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**Subject:** *On the integrality gap of the Complete Metric Steiner Tree Problem via a novel formulation*  
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Dear Professor Jong-Shi Pang,

Please find attached our manuscript titled “On the integrality gap of the Complete Metric Steiner Tree Problem via a novel formulation” for consideration in SIAM Journal on Optimization. Part of the work of this paper has already been presented as an extended abstract at the International Symposium on Combinatorial Optimization (ISCO) 2024 and as a poster in an informal poster session at Integer Programming and Combinatorial Optimization (IPCO) 2024.

In this paper, we aim to extend Benoit, Boyd, and Elliott-Magwood’s foundational work on the integrality gap in the Metric Symmetric/Asymmetric Traveling Salesman Problem to the Metric Steiner Tree Problem.

We introduce the Complete Metric formulation, specifically designed to address the weakness of the well-established bidirected cut formulation on metric instances.

Our formulation refine the polytope structure of the Metric Steiner Tree Problem, resulting in a polytope  $P_{CM}$  with a significantly reduced number of vertices compared to the existing directed cut formulation.

We introduce two new heuristic algorithms for enumerating vertices of  $P_{CM}$ , that facilitate the computation of vertices with the largest known integrality gap for instances up to 10 vertices.

Our computational results reveal notable instances, including small-node examples with high integrality gap, which realize the best-known lower bounds on the integrality gap for the directed cut formulation.

Thank you for considering our manuscript, we look forward to receiving your feedback.

Best Regards,

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