coefficient of Restitution (e)** compares the relative velocities of the interacting objects before and after the collision. It is a measure of how elastic or inelastic the collision is. An elastic collision has a coefficient of restitution of 1, for a perfectly inelastic collision, it is 0.

$$\boldsymbol{e = \frac{v_{Afinal} - v_{Bfinal}}{v_{Ainitial} - v_{Binitial}}}$$

Remember

1. The **momentum**, $\vec{p}$, of an object is defined as the product of its mass times its velocity, $\vec{p} = m\vec{v}$.
2. In terms of momentum, Newton's second law can be written as $\Sigma\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$. This is the rate of change of momentum equals the net applied force.
3. In a **collision**, two (or more) objects interact with each other over a very short time interval, and the forces between them during this time interval are very large.
4. The **impulse** of a force on an object is defined as $\vec{F}\Delta t$, where $\vec{F}$ is the average force acting during the time interval Δt. The impulse is equal to the change in momentum of the object: $\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$
5. Collisions can be classified as either elastic or inelastic. In an **Elastic Collision**, the total linear momentum and Kinetic Energy are both conserved. In an **Inelastic Collision**, only the Total Linear Momentum of the system is conserved. In a **perfectly inelastic collision**, the interacting objects will stick together after the collision, becoming a single compound object.
6. A quantity called the **Coefficient of Restitution (e)** compares the relative velocities of the interacting objects before and after the collision.

Check Your Understanding

Directions: Calculate for the coefficient of restitution for the different configurations of collisions below. Write your complete solutions in the space provided.

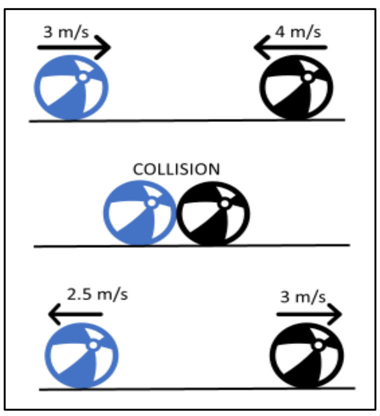


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Figure 7. Collision of two inflatable balls (B)

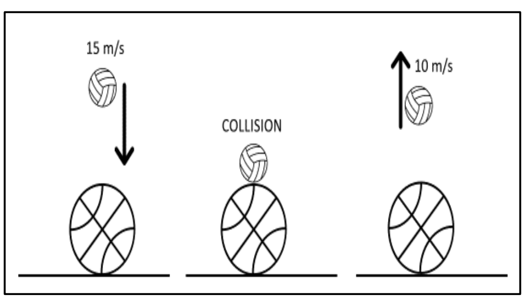

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Figure 8. Collision of two sport balls

Post-Test

Directions: Read each question below carefully. Write the letter of your best answer on the space provided before the number.

_____ 1. What is the magnitude of the momentum of a 28-g sparrow flying with a speed of 8.4 m/s?

A. 0.20 kg · m/s
B. 0.24 kg · m/s
C. 8.37 kg · m/s
D. 300 kg · m/s

_____ 2. Why is there a greater momentum in a heavy truck than a passenger car moving at the same speed? Because the truck ___________.

A. has greater mass
B. has greater speed
C. is not streamlined
D. has a large wheelbase

_____ 3. What type of collision occurs when interacting objects stick together after the collision, becoming a single compound object?

A. Elastic collision
B. Inelastic collision
C. Super elastic collision
D. Perfectly inelastic collision

_____ 4. What quantity compares the relative velocities of the interacting objects before and after the collision?

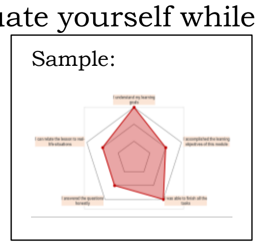
A. Coefficient of collision
B. Coefficient of friction
C. Coefficient of restitution
D. Coefficient of variation

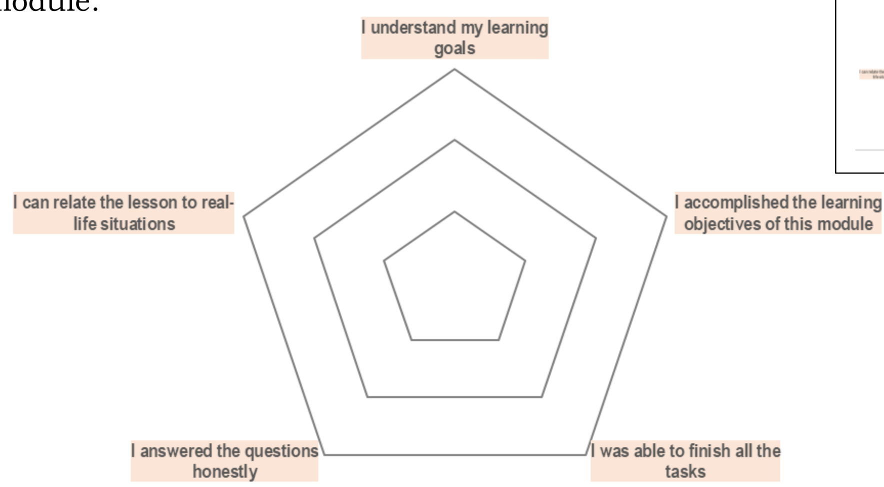
_____ 5. A 10 kg toy truck moves at 5 m/s East. It collides head-on with a 5 kg toy car moving 10 m/s moving west. What is the total momentum of the system?

A. 0 kg · m/s
B. 10 kg · m/s
C. 30 kg · m/s
D. 50 kg · m/s

Reflection

Directions: Create your personal radar chart based on how you evaluate yourself while using this module.





Based on what you have learned in this lesson, how will you apply your knowledge of "Momentum, Impulse and Collision" in real life scenarios? Cite some examples.

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References:

1. Young, H. D., Freedman, R. A., & Ford, A. L. (2012). Sears and Zemansky's University physics. San Francisco: Pearson Addison Wesley. Pp. 241-265
2. Das, B., & Giancoli, D. C. (2005). Physics: Principles with Applications, Sixth Edition. Upper Saddle River, NJ: Pearson Prentice Hall. pp. 167-187