Review of "A Novel Numerical Method for Solving Nonlinear Partial Differential Equations"

Reviewer #1

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Summary

The paper presents a new numerical method for solving nonlinear partial differential equations (PDEs) arising in fluid dynamics. The authors introduce an adaptive mesh refinement technique combined with a modified finite element approach to improve computational efficiency and accuracy. The method is tested on several benchmark problems, demonstrating superior performance compared to existing methods.

Major Comments

1. Clarity of the Numerical Method (Sections 2 and 3)

While the proposed numerical method is promising, the description lacks sufficient detail for full comprehension and reproducibility. Specifically, the algorithm for the adaptive mesh refinement is not thoroughly explained. Including pseudocode or a flowchart would greatly enhance understanding.

2. Comparative Analysis with Existing Methods (Section 5)

The paper compares the proposed method with only one existing method. To strengthen the validation, it would be beneficial to include comparisons with other standard methods, such as spectral methods or finite difference methods, especially those commonly used for similar types of PDEs.

3. Convergence Analysis (Section 4)

The convergence analysis is brief and lacks rigorous proof. Providing a detailed theoretical convergence analysis, along with error estimates, would solidify the method's reliability.

4. Applicability to Higher-Dimensional Problems

The method is tested on one-dimensional and two-dimensional problems. It would be valuable to discuss the potential challenges and modifications required to extend the method to three-dimensional PDEs.

Minor Comments

1. Abstract

The abstract mentions "excellent results" without specifying metrics. Consider providing quantitative results to support this claim.

2. Page 3, Paragraph 2

The notation for the function spaces is inconsistent. Sometimes H^1 is used, and other times \mathcal{H}^1 . Please standardize the notation.

3. Equation (12)

There appears to be a typographical error in the discretization term. The index should be n+1 instead of n.

4. Figure 2

The labels on the axes are too small to read clearly. Increasing the font size would improve readability.

5. References

Several key references on adaptive mesh refinement are missing, such as those by Smith et al. (2015) and Lee and Wong (2017). Including these would provide a more comprehensive background.

Suggestions for Improvement

- **Enhance Method Description**: Include detailed pseudocode for the adaptive mesh refinement algorithm and the modified finite element approach. This will aid in reproducibility and allow other researchers to implement the method.
- **Expand Comparative Studies**: Add comparisons with additional numerical methods, and include performance metrics such as computation time, accuracy, and convergence rates.
- **Detailed Convergence Proof**: Provide a thorough convergence analysis with mathematical proofs and discuss the conditions under which the method is guaranteed to converge.
- **Three-Dimensional Extension**: Discuss how the method could be adapted for three-dimensional problems, including any potential computational challenges.
- **Language and Grammar**: Review the manuscript for minor grammatical errors and improve the overall language flow. For example, on page 5, "the method prove to be effective" should be "the method proves to be effective."

Recommendation

I recommend **major revision** of the paper. The proposed numerical method has the potential to make a significant contribution to the field of computational fluid dynamics. However, the manuscript requires substantial improvements in the clarity of the method description, a more rigorous convergence analysis, and expanded comparative studies to fully demonstrate its effectiveness.