Stucture-preserving constraints aggregations in LP problems:

Application to Renewable Energy Grids with Hydrogen Storage

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28 February 2025

Part I

Motivation

The Problem

The Problem
(it's a big one)

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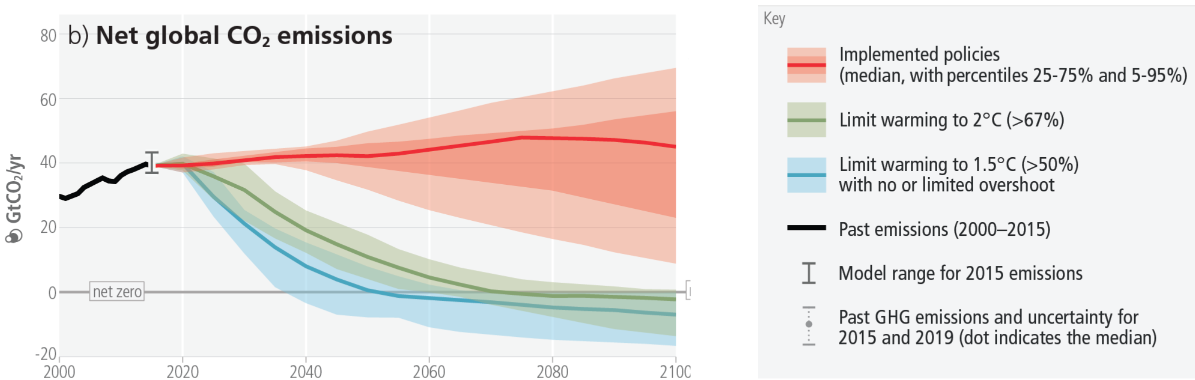

Figure. CO2 Emissions from the IPCC Report [1]

[1] IPCC, 2023: Climate Change 2023: Synthesis Report. Available at: `https://www.ipcc.ch/report/ar6/syr/`


Everything is Fine

Energy Grid Transition

One of the challenges of transitioning to a fully renewable energy grid comes from two Ducks:

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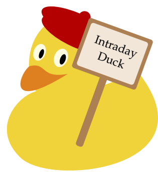

Duck one

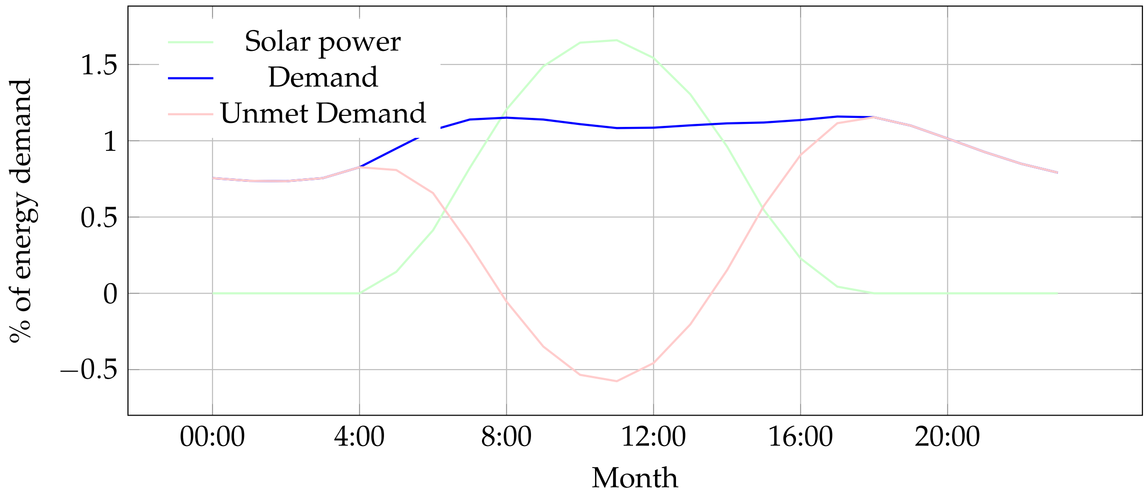

Figure. Comparison of Daily Solar Power Generation, Energy Demand, and Net Energy Demand in Italy.[2]

[2]The Demand, and Solar Power data is taken from the Ninja Dataset at: `https://www.renewables.ninja/downloads`

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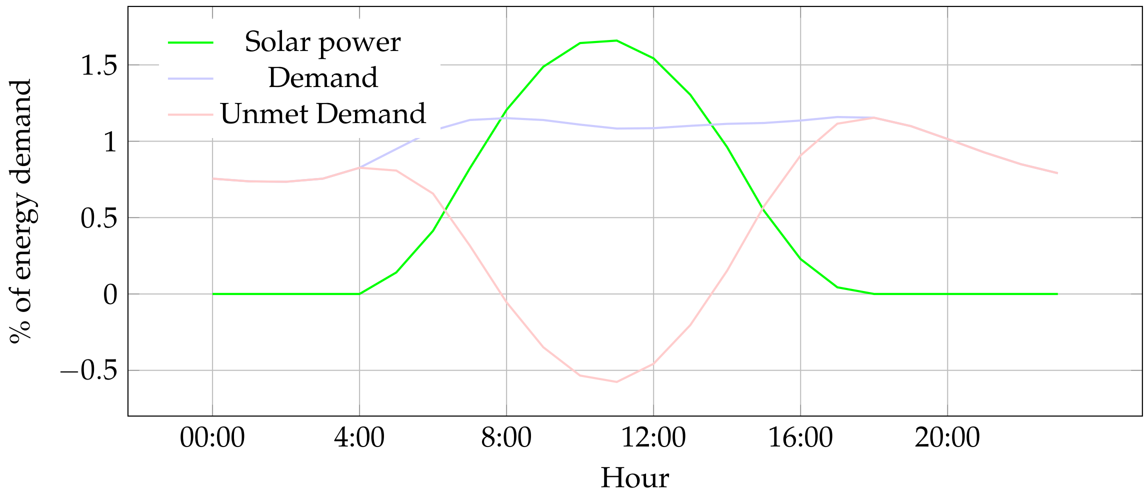

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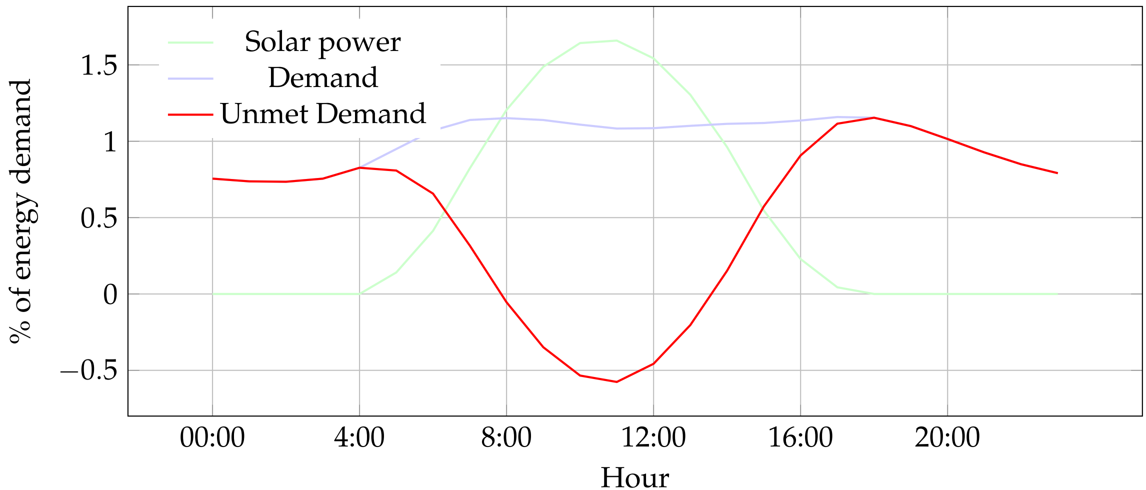

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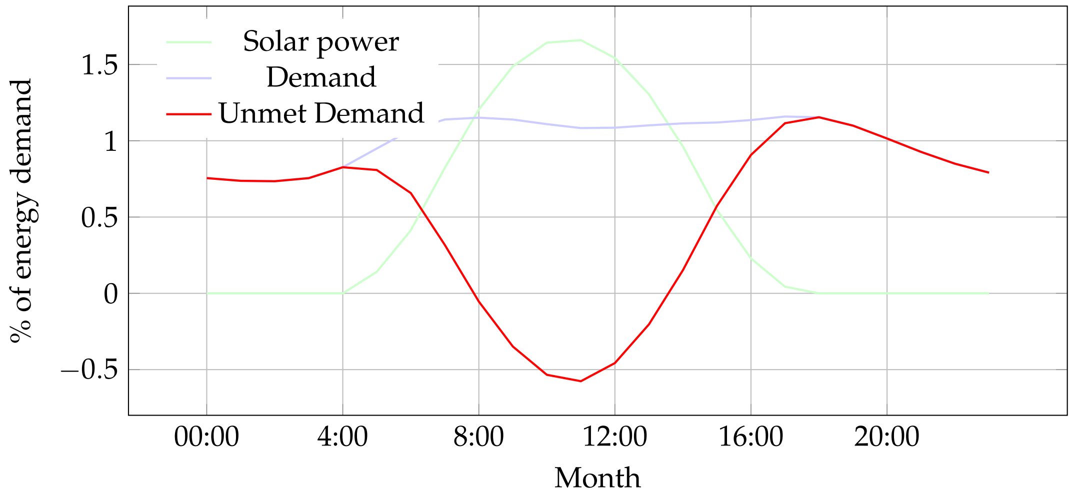

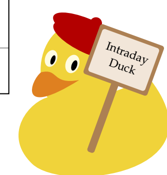

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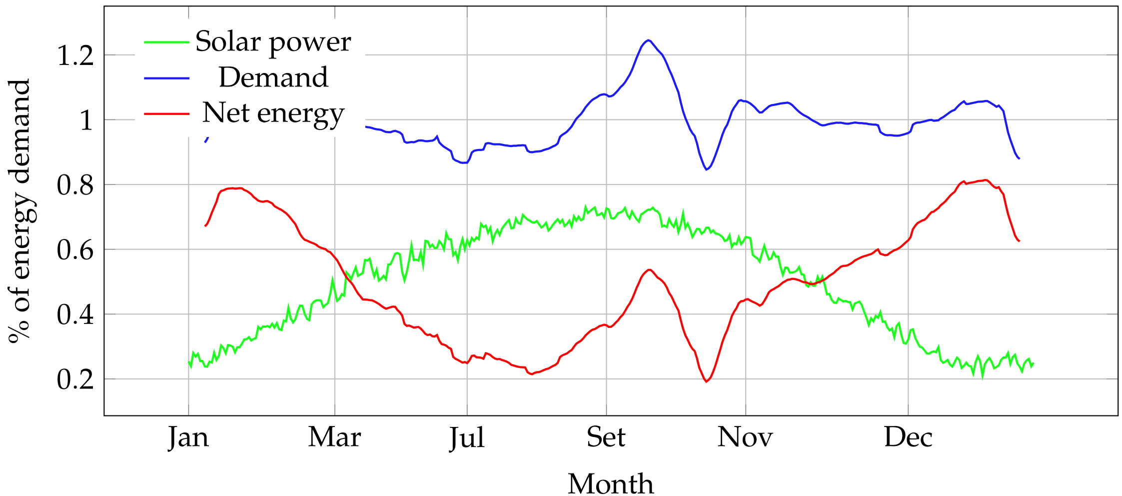

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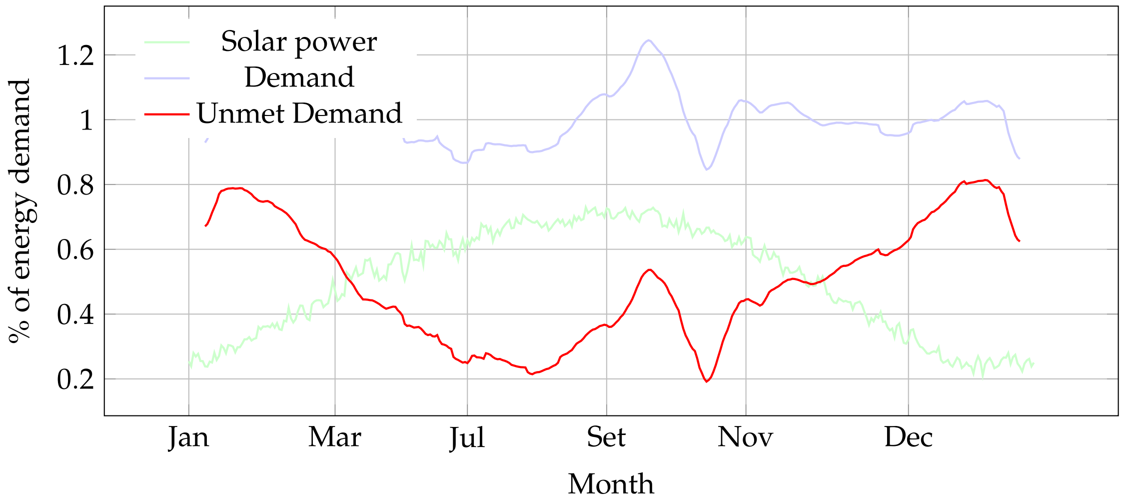

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Capacity Expansion Problem

$$\min_{x} \quad c'x$$

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$\mathcal{Z}(x)$: is the set of feasible soluzions to the Economic Dispatch problem: satisying power and hydrogen flow conservation constraints at each node, and magnitude constraints.

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- This makes the problem very large to solve
- and it cannot be easily simplified by making timesteps longer or with a shorter Time Horizon.
- Objective: Having as few time steps as possible, while capturing intra-day and seasonal variability

Time Series Aggregation

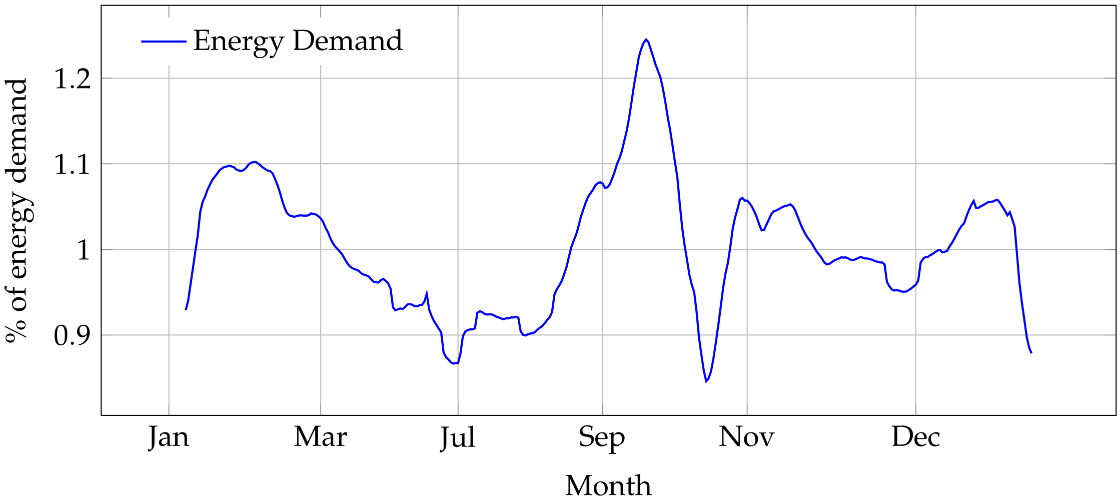

Figure. Energy Demand over a year.

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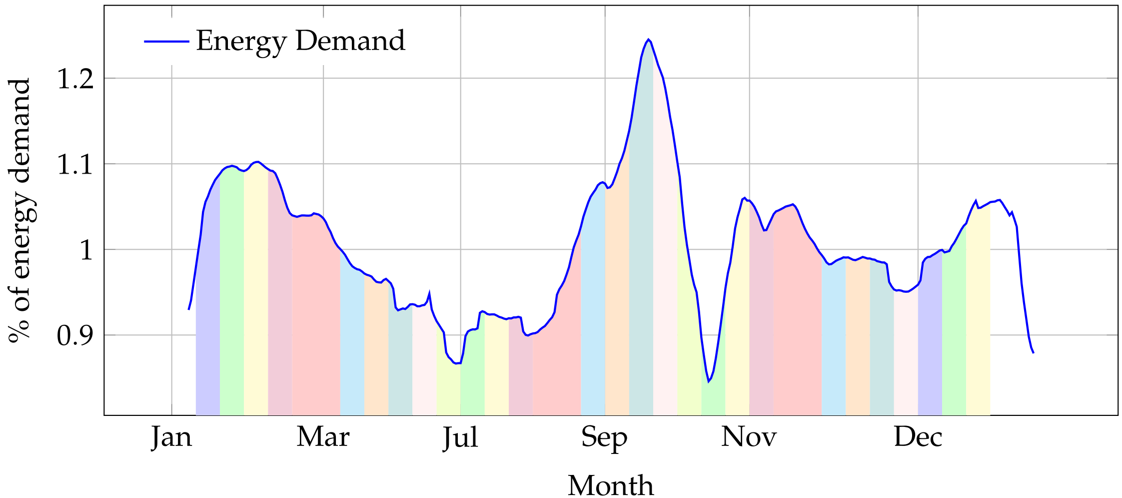

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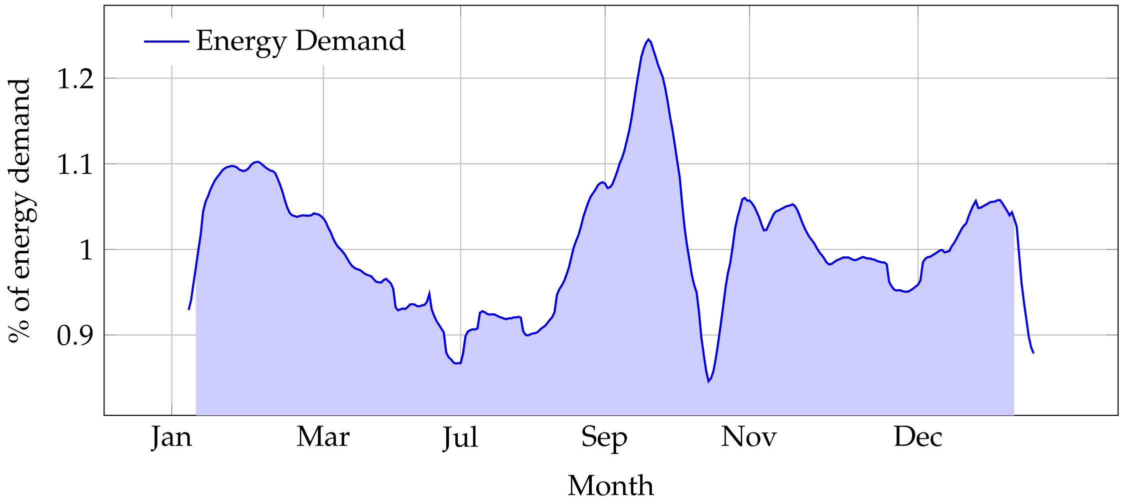

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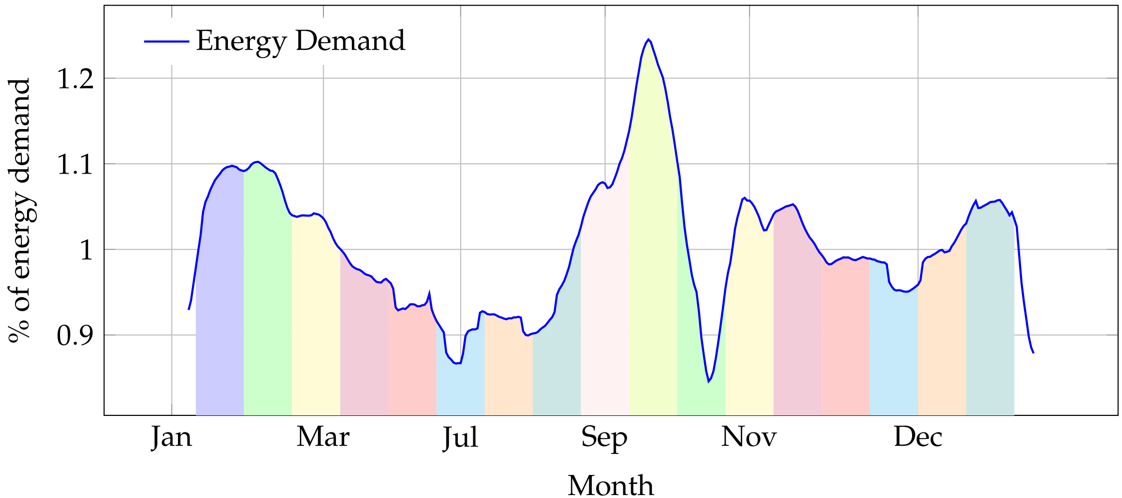

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Time Series Aggregation

Selecting the length of the time steps corresponds to picking a partition $\mathcal{P}$ of the original time horizon $\mathcal{T}$:

$$\mathcal{T} = \{1, 2, 3, \dots, T_{\max}\}$$

$$\Big\downarrow \text{aggregate time horizon}$$

$$\mathcal{P} = \Big\{\{1, \dots, t_1\}, \dots, \{t_{k-1}, \dots, T_{\max}\}\Big\}$$

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For each time partition $\mathcal{P}$ we can define the corresponding Capacity Expansion Problem CEP. For example if $\{1, 2\} \in \mathcal{P}$:

$$\mathrm{CEP}_{\mathcal{T}} \xrightarrow{\text{aggregation}} \mathrm{CEP}_{\mathcal{P}}$$

$$z_1^{(e\to h)}, z_2^{(e\to h)} \longrightarrow z_{\{1,2\}}^{(e\to h)}$$

QUESTIONS

1) In what relationships are the Capacity Expansion Problems of different partitions?

2) When can we obtain a solution of $\mathrm{CEP}_{\mathcal{T}}$ given a solution of the aggregated problem $\mathrm{CEP}_{\mathcal{P}}$?

3) Can we effectively iterate over finer time partitions to obtain a solution of $\mathrm{CEP}_{\mathcal{T}}$?

Row and Column aggregation of CEP

$$E^{(e)}_{i\omega t_1} x^{(p)} + W^{(e)}_{i\omega t_1} x^{(w)} - z^{(e\to h)}_{t_1} + z^{(h\to e)}_{t_1} + z^{(e)}_{jt_1} = L^{(e)}_{i\omega t_1}$$

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Row aggregations are always relaxations of the original problem.

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Column aggregation are relaxations[3] *if the non-zero rows of the of the aggregated columns are equal.*

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Thus given a sequence of finer time partitions $\mathcal{P}_1 \leq \mathcal{P}_2 \leq \ldots \leq \mathcal{P}_n = \mathcal{T}$ we obtain tighter and tigher relaxations $\mathrm{CEP}_{\mathcal{P}_i}$ whose optimal cost is eventually equal to the optimal cost of $\mathrm{CEP}_{\mathcal{T}}$.

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General definition of Structure Preserving Aggregation

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$$A_{r,c} = \tilde{A}_{R,f(c)} \quad \textit{for all } c \in \mathrm{supp}(A_r)_{>1}.$$

(2) When can we obtains a solution of $\text{CEP}_{\mathcal{T}}$ given a solution of the aggregated problem $\text{CEP}_{\mathcal{P}}$?

Given a solution $\tilde{x}$ of the aggregated problem $\text{CEP}_{\mathcal{P}}$, we want to extend it to a solution x of $\text{CEP}_{\mathcal{T}}$. What is a good candidate solution? We keep unaggregated variables the same, that is:

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Proposition 1

Is such a solution x of $\text{CEP}_{\mathcal{T}}$ is well defined, and $\rho_t \geq 0$, then it is optimal.

Generalization to structure preserving aggregations

Given a solution $\tilde{x}$ of the aggregated problem $\tilde{\text{LP}}$, we want to extend it to a solution x of LP.
We keep unaggregated vairiables the same, that is, if $\{c\} \in \delta$, then $x_c = \tilde{x}_c$.
Otherwise, for a given aggregated variable $c \in C$ in the support of the row r we search solution of the form:

$$x_c := \rho_r \tilde{x}_{f(c)} \tag{1}$$

Then:

$$A_r x = A_{r,\delta_{=1}} x_{\delta_{=1}} + \sum_{c \in \text{supp}(A_r)_{>1}} \mathcal{A}_{r,c} x_c = A_{r,\delta_{=1}} x_{\delta_{=1}} + \sum_{c \in \text{supp}(A_r)_{>1}} \tilde{A}_{R,f(c)} x_c \tag{2}$$

$$= A_{r,\delta_{=1}} \tilde{x}_{\delta_{=1}} + \rho_r \sum_{C \in \text{supp}(\tilde{A}_R)_{>1}} \tilde{A}_{R,C} \tilde{x}_C = b_r \tag{3}$$

Since $\sum_{C \in \text{supp}(\tilde{A}_R)_{>1}} \tilde{A}_{R,C} \tilde{x}_C = \tilde{b}_r - \tilde{A}_{R,\delta_{=1}} \tilde{x}_{\delta_{=1}}$, the equality holds if and only if:

$$\rho_r := \frac{b_r - A_{r,\delta_{=1}} \tilde{x}_{\delta_{=1}}}{\tilde{b}_r - \tilde{A}_{R,\delta_{=1}} \tilde{x}_{\delta_{=1}}}. \tag{4}$$

Generalization to structure preserving aggregations

Proposition 2

Let $\tilde{x}$ be a solution to the aggregated problem, define

$$\rho_r := \frac{b_r - A_{r,\delta_{=1}}\tilde{x}_{\delta_{=1}}}{\tilde{b}_r - \tilde{A}_{R,\delta_{=1}}\tilde{x}_{\delta_{=1}}}.$$

If $\rho_r \geq 0$ and $x \in \mathbb{R}^n$ satisfies $x_{\delta_{=1}} = \tilde{x}_{\delta_{=1}}$ and $x_c = \rho_r \tilde{x}_{f(c)}$ for all $c \in \mathrm{supp}(A_r)_{>1}$, then x satisfies the constraints $A_r x = b_r$ and $x_{\mathrm{supp}(A_r)} \geq 0$ of the original problem.

Whenever such a solution is well defined we refer to it as a ρ-solution.

(3) Can we effectively iterate over finer time partitions to obtain a solution to $\mathrm{CEP}_{\mathcal{T}}$?

1. Impose the constraints relative to an initial time partition and solve the corresponding LP.
2. Select a time interval such that either $\rho_r < 0$ or the ρ-solution is not well defined and refine it into smaller sub-intervals.
3. Add the constraints relative to each sub-interval of the selected interval. Solve the model again but using a warm-start.
4. Repeat steps 2 and 3 until a specified halting condition is met.

Halting condition:

1. ρ_r is constant over the hypergraph associated to the aggregated problem $\mathrm{CEP}_{\mathcal{P}}$and and $\rho \geq 0$.
2. A maximum number of iterations is reached

Observation 3

If the algorithm alts before the second condition is met, then the ρ-solution is well defined and is an optimal solution for $\mathrm{CEP}_{\mathcal{T}}$.

Computational Results

- 5-node network simulation
- Comparison of random vs heuristic-based refinement
- Faster convergence with structure-preserving methods

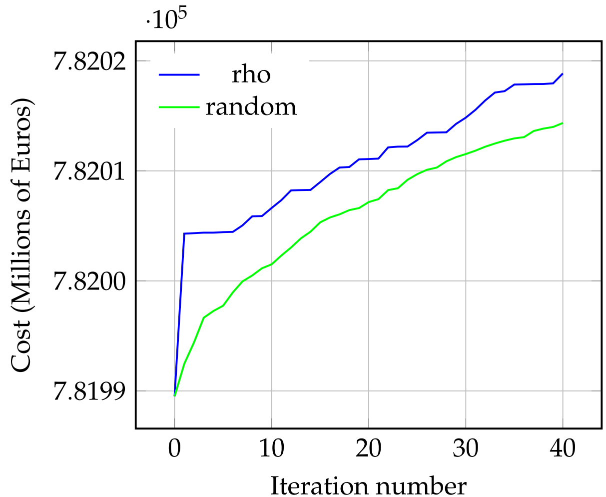

Figure. Cost over iterations of rho selection method versus random selection method

RESULTS

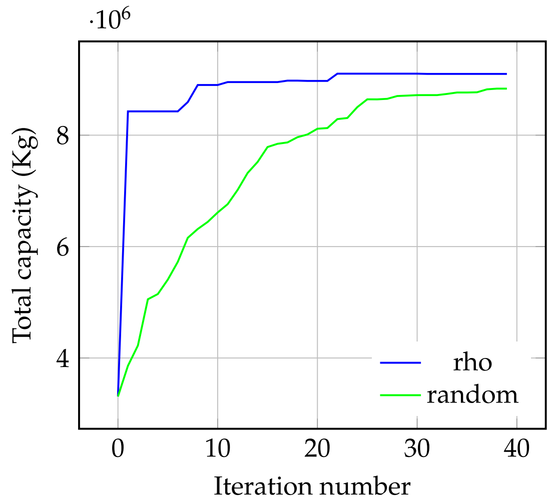

Figure. Hydrogen to Power Capacity over iterations

Conclusion

- Time series aggregation reduces computational costs
- Preserves structure and accuracy
- The index ρ can also be interpreted as a fractional net power production index.
- Future direction: Is there always a non trivial time aggregation which induces a tight relaxation of $\mathrm{CEP}_{\mathcal{T}}$?