

# A Parallelization Algorithm for Adequacy Assessment of the Electrical Grid



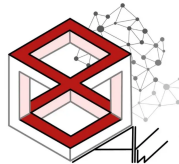
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Joint Research Centre



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**UNIVERSITY OF CALABRIA**

*"Emerging technologies for decision support  
systems and innovative optimization paradigms"*

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- The first stage determines the capacity expansion  $x_{n,g}$  for each generator  $g \in \mathcal{G}$
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Where  $\mathcal{V}(x, \omega)$  is the solution to (ED) in function of the expanded capacities  $x$  and the scenario  $\omega$ .

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$$\min_y q' y_\omega \quad (1)$$

$$\begin{aligned} s.t. \quad p_{n,g,t,\omega} + bd_{n,t,\omega} + \sum_{l \in \mathcal{L}(n)} f_{n,l,t,\omega} + ls_{n,t,\omega} + \mathcal{PV}_{n,t,\omega} + \mathcal{W}_{n,t,\omega} = \\ = \mathcal{D}_{n,t,\omega} + s_{nt,\omega} + bc_{n,t,\omega} \end{aligned} \quad (2)$$

$$v_{n,t,\omega} = v_{n,t-1,\omega} + BCE \cdot bc_{n,t,\omega} - BDE \cdot bd_{n,t,\omega} + A_{n,t,\omega} \quad (3)$$

$$(v_{n,t,\omega}, bc_{n,t,\omega}, bd_{n,t,\omega}) \leq (BV, BC, BD) \quad (4)$$

$$p_{n,g,t,\omega} \leq p_{n,g}^{\max} + x_{n,g} \quad (5)$$

$$L_{n,l}^{\min} \leq f_{n,l,t,\omega} \leq L_{n,l}^{\max} \quad (6)$$

Where  $v_{n,t,\omega}$  is the power stored at bus  $n$  at time  $t$ .

## Model relaxation description: Intermediate Economic Dispatches (ED-k)

- We divide the time horizon into  $K$  intervals,  $\{t_0 = 0 := 1, \dots, t_1\}, \{t_1 + 1, \dots, t_2\}, \dots, \{t_{K-1} + 1, \dots, t_K := T\}$

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- We refer to the (ED) problems restricted to each time interval with fixed initial and final storage values as **(ED-k)** and to its optimal values as and with optimal value  $\mathcal{V}_k(\mathbf{x}, \mathbf{v}_{t_k}, \mathbf{v}_{t_{k+1}}, \omega)$

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### Observation

$$\mathcal{V}(x, \omega) = \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^{K-1} \mathcal{V}_k(x, v_{t_k}, v_{t_{k+1}}, \omega) \quad (7)$$

## Model relaxation description: Lower Approximation of (ED)

Since each function  $\mathcal{V}_k$  is piecewise linear convex in  $x, v_{t_K}, v_{t_{K+1}}$ , it can be approximated by a collection of supporting hyperplanes  $\{\pi_{i,k}^w(x, v_{t_k}, v_{t_{k+1}})\}$  of each  $\mathcal{V}_k$ .



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$$\begin{aligned}\hat{\mathcal{V}}(x, \omega) &= \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^K \hat{\mathcal{V}}_k(x, v_{t_k}, v_{t_{k+1}}) = \\ &= \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^K \theta_k^\omega \quad \quad \quad (\text{ISP}) \\ \text{s.t. } \quad &\theta_k^\omega \geq \pi_{i,k}^\omega(x, v_{t_k}, v_{t_{k+1}}) \quad \forall i, k\end{aligned}$$

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(I know, very original)

## Model description: Relaxed Capacity Expansion(CEP-R)

$$\begin{aligned} \min_x \quad & c'x + \mathbb{E}_\omega [\mathcal{V}(x, \omega)] \\ \text{s.t.} \quad & 0 \leq x_{n,g} \leq X_{n,g} \end{aligned} \quad (\text{CEP})$$

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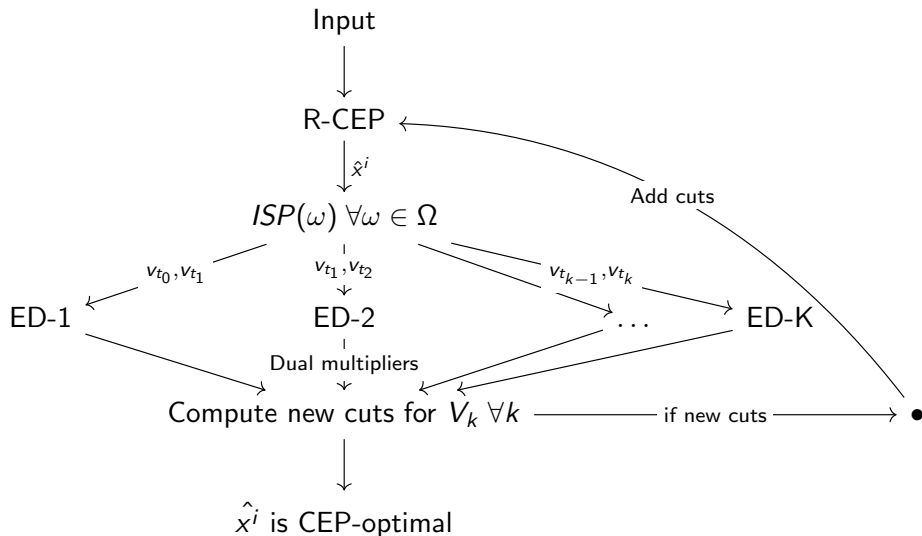
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Since calculating  $\hat{\mathcal{V}}$  is straightforward, solving (CEP-R) can be done efficiently with L-shaped or subgradient schemes.

# Algorithm



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**Remark 1:** It is sufficient to prove that after a finite number of steps ( $i$ ) of the algorithm we have:

$$\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega) \text{ for all } \omega \in \Omega \quad (8)$$

## Convergence results 3

### Observation

*After a finite number of iterations no new cuts are found for  $\mathcal{V}_k$ .*

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$$\begin{aligned} \#\{p \mid p \text{ is a normal vector of a supporting hyperplane of } \mathcal{V}_k\} &\leq \\ \#\{\text{dual solutions } p = q' B^{-1} \text{ of (ED-k) for varying } x, v_{t_k}, v_{t_{k+1}}\} &\leq \end{aligned} \tag{9}$$

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- new cut:  $\bar{c}(x, v) = p'(x, v) + b$

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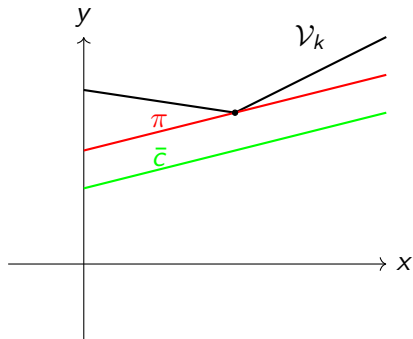
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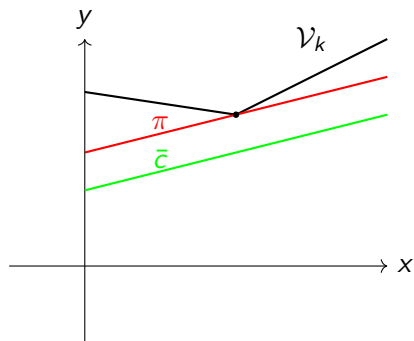




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Since both are supporting hyperplanes it follows that  $b = \bar{b}$   
(and therefore  $\bar{c}$  is not a new cut).

## Convergence results 4

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*If after the  $i$ -iteration no new cuts are added for some  $i$  and  $k$  then*

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### Proof.

Let  $\bar{c}_k^\omega(x, v_{t_k}) := p'(x - \hat{x}^i, v_{t_k} - \hat{v}_{t_k}) + V_k(\hat{x}^i, \hat{v}_{t_k})$  be the new cut found after the  $i$ -th iteration.

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which concludes the proof. □



## Convergence results 5

In conclusion, we have  $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$  for all  $\omega, k$ .

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Thus  $\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega)$ .

### Proposition

*The algorithm converges after a finite number of iterations and  $\hat{x}^i$  is an optimal solution for (CEP).*

## Future implementation in Pypsa

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- We expect improved convergence speed respect to the L-shaped method, especially when leveraging parallel processing capabilities.

## Conclusions and future perspectives.

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Thank you for your attention.

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`https://www.compopt.it`

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# Adequacy Assessment of the Electrical Grid

- Measuring the ability of the electric power system to react to adverse uncertain condition has become increasingly important.
- Member States wishing to introduce capacity mechanisms can do so if an adequacy concern is identified in the ERAA study, a pan-European adequacy assessment for up to 10 years ahead.
- Due to the scale of the ERAA study, ERAA 2022 considered a reduced stochastic problem with three scenarios.
- In [Ávi+23], Daniel A'vila introduced a decomposition algorithm based on subgradient approximations was introduced

# Power Grid Optimization

Stochasticity —  
Time / Exactness ↑

- Optimal Power Flow (OPF) [Bie+20]
  - AC OPF: exact physical model
  - Security-Constrained OPF (SCOPF) – Includes contingencies to guarantee system security under failures.
  - DC OPF and other linearized models [BM14]
  - other relaxations.
- Unit Commitment – Determines on/off status of power units, ignoring grid constraints.
- Economic Dispatch (ED) – Minimizes generation cost, ignoring grid constraints.

**Capacity expansion problem:** Based on Economic Dispatch models with added flow balance at bus nodes and various scenarios.