# Interdependent Evolution of Non-Spectral Opinions and Social Networks

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### **OVERVIEW**

INTRODUCTION

# INTRODUCTION Background and Motivation

THE MODEL

Initial Setup Time Evolution Algorithm

#### RESULTS

Cluster Size Distribution Phase Transition and Critical Point Convergence Time Comparisons to Empirical Data

#### CONCLUSION

Summary References

#### **BACKGROUND AND MOTIVATION**

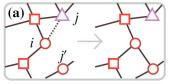
- ► Opinion Formation (e.g. voter models) is a common and very fundamental problem in the social sciences
- ► Goal: Modelling the *co*evolution of both opinions and the underlying social network
- Does our social network shape the opinion we hold or does our opinion determine who is part of our network?
- "Opinion" must be mutually exclusive and "non-spectral", e.g. brand preference, religious views...
- Preview: Analogies to statistical physics, e.g. phase transitions can be identified

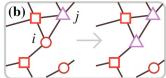
#### INITIAL SETUP

- ► Random graph with *N* nodes (opinion holder) and *M* edges (social connection)
- ▶ Random opinion  $g_i$  ∈ G assigned to node i
- Nodes exchange information (opinion) via undirected edges
- Externally set parameters:
  - ► *N* number of nodes
  - $\gamma = \frac{N}{G}$  average number of nodes per opinion
  - $k_{avg} = \frac{2M}{N}$  average degree
  - $\Phi$  reconnection probability

INTRODUCTION

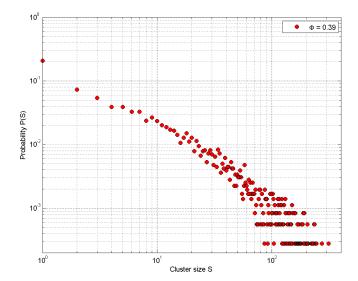
- 1. Pick a random node i with opinion  $g_i$ .
- 2. (a) With probability  $\Phi$  select at random one of the nodes j that i is connected to.
  - ▶ If  $g_i = g_i$ , start over at step 1.
  - ▶ Otherwise, reconnect to a randomly chosen j' of same opinion, i.e.  $g_{j'} = g_i$ .
- 3. (**b**) Otherwise, with probability  $1 \Phi$  randomly select one of the neighboring vertices j and change  $g_i$  to  $g_j$ .
- 4. Repeat until *consensus state* is achieved.





# **CLUSTER SIZE DISTRIBUTION**

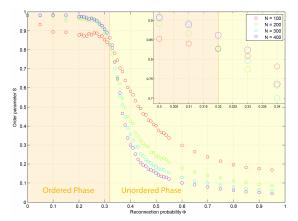
# CONTINUOUS PHASE TRANSITION?



#### **CLUSTER SIZE DISTRIBUTION**

- Ordered phase
  - ▶ Low  $\Phi$ , i.e. tendency to change opinion
  - Small clusters follow power law distribution
  - ► Existence of giant cluster
- ► Unordered phase
  - ▶ High  $\Phi$ , i.e. tendency to keep opinion
  - ► Clusters follow Poisson-like distribution
  - ► No giant cluster!
- ► Phase transition
  - First guess:  $\Phi_c = 0.35 \pm 0.05$
  - ▶ Power law behavior over the whole *s*-range

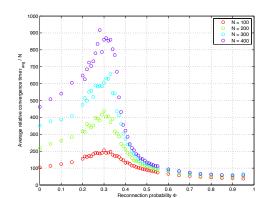
## PHASE TRANSITION & CRITICAL POINT



- ► Really continuous phase transition
- ▶ Bigger N → more dramatic transition
- $\Phi_c = 0.32 \pm 0.02$  independent of system size *N*
- Weak agreement with  $\Phi_c = 0.39 \pm 0.05$

# **CONVERGENCE TIME**

- ▶ Iterations per node to reach consensus as function of  $\Phi$ :
- ▶ "Divergence" at some  $\Phi_c$  for different N
- ► Similar to divergent response functions in physics
- ► Supporting phase transition interpretation, but difficult to find direct analogy to  $\tau_{avg}$



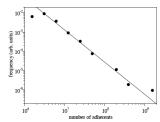


#### COMPARISONS TO EMPIRICAL DATA

- ▶ Idea: Compare distributions of some "opinion" in real world to the model  $\rightarrow$  identify and interpret corresponding  $\Phi$
- ► Religion:

INTRODUCTION

► Worldwide distribution of religions follows power law: Neither adaptation nor reconnection dominate in the formation?



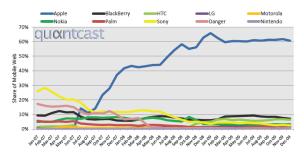
▶ Interpret  $\Phi$  as an "intolerance indicator"?

#### COMPARISONS TO EMPIRICAL DATA

► Mobile Web Browsers:

INTRODUCTION

- ► An example for opinion = brand preference
- ► Contrast between giant cluster and "softer" distribution
- ▶ Note: Plot is not a cluster size histogram!



Interpret Φ as a "brand loyalty indicator"?





- Interdependent evolution of opinions and networks, combining two mechanisms of adaption and reconnection determined by  $\Phi$
- Holme's and Newman's work could be reproduced with more realistic assumptions
- Continuous phase transition
  - *N*-independent critical value  $\Phi_c = 0.32 \pm 0.02$
  - ▶ Divergent convergence time at  $\Phi_c$

#### Outlook

- ▶ Variation of  $\gamma$  (diversity) and  $k_{avg}$  (density)
- ► Include analogue of "magnetic field" in model: "informed agents"?
- ► Make opinions *spectral*



INTRODUCTION

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