



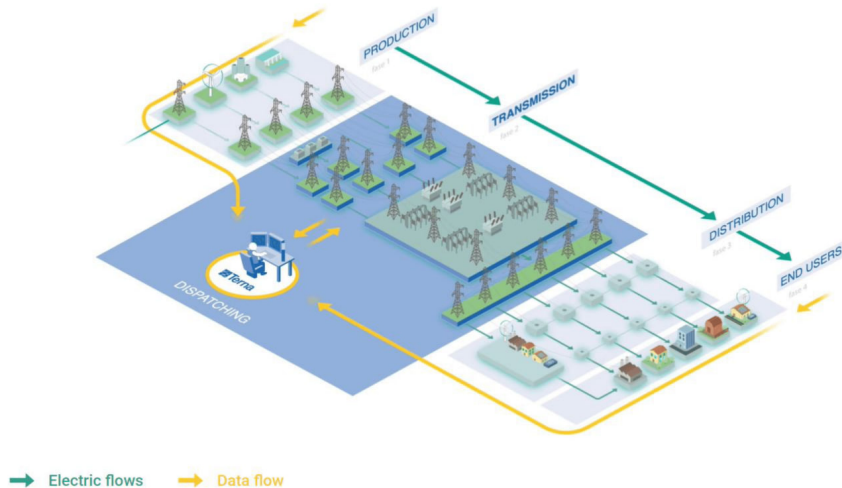
Optimization problems in Power Systems

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Optimal Power Flow problems



AC-OPF formulations

Optimization methods

Compact formulation

$$\inf_{\mathbf{u}, \mathbf{S} \in \mathbb{C}^{\mathcal{B}}} c(\mathbf{u}, \mathbf{S}) \quad (1)$$

s.t.

$$\text{diag}(\bar{\mathbf{u}}) \mathbf{Y} \mathbf{u} = \mathbf{S} \quad (2)$$

$$\underline{\mathbf{u}} \leq \bar{\mathbf{u}} \quad (3)$$

$$\underline{\mathbf{S}} \leq \bar{\mathbf{S}} \quad (4)$$

Polar OPF formulation

$$\inf_{\substack{P_g^G, Q_g^G, \delta_k \\ |V|_k, S_{km}}} \sum_{g \in \mathcal{G}} F_g(P_g^G) \quad (5)$$

Subject to:

Voltage to Power Flow constraints: $\forall km \in \mathbf{L}$

$$S_{km} = (G_{kk} - iB_{kk})|V_k|^2 + (G_{km} - jB_{km})|V_k||V_m|(\cos(\delta_k - \delta_m) + j \sin(\delta_k - \delta_m)) \quad (6)$$

Power balance constraints:

$$\sum_{km \in \delta(k)} S_{km} = \left(\sum_{g \in \mathcal{G}(k)} P_g^g - P_k^d \right) + i \left(\sum_{g \in \mathcal{G}(k)} Q_g^G - Q_k^L \right) \quad (7)$$

Power flow, Voltage, and Power generation limits:

$$|S_{km}|^2 \leq U_{km}, \quad V_k^{\min} \leq |V_k| \leq V_k^{\max}, \quad P_g^{\min} \leq P_g^G \leq P_g^{\max} \quad (8)$$

$$\theta_{km}^{\min} \leq \delta_k - \delta_m \leq \theta_{km}^{\max} \quad (9)$$

Methods for Solving AC-OPF

1. Classical Methods

- ▶ **Newton-Raphson:** Iterative, relies on solving nonlinear equations.
- ▶ **Interior Point Methods (IPMs):**
 - ▶ Solve Karush-Kuhn-Tucker (KKT) conditions.
 - ▶ Efficient for large-scale systems.

2. Relaxation Techniques

- ▶ **Semidefinite Programming (SDP):**
 - ▶ Convex relaxation.
 - ▶ Provides bounds on global optimum.
- ▶ **Second-Order Cone Programming (SOCP):**
 - ▶ Weaker relaxation than SDP.
 - ▶ Faster, scalable for large systems.
- ▶ **Quadratic Programming (QP):**
 - ▶ Linearizes power flow equations.

Global Optimization Techniques

3. Global Optimization Methods

▶ **Branch-and-Bound:**

- ▶ Systematically explores subproblems.
- ▶ Guarantees global solution.

▶ **Heuristics:**

- ▶ Genetic Algorithms, Simulated Annealing.
- ▶ Useful for obtaining good feasible solutions.

Conclusion

Choosing a Method:

- ▶ **Classical methods:** Efficient but may converge to local optima.
- ▶ **Relaxations:** Provide bounds but not always feasible solutions.
- ▶ **Global methods:** "Ensure" global optimality but computationally intensive.

Trade-off between accuracy and computational effort.