Optimization problems in Power Systems

Gabor Riccardi

Università di Pavia

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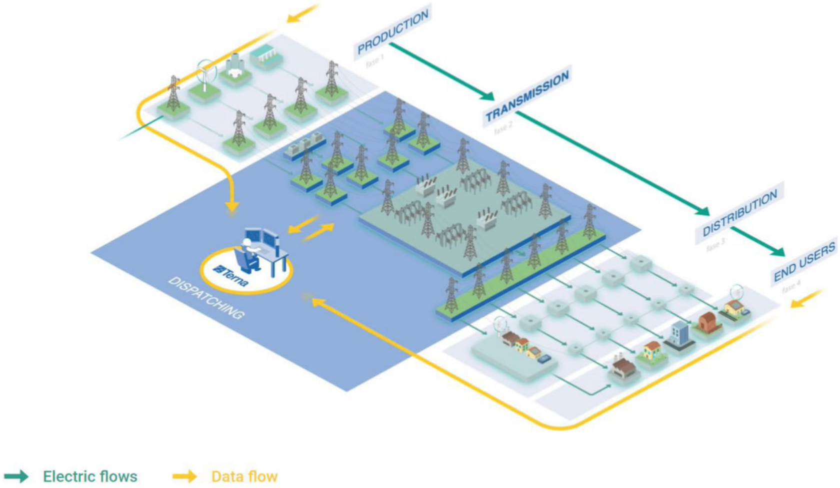
Optimal Power Flow problems



Compact formulation

$$\inf_{\mathbf{u},\mathbf{S}\in\mathbb{C}^{\mathcal{B}}} \mathrm{c}(\mathbf{u},\mathbf{S}) \tag{1}$$

s.t.

$$\mathrm{diag}(\overline{\mathbf{u}})\mathbf{Y}\mathbf{u} = \mathbf{S} \tag{2}$$

$$\underline{\mathbf{u}} \leq \overline{\mathbf{u}} \tag{3}$$

$$\underline{\mathbf{S}} \leq \overline{\mathbf{S}} \tag{4}$$

Polar OPF formulation

$$\inf_{\substack{P_g^G, Q_g^G, \delta_k \\ |V|_k, S_{km}}} \sum_{g \in \mathcal{G}} F_g(P_g^G) \tag{5}$$

Subject to:

Voltage to Power Flow constraints: $\forall km \in \mathbf{L}$

$$S_{km} = (G_{kk} - iB_{kk})|V_k|^2 + (G_{km} - jB_{km})|V_k||V_m|(\cos(\delta_k - \delta_m) + j\sin(\delta_k - \delta_m)) \tag{6}$$

Power balance constraints:

$$\sum_{km \in \delta(k)} S_{km} = (\sum_{g \in \mathcal{G}(k)} P_g^g - P_k^d) + i(\sum_{g \in \mathcal{G}(k)} Q_g^G - Q_k^L) \tag{7}$$

Power flow, Voltage, and Power generation limits:

$$|S_{km}|^2 \leq U_{km}, \quad V_k^{\min} \leq |V_k| \leq V_k^{\max}, \quad P_g^{\min} \leq P_g^G \leq P_g^{\max} \tag{8}$$

$$\theta_{km}^{\min} \leq \delta_k - \delta_m \leq \theta_{km}^{\max} \tag{9}$$

Methods for Solving AC-OPF

1. **Classical Methods**
   - **Newton-Raphson**: Iterative, relies on solving nonlinear equations.
   - **Interior Point Methods (IPMs)**:
     - Solve Karush-Kuhn-Tucker (KKT) conditions.
     - Efficient for large-scale systems.
2. **Relaxation Techniques**
   - **Semidefinite Programming (SDP)**:
     - Convex relaxation.
     - Provides bounds on global optimum.
   - **Second-Order Cone Programming (SOCP)**:
     - Weaker relaxation than SDP.
     - Faster, scalable for large systems.
   - **Quadratic Programming (QP)**:
     - Linearizes power flow equations.

Global Optimization Techniques

3. Global Optimization Methods

- **Branch-and-Bound**:
  - Systematically explores subproblems.
  - Guarantees global solution.
- **Heuristics**:
  - Genetic Algorithms, Simulated Annealing.
  - Useful for obtaining good feasible solutions.

1. **General Theory**
    - Invertibility conditions of admittance matrix are known.
    - Item NP hardness
    - What proprieties does the bilinear function diag($\overline{\mathbf{u}}$)$\mathbf{Yu}$ have? (invertibility?)
    - Can constraint bound violations give bounds on the distance of a feasible solution? (is this the continuity of the inverse)
2. **Manifold Optimization?** itemize

The space of feasible solutions of the AC OPF (without magnitude constraints) is a smooth manifold.

The tangent space of the manifold is known.

No explicit retractions are known. Feasible solution "can" be computed with Calssical Methods. Can these be used as retractions?
Trade-off between accuracy and computational effort.