Review of ”Solving the Multi-Agent Pathfinding Problem with Time-Expanded Networks”

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Summary

The paper presents a novel exact algorithm for the Multi-Agent Pathfinding Problem (MAPF) using Time-Expanded Networks (TENs) and a new heuristic approach estimating the number of timesteps T required to have a feasible solution in the time-expanded network. The algorithm is initialized with a Time-Expanded-Graph only containing arcs and vertices contained in minimal distance paths for all agents. At each iteration if a feasible solution is found, with the cost equal to the lower bound, then the algorithm has found an optimal solution. Otherwise the graph is expanded with the vertices and arcs corresponding to origin-demand paths one unit longer than the previous iteration and the lower bounds are updated. The authors prove that the iteratevely defined lower bounds are lower bounds for the optimal solution. The authors compare the algorithm with state-of-the-art algorithms and show that the algorithm is competitive.

Minor Comments

1. **[Page 5, Example 1]** When defining the actions L, R, U, D (left, right, up, down), it is currently unclear what these functions return when a suitable neighbor does not exist. For instance, if a vertex $i \in V$ has no left neighbor, $L(i)$ would be undefined. Consider adding a clarifying statement such as: *“If no suitable neighbor exists for a given action L, R, U and D, that action leaves the vertex unchanged (e.g., $L(i) = i$ in the absence of a left neighbor).”*

2. **[Page 5, Example 1, Plan decriptions in Examples]**: When illustrating a plan (e.g., $\pi_{green} = (L, L, U)$), listing out each intermediate value ($\pi_{green}[0] = 6$, $\pi_{green}[1] = 5$, $\pi_{green}[2] = 4$, etc.) can be overly detailed and may not substantially aid the reader's understanding. For clarity and conciseness, consider omitting the explicit enumeration of all vertex positions and focus instead on the general sequence of actions.

3. **[Page 6, Section 3]** When defining $\tau(i,j)$, the meaning of a "conflict-free" path between two vertices is not fully clear. The absence of vertex or edge conflicts is a property concerning the plans of all agents collectively, rather than just a single (agent's) path. It is also unclear how $\tau(i,j)$ differs from the standard quickest path between two vertices in a weighed graph. We suggest clarifying this distinction by providing an explicit definition of"conflict-free in this context, and explaining how and if $\tau(i,j)$ is computed differently from a straightforward quickest-path distance.

4. **[Page 9-10, Section 4]** It is not entirely clear how $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ is initialized. Specifically, if $\mathcal{V}$ is defined as the union of the sets $\mathcal{V}_h$ for each agent when $q = 0$, the definition of $\mathcal{V}$ becomes circular because $\mathcal{V}_h$ is itself defined as a subset of $\mathcal{V}$. To resolve this, we suggest either introducing a separate notation for the graph updated during the various iterations (e.g., $\overline{\mathcal{G}} = (\overline{\mathcal{V}}, \overline{\mathcal{A}})$) or using iteration-specific graphs (e.g., $\mathcal{G}^q = (\mathcal{V}^q, \mathcal{A}^q)$ to highlight the iteration index), and then defining $\mathcal{G}^q$ in terms of $\mathcal{G}$.

5. **[Page 9-10, Section 4]** Continuing the above point, when defining $\mathcal{G}^q$ for $q = 0$, it should be made explicit whether its vertex set is indeed the union of all $\mathcal{V}_h$.

6. **[Page 10, Section 4]** The sentence *"Before the first iteration ($q = 0$), $\mathcal{B}$ is empty"* can be omitted, as $\mathcal{B}$ is defined only for $q \in \mathbb{N}$ and, for $q = 0$, we have $\mathcal{B} = \mathcal{V}^0$.

7. **[Page 16, Subsection 5.2]** It is unclear what is meant by

   > *"...increasing the time limit allows for more accurate and comprehensive data collection..."*

   Does this imply that more solutions will be found, resulting in fewer unsolved instances and thus a more "accurate" comparison between number of proven feasibility vs proven optimality? Additionally, does the number of unsolved instances change differently for MCNF compared to CBS when increasing the time limit?

Suggestions for Improvement

For the proof of Theorem 3, for the first $\underline{y}^{(0)} - \min_{h\in N} \xi_h$ iterations the graph $\mathcal{G}$ could be expanded by a smaller number of nodes making the iterations quicker. Each agent graph $\mathcal{V}_h$ can be expanded independently. For iteration $q \leq \underline{y}^{(0)} - \min_{h\in N} \underline{\xi}_h^{(0)}$ we can expand the graphs $\mathcal{V}_h$ for only those agents where $\underline{\xi}_h^{(0)} + q < \underline{y}^{(0)}$, while leaving all other agent graphs unchanged (That is $\mathcal{V}_h$ is expanded exactly $\underline{y}^{(0)} - \underline{\xi}_h^{(0)}$ times in the first $\underline{y}^{(0)} - \min_{h\in N} \underline{\xi}_h^{(0)}$ iterations). At iteration $q' = \underline{y}^{(0)} - \min_{h\in N} \underline{\xi}_h^{(0)} \geq \underline{y}^{(0)} - \underline{\xi}_h^{(0)}$, from Theorem 1 follows that $\mathcal{V}_h$ contains every plan π_h such that:

$$|\pi_h| \leq \underline{\xi}_h^{(0)} + (\underline{y}^{(0)} - \underline{\xi}_h^{(0)}) = \underline{y}^{(0)}.$$

Thus, $\mathcal{G}$ contains every sequence of n plans with a makespan $y^{(q')} \leq \underline{y}^{(q')}$. Then at each iteration $q > \underline{y}^{(0)} - \min_{h \in N} \xi_h$, every $\mathcal{V}_h$ can be expanded for every agent h, and by the same reasoning we have that $\mathcal{G}$ contains every sequence of n plans with a makespan $y^{(q)} \leq \underline{y}^{(q)}$. Therefore Theorem 3 still holds with the same definition of the lower bound $\underline{y}^{(q)}$ but adding fewer nodes at each iteration.

Minor Editorial Remarks

1. [**Page 8, Section 4**]: There is a small typo in the sixth line of this section: "To easy" should be corrected to "To ease".

2. [**Page 13, Subection 4.2**] At the last line of the proof of Theorem 2, a comma is missing at the start of the sentence: *"q is incremented by 1, and the procedure continues.*

3. [**Page 17, Subsection 5.2**] In the description of Figure 8, in the sentence:

   > *"It is worth noting that the worst performance is obtained for 25 agents..."*

   consider adding "of the MCNF algorithm respect to CBS" since the absolute unsuccess rate for 25 agents is "low" for both algorithms.

4. [**Page 18, Figure 8**] Consider adding a label for the y-axis to make it easier to read.

5. [**Inconsistent Figure Labels**]: Some figure labels end with a full stop (Figures 1-4), while others do not.

Recommendation

In conclusion, we find that the paper is well-written and recommend acceptance pending minor revisions.