

CSC476 Introduction to Computer Vision

Lecture 4

More on Histogram Equalization, Border
Effect, Image Derivatives, Edge Detection

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Last lecture

- Linear Algebra Review
- Gaussian Kernel
- Histogram Equalization

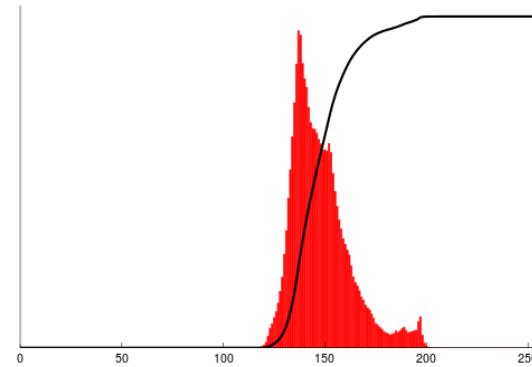
Today's Lecture

- More on image histograms
- Border effect and padding
- Image Derivatives/gradients
- Canny edge detection

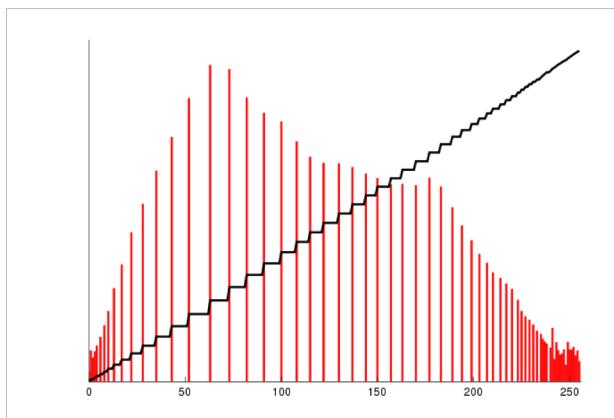
Learning objective check

- You should be so far:
- 1) fluent with convolving images with filter
- 2) fluent with convolving matrix with another matrix.
- 3) Know basic Numpy matrix manipulation.
- 4) Create box, 2D Gaussian and 1D Gaussian filters of all sizes.

Image Histogram Equalization



Notice the “shape” of the histogram is preserved!



Same number of pixels
In each bin!

Questions

1. Why don't we do the equalization directly on the histograms? What will happen to the image if it has a perfectly flat histograms?
2. What is CDF? Why are there steps (zig-zags) on the CDF?
3. How to make sure the “shape” of the PDF is preserved in the transformation?

Let's look at in more details

We start with a gray-scale jpeg image of 8 by 8

52	55	61	66	70	61	64	73
63	59	55	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

https://en.wikipedia.org/wiki/Histogram_equalization

Let's look at in more details

The histograms for this image (8 by 8 pixels) is shown in the following table (intensity that has zero pixels are skipped:

Value	Count								
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Let's look at in more details

The cumulative distribution function (CDF) is shown below

Value	cdf	scaled cdf	
52	1	0	This cdf shows that the min value in the subimage is 52 and max is 154. The cdf of value 154 corresponding to the total number of pixels (64)
55	4		
58	6		
59	9		
60	10		
61	14		
62	15		
...	...		
154	64	255	

Number of pixels



Let's look at in more details

How do we compute the normalized CDF?

$$h(v) = \text{round} \left(\frac{cdf(v) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L - 1) \right)$$

Cdf(v): original cdf of pixel v

Cdfmin: minimum non-zero vale of the cdf

M×N: number of pixels, e.g. 64 (8x8)

L: 256

Quiz:

How do we compute normalized CDF of pixel 62?

Value	cdf	scaled cdf
52	1	0
55	4	
58	6	
59	9	$h(v) = \text{round} \left(\frac{cdf(v) - cdf_{min}}{(M \times N) - cdf_{min}} \times (L - 1) \right)$
60	10	
61	14	
62	15	?
...	...	
154	64	255

Quiz:

How do we compute normalized CDF of pixel 62?

Value	cdf	scaled cdf
52	1	0
55	4	
58	6	
59	9	
60	10	
61	14	
62	15	?
...	...	
154	64	255

$$H(62) = \text{round}((15-1)/63 * 255) \\ = 57$$

Let's look at in more details

The cumulative distribution function (CDF) is shown below

Value	cdf	scaled cdf
52	1	0
55	4	12
58	6	22
59	9	32
60	10	36
61	14	53
62	15	57
...
154	64	255

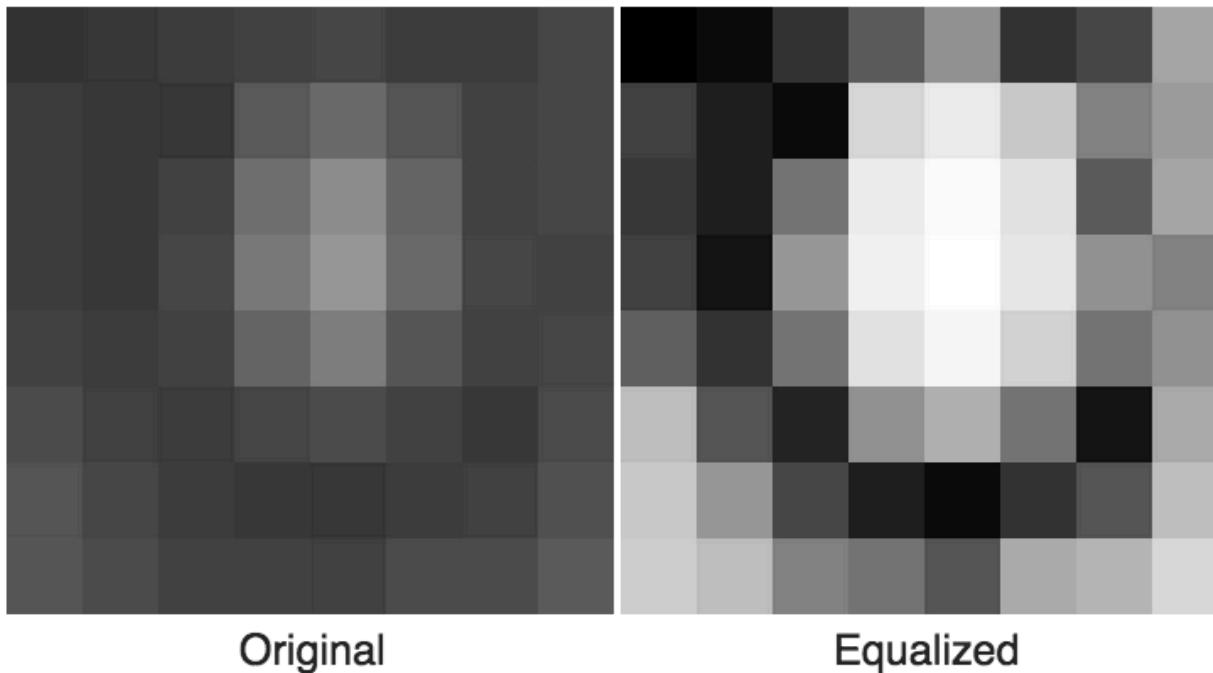
Let's look at in more details

Now we can directly map the scaled cdf back to pixel values using the look up table we derived above

	52	55	61	66				
0	12	53	93	146	53	73	166	
65	32	12	215	235	202	130	158	
57	32	117	239	251	227	93	166	
65	20	154	243	255	231	146	130	
97	53	117	227	247	210	117	146	
190	85	36	146	178	117	20	170	
202	154	73	32	12	53	85	194	
206	190	130	117	85	174	182	219	

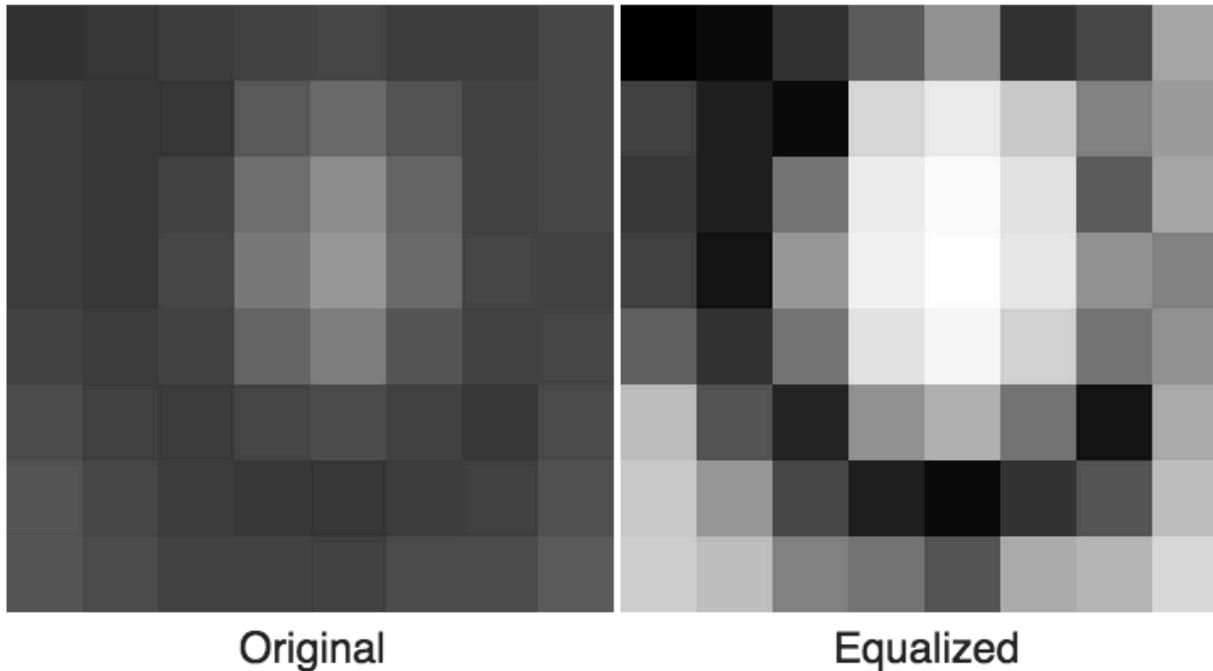
Let's look at in more details

Notice that the minimum value (52) is now 0 and the maximum value (154) is now 255.



Let's look at in more details

Notice that the minimum value (52) is now 0 and the maximum value (154) is now 255.



Homework 2: histogram equalization

Hint:

```
# read in the image  
im = misc.imread('lowcontrast.jpg', flatten=1)
```

#Compute cdf in Python:

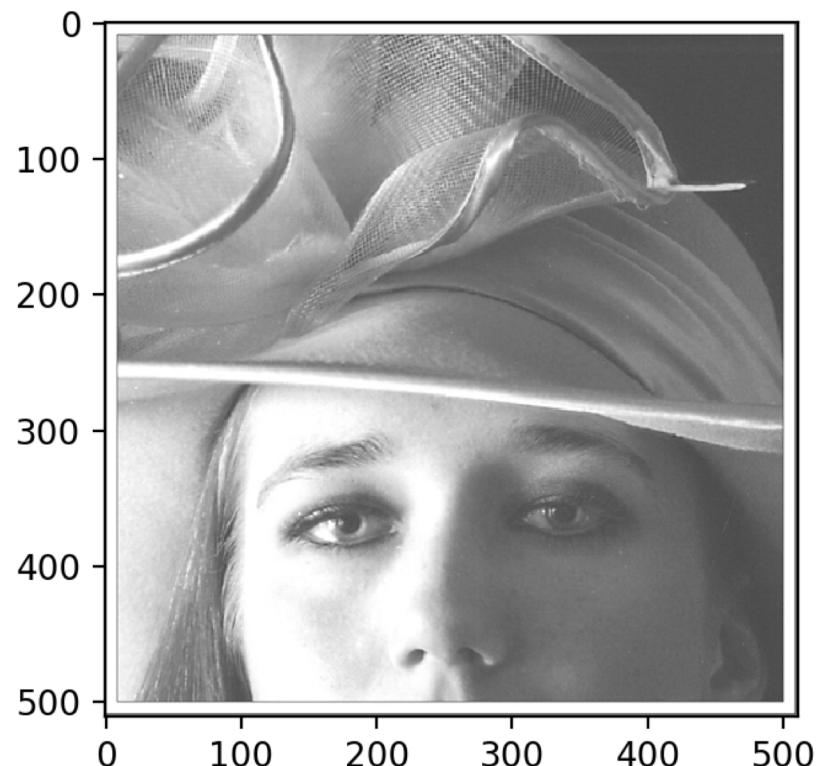
```
np.histogram(im.flatten(), nbr_bins, normed=True)
```

```
cdf = imhist.cumsum()
```

```
# normalize  
cdf = 255 * cdf / cdf[-1]
```

```
# Using linear interpretation of cdf to find new pixels.  
im2 = np.interp(im.flatten(), bins[:-1], cdf)
```

Homework 2: histogram equalization



Back to Convolution

- Let's blur a flat gray image:



How should it look like?

Back to Convolution

- Convolve with a box kernel

1/9	1/9	1/9			
1/9	1/9	1/9	20	20	20
1/9	1/9 20	1/9 20	20	20	20
	20	20	20	20	20
	20	20	20	20	20

Border Handling

Depending on how you do the convolution, you could end up with 3 different images.



266x266 Image

full



256x256 Image

same



246x246 Image

valid

Back to Convolution

- Zero padding:

Filled in borders with zeros, computed everywhere the kernel touches.

`Numpy.convolve(a,v,mode ='full' or 'same')`

This returns the convolution at each point of overlap, with an output shape of $(N+M-1,)$. At the end-points of the convolution, the signals do not overlap completely, and boundary effects may be seen.

Back to Convolution

- Convolve with a box kernel, suppose there are zeros outside the image matrix.

1/9	1/9	1/9			
1/9	1/9	1/9			
1/9	20	20	20	20	
1/9	20	1/9	20	20	
	20	20	20	20	
	20	20	20	20	



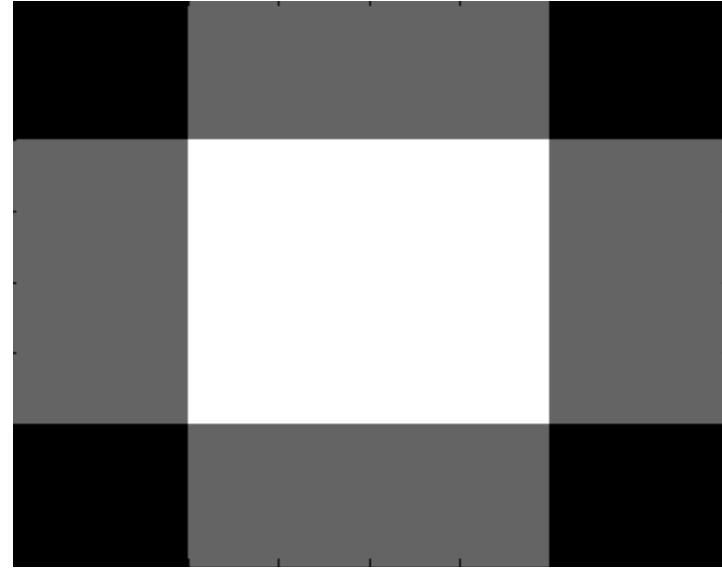
8.9	13.3	13.3	8.9
13.3	20.0	20.0	13.3
13.3	20.0	20.0	13.3
8.9	13.3	13.3	8.9

Back to Convolution

- This is called “same” in MATLAB/Numpy



4×4



4×4

Back to Convolution

1/9	1/9	1/9				
1/9	1/9	1/9				
1/9	1/9	1/9				
		1/9	20	20	20	
			20	20	20	20
			20	20	20	20
			20	20	20	20



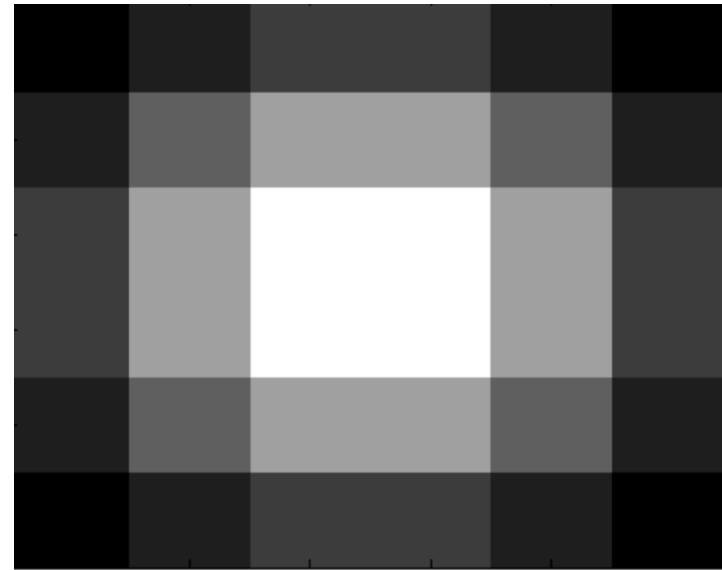
2.2	4.4	6.6	6.6	4.4	2.2
4.4	8.9	13.3	13.3	8.9	4.4
6.6	13.3	20.0	20.0	13.3	6.7
6.6	13.3	20.0	20.0	13.3	6.7
4.4	8.9	13.3	13.3	8.9	4.4
2.2	4.4	6.6	6.6	4.4	2.2

Back to Convolution

- This is called “full” in MATLAB/Numpy.



4×4



6×6

Back to Convolution

- Only compute at places where the kernel fits the image

$\frac{1}{9}$ 20	$\frac{1}{9}$ 20	$\frac{1}{9}$ 20		20
$\frac{1}{9}$ 20	$\frac{1}{9}$ 20	$\frac{1}{9}$ 20		20
$\frac{1}{9}$ 20	$\frac{1}{9}$ 20	$\frac{1}{9}$ 20		20
20	20	20	20	



20	20
20	20

Back to Convolution

- This is called “valid” in MATLAB/Numpy.



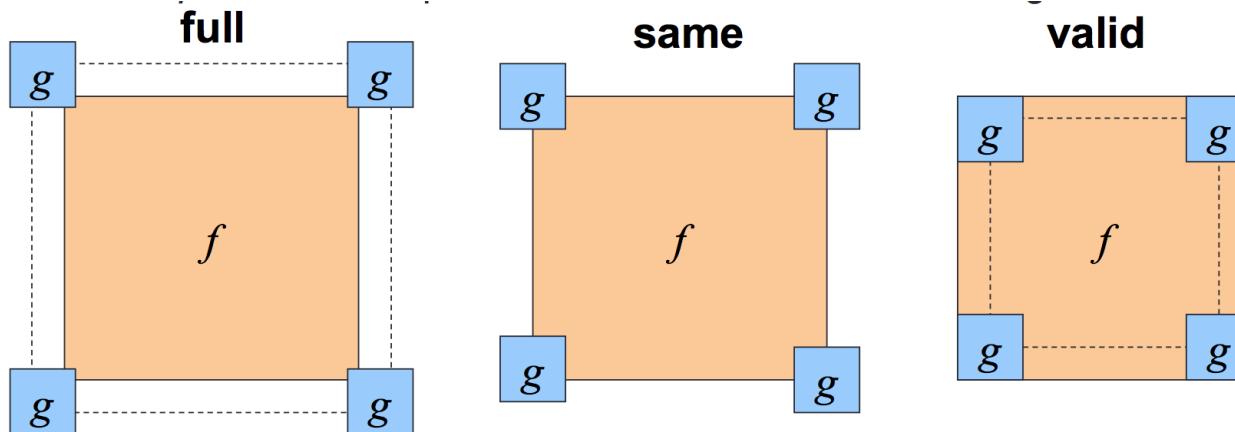
4×4



2×2

To summarize

- Python:
 - [Numpy.convolve\(a,v,mode="full"\)](#)
 - [Scipy.signal.convole2d\(a,v,mode, boundary, value\)](#)



There are other methods

- The first two methods that I described fill missing values in by substituting zero
- Can fill in values with different methods
 - Reflect image along border
 - Pull values from other side
- Read Chapter 3.2 “padding”.
- [scipy.ndimage.convolve\(input, weights, output=None, mode =“reflect”\)](#)

There are other methods

zero



valid

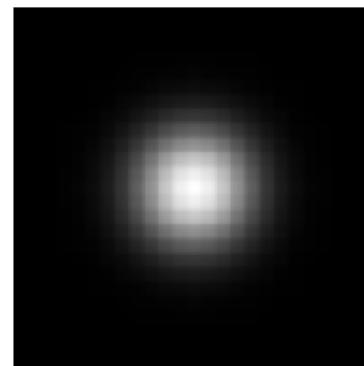
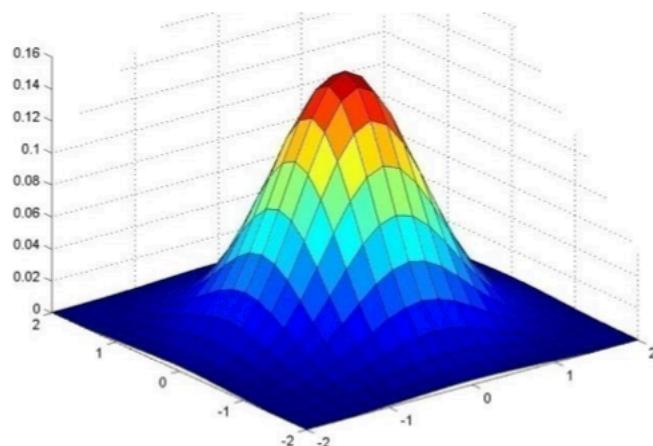


Nearest neighbor



Gaussian Filter

- Gaussian = normal distribution function



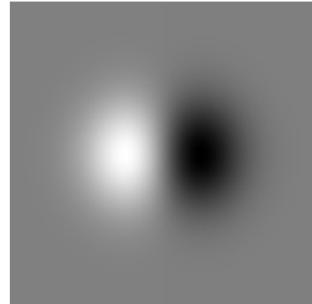
$$K(i, j) = \frac{1}{Z} \exp\left(-\frac{i^2 + j^2}{2\sigma^2}\right)$$

Derivative of Gaussian

- Take the derivative of the filter with respect to i :

$$\frac{\partial K(i, j)}{\partial i} = \frac{-i}{\sigma^2 Z} \exp\left(-\frac{i^2 + j^2}{2\sigma^2}\right)$$

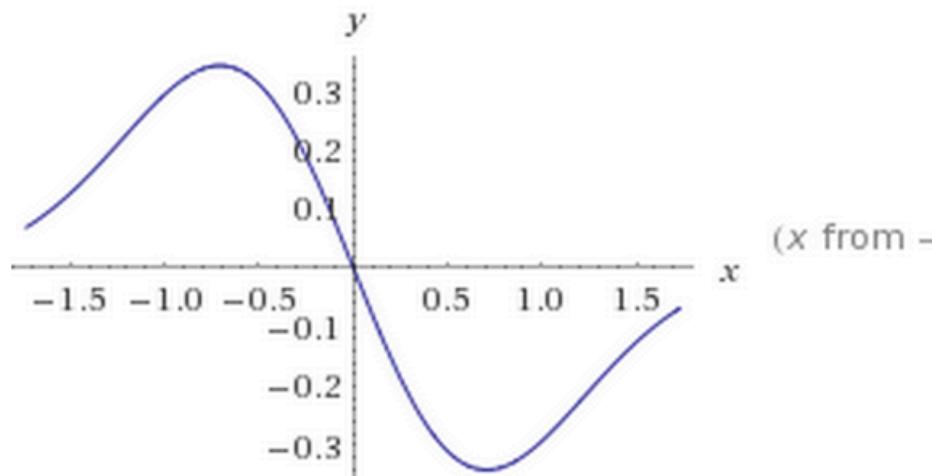
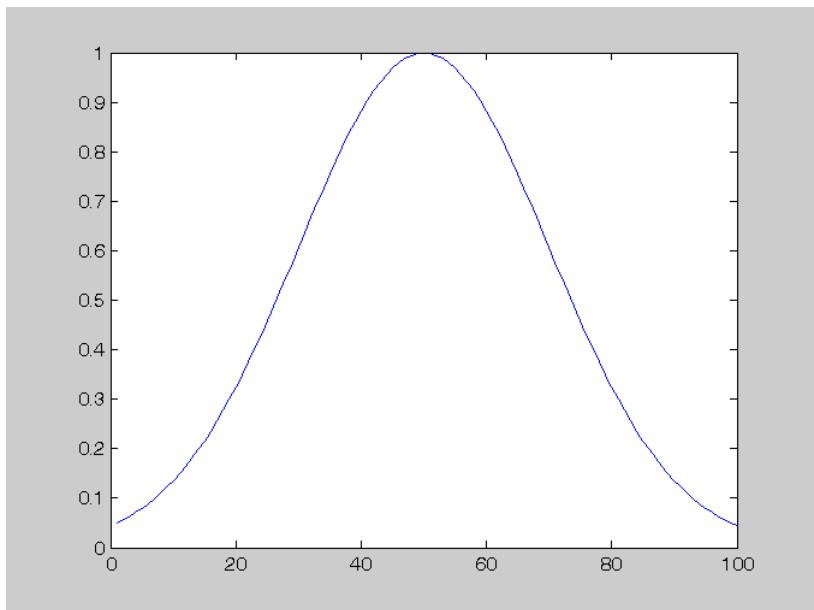
- Filter looks like:



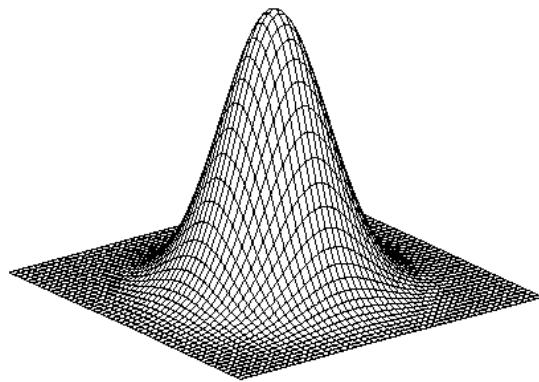
- Basically blur then take the derivative

Source: M. Tappen

Derivative of Gaussian

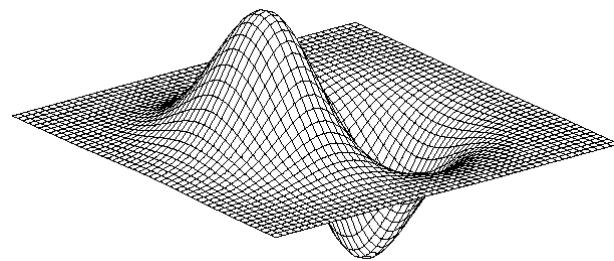


2D edge detection filters



Gaussian

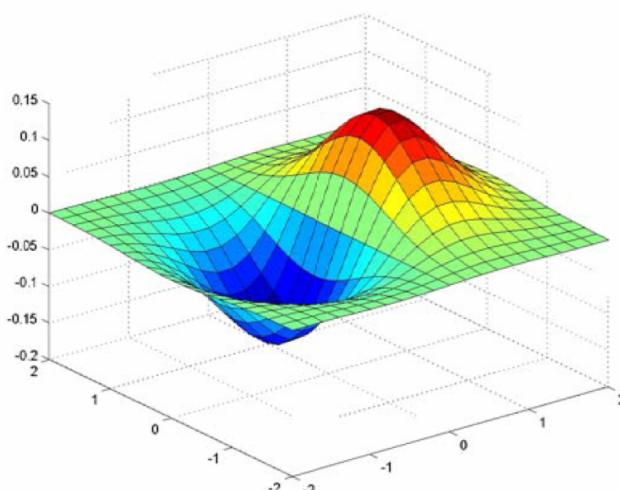
$$h_\sigma(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



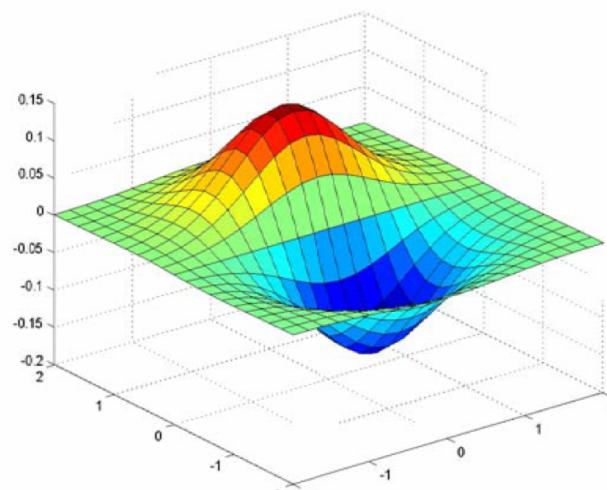
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_\sigma(u, v)$$

Derivative of Gaussian filter



x-direction



y-direction

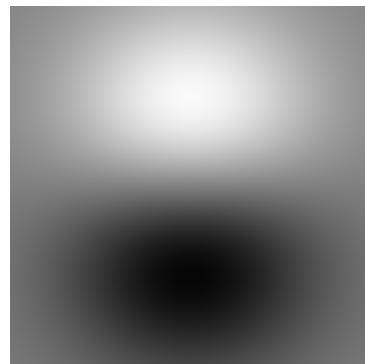
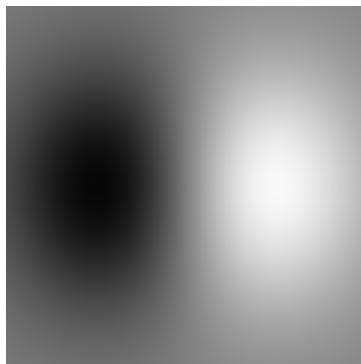
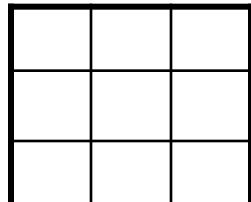


Image derivatives

- How can we differentiate a *digital* image $F[x,y]$?
 - Option 1: reconstruct a continuous image, f , then compute the derivative
 - Option 2: take discrete derivative (finite difference)

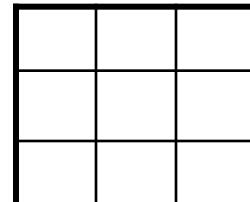
How would you implement this as a linear filter?

$$\frac{\partial f}{\partial x}:$$



H_x

$$\frac{\partial f}{\partial y}:$$

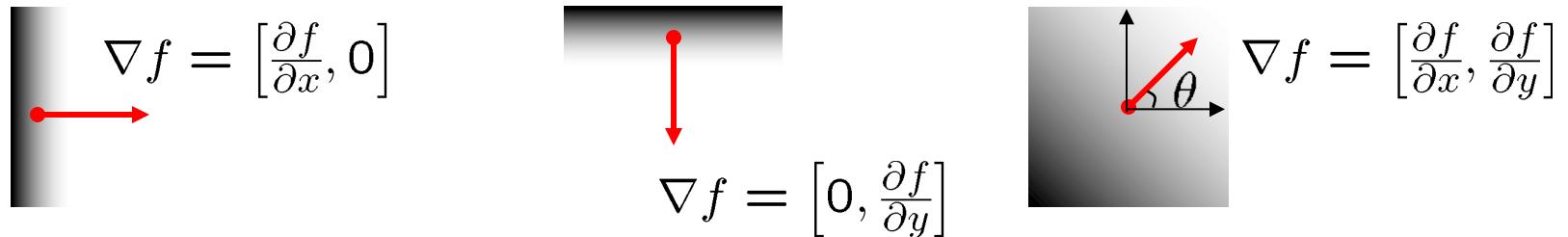


H_y

Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

The gradient points in the direction of most rapid increase in intensity



The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

- how does this relate to the direction of the edge?

The Sobel operator

- Common approximation of derivative of Gaussian

$$\frac{1}{8} \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$$

s_x

$$\frac{1}{8} \begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

s_y

- The standard defn. of the Sobel operator omits the 1/8 term
 - doesn't make a difference for edge detection
 - the 1/8 term **is** needed to get the right gradient value

Example of Sobel filtered image

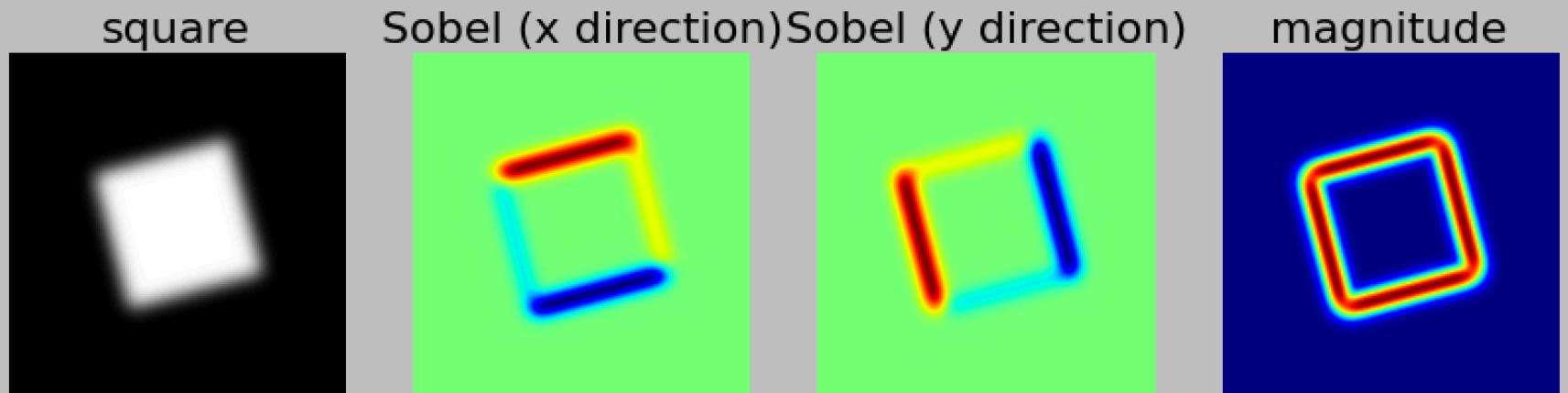
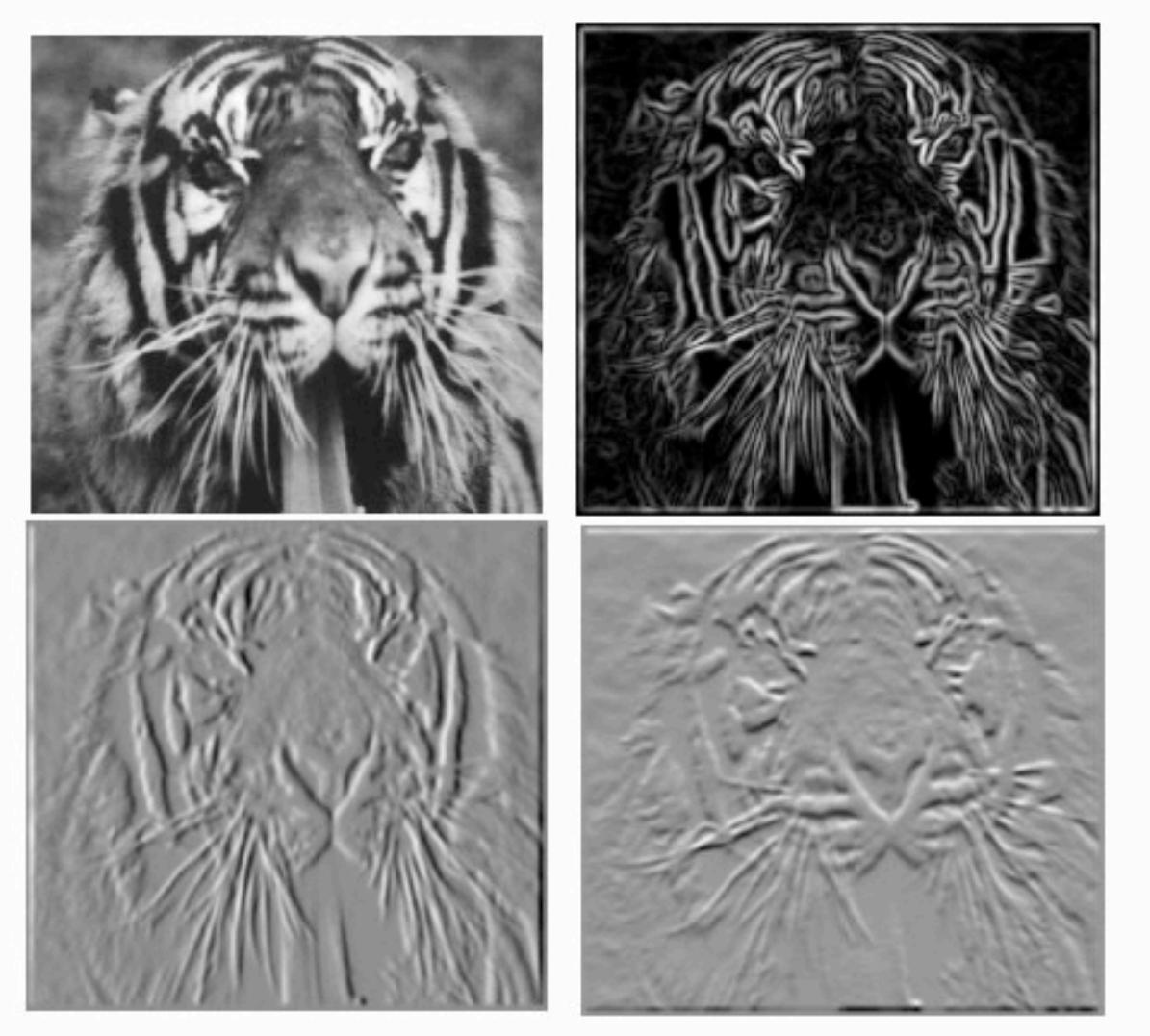
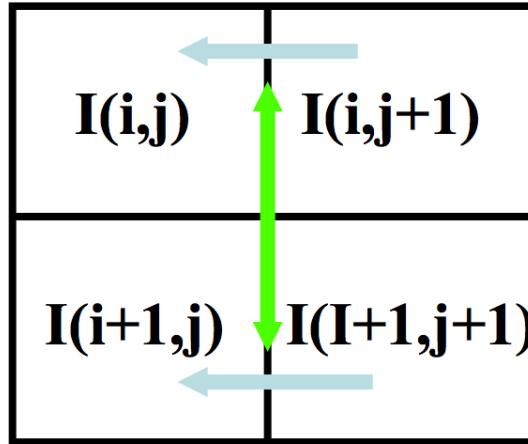
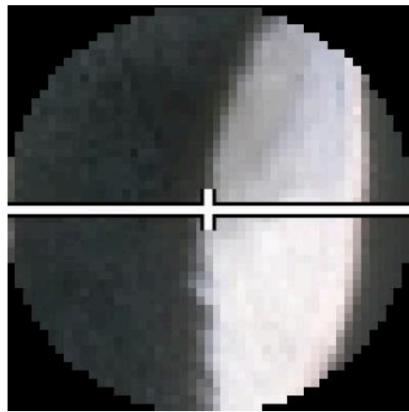


Image gradient



Source: L. Lazebnik

Compute gradient: first order derivatives



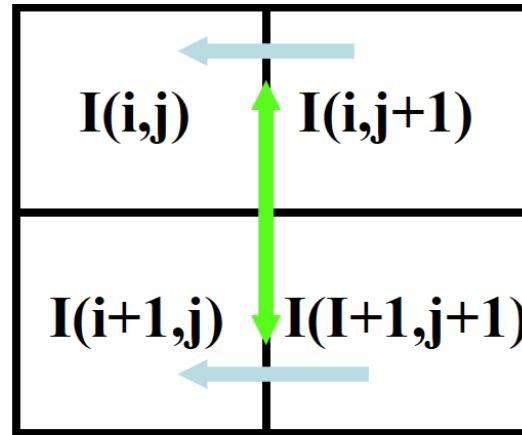
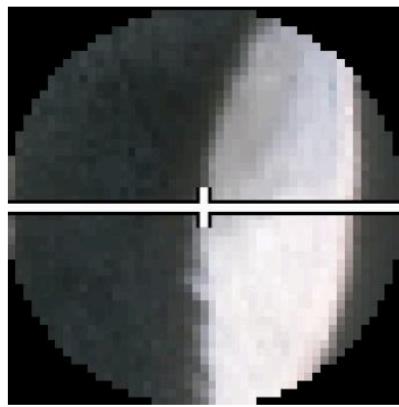
Compute gradient in the X-direction:

- 1) Take the image intensity difference in the X-direction
- 2) Average the difference in the Y-direction(smoothing)

$$\frac{\delta I}{\delta x}(i, j) = \frac{1}{2} \left((I(i, j + 1) - I(i, j)) + (I(i + 1, j + 1) - I(i + 1, j)) \right)$$

Compute gradient: first order derivatives

$$\frac{\delta I}{\delta x}(i, j) = \frac{1}{2}((I(i, j + 1) - I(i, j)) + (I(i + 1, j + 1) - I(i + 1, j)))$$



```
[nr,nc] = size(Is);
```

```
Ix = zeros(nr,nc); % generate an empty matrix of size nr by nc
```

```
for i=1:nr-1,
```

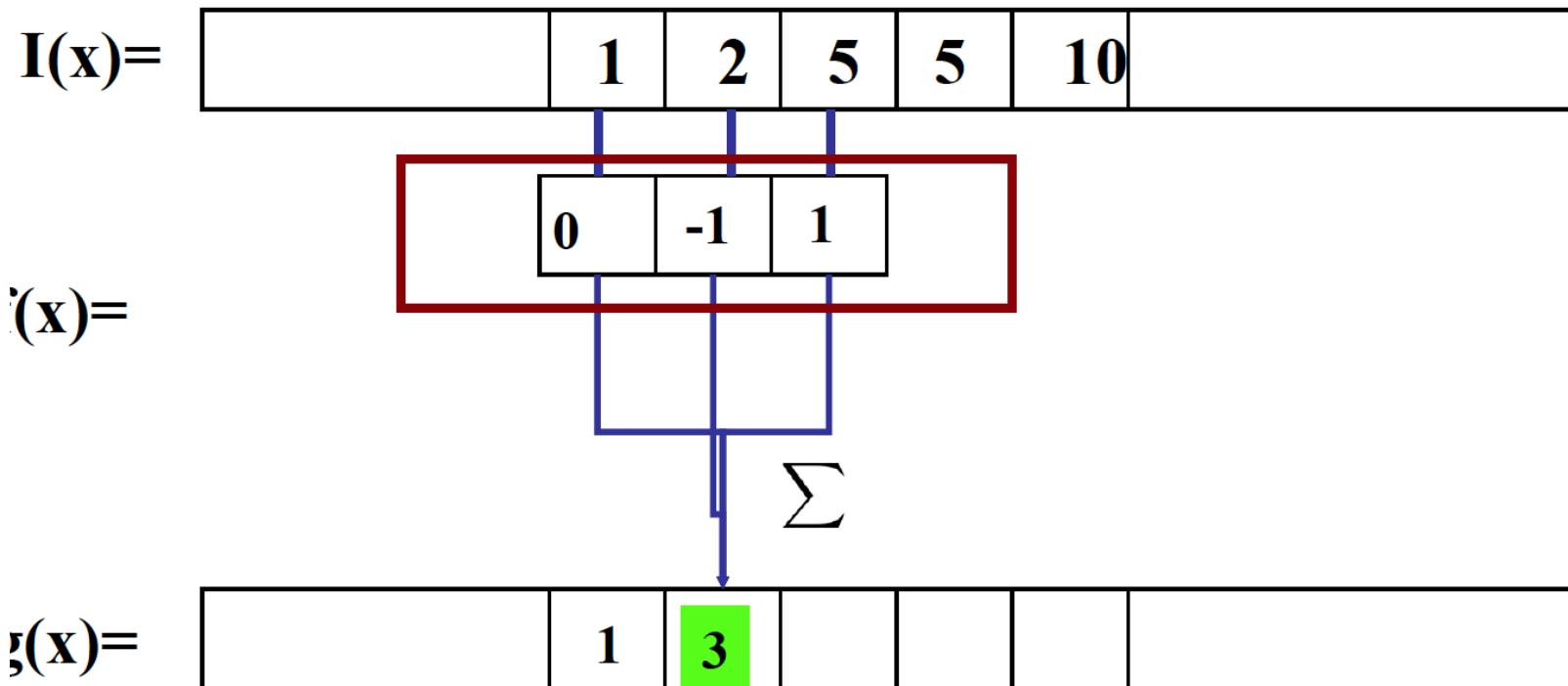
```
    for j=1:nc-1,
```

```
        Ix(i,j) = 0.5*( (Is(i,j+1) - Is(i,j)) + (Is(i+1,j+1) - Is(i+1,j)));
```

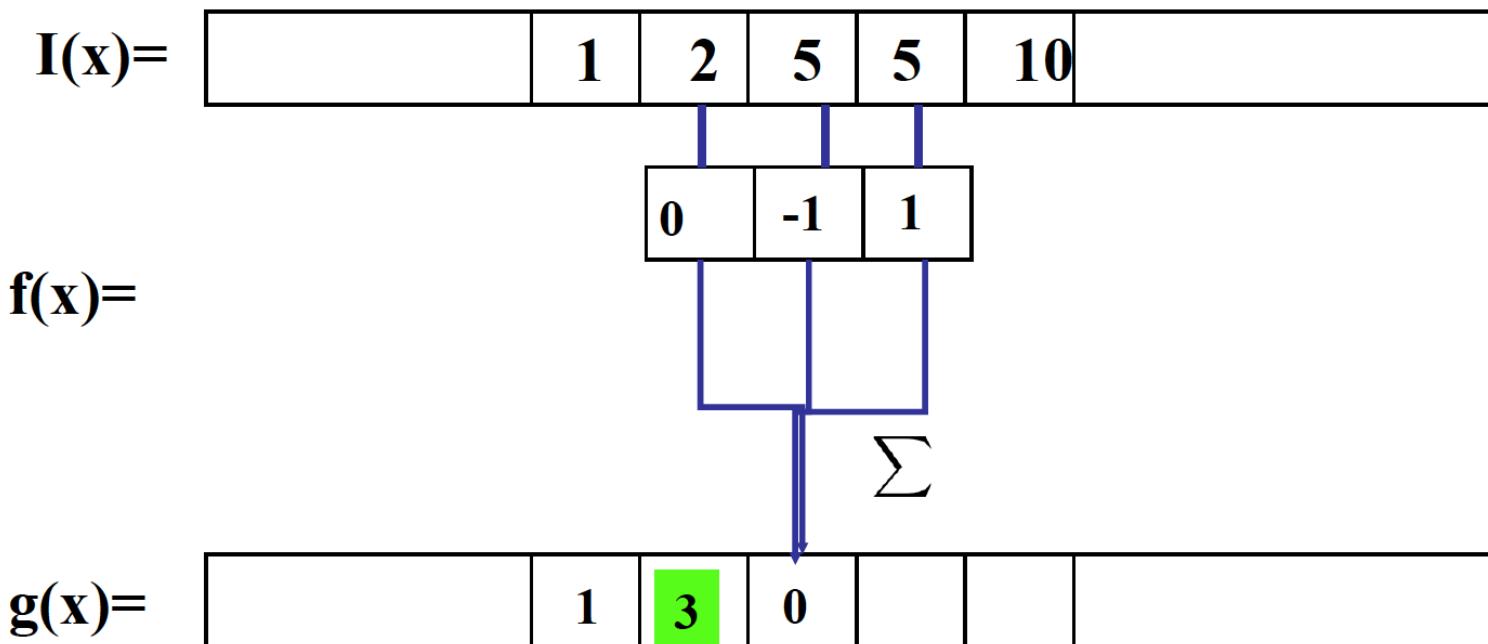
```
    end
```

Slide source: Jianbo Shi

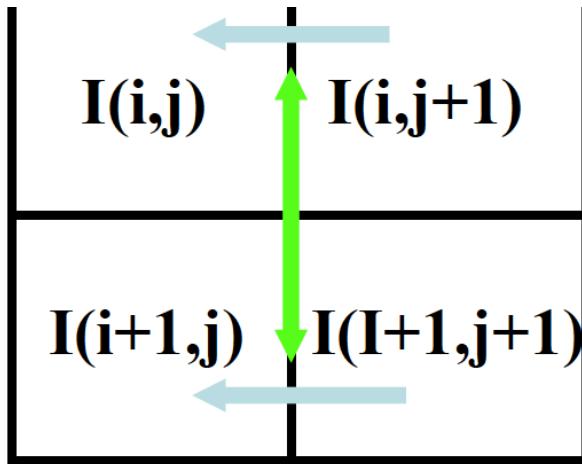
Compute gradient as convolution operation!



Compute gradient as convolution operation!



Compute gradient: first order derivatives

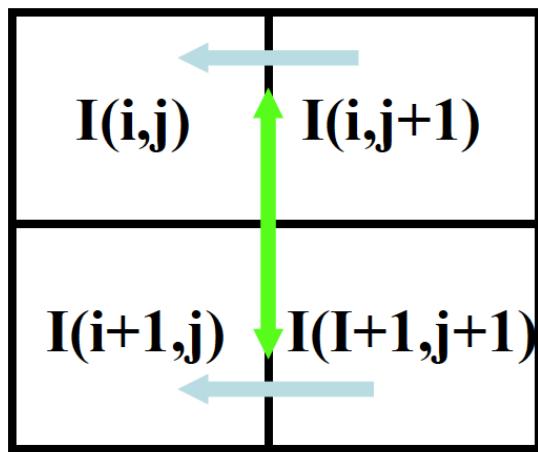


$$\frac{\delta}{\delta x} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\frac{\delta I}{\delta x}(i, j) = (I(i, j + 1) - I(i, j));$$

$$= I \otimes \left(\frac{\delta}{\delta x} \right)$$

Compute gradient: first order derivatives



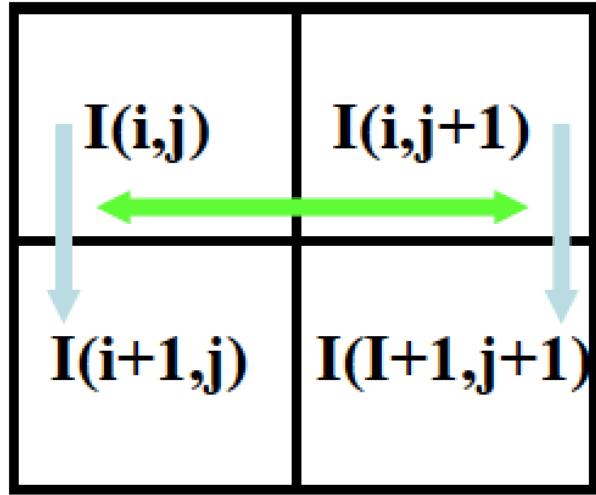
$$\frac{\partial}{\partial x} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{\delta I}{\delta x}(i, j) = \frac{1}{2}((I(i, j + 1) - I(i, j)) + (I(i + 1, j + 1) - I(i + 1, j)))$$

$$= (I \otimes \frac{\delta}{\delta x}) \otimes S$$

Compute gradient: first order derivatives



$$\frac{\delta}{\delta y} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\frac{\delta I}{\delta y}(i, j) = \frac{1}{2}((I(i + 1, j) - I(i, j)) + (I(i + 1, j + 1) - I(i, j + 1)))$$

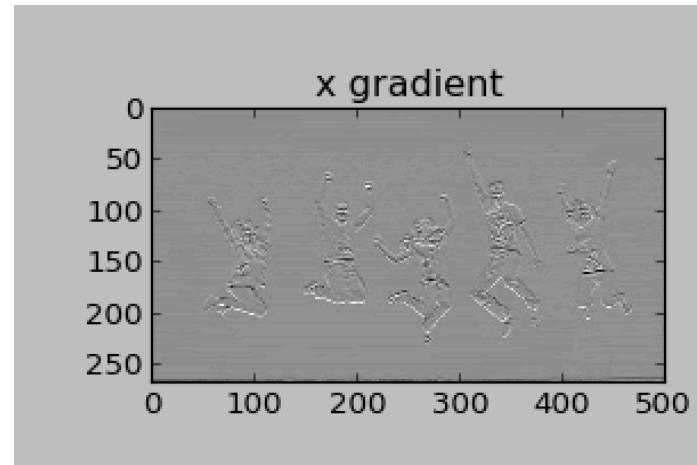
$$I_y = (I \otimes \frac{\delta}{\delta y}) \otimes S'$$

Example



Exercise in Python

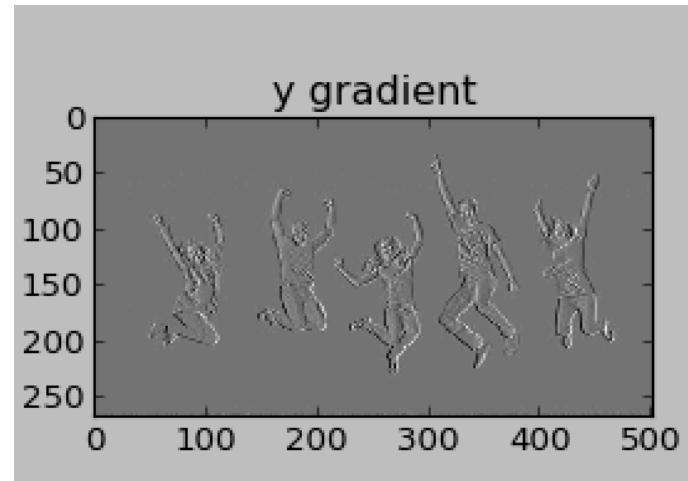
- `s1 = np.array([1,1])`
- `dx = np.array([1,-1])`
- `dy = np.array([1,-1])`
- `x = ndimage.convolve1d(l,dx, axis= 0)`
- `gx_l = ndimage.convolve(x,s)`



$$I_x = \left(I \otimes \frac{\delta}{\delta x} \right) \otimes S$$

Usage in Python

- `s1 = np.array([1,1])`
- `dx = np.array([1,-1])`
- `dy = np.array([1,-1])`
- `y = ndimage.convolve1d(l,dx, axis= 1)`
- `gy_l = ndimage.convolve(y,s)`



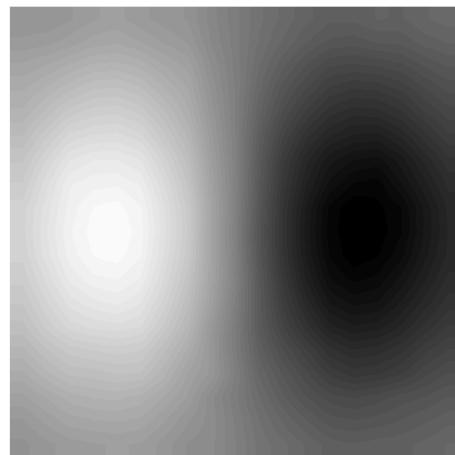
Or : `gx_l,gy_l = np.gradient(l)[:2]`

$$I_y = \left(I \otimes \frac{\delta}{\delta y} \right) \otimes S'$$

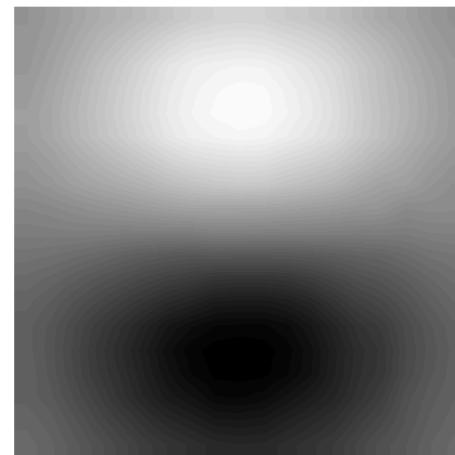
Smoothed derivative filter

$$\frac{\delta}{\delta x} \otimes G = \frac{\delta G}{\delta x} \longrightarrow \frac{\delta G}{\delta x} = -\frac{2x}{\sigma_x^2} G(x, y)$$

G_x



G_y



Sobel Filter

- Product of averaging and gradient.
- An cross product of two 1 d filter, Gaussian and gradient

$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & +1 \end{bmatrix}$$

Exercise: Create image derivatives using Sobel filter

1. Generate an image of a rotated rectangle

```
import numpy as np  
from scipy import ndimage  
import matplotlib.pyplot as plt
```

```
im = np.zeros((256, 256))  
im[64:-64, 64:-64] = 1  
im = ndimage.rotate(im, 15, mode='constant')
```

2. Blur the image using a Gaussian filter

```
im = ndimage.gaussian_filter(im, 8)
```

3. Apply Sobel filter to both x and y direction.

```
sx = ndimage.sobel(im, axis=0, mode='constant')
```

4. Display the original image, x-derivatives, y-derivatives, and the gradient magnitude. You can use `np.hypot` to compute magnitude.

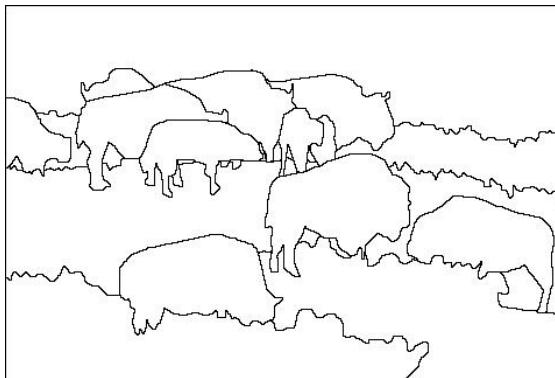
See here: <http://docs.scipy.org/doc/scipy/reference/generated/scipy.ndimage.sobel.html>)

Where do humans see boundaries?

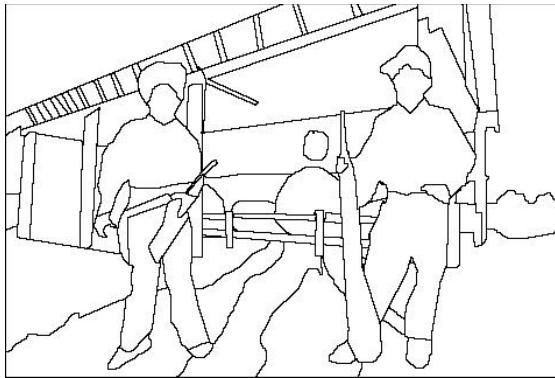
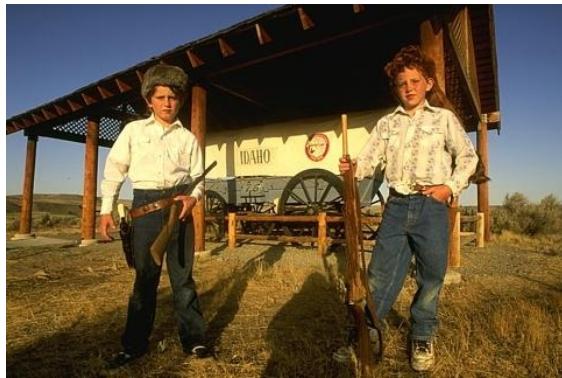
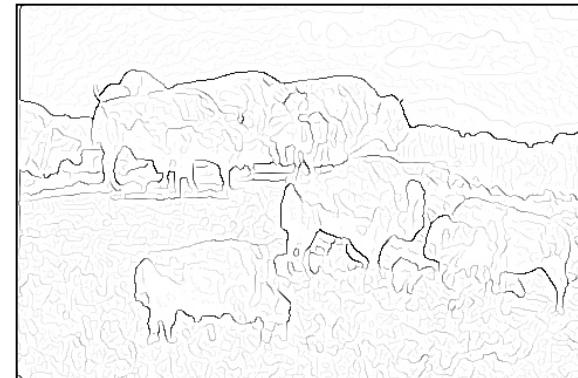
image



human segmentation



gradient magnitude



- Berkeley segmentation database:

<http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/>

Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
 - Intuitively, most semantic and shape information from the image can be encoded in the edges
 - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

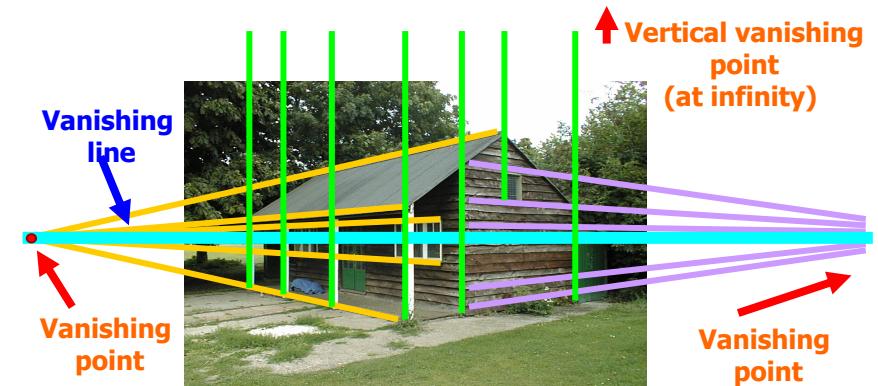


Why do we care about edges?

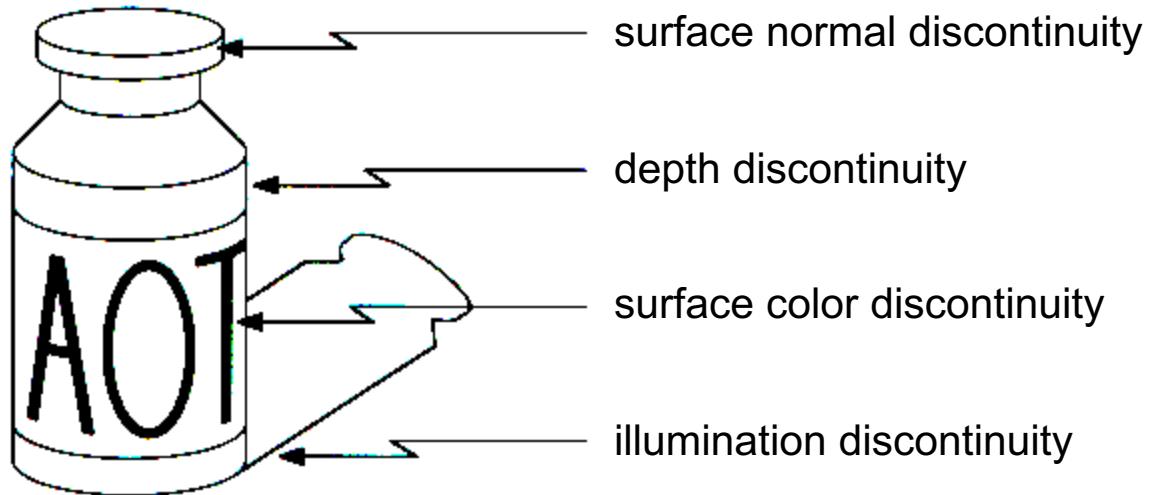
- Extract information,
recognize objects



- Recover geometry and
viewpoint



Origin of Edges



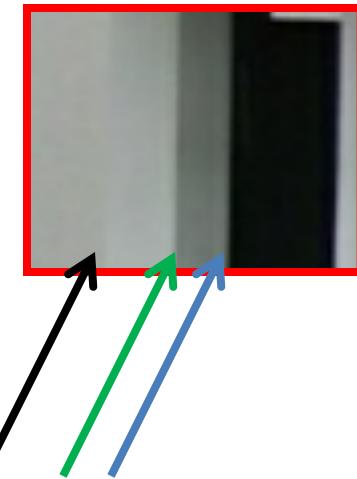
- Edges are caused by a variety of factors

Closeup of edges



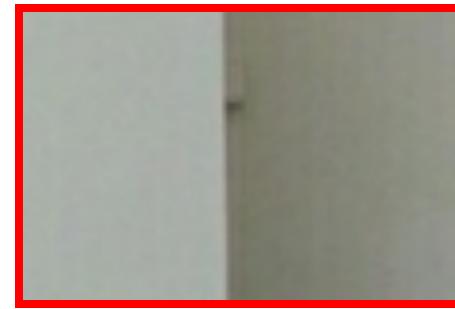
Source: D. Hoiem

Closeup of edges



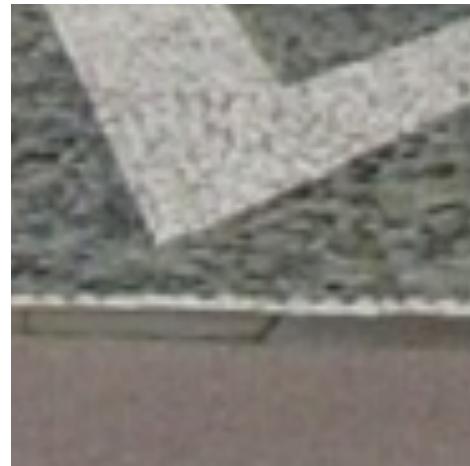
Source: D. Hoiem

Closeup of edges



Source: D. Hoiem

Closeup of edges



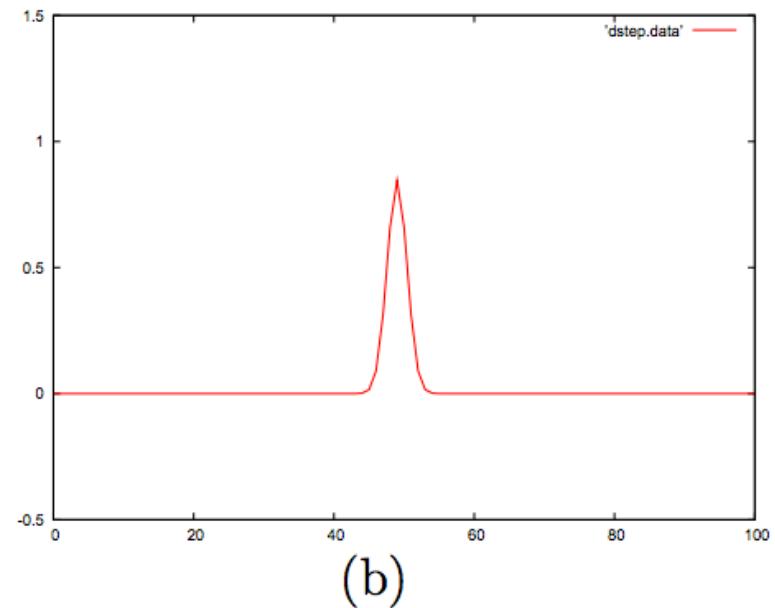
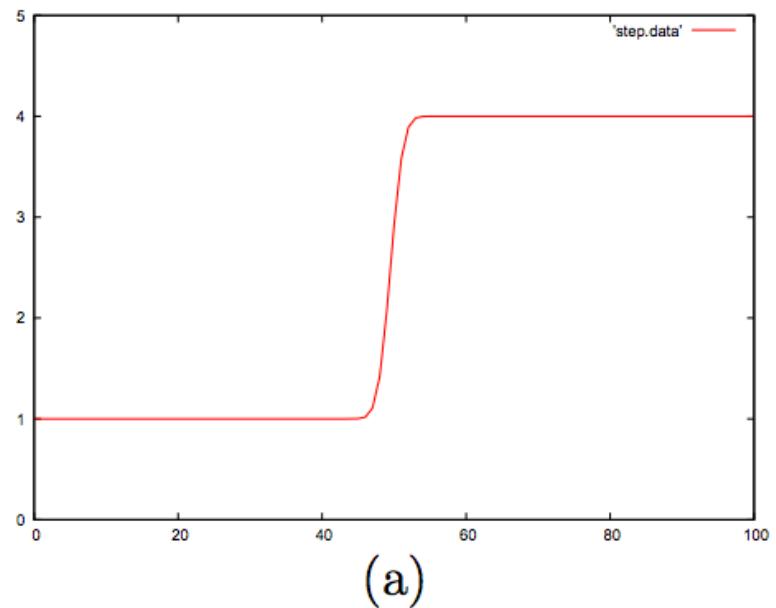
Source: D. Hoiem

Edge is Where Change Occurs

- Change is measured by derivative in 1D
- Biggest change, derivative has maximum magnitude
- Or 2nd derivative is zero.



Edge is Where Change Occurs



Characterizing edges

- An edge is a place of rapid change in the image intensity function

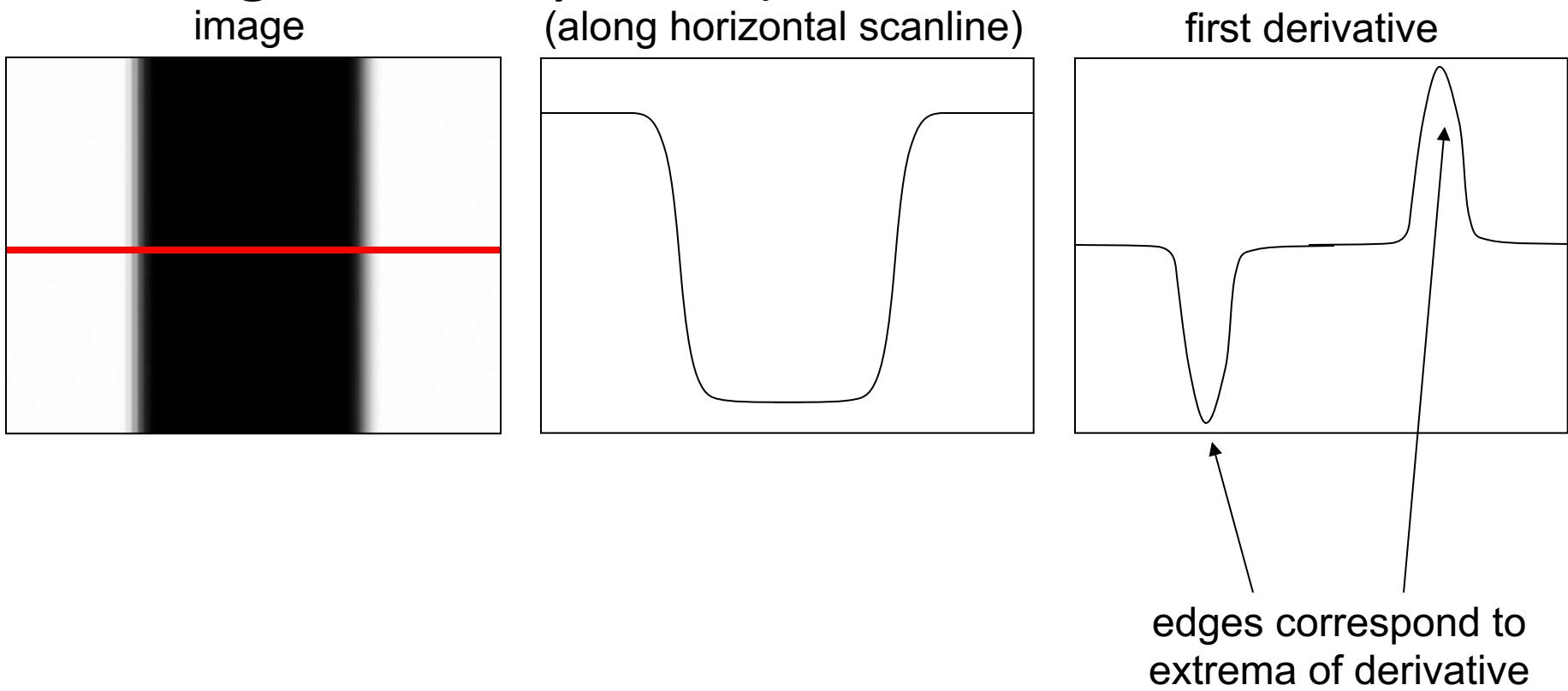
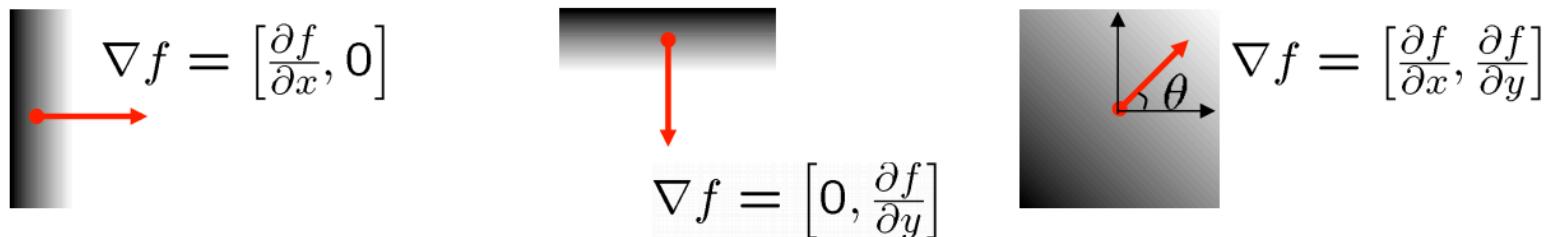


Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

The gradient points in the direction of most rapid increase in intensity



The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

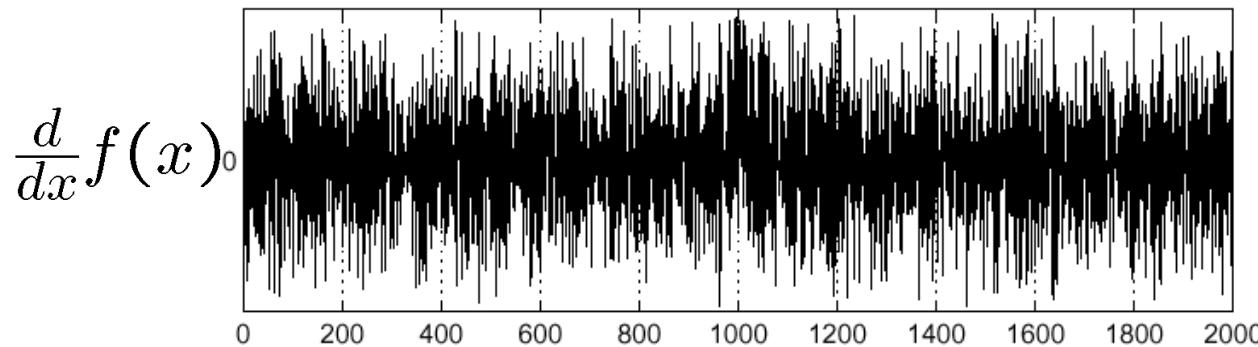
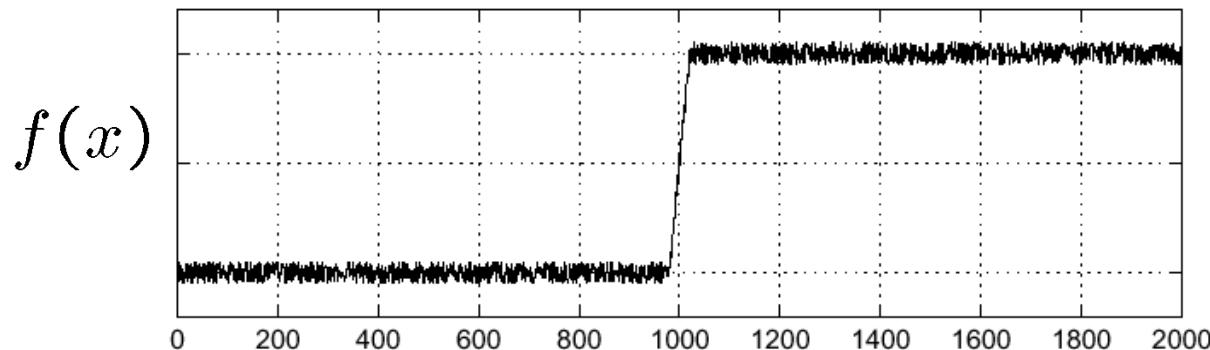
The gradient direction is given by:

$$\theta = \tan^{-1} \left(\frac{\partial f / \partial y}{\partial f / \partial x} \right)$$

- how does this relate to the direction of the edge?

Effects of noise

- Consider a single row or column of the image
 - Plotting intensity as a function of position gives a signal



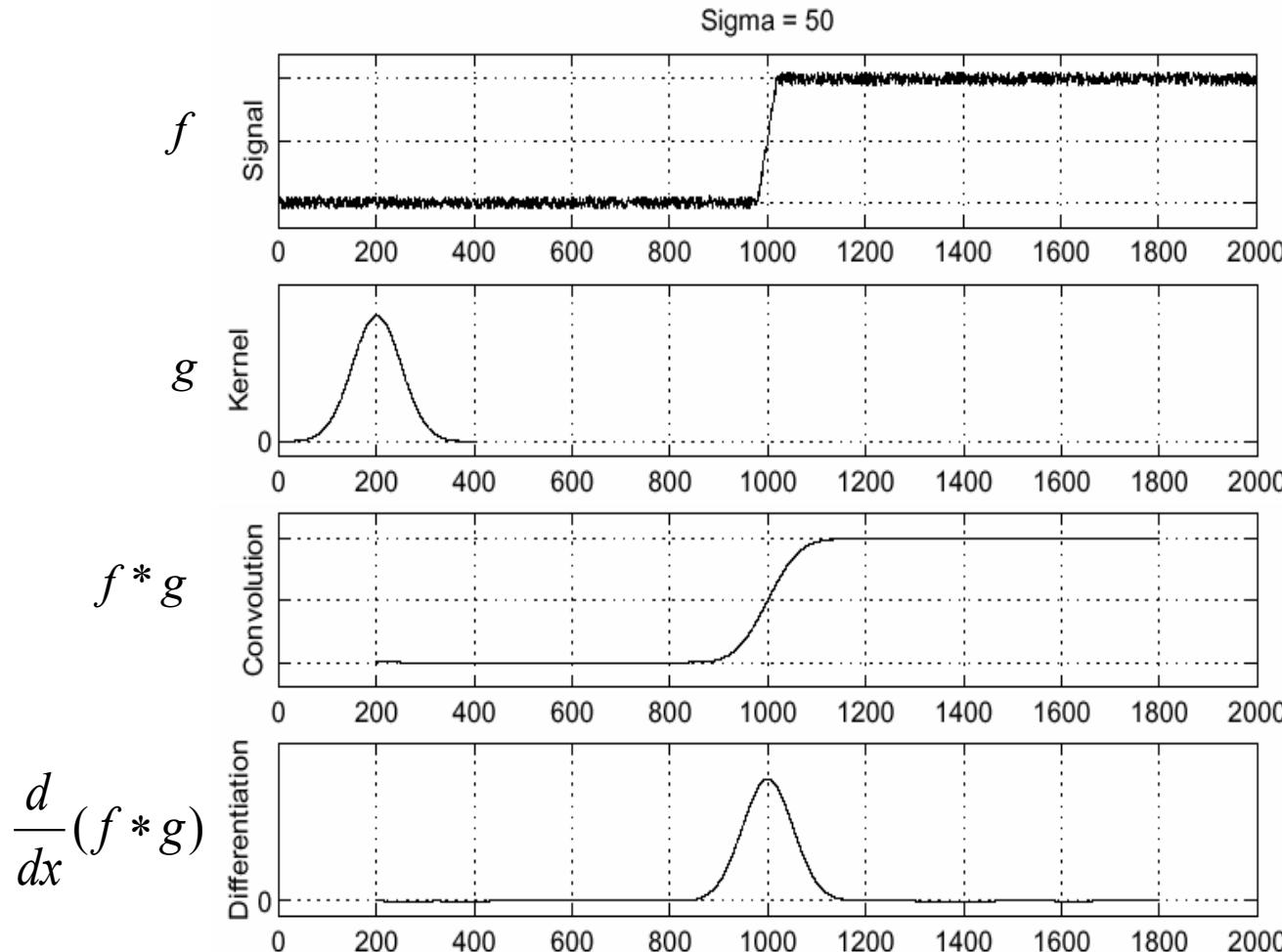
Where is the edge?

Source: S. Seitz

Effects of noise

- Difference filters respond strongly to noise
 - Image noise results in pixels that look very different from their neighbors
 - Generally, the larger the noise the stronger the response
- What can we do about it?

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx}g$$

- This saves us time

$$f$$
$$\frac{d}{dx}g$$
$$f * \frac{d}{dx}g$$

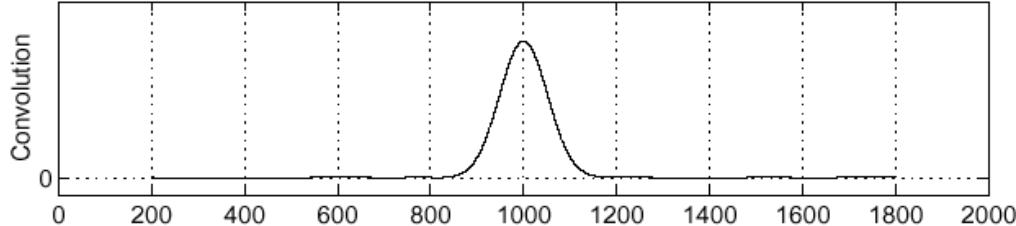
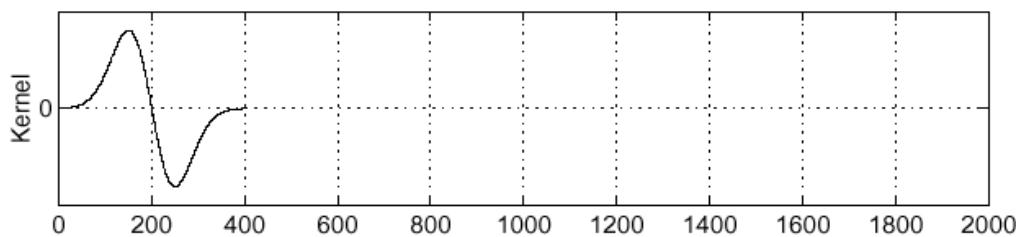
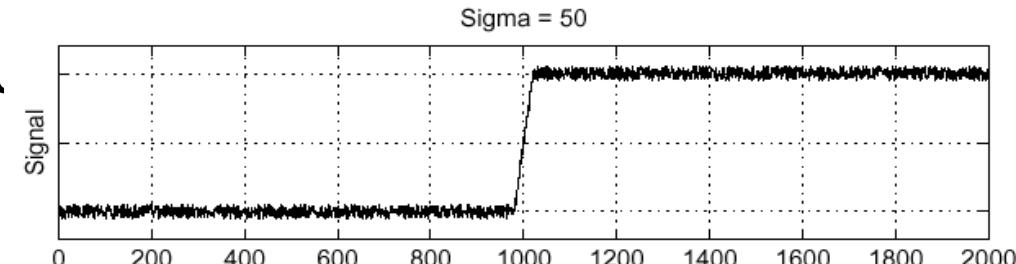
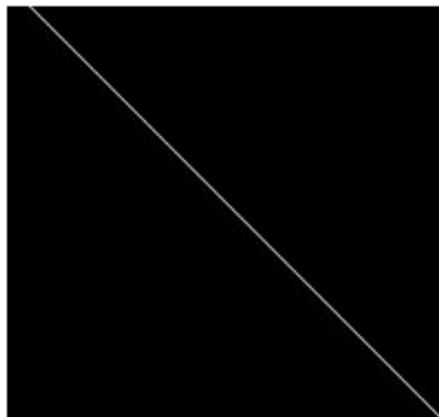




Image with Edge



Edge Location

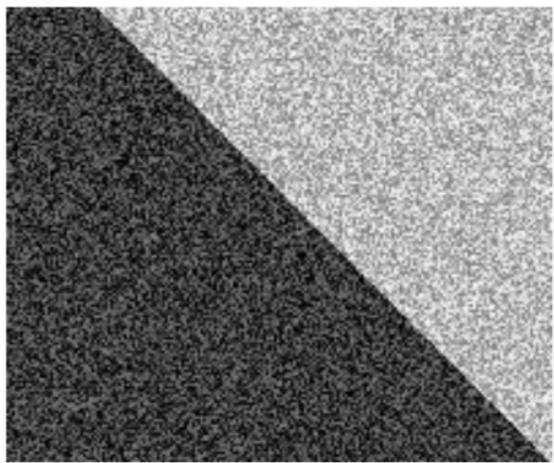
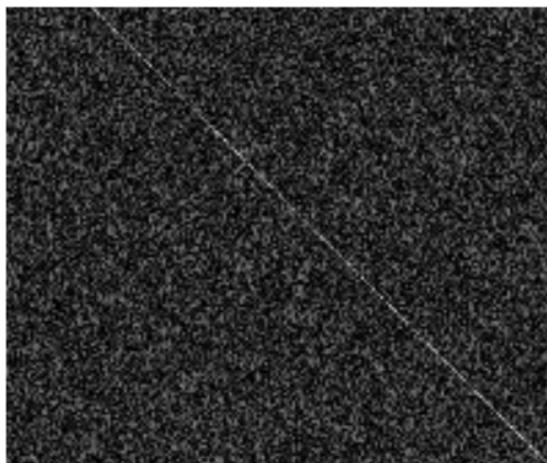


Image + Noise

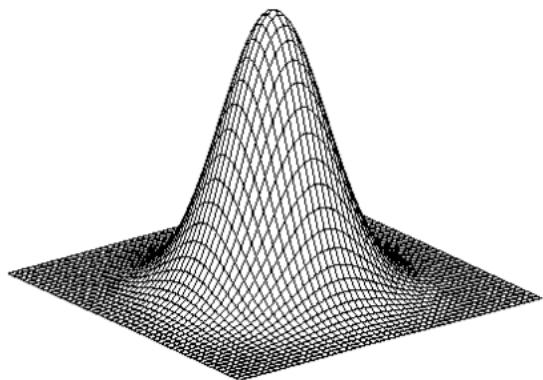


Derivatives detect
edge *and* noise

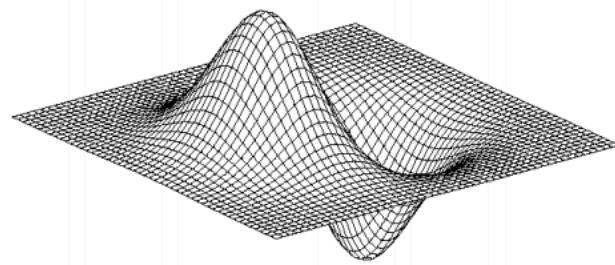


Smoothed derivative removes
noise, but blurs edge

2D edge detection filters



* [1 -1] =



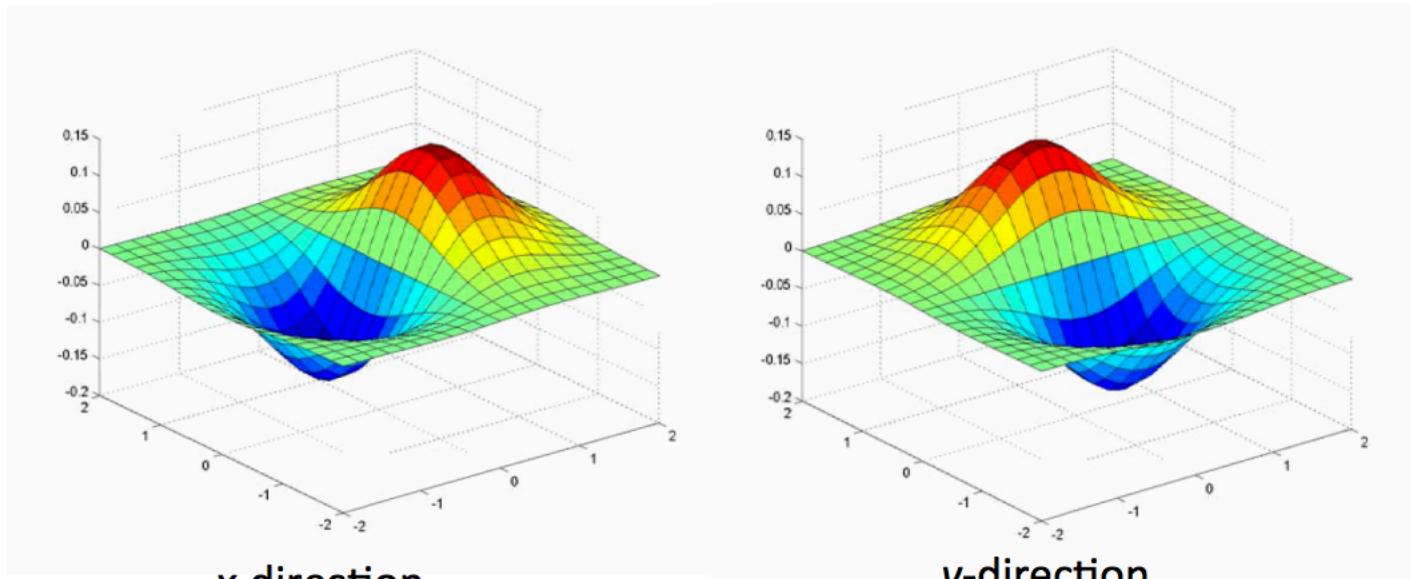
Gaussian

$$h_\sigma(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

derivative of Gaussian (x)

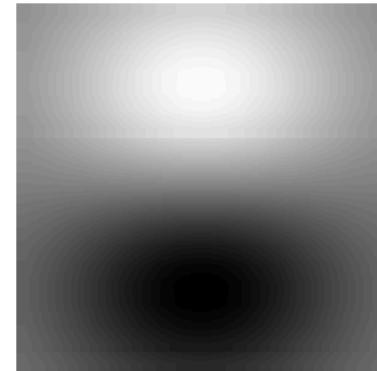
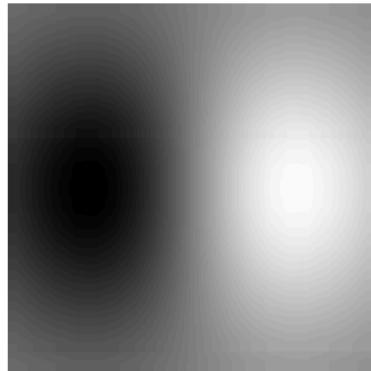
$$\frac{\partial}{\partial x} h_\sigma(u, v)$$

Derivative of Gaussian filter



x-direction

y-direction



The Sobel operator

- Common approximation of derivative of Gaussian
 - A mask (not a convolution kernel)

$$\frac{1}{8} \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$$

s_x

$$\frac{1}{8} \begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

s_y

- The standard defn. of the Sobel operator omits the 1/8 term
 - doesn't make a difference for edge detection
 - the 1/8 term **is** needed to get the right gradient magnitude

What kind of filters are those?

Sobel filter:

$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A} \quad \text{and} \quad \mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * \mathbf{A}$$

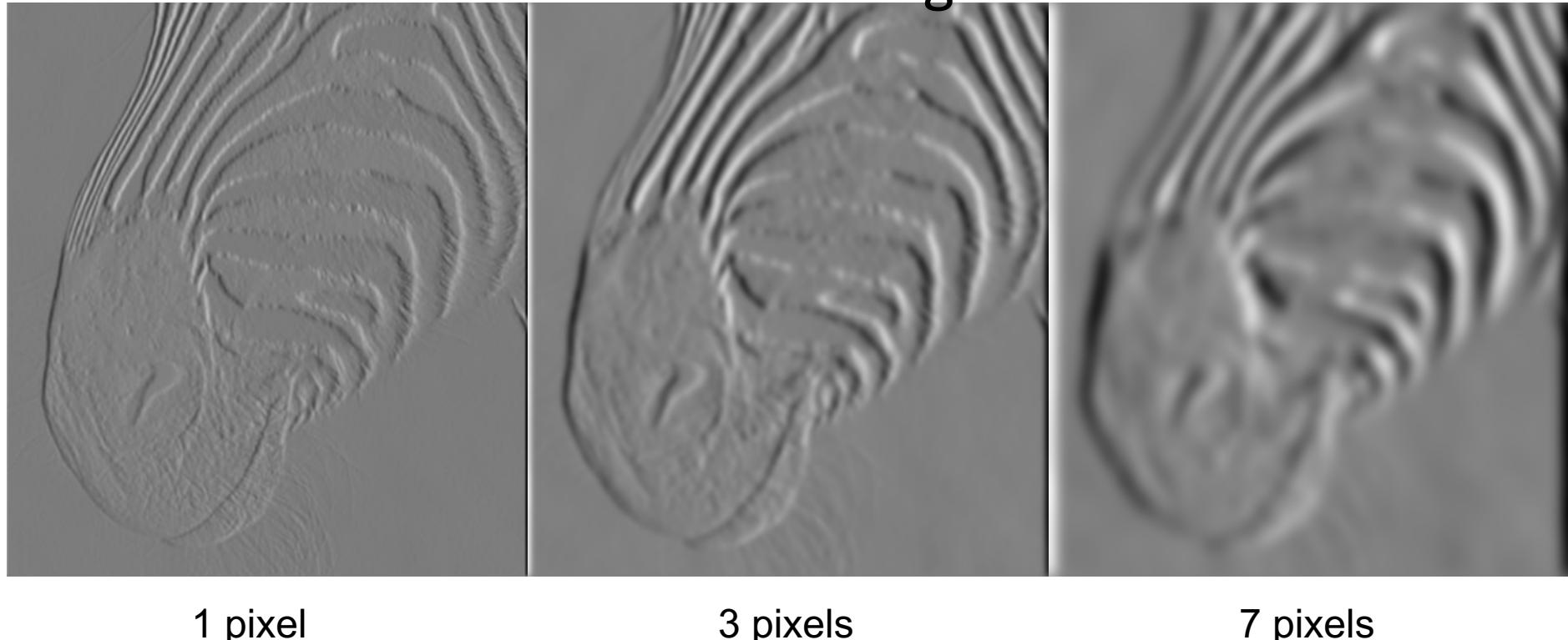
Magnitude:

$$\mathbf{G} = \sqrt{\mathbf{G}_x^2 + \mathbf{G}_y^2}$$

Direction:

$$\Theta = \text{atan2}(\mathbf{G}_y, \mathbf{G}_x)$$

Tradeoff between smoothing and localization



1 pixel

3 pixels

7 pixels

FIGURE 5.3: The scale (i.e., σ) of the Gaussian used in a derivative of Gaussian filter has significant effects on the results. The three images show estimates of the derivative in the x direction of an image of the head of a zebra obtained using a derivative of Gaussian filter with σ one pixel, three pixels, and seven pixels (**left to right**). Note how images at a finer scale show some hair, the animal's whiskers disappear at a medium scale, and the fine stripes at the top of the muzzle disappear at the coarser scale.

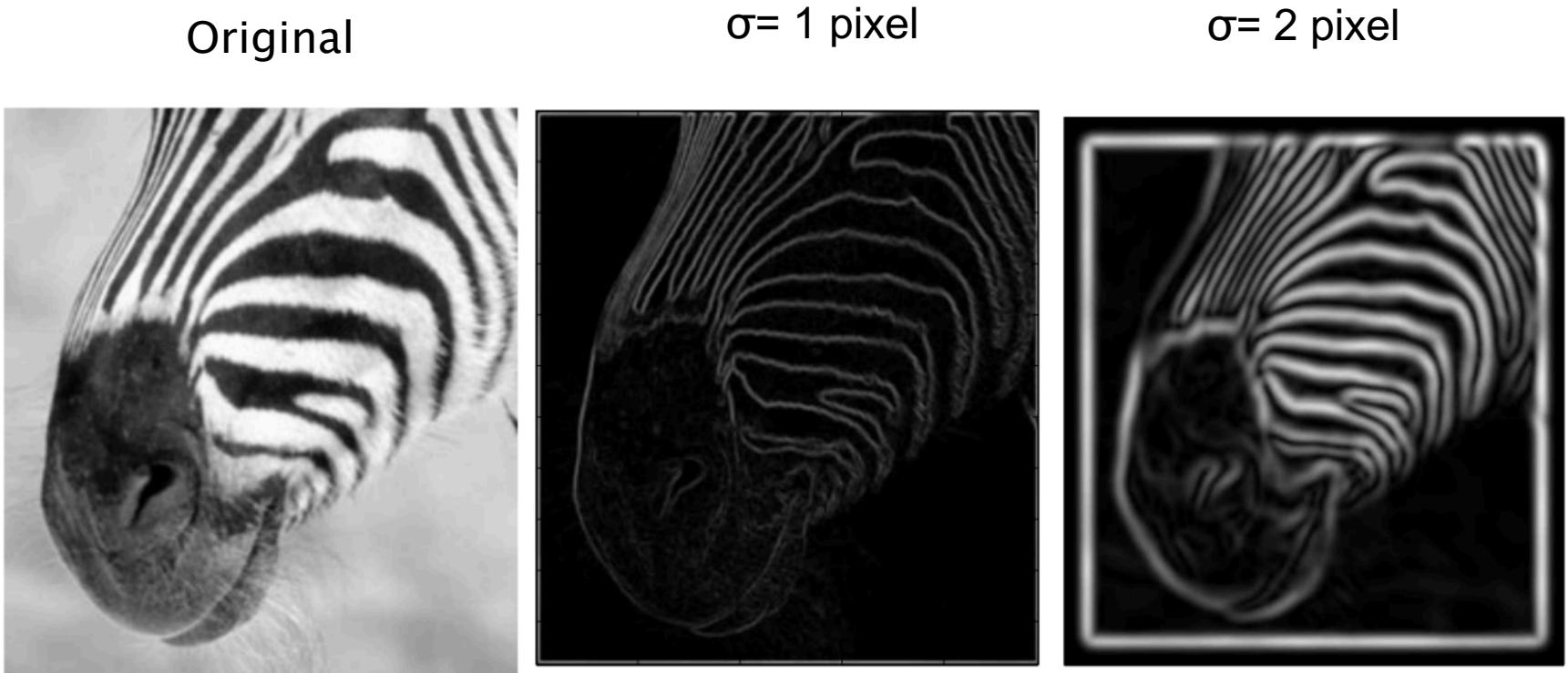


FIGURE 5.4: The gradient magnitude can be estimated by smoothing an image and then differentiating it. This is equivalent to convolving with the derivative of a smoothing kernel. The extent of the smoothing affects the gradient magnitude; in this figure, we show the gradient magnitude for the figure of a zebra at different scales. At the **center**, gradient magnitude estimated using the derivatives of a Gaussian with $\sigma = 1$ pixel; and on the **right**, gradient magnitude estimated using the derivatives of a Gaussian with $\sigma = 2$ pixel. Notice that large values of the gradient magnitude form thick trails.

Implementation issues



- The gradient magnitude is large along a thick “trail” or “ridge,” so how do we identify the actual edge points?
- How do we link the edge points to form curves?

Designing an edge detector

- Criteria for a good edge detector:
 - **Good detection:** the optimal detector should find all real edges, ignoring noise or other artifacts
 - **Good localization**
 - the edges detected must be as close as possible to the true edges
 - the detector must return one point only for each true edge point
- Cues of edge detection
 - Differences in color, intensity, or texture across the boundary
 - Continuity and closure
 - High-level knowledge

Canny edge detector

- This is probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization

J. Canny, [*A Computational Approach To Edge Detection*](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Canny edge detector

- Minimizes the probability of multiply detecting an edge
- Minimizes the probability of failing to detect an edge
- Minimizes the distance of the reported edge to the true edge.

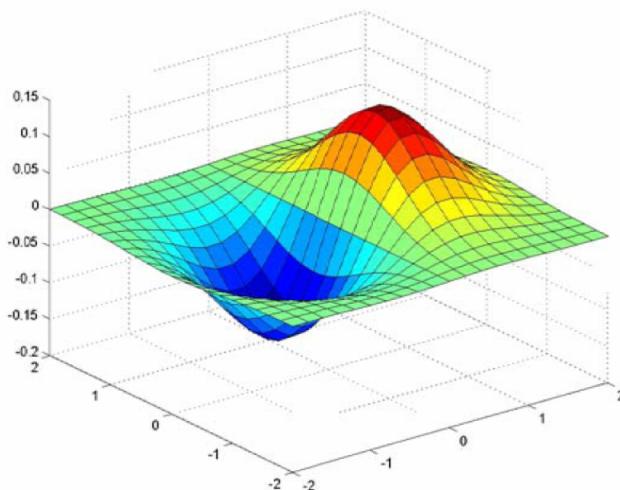
J. Canny, [*A Computational Approach To Edge Detection*](#), IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Example

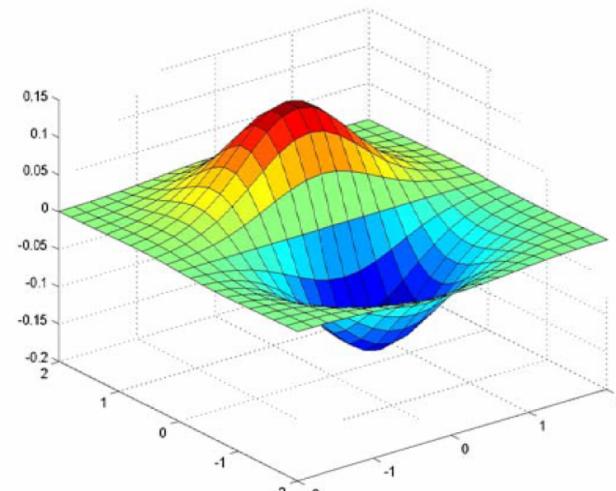


original image (Lena)

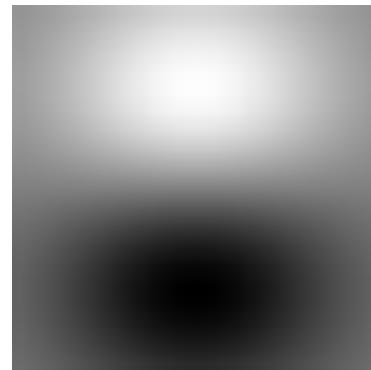
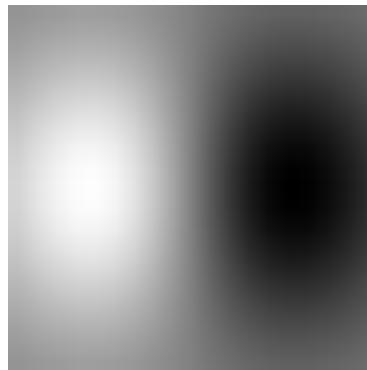
Derivative of Gaussian filter



x-direction



y-direction



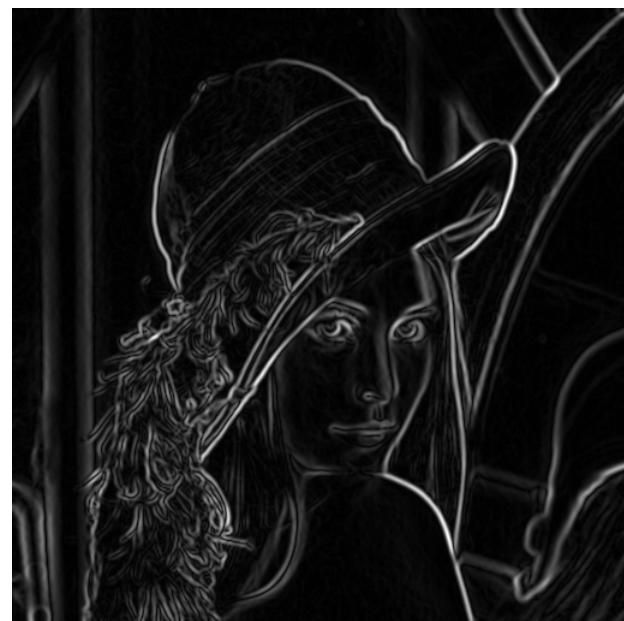
Compute Gradients (DoG)



X-Derivative of Gaussian



Y-Derivative of Gaussian



Gradient Magnitude

The Canny edge detector



- original image (Lena)

The Canny edge detector



norm of the gradient

The Canny edge detector



thresholding

The Canny edge detector

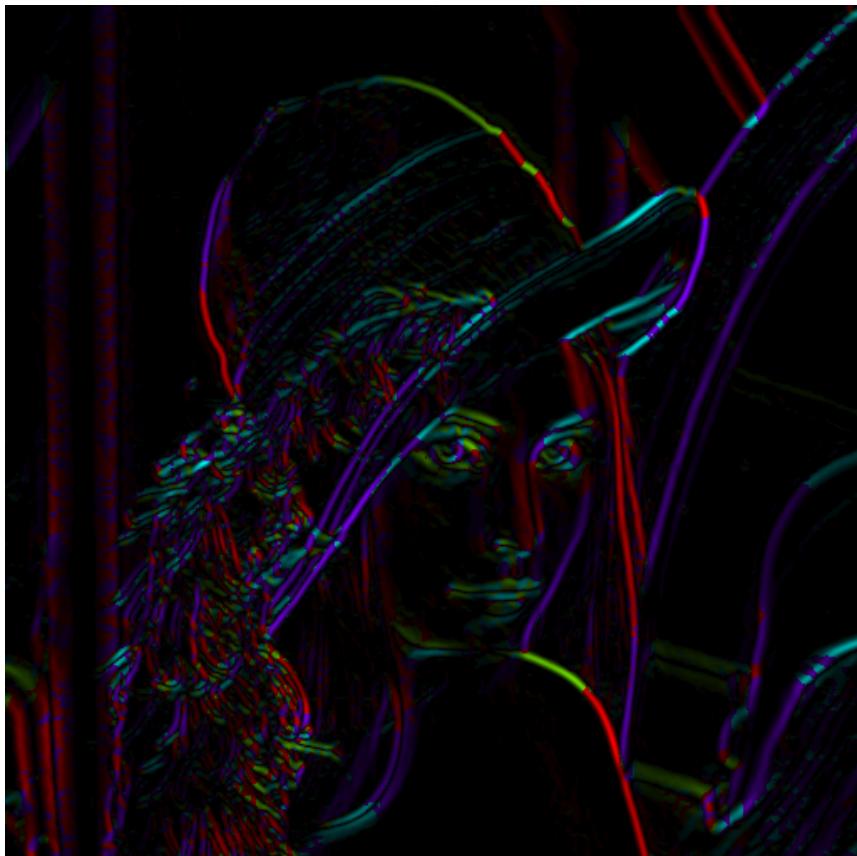


thinning
(non-maximum suppression)

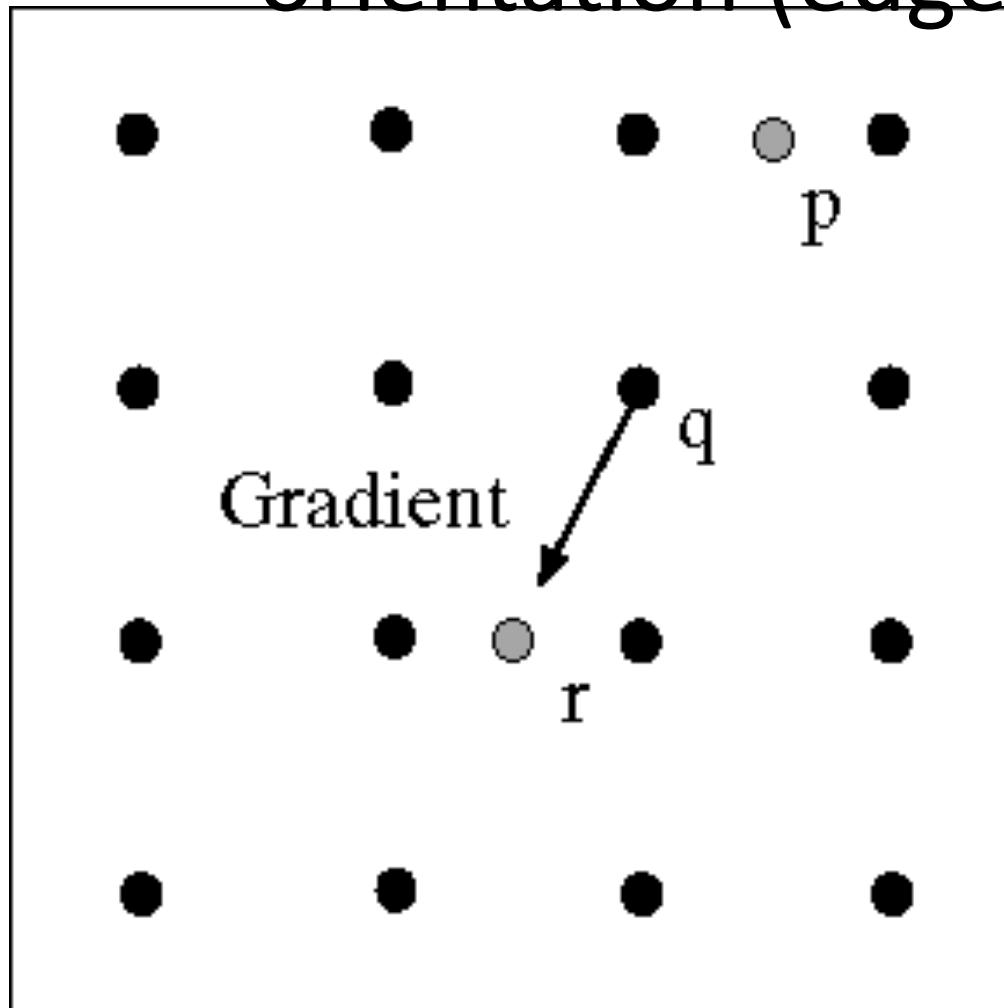
Get Orientation at Each Pixel

- Threshold at minimum level
- Get orientation

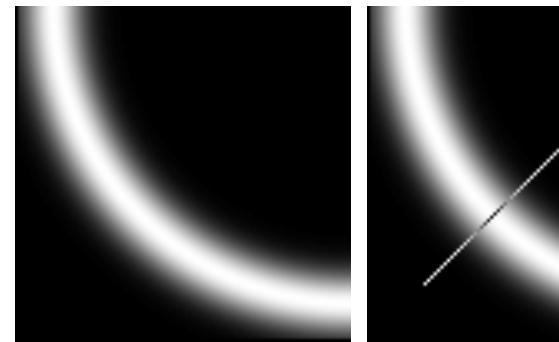
$$\theta = \text{atan2}(gy, gx)$$



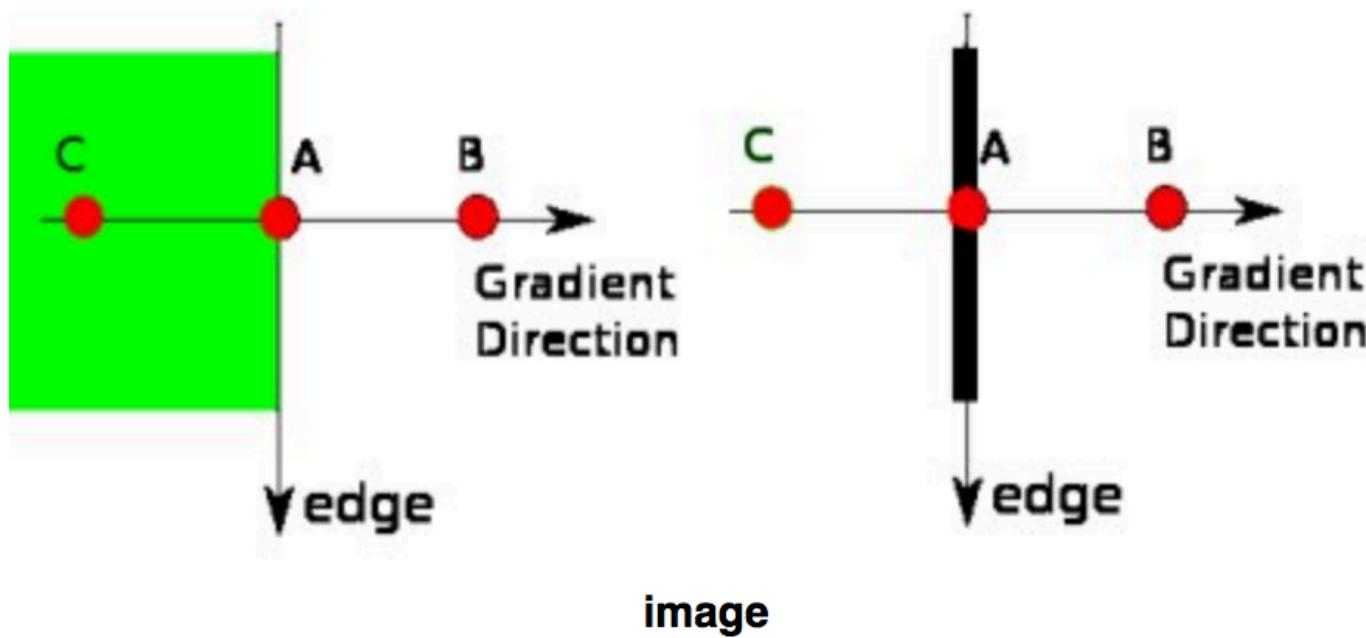
Non-maximum suppression for each orientation (edge thinning)



At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.



Non-maximum suppression for each orientation



Non-maximum suppression

- $m(x,y)$ is the local peak
- Canny calls local peak detection non-maximum suppression

$$m(x, y) > m(x + \delta x, y + \delta y),$$

$$m(x, y) > m(x - \delta x, y - \delta y).$$

$M(x,y)$ is a local peak whenever it is greater than the values in the gradient direction and the opposite of the gradient direction.

Non-maximum suppression

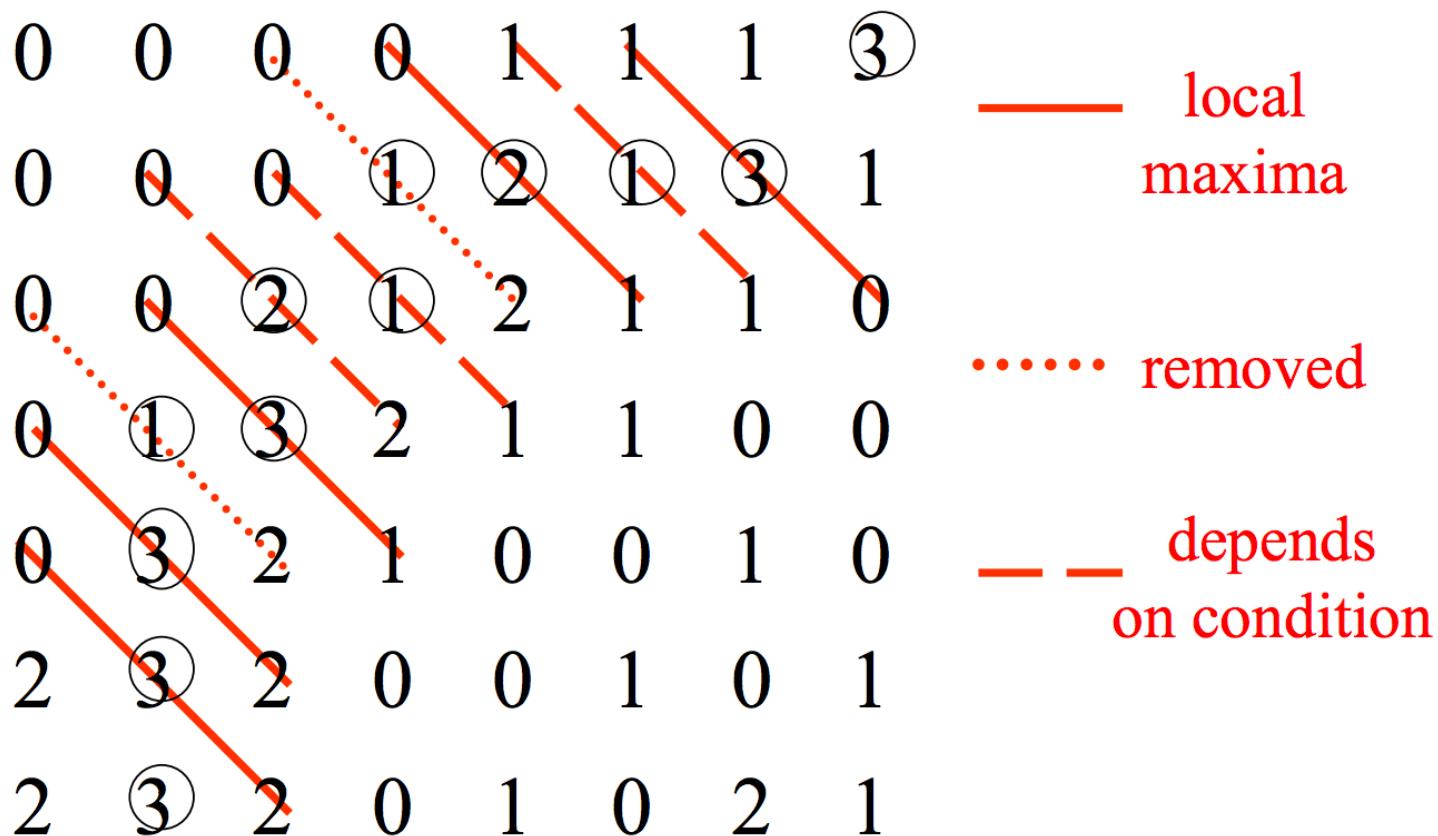
- Thin the broad ridges in $M[i,j]$ into ridges that are **only one pixel wide**
- Find local maxima in $M[i,j]$ by suppressing all values along the line of the Gradient that are not peak values of the ridge

0	0	0	0	1	1	1	(3)	
3	0	0	1	2	1	(3)	1	
0	0	2	1	2	1	1	0	
0	1	(3)	2	1	1	0	0	
0	(3)	2	1	0	0	1	3	
2	(3)	2	0	0	1	0	1	
2	(3)	2	0	1	0	2	1	

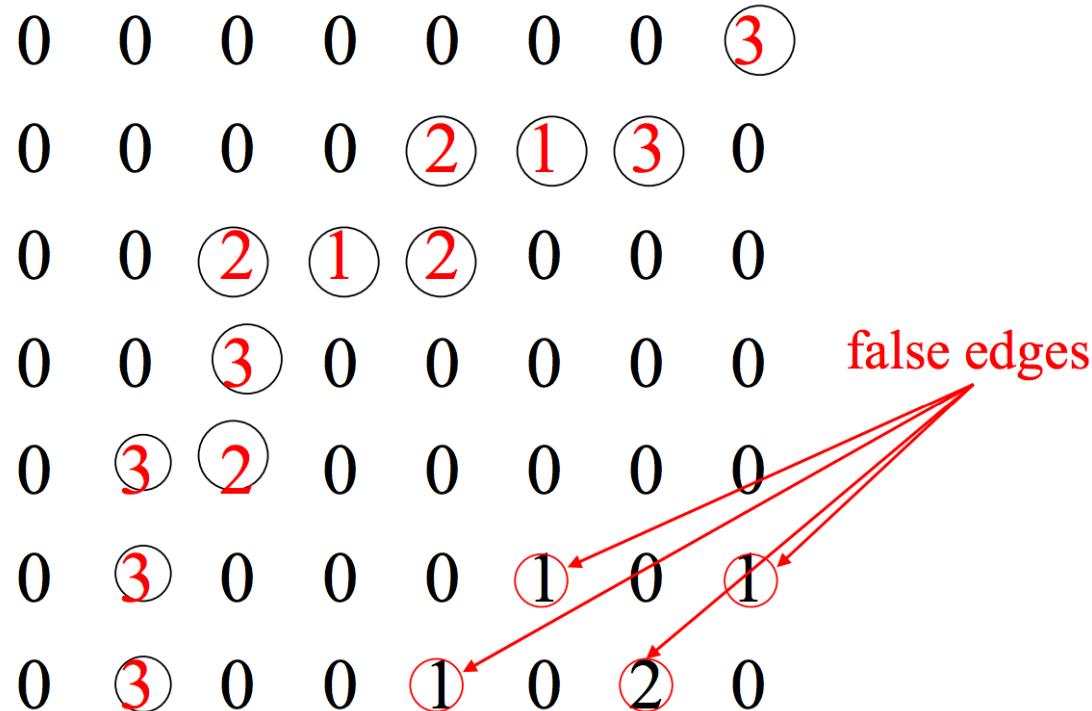
false edges

gaps

Non-maximum suppression

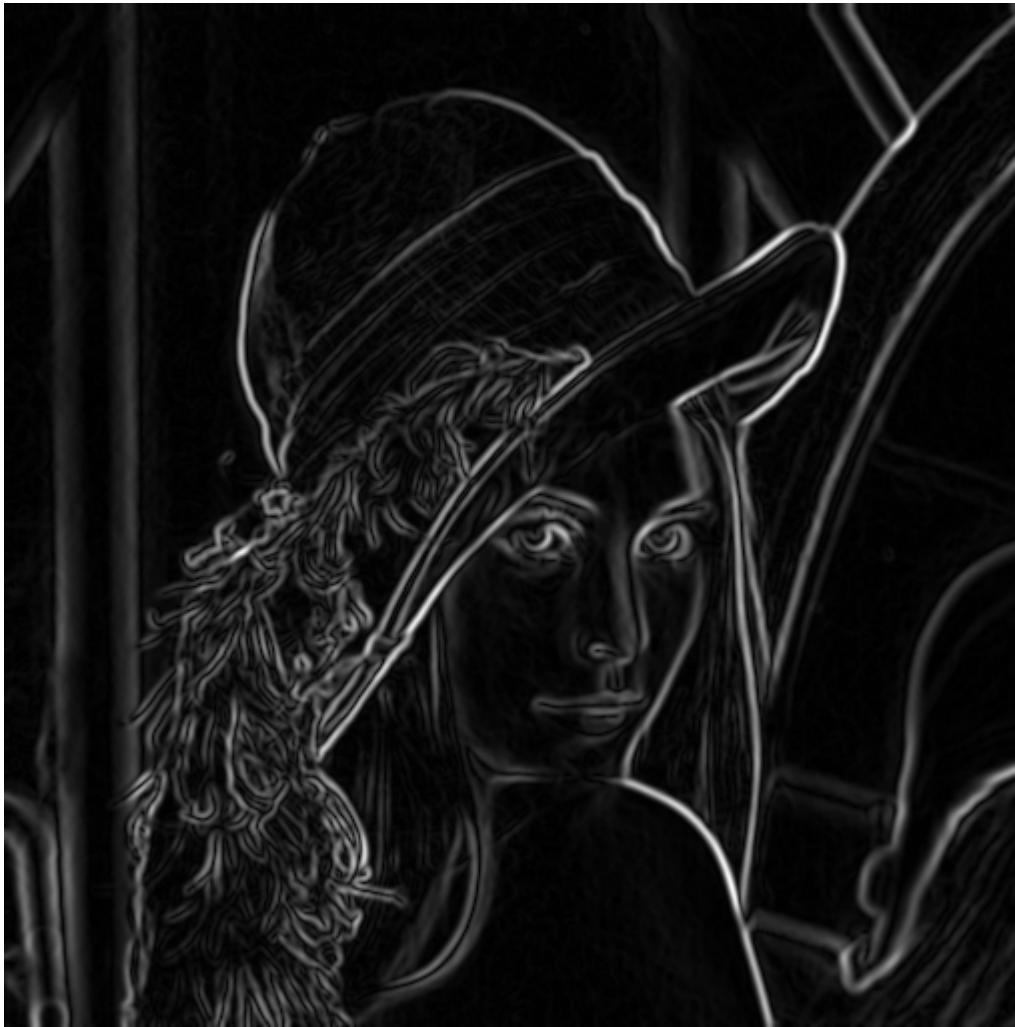


Non-maximum suppression



- The suppressed magnitude image will contain many false edges caused by noise or fine texture

Before Non-max Suppression



After non-max suppression



Double thresholding

- The remaining pixels are then categorized into strong, weak and non-edges.
- Above a high threshold, mark as strong.
- Between high and lo, mark as weak.
- Below lo threshold, mark as non-edges

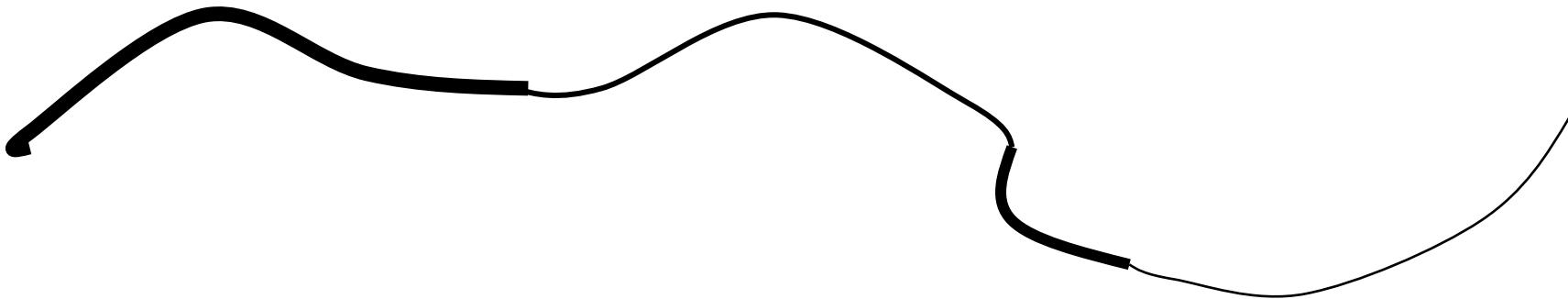
Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

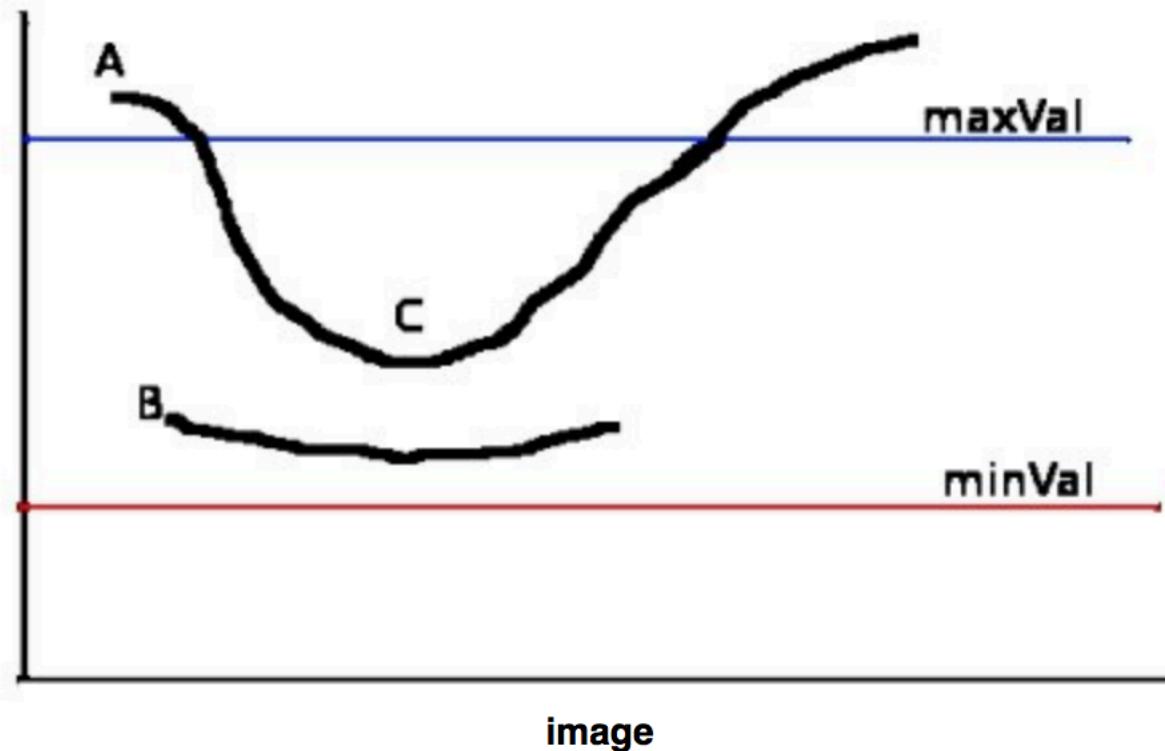


Hysteresis thresholding

- Check that maximum value of gradient value is sufficiently large
 - drop-outs? use **hysteresis**
 - use a high threshold to start edge curves and a low threshold to continue them.



Hysteresis thresholding



Final Canny Edges



Canny Edge Operator

- Smooth image I with 2D Gaussian: $G * I$
- Find local edge normal directions for each pixel

$$\bar{\mathbf{n}} = \frac{\nabla(G * I)}{|\nabla(G * I)|}$$

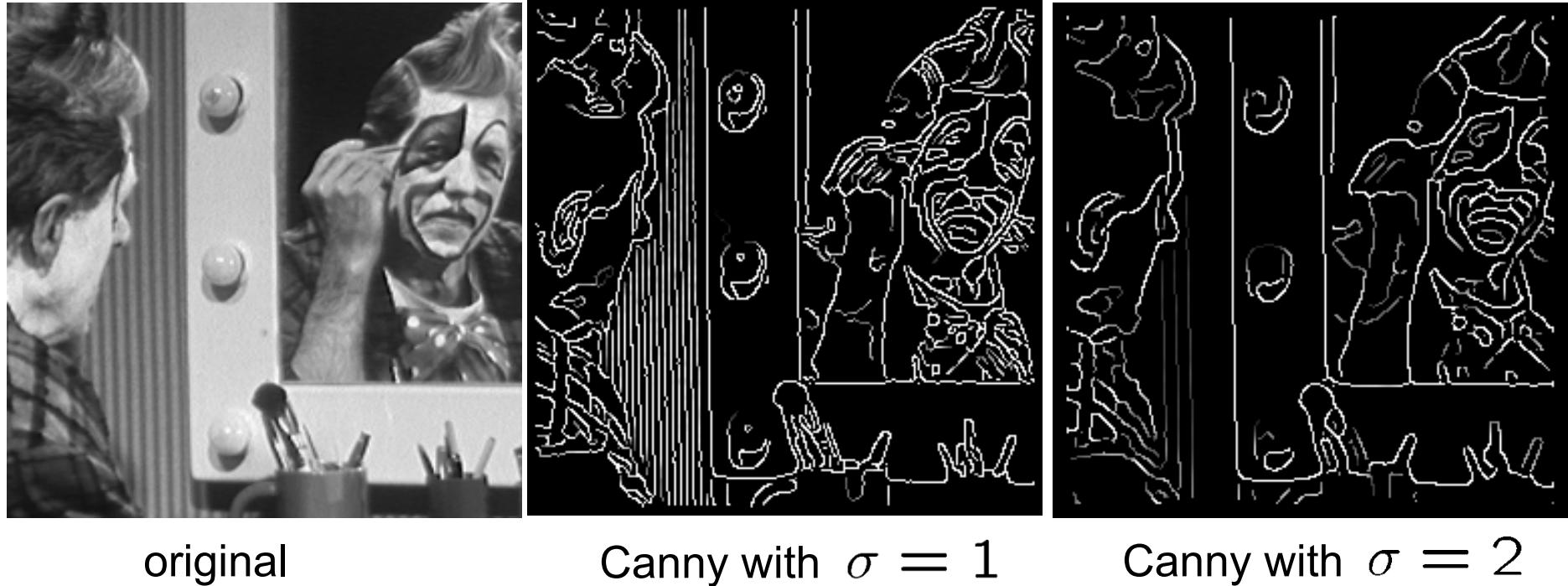
- Compute edge magnitudes $|\nabla(G * I)|$
- Find the location of the edges by finding zero-crossings along the edge normal directions (**non-maximum suppression**)

$$\frac{\partial^2(G * I)}{\partial \bar{\mathbf{n}}^2} = 0$$

- Threshold edges in the image with **hysteresis** to eliminate spurious responses

Read Canny's original paper for further details

Effect of σ (Gaussian kernel spread/size)



The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Canny detection with Python

1. Read in an image and filter with a Gaussian filter.
 2. Apply sobel filter to x and y direction.
 3. Computer gradients magnitudes and orientation.
 4. Performing non-maximum suppression upper threshold
 5. Perform threading ad labeling edge maps.
-
1. Compare your result with :

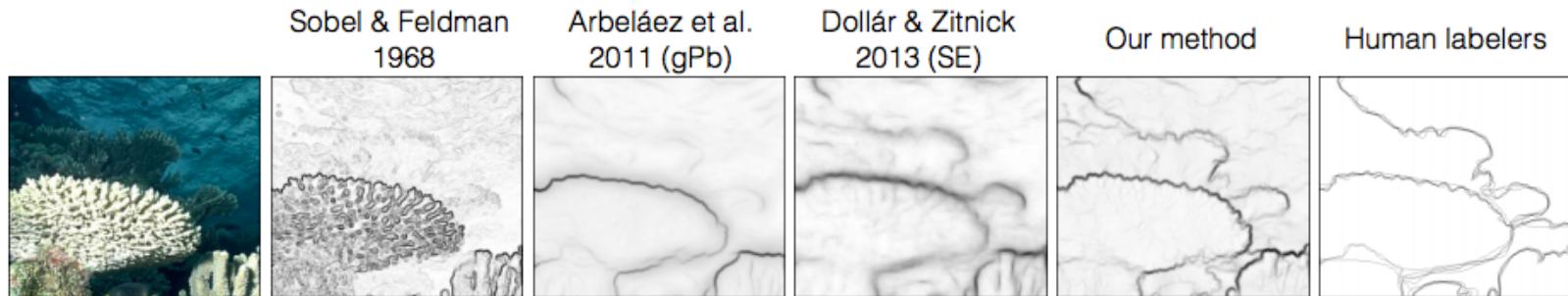
`cv2.Canny2()`

http://dasl.unlv.edu/daslDrexel/alumni/bGreen/www.pages.drexel.edu/_weg22/can_tut.html

Take-home reading

- Szeliski Chapter 4.2 Edges
- A very good introduction of Canny edge detection:
- <http://www.classes.cs.uchicago.edu/archive/2005/fall/35040-1/edges.pdf>
- A youtube video is also helpful:
- <https://www.youtube.com/watch?v=-Z3kr26Eci4>

Edge detection vs. boundary detection



1. Classical methods use local derivative filters with fixed scales and only a few orientations. Tend to emphasize small and unimportant edges.
2. Contemporary methods use multiple scales, multiple feature from image patches (color, textures, intensity). Using statistical methods (give each pixel a probability of being a boundary) to learn boundaries.
3. Isola et al. (2014) uses mutual information between pixels to detect boundary.



Key observation: *Pixels belonging to the same object have higher statistical association than pixels belonging to different objects.*

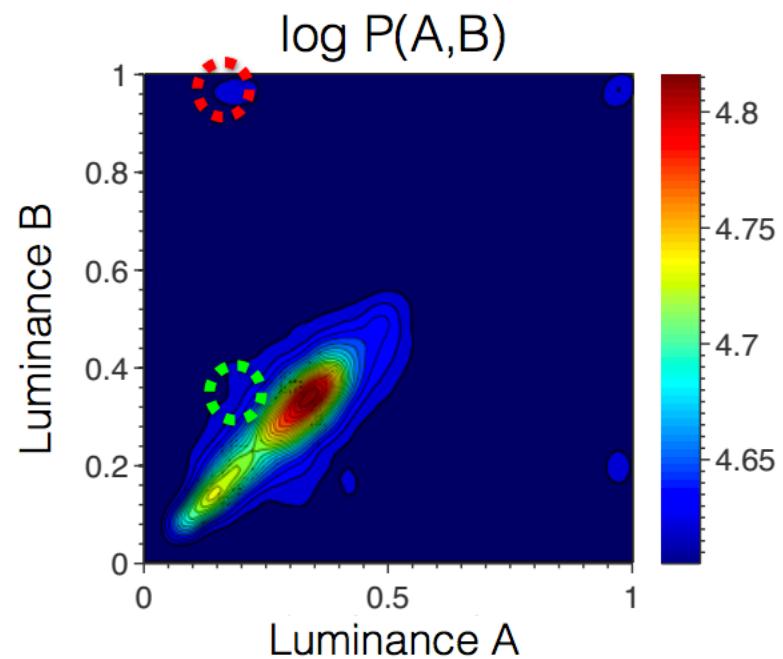
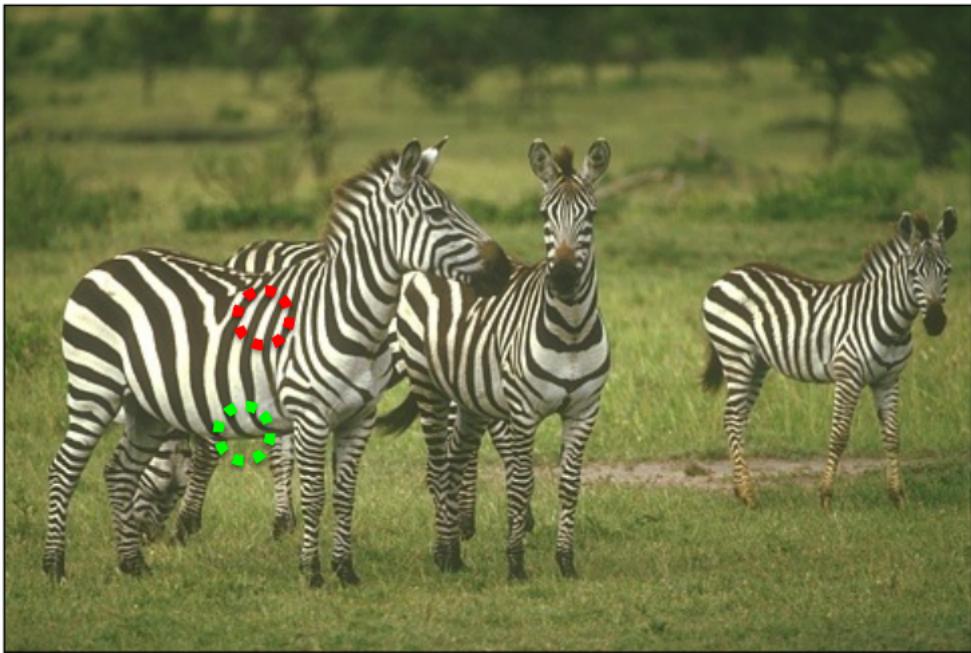
Slide courtesy from Philip Isola

Point-wise mutual information reveals object structure

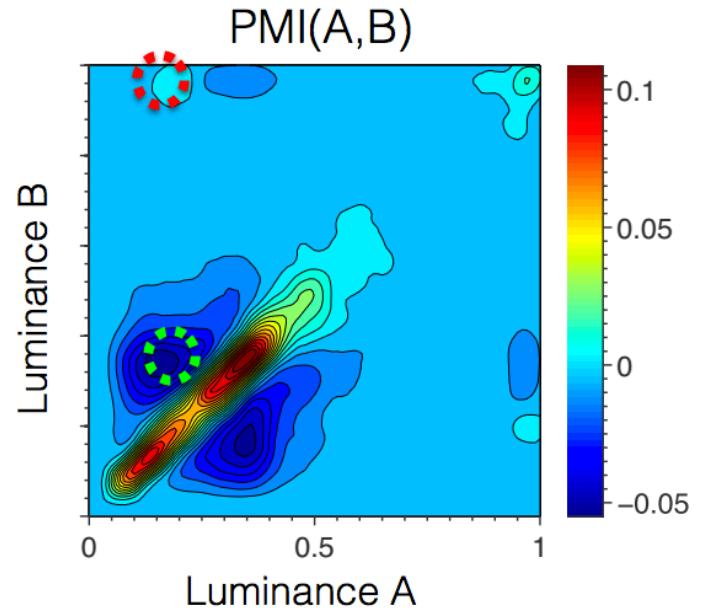


Above, black-next-to-white occurs over and over again. This pattern shows up in the image's statistics as a *suspicious coincidence* — these colors must be part of the same object!

How do we distinguish the red and the green patches?



$P(A, B) = \text{how often each color } A \text{ occurs next to each color } B$
within this image.



Pointwise mutual information (PMI)

$$\text{PMI}_\rho(A, B) = \log \frac{P(A, B)^\rho}{P(A)P(B)}$$

Use PMI as affinity measure for affinity-based pixel grouping.

How much more likely is observing A given that we saw B in the same local region, compared to the base rate of observing A in the image.

nB boundary detector



pB Boundary Detector

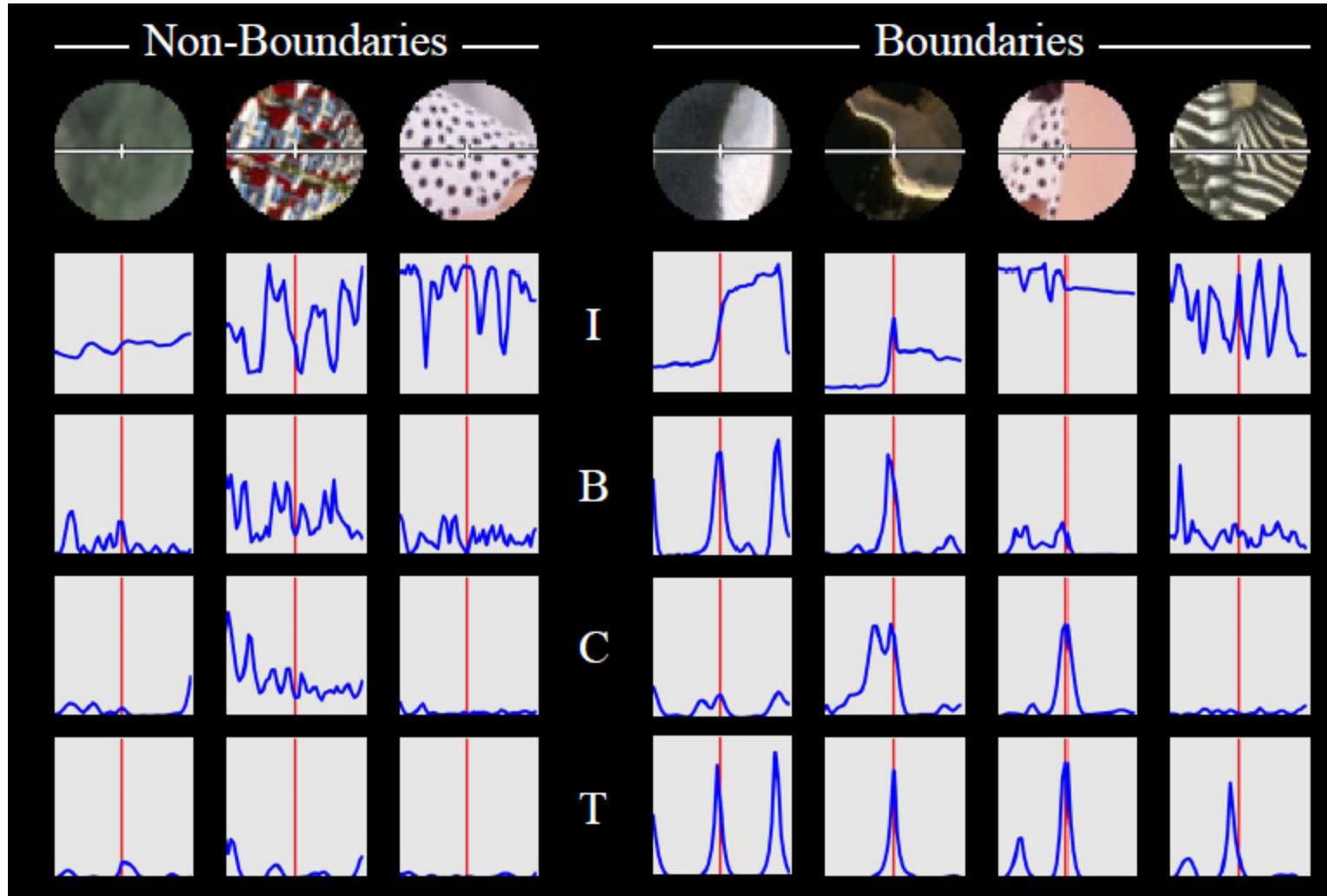
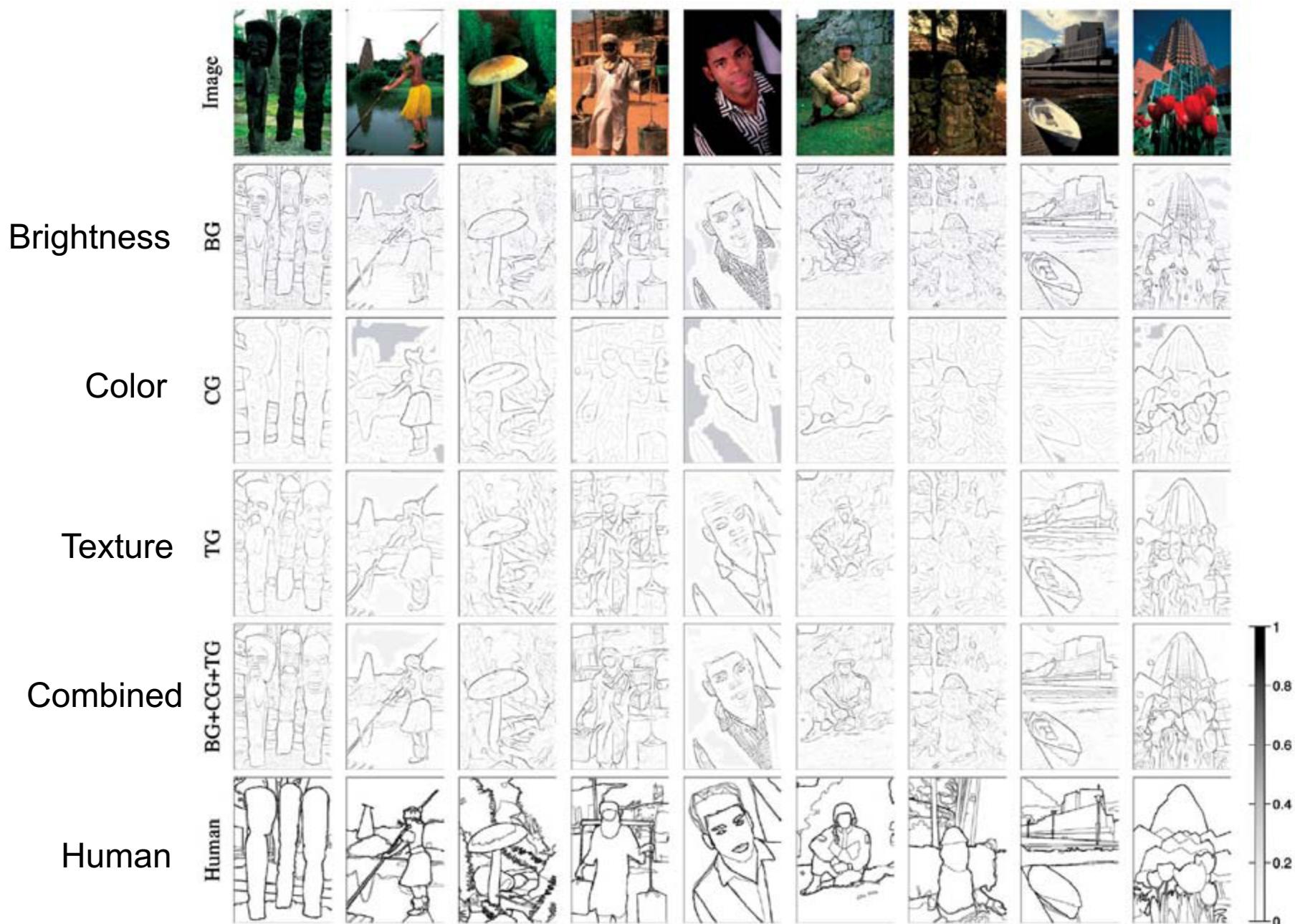
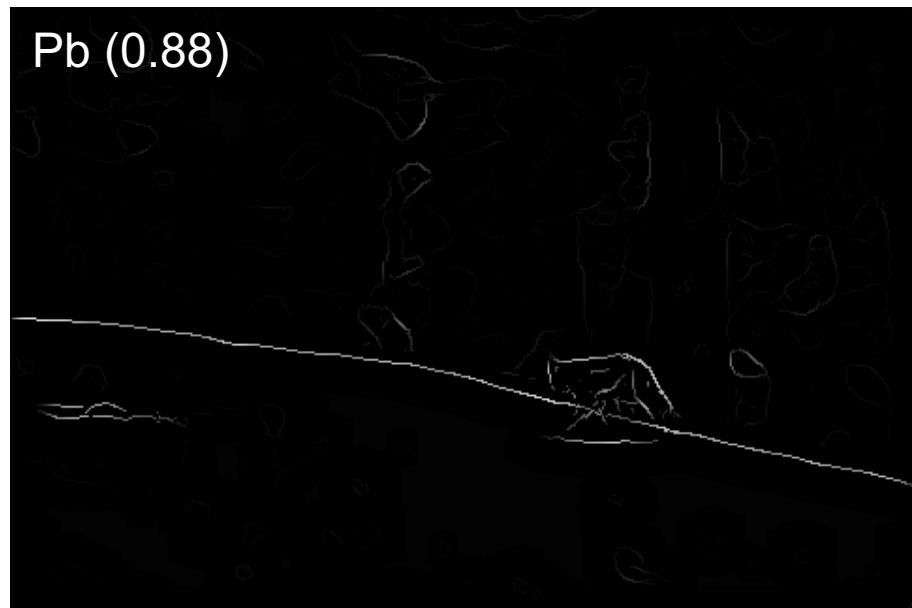
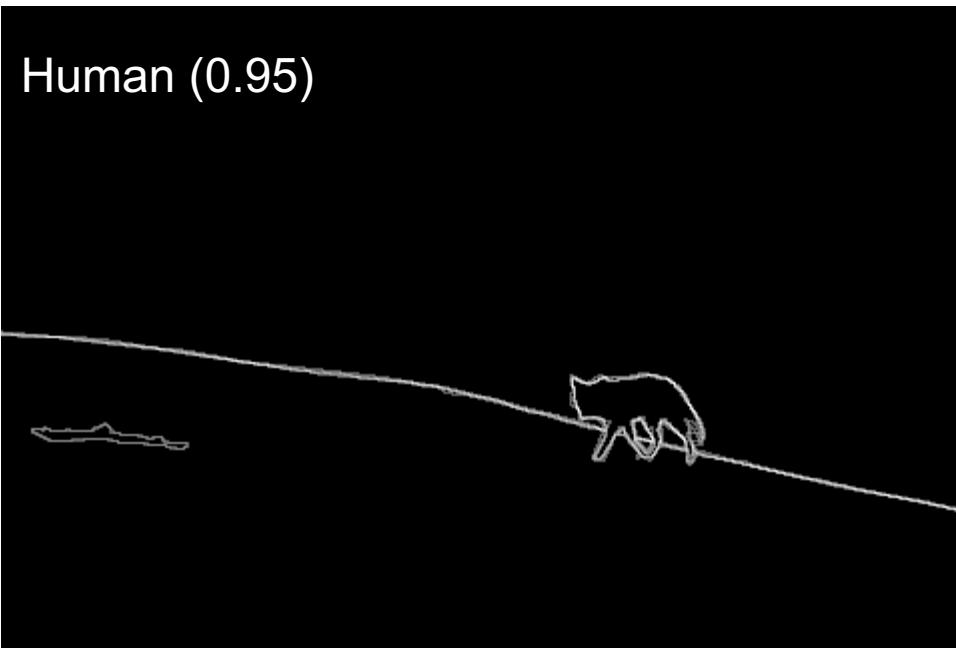


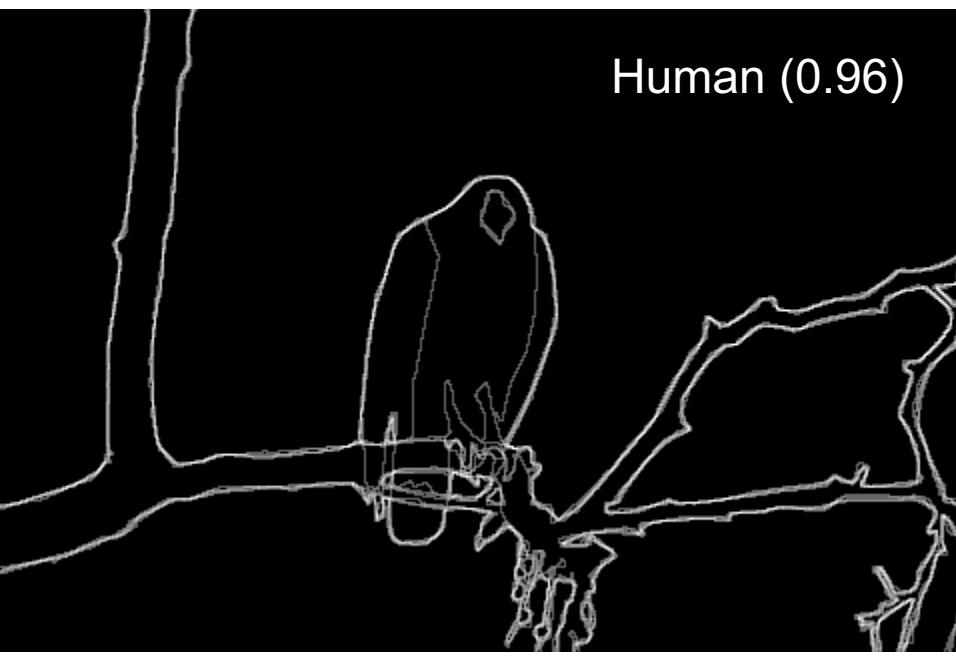
Figure from Fowlkes



Results



Results



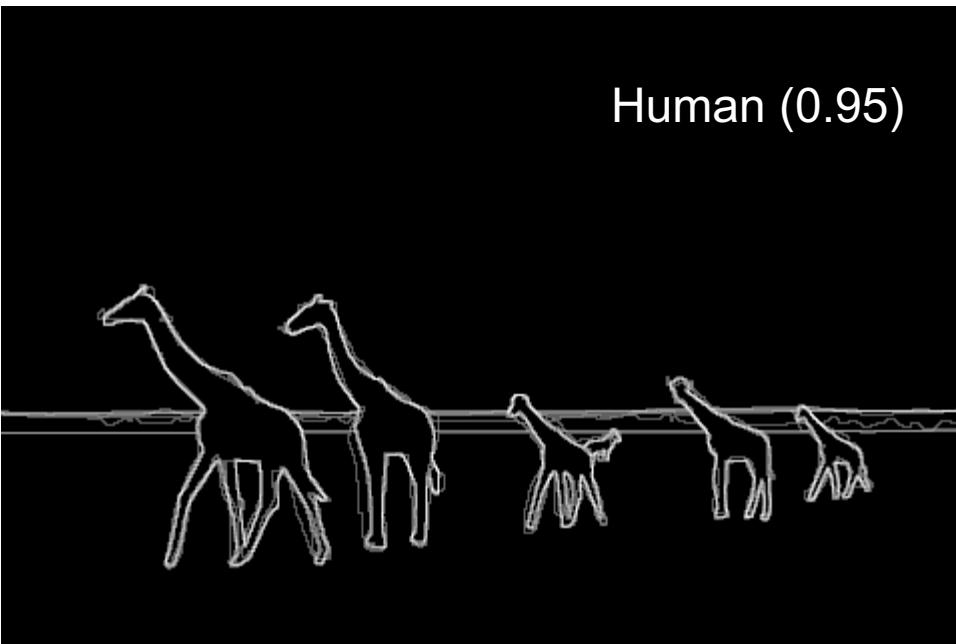
Human (0.96)



Pb (0.88)

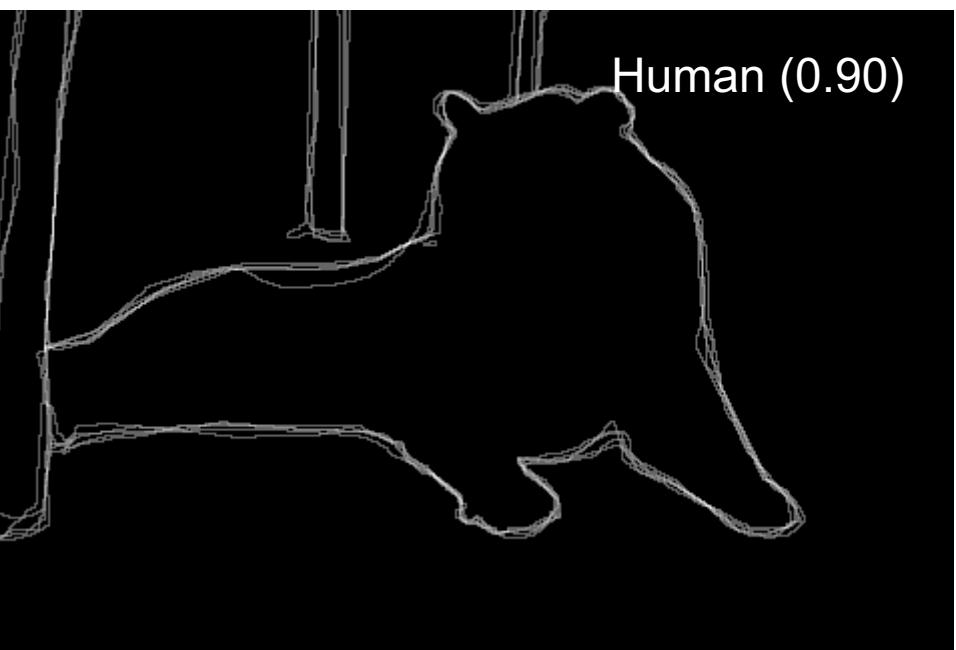


Human (0.95)



Pb (0.63)

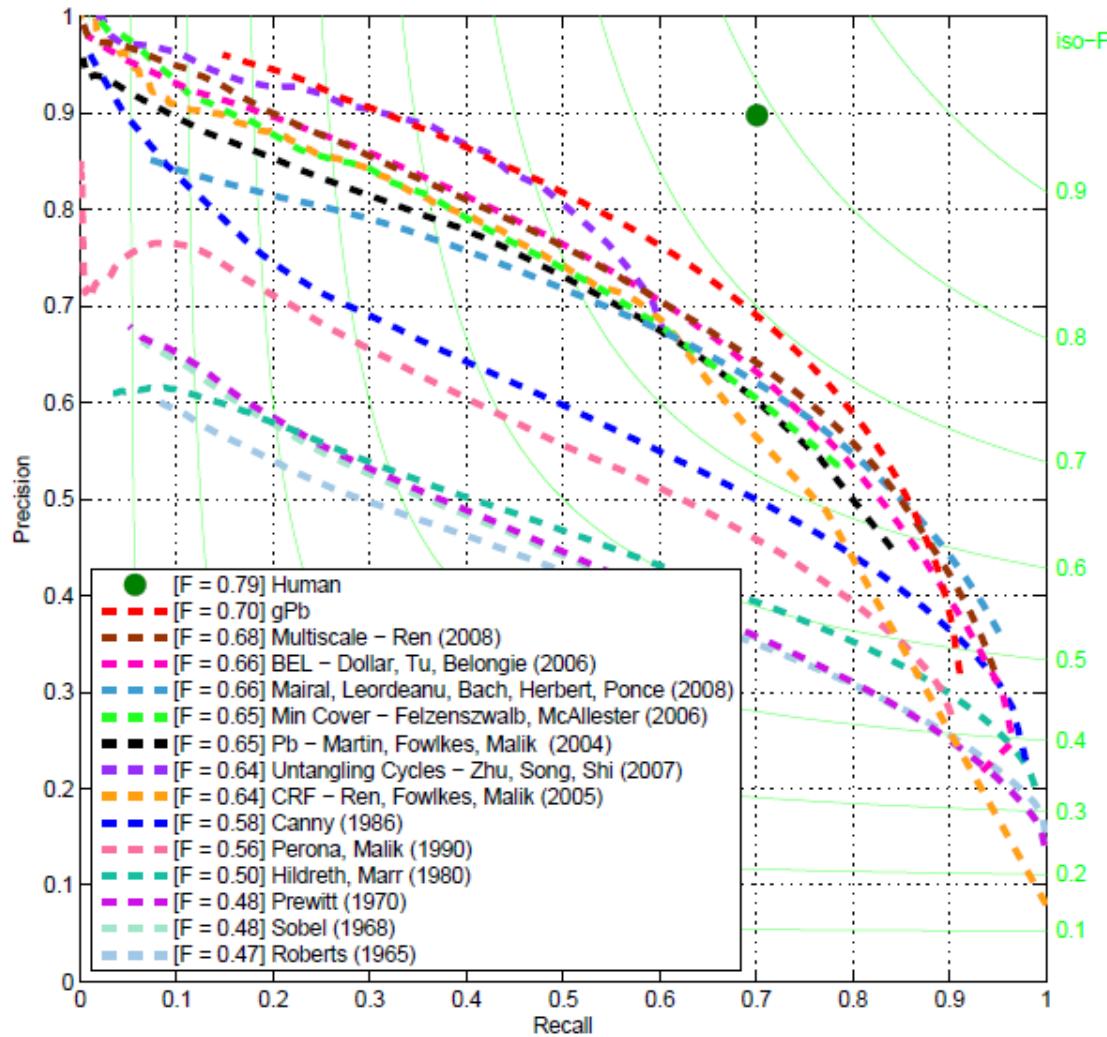




For more:

<http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/bench/html/108082-color.html>

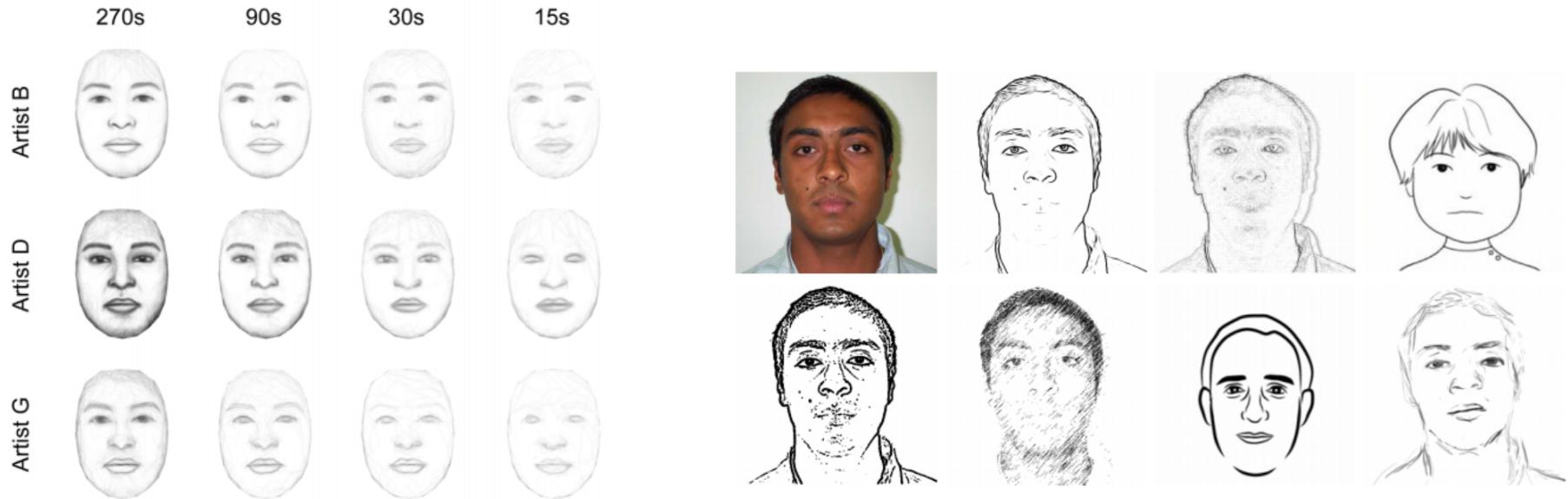
45 years of boundary detection



State of edge detection

- Local edge detection works well
 - But many false positives from illumination and texture edges
- Some methods to take into account longer contours, but could probably do better
- Few methods that actually “learn” from data. Your project 5, Sketch Tokens, will do so.
- Poor use of object and high-level information

Style and abstraction in portrait sketching, Berger et al. SIGGRAPH 2013



- Learn from artist's strokes so that edges are more likely in certain parts of the face.

Next class

- Laplacian of Gaussian
- Steerable filter
- Fourier transform
- Reading: Chapter 3.1-3.3! Very important to keep up the reading.
- You should have read Chapter 1. Skip Chapter 2 since we haven't covered it.