A Parallelization Algorithm for Adequacy Assessment of the Electrical Grid

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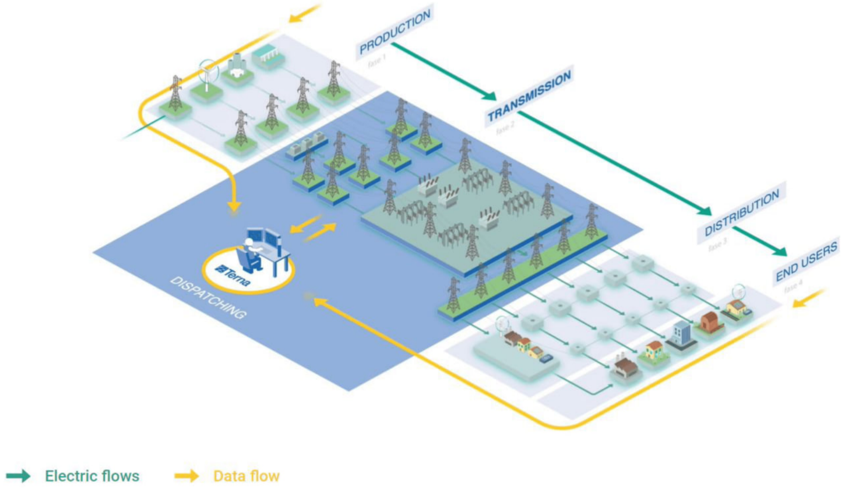
Power Grid Optimization problems 1



Power Grid Optimization: From Simple to Exact Models

Stochasticity

Time / Exactness

- Optimal Power Flow (OPF) [Bie+20]
  - AC OPF: exact physical model
  - Security-Constrained OPF (SCOPF) – Includes contingencies to guarantee system security under failures.
  - DC OPF and other linearized models [BM14]
  - other relaxations.
- Unit Commitment – Determines on/off status of power units, ignoring grid constraints.
- Economic Dispatch (ED) – Minimizes generation cost, ignoring grid constraints.

Adequacy Assessment Model: Based on Economic Dispatch models with added flow balance at bus nodes and various scenarios.

Economic Dispatch (ED) model*

For a fixed scenarios w let $y_w = (p_w, f_w, ls_w)'$ be the vector containing the power genaration, power flows and line shedding variables.

$$\min_{y} \; q' y_w \tag{1}$$

$$s.t. \; p_{n,g,t,w} \leq x_{n,g} \tag{2}$$

$$L_{n,l}^{\min} \leq f_{n,l,t,w} \leq L_{n,l}^{\max} \tag{3}$$

$$v_{n,t,w} = v_{n,t-1,w} + BCE \cdot bc_{n,t,w} - BDE \cdot bd_{n,t,w} + A_{n,t,w} \tag{4}$$

$$(v_{n,t,w}, bc_{n,t,w}, bd_{n,t,w}) \leq (BV, BC, BD) \tag{5}$$

$$p_{n,g,t,w} + bd_{n,t,w} + \sum_{l \in \mathcal{L}(n)} f_{n,l,t,w} + ls_{n,t,w} + \mathcal{PV}_{n,t,w} + \mathcal{W}_{n,t,w} = \tag{6}$$

$$= \mathcal{D}_{n,t,w} + ps_{nt.w} + bc_{n,t,w} \tag{7}$$

Where $w = (\mathcal{PV}, \mathcal{W}, \mathcal{D})$ represents the scenario realization of solar power, wind power and loads

Stochastic Capacity Expansion Problem (CEP)

Let $\mathcal{V}(x, w)$ be the solution to (ED) in function of the expanded capacities x and the scenario w. We formulate the Stochastic Capacity Expansion Problem as a two-stage stochastic program.

- The first stage determines the capacity expansion $x_{n,g}$ for each generator $g \in \mathcal{G}$
- The second stage solves the (ED).

$$\begin{aligned} &\min_x c'x + \mathbb{E}_w\left[\mathcal{V}(x, w)\right] \\ &s.t.\ 0 \leq x_{n,g} \leq X_{n,g} \end{aligned} \tag{CEP}$$

Literature

- Traditional stochastic capacity expansion methods, such as the L-shaped method, may perform poorly as the number of expansion possibilities increases.
- A decomposition algorithm based on subgradient approximations was introduced by Daniel A'vila et al in [Ávi+23]
- Building upon this work, we propose another subgradient approximation algorithm to enhance the decomposition approach.
- We take advantage of the time steps' general independence, except for the constraints related to batteries and storage, which rely on adjacent time steps.
- Our approximation is refined throught iterations to ensure convergence within a finite number of steps.

Model description 1

- We divide the time horizon into K intervals, $\{t_0 = 0 := 1, \ldots, t_1\}, \{t_2, \ldots, t_3\}, \ldots, \{t_{K-1}, T\}$
- If we fix a priori the storage values adding the constraints $v_{t_k} = \bar{v}_{t_k}$ to (ED) we obtain a solution $V(x, \{v_{t_k}\}_k, w)$ also dependent on these intermediary storage values.
- Then considering the (ED) problems restricted to each interval with fixed initial and final storage values and with optimal value $V_k(x, v_{t_k}, v_{t_{k+1}, w})$.

Observation

$$V(x, w) = \min_{\{v_{t_k}\}_{k=1}^{K}} \sum_{k=0}^{K-1} V_k(x, v_{t_k}, v_{t_k+1}, w) \tag{8}$$

Model description 2

Since each function V_k is peacewise linear convex in $x, v_{t_K}, v_{t_{K+1}}$, given a collection of supporting hyperplanes $\{\pi^w_{i,k}(x, v_{t_k}, v_{t_{k+1}})\}$ of each V_k an approximation of (8) is given by:

$$\hat{V}(x, w) = \min_{\{v_{t_k}\}_{k=1}^K} \sum_{k=0}^{K} \theta^w_k \tag{9}$$

$$\text{s.t.} \quad \theta^w_k \geq \pi^w_{i,k}(x, v_{t_k, t_{k+1}}) \quad \forall i, k \tag{10}$$

Model description 3

Thus by substituting $\mathcal{V}$ with $\hat{\mathcal{V}}$ in (CEP) we obtain the following relaxation:

$$\begin{aligned}&\min_{x} c'x + \mathbb{E}_w\left[\hat{\mathcal{V}}(x, w)\right]\\&s.t.\ 0 \leq x_{n,g} \leq X_{n,g}\end{aligned} \tag{CEP-R}$$

Since calculating $\hat{\mathcal{V}}$ is straightforward, solving (CEP-R) can be done efficiently with L-shaped or subgradient schemes.

Algorithm description

INPUT: Provide a lower bound for θ_k^ω for $k = 1, \dots, K$ and $\omega \in \Omega$ and a trial action $\hat{x}^0$

1. Warm-Start: Calculate initial approximation for $\mathcal{V}$ for all $w \in \Omega$ around $\hat{x}^0$.
2. For $i = 1, \dots, N$:
   2.1. For $w \in \Omega$ (in parallel):
      2.1.1. Solve the (ED) approximation problem $\hat{\mathcal{V}}(\hat{x}^{(i)}, \omega)$ and obtain intermediate storage values $\hat{v}_k^i$ for $k = 1, \dots, K$.
      2.1.2. Solve (ED) (in parallel) for each time step $k = 0, \dots, K-1$, $V_k(\hat{x}^i, \hat{v}_k^i, \hat{v}_{k+1}^i, \omega)$.
      2.1.3. Using dual multipliers, compute a supporting hyperplane for V_k around $\hat{x}^i, \hat{v}_k^i, \hat{v}_{k+1}^i$ for $k = 0, \dots, K-1$.
      2.1.4. Add the supporting hyperplanes to the approximation problem (CEPR) $\hat{\mathcal{V}}(\hat{x}^{(i)}, \omega)$.

Convergence results 2

- Since (CEP-R) is a relaxation of (CEP), an optimal solution $\hat{x}^i$ of (CEP-R) is also optimal for (CEP) iff it's (CEP)-feasible
- Since the first stage constraints of (CEP-R) and (CEP) are the same this is true whenever, $\hat{x}^i$ is feasible for the second stage problems
- This is true whenever $\mathcal{V}(\hat{x}^i, w) = \hat{\mathcal{V}}(\hat{x}^i, w) < +\infty$ for all $\omega \in \Omega$

Remark 1: It is sufficient to prove the after a finite number of steps (i) of the algorithm we have $\hat{\mathcal{V}}(\hat{x}^i, \omega) = \mathcal{V}(\hat{x}^i, \omega)$ for all $\omega \in \Omega$

Convergence results 3

Observation

Let $\hat{\mathcal{V}}_k(x, v_{t_k}, v_{t_{k+1}}, w) = \max_i \pi^w_{i,k}(x, v_{t_k}, v_{t_{k+1}})$ *be the current supporting hyperplane approximation of* $\mathcal{V}_k$ *Let* $\omega \in \Omega$ *and* $k \in \{0, \dots, K-1\}$*. If after step (2.1.3) no new cuts are added then* $\hat{\mathcal{V}}_k(x, v_k, v_{k+1}) = \mathcal{V}_k(x, v_k, v_{k+1})$.

Proof.

Let $\bar{c}^w_k(x, v_{t_k}) := p'(x - \hat{x}^i, v_{t_k} - \hat{v}_{t_k}) + V_k(\hat{x}^i, \hat{v}_{t_k})$ be the new cut found after the i-th iteration.
Since $\bar{c}$ is not a new cut we have $\bar{c}(x, v_{t_k}) \leq \hat{\mathcal{V}}(x, v_{t_k})$.
Since $\mathcal{V}_k \geq \pi^w_{i,k}$, $\mathcal{V}_k \geq \hat{\mathcal{V}}_k$.
We have thus

$$\mathcal{V}_k(\hat{x}, \hat{v}_{t_k}) \geq \hat{\mathcal{V}}_k(\hat{x}, \hat{v}_{t_k}) \geq \bar{c}(\hat{x}, hatv_{t_k}) = \mathcal{V}(\hat{x}, \hat{v}_{t_k})$$

which concludes the proof. $\square$

Convergence results 4

Proposition

After a finite number of iterations no new cuts are found for V_k

Proof.

$$
\begin{aligned}
&\#\{p \mid p \text{ is a normal vector of a supporting hyperplane of } \mathcal{V}_k\} \leq \\
&\#\{\text{dual solutions } p = q'B^{-1} \text{ of (ED) for varying } x, v_{t_k}, v_{t_{k+1}}\} \leq \\
&\#\{\text{basis matrices of (ED)}\} < \infty
\end{aligned} \tag{11}
$$

After a finite number of steps we'll have a new cut $\bar{c}(x, v) = p'(x, v) + b$ with the same normal vector p as a previous cut $\pi(x, v) = p'(x, v) + \bar{b}$. Since both are supporting hyperplanes it follows that $b = \bar{b}$. □

Convergence results 5

In conclusion, we have $\hat{\mathcal{V}}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega) = \mathcal{V}_k(\hat{x}^i, v_{t_k}, v_{t_{k+1}}, \omega)$ for all ω, k. And from the definition of $\hat{\mathcal{V}}$ follows that $\hat{\mathcal{V}}(\hat{x}^i, w) = \mathcal{V}(\hat{x}^i, w)$. From Remark 1, it follows:

Proposition

The algorithm converges after a finite number of iterations.

Pypsa

Implementation coming soon in the PyPsa environment...

Thank you for your attention.

Some references:

[Ávi+23] Daniel Ávila, Anthony Papavasiliou, Mauricio Junca, and Lazaros Exizidis. "Applying High-Performance Computing to the European Resource Adequacy Assessment". In: *IEEE Transactions on Power Systems* (2023), pp. 1–13. DOI: 10.1109/TPWRS.2023.3304717.

[Bie+20] Daniel Bienstock, Mauro Escobar, Claudio Gentile, and Leo Liberti. "Mathematical Programming formulations for the Alternating Current Optimal Power Flow problem". In: *4OR* 18.3 (July 2020), pp. 249–292. DOI: 10.1007/s10288-020-00455-w.

[BM14] Daniel Bienstock and Gonzalo Munoz. "On linear relaxations of OPF problems". In: (Nov. 2014).

further explanation