Robustness of quantum initial conditions for inflation

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Focus I

- Investigated robustness of initial conditions set to (scalar) primordial perturbations[1]
- Power spectrum of perturbations $\mathcal{R}_k$ well into inflation is the PPS:

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2$$

- Some sets of initial conditions differ in their definitions of the ground state (not obvious on a curved, non-static spacetime!)
- Demand that i.c. are invariant under a set of transformations that are canonical, otherwise the vacuum prescription is unphysical

[1] F. J. Agocs et al. "Investigating the gauge invariance of quantum initial conditions for inflation". In: *arXiv e-prints*, arXiv:2002.07042 (Feb. 2020), arXiv:2002.07042. arXiv: 2002.07042 [gr-qc].

Focus II

- The i.c. considered:
  1. Hamiltonian diagonalisation (**HD**)[2],
  2. Danielsson vacuum [3],
  3. Minimising the local energy density in vacuo using the renormalised stress–energy tensor (**RST**) [4]
- The transformations considered:
  1. A simultaneous redefinition of the field and time in the action s.t. the equation of motion and the commutator structure is conserved
  2. Addition of surface terms to the associated action ($\equiv$ integration by parts)

[2] The robustness of HD was first questioned by S. Fulling in 1979 [2], he concluded "it should not be taken seriously"

Motivation

- A practical point:
  - Initial conditions typically phrased in terms of $v_k = \frac{\mathcal{R}_k}{z}$ (Mukhanov variable), η (conformal time), where $z(t) = \frac{a\dot{\phi}}{H}$
  - But computationally a different choice of variables may be better, e.g. $\mathcal{R}_k$, $N = \ln a$
  - Imagine running code that calculates $P_{\mathcal{R}}(k)$, but works in terms of ($\mathcal{R}_k$, τ)
  - If initial conditions are given for (v_k,η), then how to set i.c. for ($\mathcal{R}_k$, τ)?

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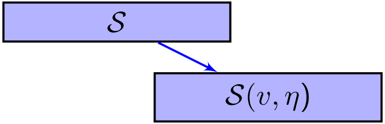

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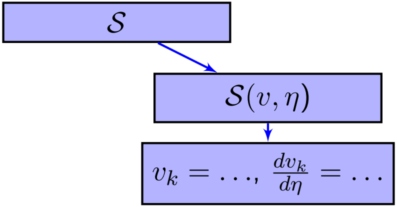

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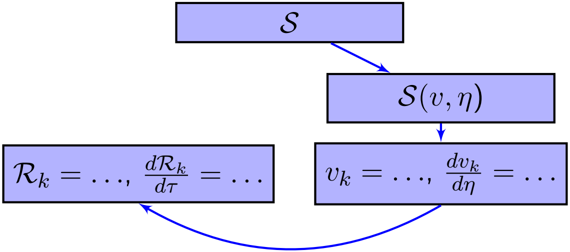

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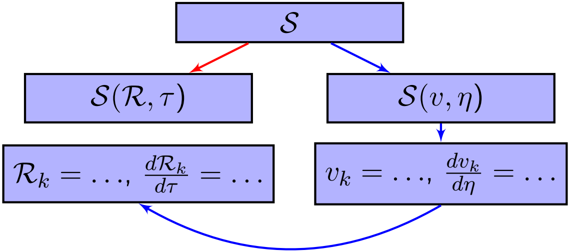

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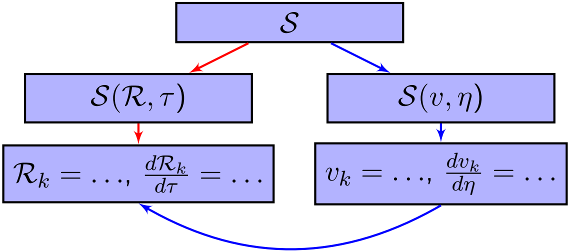

Motivation II

- A practical point:
  - The two paths should give identical results
  - otherwise theoretically they are equally correct, but may give rise to different observations[3]
- A theoretical point:
  - If the two actions $\mathcal{S}(v, \eta)$ and $\mathcal{S}(\mathcal{R}, \tau)$ give identical equations of motion and commutation relations, they are physically equivalent and should give identical vacua

[3] Under a non-standard inflationary model, e.g. 'just enough inflation'

Motivation III: A non-standard inflation model

- Single scalar field
- If a period of kinetic dominance [5, 6, 7, 8] precedes slow-roll inflation, Hubble horizon grows then shrinks
- We use the 'just enough inflation' model, total e-folds of inflation ≈ 60, $N_{*} \approx N_{\mathrm{tot}} \approx 60$
- Perturbation modes initialised simultaneously, some will not start from 'deep within the horizon' $\rightarrow$ will see features in the PPS
- Perturbation initial conditions affect the PPS, and in turn the angular spectra of CMB

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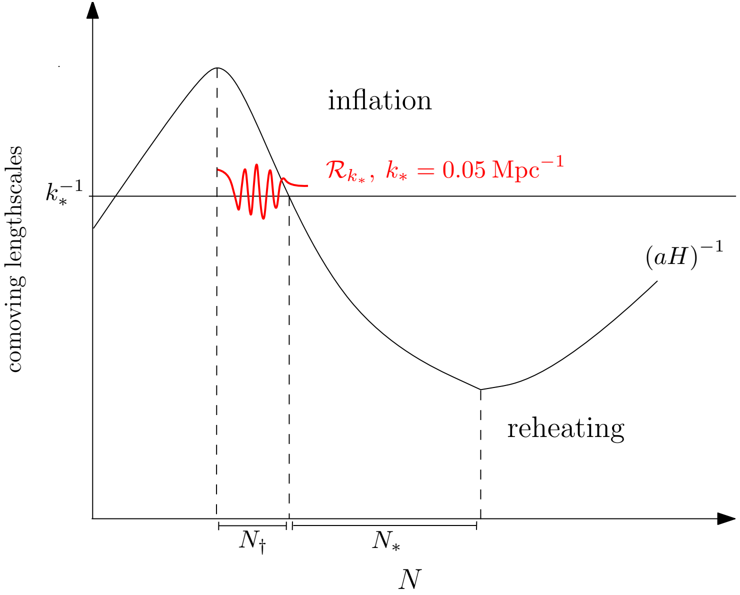

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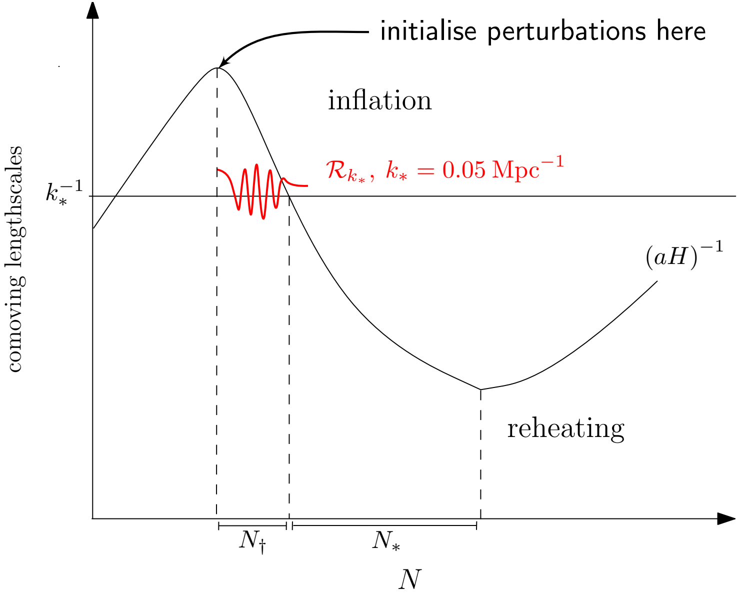
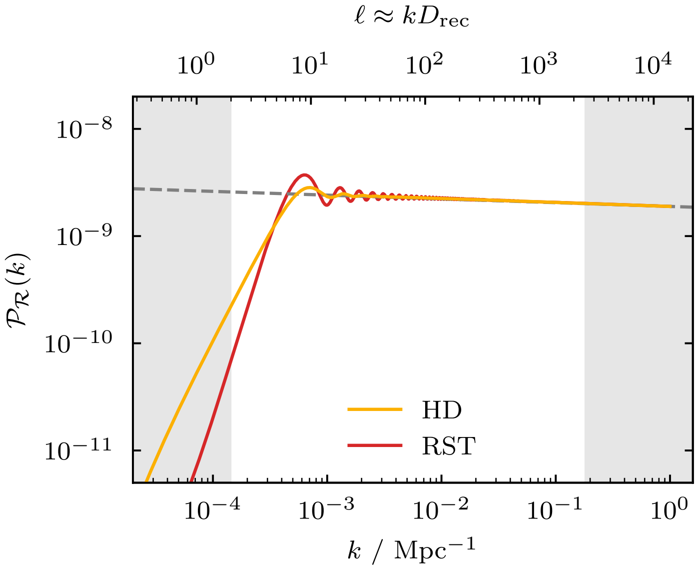
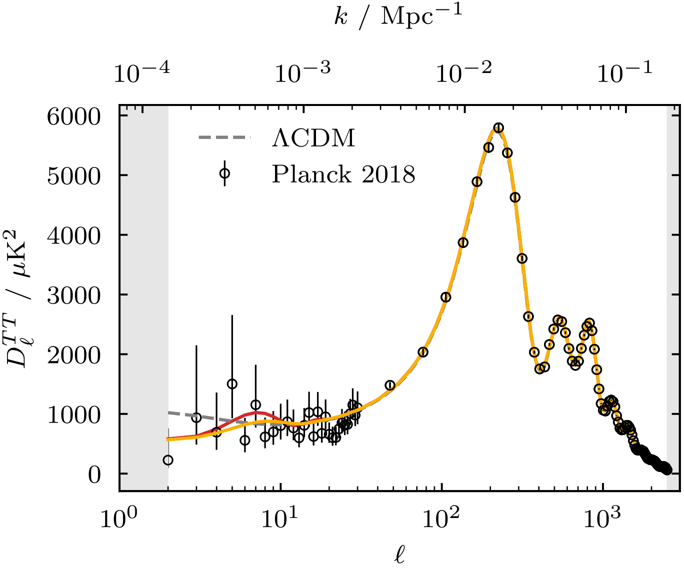


Example PPS with features



And their CMB counterparts



Methods summary

- For each of the vacuum considerations,
  1. We subject the relevant action to the transformation in question,
  2. Ensure the equation of motion and commutation relations are conserved,
  3. Derive the i.c. (typically involves minimising a quantity) for (v_k, η),
  4. This may either be invariant under the parametrisation of the transformation or will yield a set of i.c.,
  5. In the latter case, show the range of observations the different i.c. would result in

Methods example: HD + field-redefinition

- Hamiltonian diagonalisation considers the ground state as the minimum vacuum expectation value of the Hamiltonian density, i.e. minimising $\langle 0| \mathcal{H} |0\rangle$,
- Relevant action is written in terms of Mukhanov variable v and conformal time η:

$$\mathcal{S} = \frac{1}{2} \int d\eta d^3x \Big[(v')^2 - (\partial_i v)^2 + \frac{z''}{z} \Big]$$

- Action has canonically normalised form $\rightarrow$ can quantise v as quantum harmonic oscillator (hence the choice of (v, η))

Methods example: HD + field-redefinition

- Why not quantise a different, canonically normalised field?
- Field redefinition + change of time:

$$t \to \tau(t) \tag{1}$$

$$v \to \chi = \frac{\mathcal{R}}{h(t)} = \frac{v}{zh(t)}, \tag{2}$$

where $h(t)$ is a time-dependent homogeneous field,

- Gives:

$$\mathcal{S} = \int d^3x d\tau \frac{C}{2} \left[(\partial_\tau \chi)^2 + \chi^2(\dots) - (\nabla_i \chi)^2(\dots) \right]$$

- if $C = h^2 \dot{\tau} a z^2 = 1$, this action is canonically normalised $\to$ can quantise χ
- Get extra constraint from preserving the commutator structure

Results

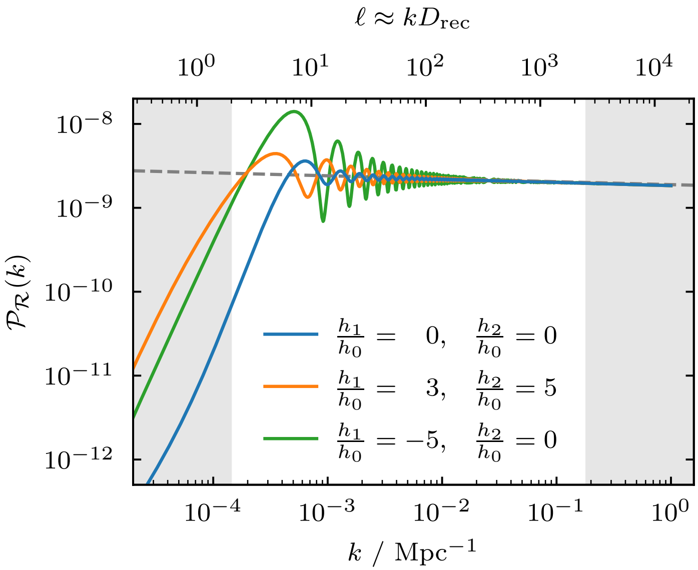
- Hamiltonian diagonalisation and Danielsson vacuum: **not invariant**
  - Functional degree of freedom $h(t)$ propagates into the initial conditions
  - HD under field-redefinition:

$$|v_k|^2 = \frac{1}{2\sqrt{k^2 + \frac{h''}{h} + 2\left(\frac{h'z'}{hz}\right)}},$$

$$v_k' = \left(-i\sqrt{k^2 + \frac{h''}{h} + 2\frac{h'z'}{hz}} + \frac{h'}{h} + \frac{z'}{z} \right) v_k,$$

  - $f' = \frac{df}{d\eta}$, and if perturbations are set simultaneously at η_0, then replace $h|_{\eta=\eta_0} = h_0$, $h'|_{\eta=\eta_0} = h_1$, $h''|_{\eta=\eta_0} = h_2$
- Minimising the local energy density with RST: **invariant**

Examples of primordial power spectra obtainable by HD



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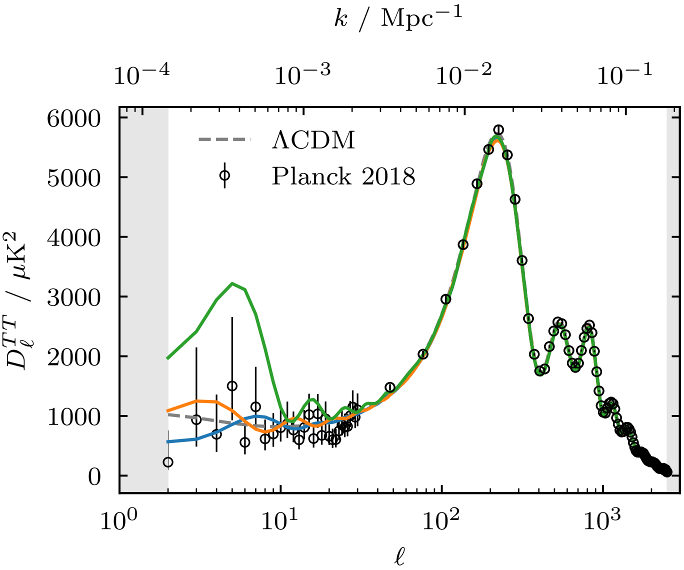
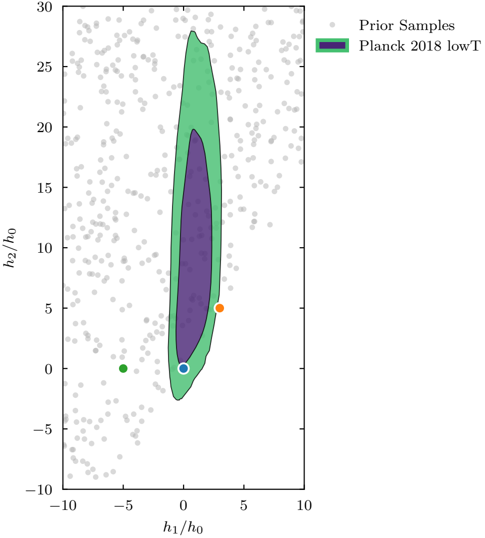


Posterior probability of the parameters h_1/h_0,h_2/h_0 from Planck 2018 data



Summary and future work

- Investigated how initial conditions for primordial perturbations, derived from different definitions of the ground state, behave under two types of transformations,
  - Vacuum prescriptions: Hamiltonian diagonalisation, Danielsson vacuum, minimising the 00-component of the renormalised stress–energy tensor
  - Transformations: field-redefinitions and addition of surface terms (both canonical)
- Found that HD and the Danielsson vacuum suffer from an ambiguity and give a range of initial conditions that would be distinguishable by observations *under certain inflationary models*, e.g. 'just enough inflation'
- RST initial conditions are invariant under the transformations

Summary and future work

- Future work
  - Are RST initial conditions unique in their invariance?
  - Generalising to all canonical transformations
  - How do other initial conditions behave?

Bibliography I

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