MATH 4760 Assignment 3

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1) Explain what causal and invertible processes are. Why do we need them?

Causal Process

A process that does not depend on the future. Causal make it possible to forcast future values.

Invertible Process

A process represented by an infinite AR model. Invertibility makes it possible to estimate parameters.

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2) Determine if the following processes are causal.

a) $X_t = 0.8X_{t-1} - 0.9X_{t-2} + Z_t$

$(1 - 0.8B + 0.9B^2)X_t$

$\phi(z) = 1 - 0.8z + 0.9z^2 = 0$

```
abs(polyroot(c(1, -0.8, 0.9)))
```

```
## [1] 1.054093 1.054093
```

$z = 1.054 > 1$ Therefore X_t is causal.

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b) $X_t = 1.2X_{t-1} + 0.25X_{t-2} + Z_t$

$(1 - 1.2B - 0.25B^2)X_t$

$\phi(z) = 1 - 1.2z - 0.25z^2 = 0$

```
abs(polyroot(c(1, -1.2, -0.25)))
```

```
## [1] 0.7240999 5.5240999
```

$z = 0.724 < 1$ Therefore X_t is not causal.

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c) $X_t = 0.8X_{t-1} - 0.5X_{t-2} + 0.4X_{t-3} + Z_t$

$(1 - 0.8B + 0.5B^2 - 0.4B^3)X_t$

$\phi(z) = 1 - 0.8z + 0.5z^2 - 0.4z^3 = 0$

```
abs(polyroot(c(1, -0.8, 0.5, -0.4)))
```

```
## [1] 1.250000 1.414214 1.414214
```

$z = 1.250 > 1$ and $z = 1.414 > 1$ Therefore X_t is causal.

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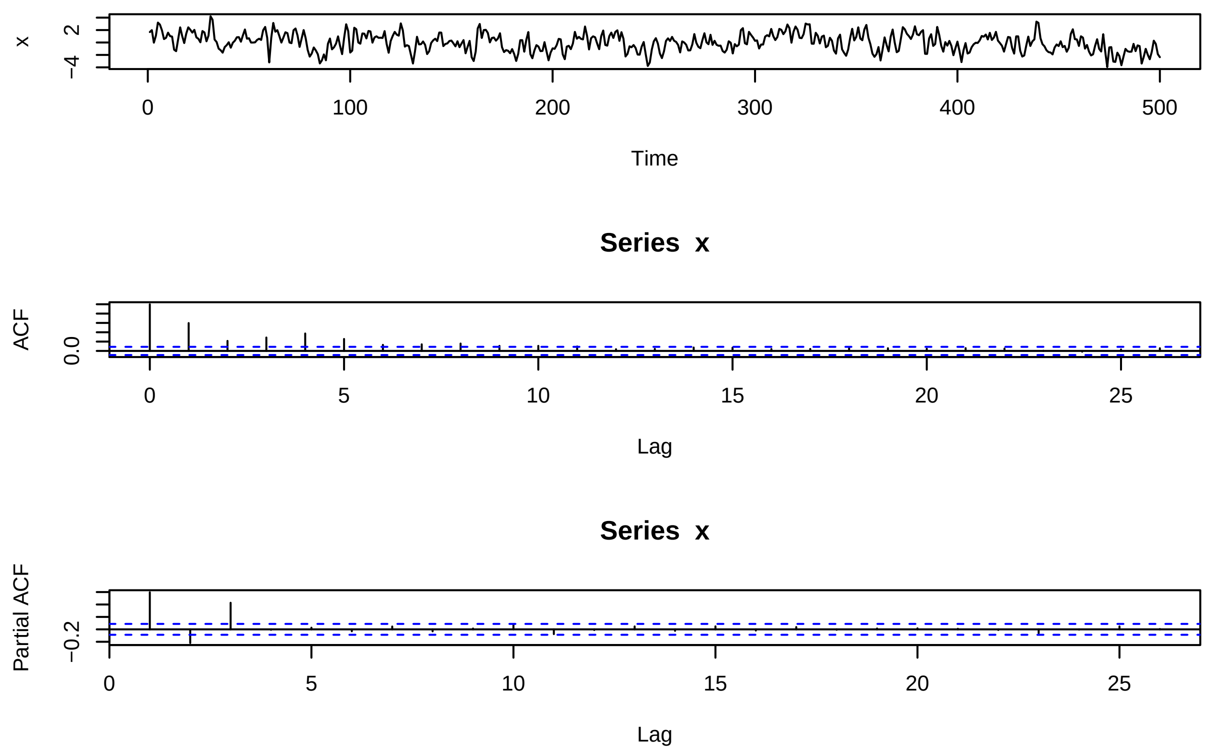
3) Simulate 500 data points of the AR(3) process of question 2c, draw the time series plot, ACF, and PACF.

```
# X_t = 0.8X_t-1 - 0.5X_t-2 + 0.4X_t-3 + Z_t
x = arima.sim(list(order=c(3,0,0), ar=c(0.8, -0.5, 0.4)), n=500)
par(mfrow=c(3,1))
plot.ts(x)
acf(x)
pacf(x)
```



Discuss what you can conclude from the ACF and PACF pattern.

The PACF cuts off after lag = 3 so because it is an AR function p = 3. Also because the ACF decreases exponentially the process is causal. Thus it is also stationary.

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4) Determine if the following processes are invertible.

a) $X_t = Z_t - 0.5Z_{t-1} + 0.8Z_{t-2}$

$(1 - 0.5B + 0.8B^2)Z_t$

$\theta(z) = 1 - 0.5z + 0.8z^2$

```
abs(polyroot(c(1, -0.5, 0.8)))
```

```
## [1] 1.118034 1.118034
```

$z = 1.118 > 1$ Therefore X_t is invertible.

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b) $X_t = Z_t + 1.5Z_{t-2}$

$(1 + 1.5B^2)Z_t$

$\theta(z) = 1 + 1.5z^2$

```
abs(polyroot(c(1, 0, 1.5)))
```

```
## [1] 0.8164966 0.8164966
```

$z = 0.817 < 1$ Therefore X_t is not invertible.

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c) $X_t = Z_t - 0.8Z_{t-1} + 1.5Z_{t-2} - 1.8Z_{t-3}$

$(1 - 0.8B + 1.5B^2 - 1.8B^3)Z_t$

$\theta(z) = 1 - 0.8z + 1.5z^2 - 1.8z^3$

```
abs(polyroot(c(1, -0.8, 1.5, -1.8)))
```

```
## [1] 0.7577791 0.7577791 0.9674805
```

$z = 0.758 < 1$ and $z = 0.968 < 1$ Therefore X_t is not invertible.

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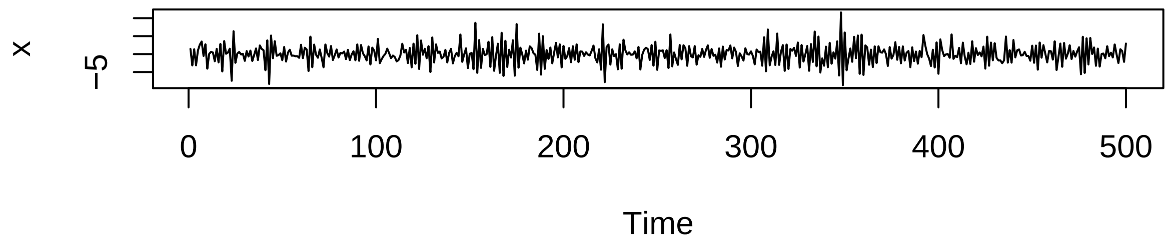
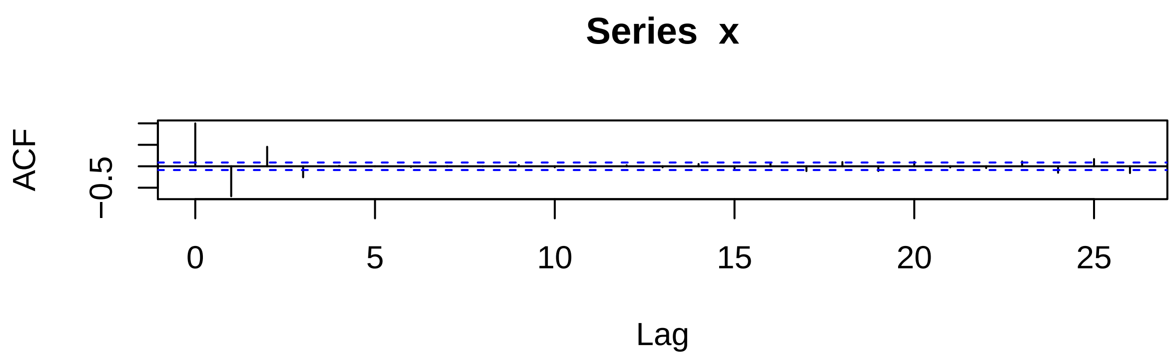
5) Simulate 500 data points of the MA(3) process of question 4c. Plot the data and its ACF.

```
# X_t = Z_t - 0.8X_t-1 + 1.5X_t-2 - 1.8X_t-3
x = arima.sim(list(order=c(0,0,3), ma=c(-0.8, 1.5, -1.8)), n=500)
par(mfrow=c(2,1))
plot.ts(x)
acf(x)
```





Explain the behavior through ACF.

Since it is a moving average process q = 3 because the ACF cuts off at 3.

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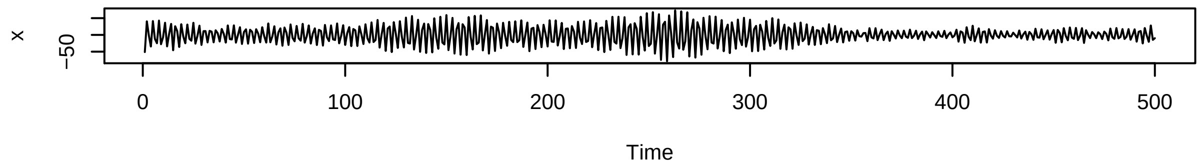
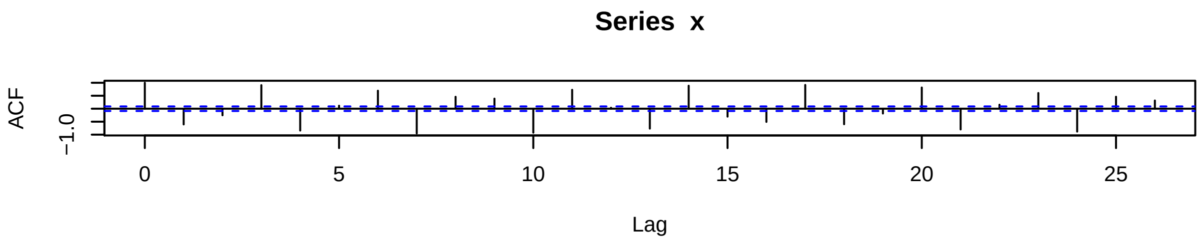
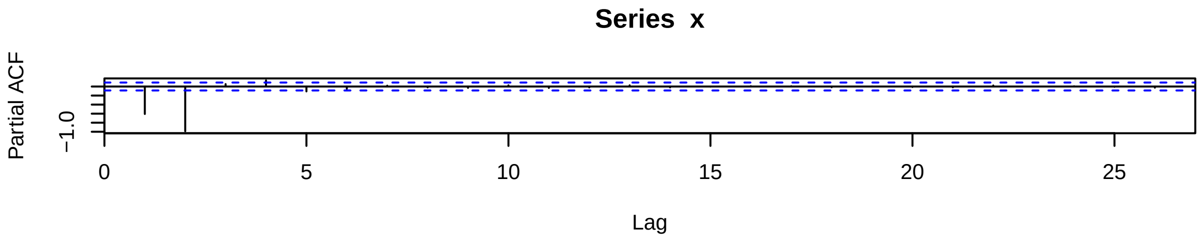
6) Simulate 500 data points of the ARIMA process $X_t = 0.8X_{t-1} - 0.5X_{t-2} + 0.4X_{t-3} + Z_t - 0.8Z_{t-1} + 1.5Z_{t-2} - 1.8Z_{t-3}$**. Draw its plot, ACF, and PACF.**

```
x = arima.sim(list(order=c(3,0,3), ar=c(-0.8, -0.5, 0.4), ma=c(-0.8, 1.5, -1.8)), n=500)
par(mfrow=c(3,1))
plot.ts(x)
acf(x)
pacf(x)
```







Explain the order of the ARIMA through its ACF and PACF.

The order of the ARIMA should be p = 3 because the PACF appears to cut off at 3.