The RKWKB method and the primordial Universe

F. Agocs

Gonville & Caius college
University of Cambridge

Part III project, 2016–17

The RKWKB

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{F}(\boldsymbol{y}(t), t)$$

RK stepper (Euler's method)

$$\boldsymbol{y}(t+h) = \boldsymbol{y}(t) + h\boldsymbol{F}(\boldsymbol{y}(t), t),$$

$$t \mapsto t + h.$$

WKB approximation

$$f_{\pm}(t) = \frac{1}{\sqrt{\omega(t)}} \exp\left(\pm i \int \omega(\tau) d\tau \pm S_2(\ddot{\omega}) + \dots \right)$$

RKWKB

$$y(t+h) = A_+ f_+(t+h) + A_- f_-(t+h),$$

$$\dot{y}(t+h) = B_+ \dot{f}(t+h) + B_- \dot{f}(t+h),$$

$$t \mapsto t + h$$

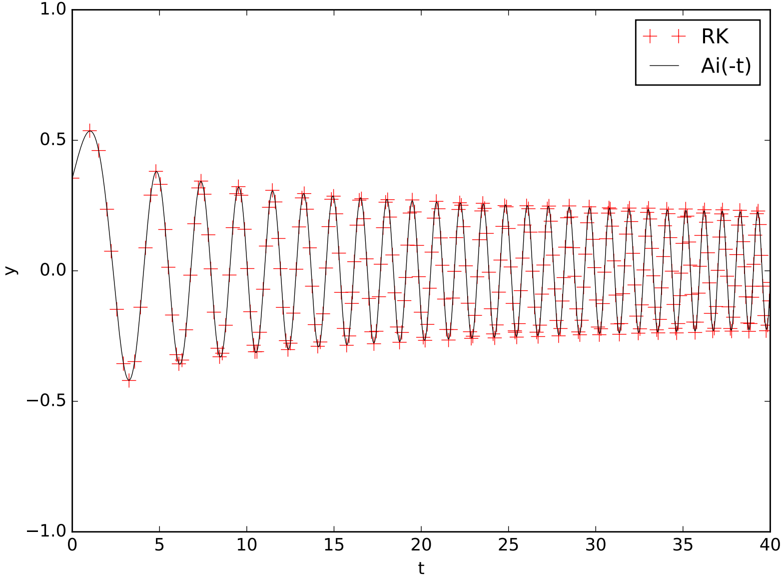
Numerical methods

- Adaptive stepsize control
- Dynamical switching
- Automatic differentiation

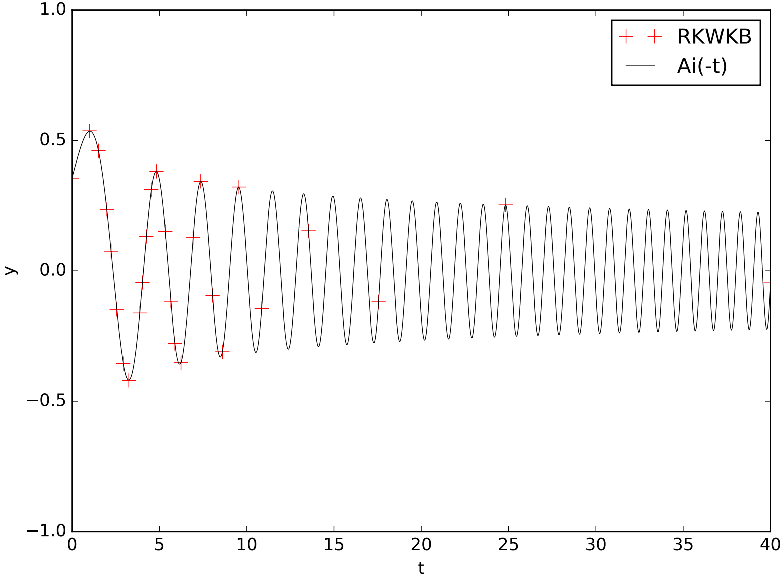
$$\omega = \omega(\boldsymbol{y}(t))$$

$$\dot{\omega} = \dot{\boldsymbol{y}} \cdot \nabla\omega = \boldsymbol{F} \cdot \nabla\omega$$

Example: Airy equation, $\ddot{y} + t \cdot y = 0$



Example: Airy equation II



The primordial Universe

Mukhanov–Sasaki equation

$$0 = \ddot{\mathcal{R}}_k + \left(2\frac{\dot{z}}{z} + H\right)\dot{\mathcal{R}}_k + \frac{k^2}{a^2}\mathcal{R}_k, \quad z = \frac{a\dot{\phi}}{H}$$

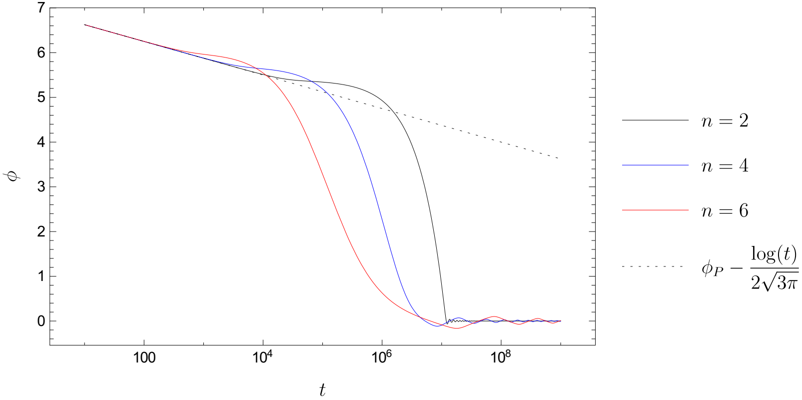
Cosmological field equations

$$\dot{H} + H^2 = -\frac{1}{3m_P^2}\left[\dot{\phi}^2 - V(\phi)\right], \quad \text{Raychaudhuri}$$

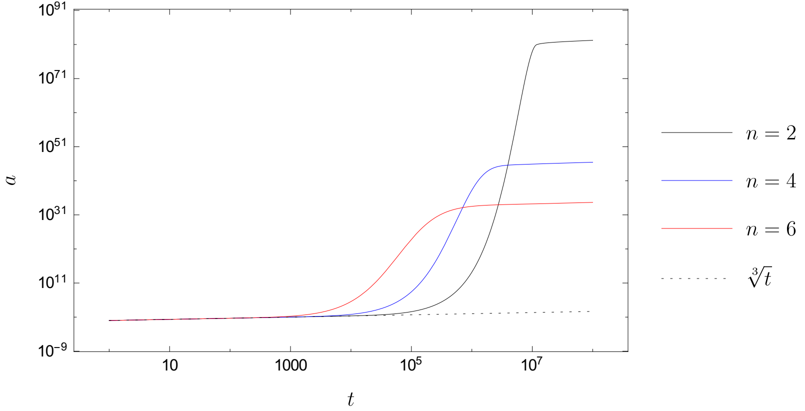
$$H^2 = \frac{1}{3m_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right], \quad \text{Friedmann}$$

$$0 = \ddot{\phi} + 3\dot{\phi}H + \frac{dV}{d\phi}, \quad \text{continuity}$$

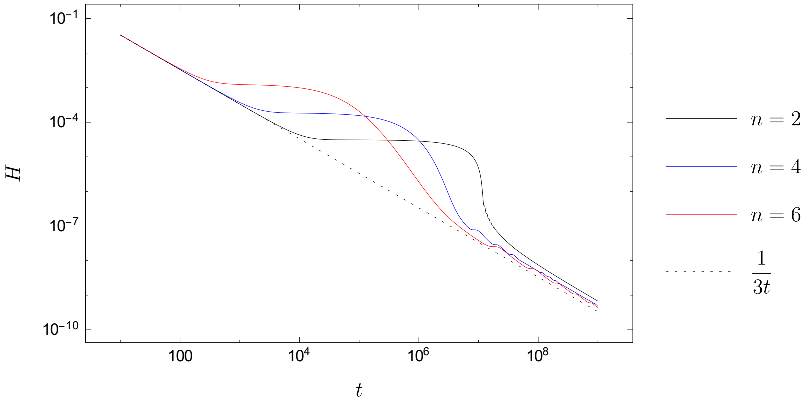
Cosmological background evolution I



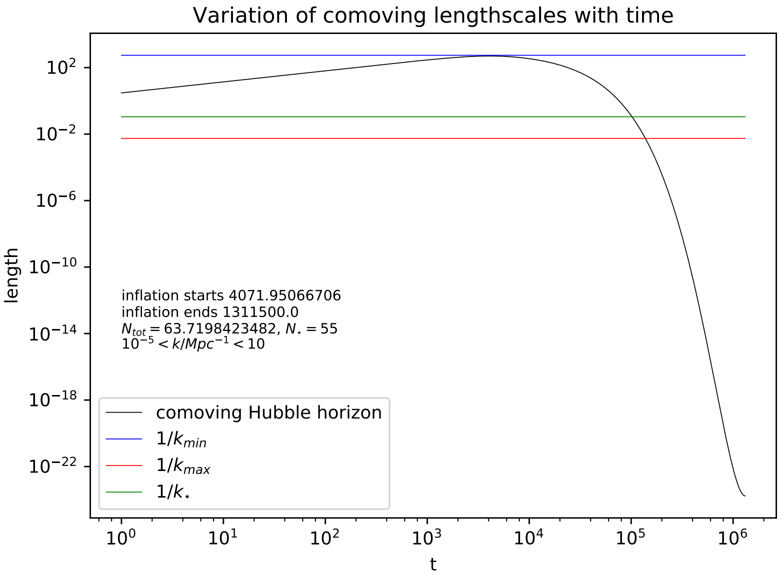
Cosmological background evolution II



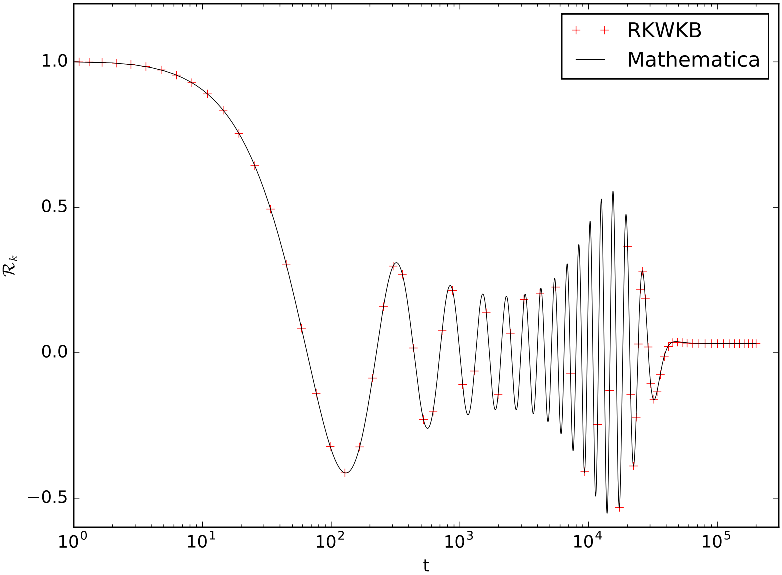
Cosmological background evolution III



Change of comoving lengthscales



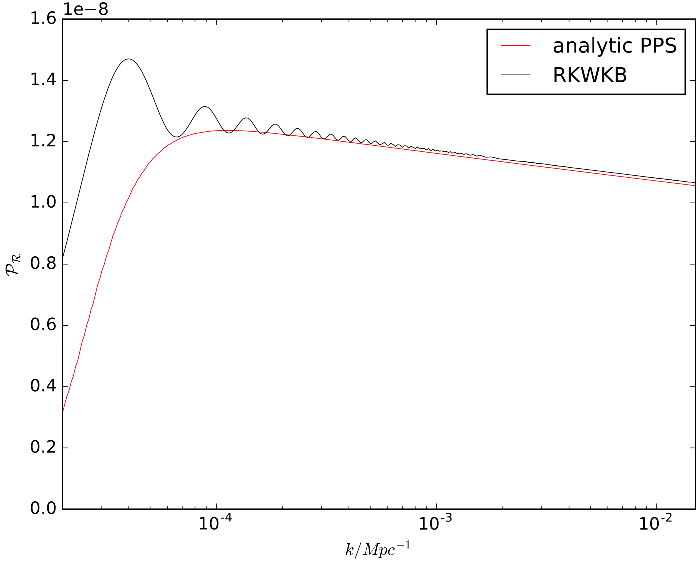
Evolution of a perturbation



Initial conditions

- Hamiltonian
- Renormalised
- Singularity

PPS I


1e−8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\mathcal{P}_\mathcal{R}$
analytic PPS
RKWKB
10^{-4}
10^{-3}
10^{-2}
k/Mpc^{-1}

Summary

- We've shown that the RKWKB method could be more efficient whilst being accurate than RK steppers
- Confirmed that KD predicts a generic cutoff in the PPS at large lengthscales, responsible for the suppression of low-l multipole components in CMB power spectrum
- Outlook
  - So far, only produced a primordial power spectrum. Need to extrapolate to the time of recombination to compare to observed power spectrum;
  - Use of AD limits the speed of RKWKB;
  - Try other two sets of initial conditions.

Title

- First point I'm making

Title

- First point I'm making
- Second point I'm making (after pause)

Title

- First point I'm making
- Second point I'm making (after pause)
- important point