Measurement of Youngs Modulus using Strain Gauges on a Cantilevered Beam

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I Abstract

This paper describes the experimental determination of Young's modulus using strain gauges on a cantilevered steel beam. The experimental apparatus consisted of a 36.8 cm long cantilevered steel beam with metallic strain gauges mounted to the top and bottom of the beam at the same distance from the the fixed end. These two strain gauges were used in a half-arm Wheatstone bridge circuit to measure the strain from bending stress caused by hanging weights from the free end of the beam. At each loading of the beam, bending stress was calculated at the location of the strain gauges. Multiple loadings were performed, and the recorded data was used to create a stress vs. strain plot. A line was fit to the data using least-squares regression. The slope of this line, known as Youngs Modulus, was determined to be 195.77 ± 3.0 MPa, which is consistent with the standard published value of 200 GPa [1].

II Introduction

The mechanical design process refers to the application of mathematics, material science, and engineering-mechanics to the design of mechanical components and systems [2]. Critical to proper mechanical design is accurate knowledge of material properties. One such important property is the mechanical stiffness of the material.

According to Hookes law, stress is proportional to strain over the elastic region of a material [2]. The ratio of stress to strain in this region is known as the modulus of elasticity, or Youngs modulus. In most cases, components are designed to stay within the elastic range of the material. Thus, knowing Youngs modulus for a material allows a designer to predict how a component might elongate or deflect when subjected to a given load or stress.

There are multiple ways to determine Youngs modulus of a material. The most common method is tensile testing, in which a material sample is loaded directly in tension. An alternative method is presented here. Specifically, we describe the measurement of Youngs modulus using metallic strain gauges mounted to a cantilevered beam. By moment-loading a cantilevered beam with multiple weights, the calculated stress may be compared to the measured strain at the location of the gauges, producing strain corresponding to the stress-strain ratio known as Youngs modulus.

Metallic strain gauges are passive electrical resistive components whose resistance increases when subjected to a load, a phenomenon first explored by Lord Kelvin in 1856 [3]. Although the design of strain gauges can amplify the effect, the sensitivity of resistance strain gauges remains low and difficult to directly measure. To solve this problem, a Wheatstone bridge circuit, which is highly sensitive to resistance changes, is typically included as part of the strain measurement system [5].

This paper describes the measurement of Youngs modulus using metallic strain gauges mounted to a cantilevered beam. The strain gauges are mounted to the steel beam so that they measure the strain at a known distance from the end of the beam where the load will be placed. Weights are then added and removed from the end of the beam to create a known stress in the steel. The corresponding strain is recorded from the metallic strain gauges mounted to the beam and the relationship between the stress applied to the beam and the strain measured by the gauges is used to mathematically determine the elastic modulus of the steel in the beam[4].

III Methods

The Cantilevered Beam

Figure 1: Cantilevered beam used for experimentally determining Youngs modulus. Near the top of the photo is the beam used in the experiment. Also shown is some of the lab equipment used in the experiment.

Distance (in)	0	5	10	14.5	Average
Thickness (in)	0.2508	0.2510	0.2507	0.2505	0.25075
Width (in)	1.499	1.500	1.501	1.502	1.500

In our experiment we used a cantilevered steel beam to test the elastic modulus of the beam material. The length was measured with a standard tape measure as 14.5 inches. The width and thickness of the beam were measured at 4 points along the length of the beam, and those measurements are shown below.

Calculated Stress

Using the measured dimensions of our beam, and the known weights used to load the beam, we can find the approximate stress in the beam at the location of the strain gauges. This can be shown as [4, 2]:

$$\sigma = \frac{6\, m\, g\, L}{w\, t^2} \tag{1}$$

In this equation σ represents stress, m represents the mass on the end of the beam, and g is the gravitational constant which is known to be $9.8051 \frac{m}{s^2}$ where our experiment was conducted. Additionally L, w, t represent length, width, and thickness of the beam respectively. For the beam dimensions the averages of our measurements were used.

Strain Measurement

Standard metallic strain gauges were used to measure the strain in the beam during the experiment. The gauges produce a change in electrical resistance under strain, but the effect is small so it was magnified through a Wheatstone Bridge circuit and an instrumentation amplifier for the purposes of the experiment. The following equation was used to correlate the output of the circuit in volts into a strain measurement in inches per inch[2]:

$$\frac{\partial V_o}{V_i G} = \frac{(GF)\ \varepsilon}{2} \tag{2}$$

In this equation we are representing the change in voltage out of the Wheatstone Bridge circuit as ∂V_o, V_i represents the voltage fed into the measurement circuit, G represents the gain of the instrumentation amplifier used, GF represents the gauge factor of our strain gauges, and ε represents the measured strain of the beam. Values for these variables can be found in Figure IV.

In this experiment the instrumentation amplifier used was the AD622 with a gain of 500, and we measured the voltage output of the circuit with a Model #34401A Digital Multimeter.

Experimental Protocol

Using the above described experimental set-up data was collected on the stress and strain on the beam for several weights. The voltage outputed by the measurement circuit was recorded and the weight loaded on the end of the beam. The new voltage was then recorded and the weight removed. After unloading the beam the voltage was measured again, creating two sets of data, for both the loading and unloading of the beam. This was an attempt to remove hysteresis error from the results of the experiment. The difference in voltage for each data point was then calculated and converted to a strain in the beam.

Regression

After data has been collected for a range of stresses and strains they can be correlated using a standard simple linear regression model. Given that strain is the dependent value, a linear equation relating stress and strain should be regressed from your data. The slope in this equation represents the elastic modulus of the material, literally the "stress per strain." The bias in the equation is expected to be near 0 because we know that under 0 stress we shouldn't expect any strain in the material. The regression model is based on the linear fit produced:

$$y = 0.19577x + 0.04618 \tag{3}$$

Uncertainty Analysis

The uncertainty analysis for this experiment begins with a measurement of error in each of the variables involved in calculating Young's Modulus. This involves a root sum of squares calculation for each variable. This combines factors such as linearity, resolution, hysteresis, and bias errors for each individual measurement which are shown under "Propagated" in Table IV. These are then combined using Eqn.(4), where P represents any variable, to find the total uncertainty.

$$U = \sum (\partial P \cdot u_P) \tag{4}$$

IV Results

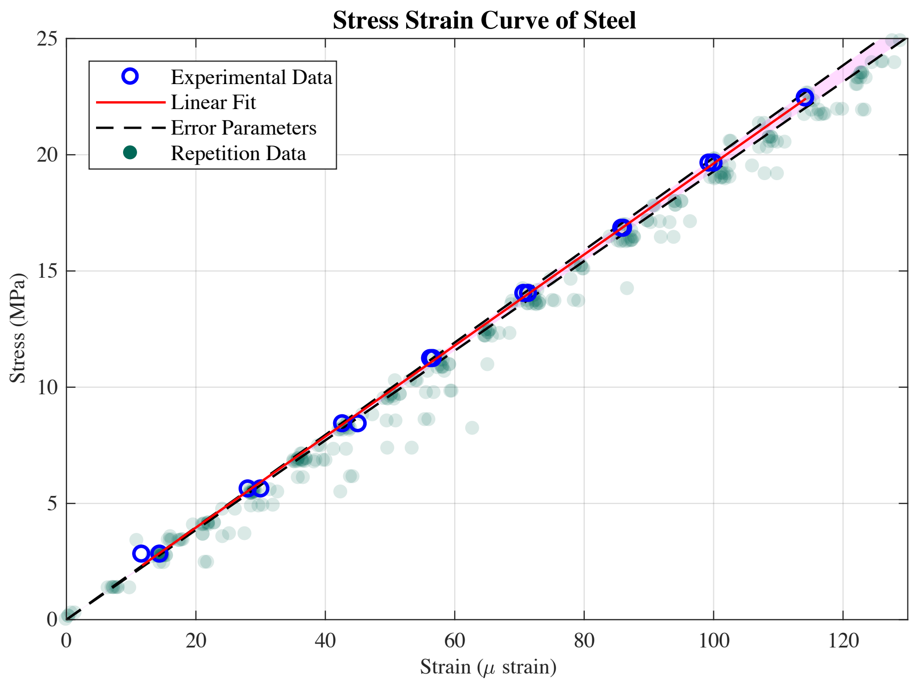

Figure 2: Plot of experimental results. This shows the data points taken during our experiment as well as our linear regression line and approximate error. The green points represent data taken by other groups during similar tests and show a strong similarity to the results presented in this report.

The equation we regress from our plot is $\sigma = 195.77\varepsilon \times 10^6$ which gives us our result:

$$\mathrm{E} = 195.77\mathrm{GPa}$$

Error Analysis

The error in our experiment is calculated in Table IV, a somewhat conservative approach seeing as we calculated max error for every variable. It is possible that there exists error unaccounted for in electrical noise or other unknowns. The final is error is less than 2%:

$$\mathrm{U_e} = 2.93\,\mathrm{GPa}$$

	m	g	L (m)	w (m)	t (m)
mean	0.9	9.8051	0.3683	0.0381	6.3969e(-3)
uncertainty ($\pm$)	0.009	1.4e(5)	7.938e(-4)	1.27e(-5)	1.27e(-6)
propogated	1.93e(9)	2.76e(5)	4.17e(8)	6.45e(7)	7.71e(7)
	GF	G	V_i (Volts)	∂V_o (Volts)	
mean	2.012	500	10	0.3281	
uncertainty ($\pm$)	1.27e(-6)	0.02012	1	0.0005	0.0005
propogated	1.93e(9)	9.67e(8)	9.67e(6)	2.94e(8)	

Table 1: Data collected during course of experiment. This table also includes uncertainty data for every variable.

V Discussion and Conclusions

The values produced in this experiment were off by 2.1 % from the published value of 200 GPa which is fairly accurate and a lower uncertainty than expected. The two largest contributors to error were the gauge factor of the strain gauges used as well as the measurement of length of the beam. The GF values would need to be improved by possibly buying gauges with more accurate documentation, while the length measurement would require more accurate tools to improve.

The experiment could also be improved either through more extensive testing, or through using more accurate methodologies such as tensile testing of the material which allows for the use of more exact equipment, although more expensive.

References

[1] Miner, D., & Seastone, John B. (1955). *Handbook of engineering materials* (1st ed., Wiley engineering handbook series). New York: John Wiley & Sons.

[2] Shigley, J. E., C. R. Mischke and R. G. Budynas (2004). *Mechanical engineering design,* McGraw-Hill.

[3] Thomson, W. L. K. (1856). *"On the electrodynamic qualities of metals."* Philosophical transactions of the Royal Society (London) 146: 649-751.

[4] Eric Wolbrecht (2018) *Experiment 7: Measurement of Youngs Modulus using Strain Gauges*

[5] Figliola, R. S. and D. Beasley (2015). Theory and design for mechanical measurements, John Wiley & Sons.