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Joint Research Centre

A Parallelization Algorithm for Adequacy Assessment of the Electrical Grid

Gabor Riccardi

Università di Pavia

16/02/24



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$$\min_{x} c'x + \mathbb{E}_{\omega} [\mathcal{V}(x, \omega)]$$

$$s.t. \ 0 \le x_{n,g} \le X_{n,g}$$
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- The first stage determines the capacity expansion $x_{n,g}$ for each generator $g \in \mathcal{G}$
- The second stage solves the Economic Dispatch (ED).

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- lacktriangle The first stage determines the capacity expansion $x_{n,g}$ for each generator $g\in\mathcal{G}$
- The second stage solves the Economic Dispatch (ED).

Where $\mathcal{V}(x,\omega)$ is the solution to (ED) in function of the expanded capacities x and the scenario ω .

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Economic Dispatch (ED) model Scary Slide

For a fixed scenarios $\omega = (\mathcal{PV}, \mathcal{W}, \mathcal{D})$ comprising of respectively solar power, wind power and loads.

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$$\min_{y} q' y_{\omega} \tag{1}$$

$$s.t. p_{n,g,t,\omega} + bd_{n,t,\omega} + \sum_{l \in \mathcal{L}(n)} f_{n,l,t,\omega} + ls_{n,t,\omega} + \mathcal{PV}_{n,t,\omega} + \mathcal{W}_{n,t,\omega} =$$
(2)

$$=\mathcal{D}_{n,t,\omega}+s_{nt,\omega}+bc_{n,t,\omega}$$

$$v_{n,t,\omega} = v_{n,t-1,\omega} + BCE \cdot bc_{n,t,\omega} - BDE \cdot bd_{n,t,\omega} + A_{n,t,\omega}$$
(3)

$$(v_{n,t,\omega}, bc_{n,t,\omega}, bd_{n,t,\omega}) \le (BV, BC, BD) \tag{4}$$

$$p_{n,g,t,\omega} \le p_{n,g}^{\mathsf{max}} + x_{n,g} \tag{5}$$

$$L_{n,l}^{\min} \le f_{n,l,t,\omega} \le L_{n,l}^{\max} \tag{6}$$

Where $v_{n,t,w}$ is the power stored at bus n at time t.

• We divide the time horizon into K intervals, $\{t_0 = 0 := 1, \ldots, t_1\}, \{t_1 + 1, \ldots, t_2\}, \ldots, \{t_{K-1} + 1, \ldots, t_K := T\}$

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- We refer to the (ED) problems restricted to each time interval with fixed initial and final storage values as (ED-k) and to its optimal values as and with optimal value $V_{\mathbf{k}}(\mathbf{x}, \mathbf{v_{t_k}}, \mathbf{v_{t_{k+1},\omega}})$

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Observation

$$\mathcal{V}(x,\omega) = \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^{K-1} \mathcal{V}_k(x, v_{t_k}, v_{t_k+1}, \omega)$$
 (7)

Since each function \mathcal{V}_k is piecewise linear convex in $x, v_{t_K}, v_{t_{K+1}}$, it can be approximated by a collection of supporting hyperplanes $\{\pi_{i,k}^w(x, v_{t_k}, v_{t_{k+1}})\}$ of each \mathcal{V}_k .

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$$\hat{\mathcal{V}}(x,\omega) = \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^K \hat{\mathcal{V}}_k(x, v_{t_k}, v_{t_{k+1}}) =$$

$$= \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^K \theta_k^{\omega}$$

$$\text{s.t.} \quad \theta_k^{\omega} \ge \pi_{i,k}^{\omega}(x, v_{t_k}, v_{t_{k+1}}) \quad \forall i, k$$
(ISP)

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$$\begin{split} \hat{\mathcal{V}}(x,\omega) &= \min_{\left\{v_{t_k}\right\}_{k=1}^K} \sum_{k=0}^K \hat{\mathcal{V}}_k(x,v_{t_k},v_{t_{k+1}}) = \\ &= \min_{\left\{v_{t_k}\right\}_{k=1}^K} \sum_{k=0}^K \theta_k^{\omega} \\ \text{s.t.} \quad \theta_k^{\omega} \geq \pi_{i,k}^{\omega}(x,v_{t_k},v_{t_{k+1}}) \quad \forall i,k \end{split} \tag{ISP}$$

We refer to this problem as the **Intermediate Storage Problem (ISP)** (I know, very original)

Model description: Relaxed Capacity Expansion(CEP-R)

$$\min_{x} c'x + \mathbb{E}_{\omega} [\mathcal{V}(x, \omega)]$$

$$s.t. \ 0 \le x_{n,g} \le X_{n,g}$$
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 $s.t. \ 0 \le x_{n,g} \le X_{n,g}$ (CEP)

$$\min_{x} c'x + \mathbb{E}_{w} \left[\hat{\mathcal{V}}(x, w) \right]$$

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Gabor Riccardi 16/02/24

(CEP-R)

Model description: Relaxed Capacity Expansion(CEP-R)

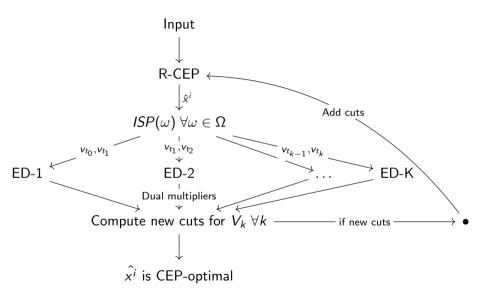
$$\min_{x} c'x + \mathbb{E}_{\omega} [\mathcal{V}(x,\omega)]$$
 $s.t. \ 0 \le x_{n,g} \le X_{n,g}$ (CEP)

$$\min_{x} c'x + \mathbb{E}_{w} \left[\hat{\mathcal{V}}(x, w) \right]$$
 $s.t. \ 0 \le x_{n,g} \le X_{n,g}$ (CEP-R)

Since calculating $\hat{\mathcal{V}}$ is straightforward, solving (CEP-R) can be done efficiently with L-shaped or subgradient schemes.

Gabor Riccardi $16/02/24 - 6 \ / \ 1$

Algorithm



■ Since $(CEP - R) \le (CEP)$ if a (CEP - R) optimal solution has the same cost for (CEP) then it's also (CEP)-optimal.

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Remark 1: It is sufficient to prove that after a finite number of steps (i) of the algorithm we have:

$$\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega) \text{ for all } \omega \in \Omega$$
 (8)

Observation

After a finite number of iterations no new cuts are found for V_k .

Proof.

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\#\{p \mid p \text{ is a normal vector of a supporting hyperplane of } \mathcal{V}_k\} \leq
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Gabor Riccardi

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After a finite number of steps:

• new cut: $\bar{c}(x, v) = p'(x, v) + b$

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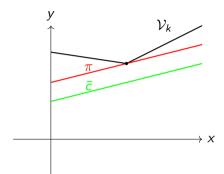
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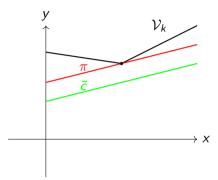
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After a finite number of steps:

- new cut: $\bar{c}(x,v) = p'(x,v) + b$
- an old cut: $\pi(x, v) = p'(x, v) + \bar{b}$





Since both are supporting hyperplanes it follows that $b=\bar{b}$ (and therefore \bar{c} is not a new cut).

Observation

If after the i-iteration no new cuts are added for some i and k then $\hat{\mathcal{V}}_k(\hat{x}^i,\hat{v}_k,\hat{v}_{k+1}) = \mathcal{V}_k(\hat{x}^i,\hat{v}_k,\hat{v}_{k+1})$.

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Let $\bar{c}_k^{\omega}(x, v_{t_k}) := p'(x - \hat{x}^i, v_{t_k} - \hat{v}_{t_k}) + V_k(\hat{x}^i, \hat{v}_{t_k})$ be the new cut found after the *i*-th iteration.

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$$\mathcal{V}_k(\hat{x}^i, \hat{v}_{t_k}) \geq \hat{\mathcal{V}}_k(\hat{x}^i, \hat{v}_{t_k}) \geq \bar{c}(\hat{x}^i, \hat{v}_{t_k}) = \mathcal{V}_k(\hat{x}^i, \hat{v}_{t_k})$$

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which concludes the proof.

In conclusion, we have $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$ for all ω, k .

In conclusion, we have $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$ for all ω, k . Thus $\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega)$.

In conclusion, we have
$$\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$$
 for all ω, k . Thus $\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega)$.

Proposition

The algorithm converges after a finite number of iterations and \hat{x}^i is an optimal solution for (CEP).

Future implementation in Pypsa

We are currently implementing this and other stochastic methods within the Pypsa [PyPSA] framework using the Linopy [Hofmann2023] modeling package in Python.

Future implementation in Pypsa

- We are currently implementing this and other stochastic methods within the Pypsa [PyPSA] framework using the Linopy [Hofmann2023] modeling package in Python.
- We expect improved convergence speed respect to the L-shaped method, especially when leveraging parallel processing capabilities.

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Thank you for your attention.

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https://www.compopt.it

Some references:

Adequacy Assessment of the Electrical Grid

- Measuring the ability of the electric power system to react to adverse uncertain condition has become increasingly importan.
- Member States wishing to introduce capacity mechanisms can do so if an adequacy concern is identified in the ERAA study, a pan-European adequacy assessment for up to 10 years ahead.
- Due to the scale of the ERAA study, ERAA 2022 considered a reduced stochastic problem with three scenarios.
- In [DecompAlg], Daniel A'vila introduced a decomposition algorithm based on subgradient approximations was introduced

Power Grid Optimization

Stochasticity ——Time / Exactness —

Optimal Power Flow (OPF)

[MathProgForm]

- AC OPF: exact physical model
- Security-Constrained OPF (SCOPF) Includes contingencies to guarantee system security under failures.
- DC OPF and other linearized models

 $[\mathsf{LinRelBien}]$

- other relaxations.
- Unit Commitment Determines on/off status of power units, ignoring grid constraints.
- Economic Dispatch (ED) Minimizes generation cost, ignoring grid constraints.

Capacity expansion problem: Based on Economic Dispatch models with added flow balance at bus nodes and various scenarios.