Interdependent Evolution of Non-Spectral Opinions and Social Networks

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OVERVIEW

Background and Motivation

- Opinion Formation (e.g. voter models) is a common and very fundamental problem in the social sciences
- Goal: Modelling the *co*evolution of both opinions and the underlying social network
- **Does our social network shape the opinion we hold or does our opinion determine who is part of our network?**
- "Opinion" must be mutually exclusive and "non-spectral", e.g. brand preference, religious views...

- Preview: Analogies to statistical physics, e.g. *phase transitions* can be identified

INITIAL SETUP

- Random graph with N nodes (opinion holder) and M edges (social connection)
- Random opinion $g_i \in G$ assigned to node i
- Nodes exchange information (opinion) via undirected edges
- Externally set parameters:
  - N - number of nodes
  - $\gamma = \frac{N}{G}$ - average number of nodes per opinion
  - $k_{avg} = \frac{2M}{N}$ - average degree

Time Evolution Algorithm

1. Pick a random node i with opinion g_i.
2. (**a**) With probability Φ select at random one of the nodes j that i is connected to.
   - If $g_i = g_j$, start over at step 1.
   - Otherwise, reconnect to a randomly chosen j' of same opinion, i.e. $g_{j'} = g_i$.
3. (**b**) Otherwise, with probability $1 - \Phi$ randomly select one of the neighboring vertices j and change g_i to g_j.
4. Repeat until *consensus state* is achieved.

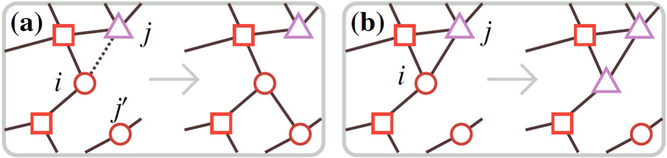
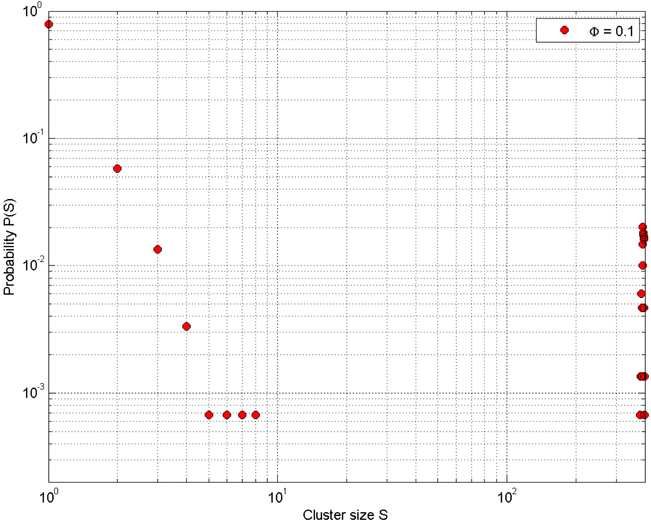
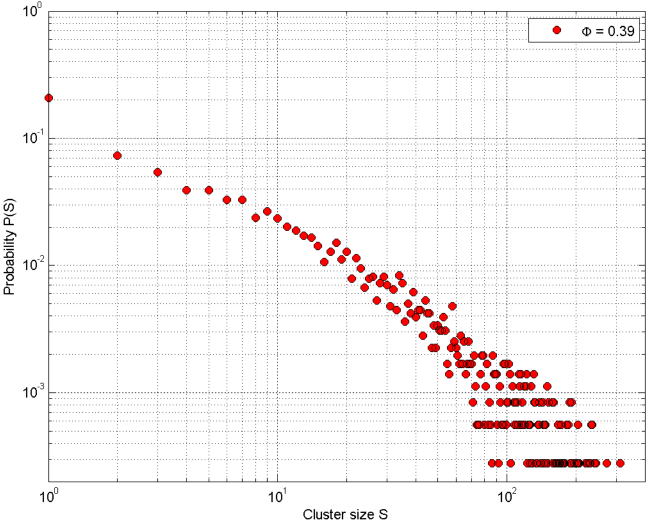


Figure: Holme and Newman, 2006 [5]

CLUSTER SIZE DISTRIBUTION



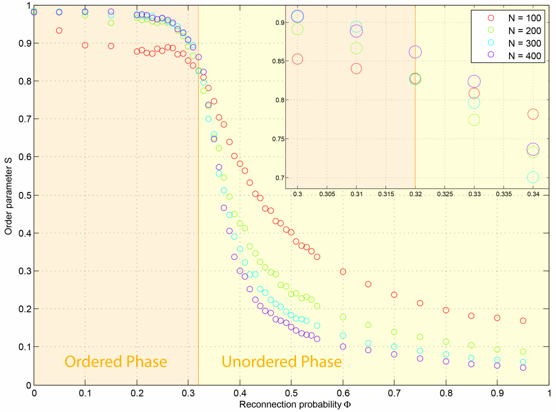
CONTINUOUS PHASE TRANSITION?



Cluster Size Distribution

- Ordered phase
  - Low Φ, i.e. tendency to change opinion
  - Small clusters follow power law distribution
  - Existence of giant cluster
- Unordered phase
  - High Φ, i.e. tendency to keep opinion
  - Clusters follow Poisson-like distribution
  - No giant cluster!
- Phase transition
  - First guess: $\Phi_c = 0.35 \pm 0.05$
  - Power law behavior over the whole s-range

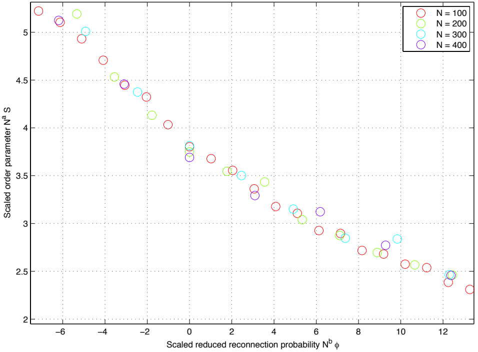
CRITICAL POINT



- Really continuous phase transition
- N limits the range of interaction $\rightarrow$ different slopes
- $\Phi_c = 0.32 \pm 0.02$ independent of system size N
- Weak agreement with $\Phi_c = 0.39 \pm 0.05$

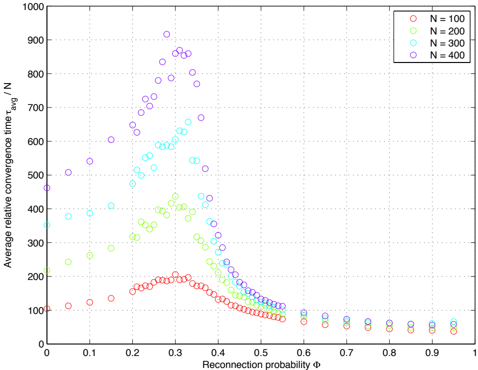
RESCALING

- $N^a S(\varphi) = S(N^b \varphi)$
- N-independence *around* critical point
- Determination of critical exponent $S(\varphi) \sim |\varphi|^{\frac{a}{b}}$
- Possible but rather arbitrary!



Convergence Time

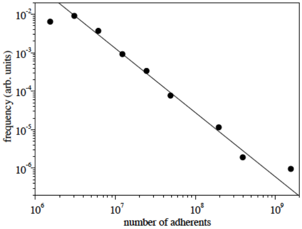
- Iterations per node to reach consensus as function of Φ:
- "Divergence" at some Φ_c for different N
- Similar to divergent response functions in physics
- Supporting phase transition interpretation, but difficult to find direct analogy to τ_{avg}



Comparisons to Empirical Data

- Idea: Compare distributions of some "opinion" in real world to the model $\rightarrow$ identify and interpret corresponding Φ

- **Religion:**
- Worldwide distribution of religions follows power law: Neither adaptation nor reconnection dominate in the formation?

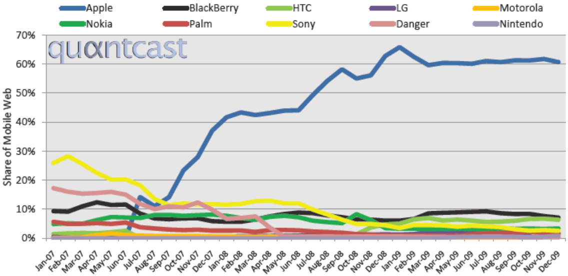


- Interpret Φ as an "intolerance indicator"?

Zanette and Manrubia, 2000 [9]

COMPARISONS TO EMPIRICAL DATA

- **Mobile Web Browsers:**
- An example for opinion = brand preference
- Contrast between giant cluster and "softer" distribution
- Note: Plot is not a cluster size histogram!



- Interpret Φ as a "brand loyalty indicator"?

Quantcast Review, 2009 [7]

SUMMARY

- Interdependent evolution of opinions and networks, combining two mechanisms of adaption and reconnection determined by Φ
- Holme's and Newman's work could be reproduced with more realistic assumptions
- Continuous phase transition
  - N-independent critical value $\Phi_c = 0.32 \pm 0.02$
  - Divergent convergence time at Φ_c
- Rescaling and calculation of critical exponent rather arbitrary.

Outlook

- Variation of γ (diversity) and k_{avg} (density)
- Include analogue of "magnetic field" in model: *"informed agents"*?
- Make opinions *spectral*

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