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Idea

For the OPF model construction it is convenient to model the network as directed graph $(\mathbf{B}, \mathbf{L})$ where $\mathbf{B}$ is the set of Buses and and $\mathbf{L} \subset \mathbf{B} \times \mathbf{B}$ is the set of branches of the network and for each adjacent buses k, m both (k, m) and (m, k) are in $\mathbf{L}$. So the line l adjacent to k, m is modeled by two edges in the arc $\{(k, m), (m, k)\}$. L can be partitioned in L_0 and L_1 with $|L_0| = |L_1|$ where every line l, adjacent to the buses k, m and with a transformer at k, is oriented so that $(k, m) \in L_0$ and $(m, k) \in L_1$. We also consider a set $\mathcal{G}$ of generators, partitioned into (possibly empty) subsets $\mathcal{G}_k$ for every bus $k \in \mathbf{B}$. We consider the following convex Jabr relaxation of the OPF problem:

$$\inf_{\substack{P_g^G, Q_g^G, c_{km}, \\ s_{km}, S_{km}, P_{km}, Q_{km}}} \sum_{g \in \mathcal{G}} F_g(P_g^G) \tag{1}$$

Subject to: $\forall km \in \mathbf{L}$

$$c_{km}^2 + s_{mk}^2 \leq c_{kk} c_{mm} \quad \text{Jabr constraint} \tag{2}$$

$$P_{km} = G_{kk} c_{kk} + G_{km} c_{km} + B_{km} s_{km} \tag{3}$$

$$Q_{km} = -B_{kk} c_{kk} - B_{km} c_{km} + G_{km} s_{km} \tag{4}$$

$$S_{km} = P_{km} + j Q_{km} \tag{5}$$

Power balance constraints: $\forall k \in \mathbf{B}$

$$\sum_{km \in L} S_{km} + P_k^L + i Q_k^L = \sum_{g \in \mathcal{G}(k)} P_g^G + i \sum_{g \in \mathcal{G}(k)} Q_g^G \tag{6}$$

Power flow, Voltage, and Power generation limits:

$$P_{km}^2 + Q_{km}^2 \leq U_{km} \tag{7}$$

$$V_k^{\min^2} \leq c_{kk} \leq V_k^{\max^2} \tag{8}$$

$$P_g^{\min} \leq P_g^G \leq P_g^{\max} \tag{9}$$

$$c_{kk} \geq 0 \tag{10}$$

$$c_{km} = c_{mk}, \; s_{km} = -s_{mk}. \tag{11}$$

Such relaxation is exact on tree Networks (also known as radial networks). Our objective is, given a network $\mathcal{N} = (\mathbf{B}, \mathbf{L})$ which can also not be a tree, consider a radial subnetwork $\mathcal{N}' = (\mathbf{B}, \mathbf{L}')$, with $\mathbf{L}' \subset \mathbf{L}$ and consider the Jabr model on $\mathcal{N}'$. This solution is not necessarily feasible for the original problem $\mathcal{N}$, our objective is to iteratively recover a feasible solution for $\mathcal{N}$. Since the Jabr relaxation is exact on $\mathcal{N}'$ it follows that the constraint 2