MATH 4760 Assignment 2

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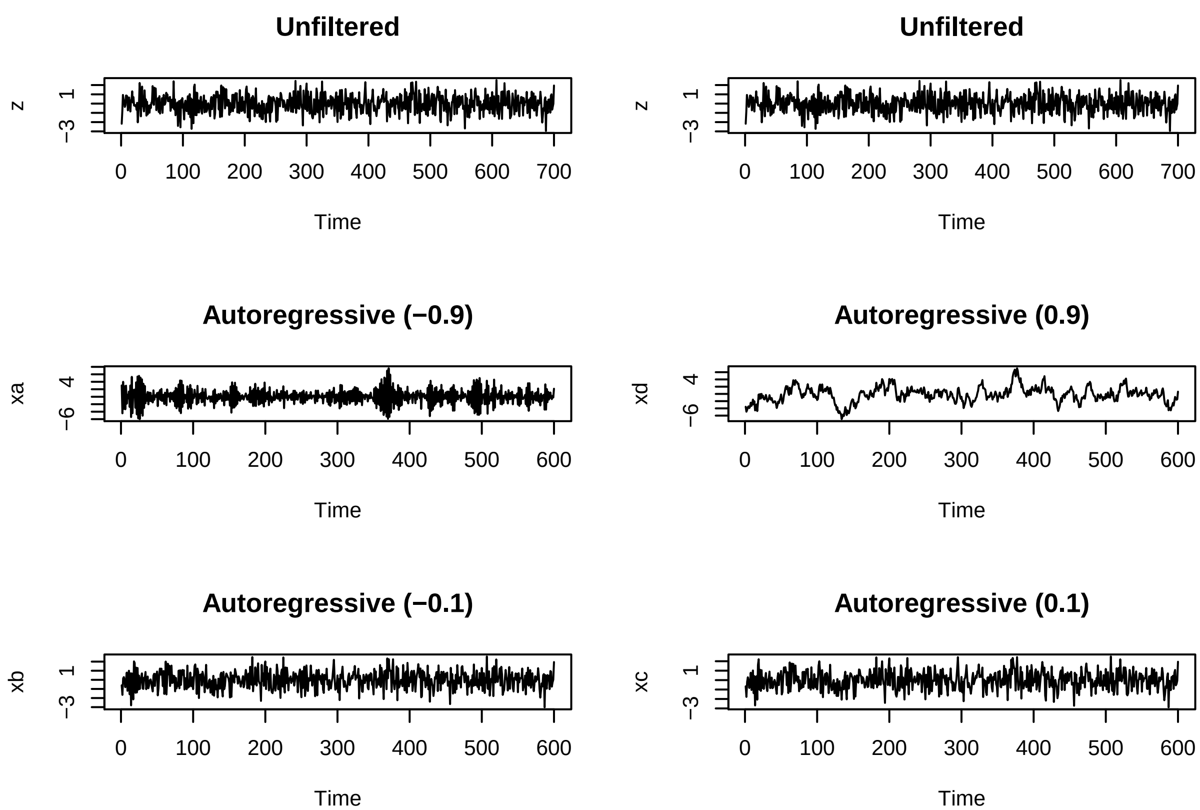
1) Simulate 700 observations of AR(1) process, remove first 100 observations

$X_t = \phi X_{t-1} + Z_t$ for $\phi = -0.9, -0.1, 0.1, 0.9$

```
z = rnorm(700, 0, 1)
xa = filter(z, filter=c(-0.9), method="recursive")[-(1:100)]
xb = filter(z, filter=c(-0.1), method="recursive")[-(1:100)]
xc = filter(z, filter=c(0.1), method="recursive")[-(1:100)]
xd = filter(z, filter=c(0.9), method="recursive")[-(1:100)]
```

a) Draw the time series plots and their autocorrelation functions

```
par(mfrow=c(3, 2))
plot.ts(z, main="Unfiltered")
plot.ts(z, main="Unfiltered")
plot.ts(xa, main="Autoregressive (-0.9)")
plot.ts(xd, main="Autoregressive (0.9)")
plot.ts(xb, main="Autoregressive (-0.1)")
plot.ts(xc, main="Autoregressive (0.1)")
```



b) Comment on the structures of the data and the autocorrelation function

The further negative the autoregressive function the more noisey the data appears. Near zero the autoregressive function does near nothing. The further positve the autoregressive function the less noisey the data appears.

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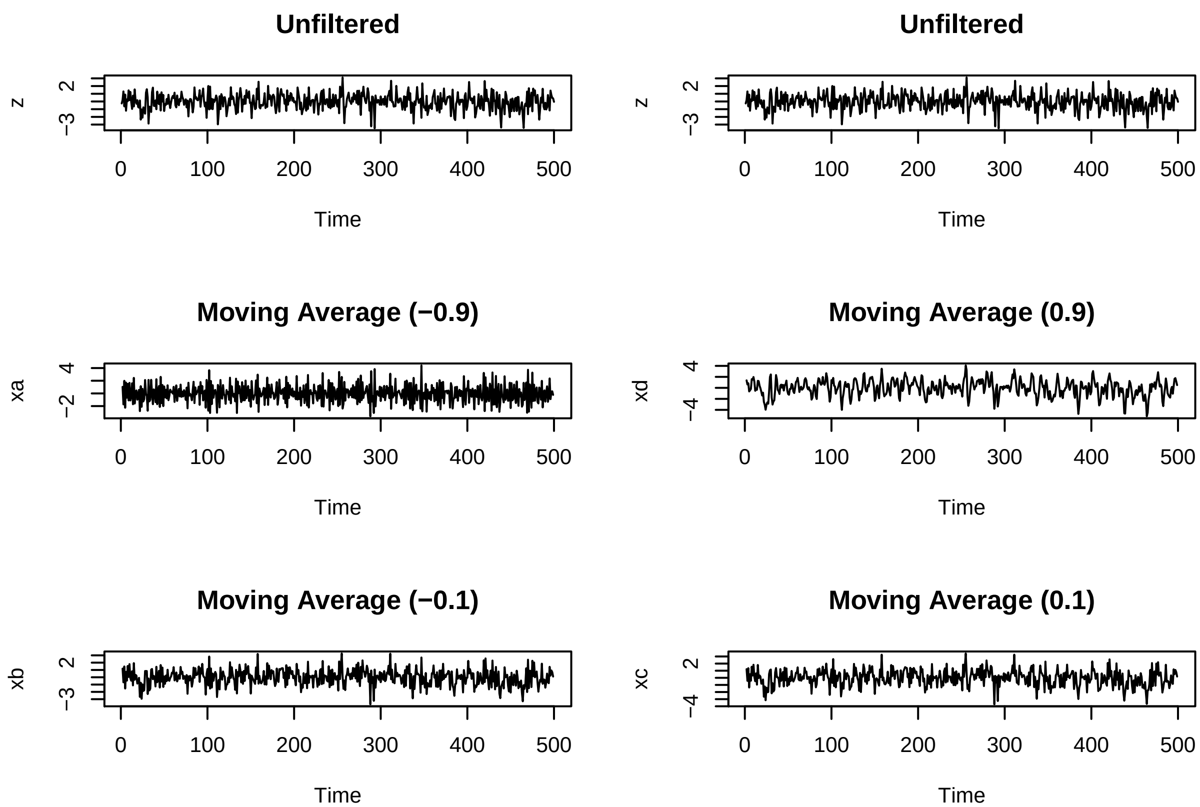
2) Simulate 500 observations of MA(1) process

$X_t = \phi Z_{t-1} + 0.4 Z_{t-2} + Z_t$ for $\phi = -0.9, -0.1, 0.1, 0.9$

```
z = rnorm(500, 0, 1)
xa = filter(z, filter=c(1, -0.9, 0.4), method="convolution")
xb = filter(z, filter=c(1, -0.1, 0.4), method="convolution")
xc = filter(z, filter=c(1, 0.1, 0.4), method="convolution")
xd = filter(z, filter=c(1, 0.9, 0.4), method="convolution")
```

a) Draw the time series plots and their moving average functions

```
par(mfrow=c(3, 2))
plot.ts(z, main="Unfiltered")
plot.ts(z, main="Unfiltered")
plot.ts(xa, main="Moving Average (-0.9)")
plot.ts(xd, main="Moving Average (0.9)")
plot.ts(xb, main="Moving Average (-0.1)")
plot.ts(xc, main="Moving Average (0.1)")
```



b) Comment on the structures of the data and the moving average function

The further negative the moving average function the more noisey the data appears. Near zero the moving average function does near nothing. The further positve the moving average function the less noisey the data appears.

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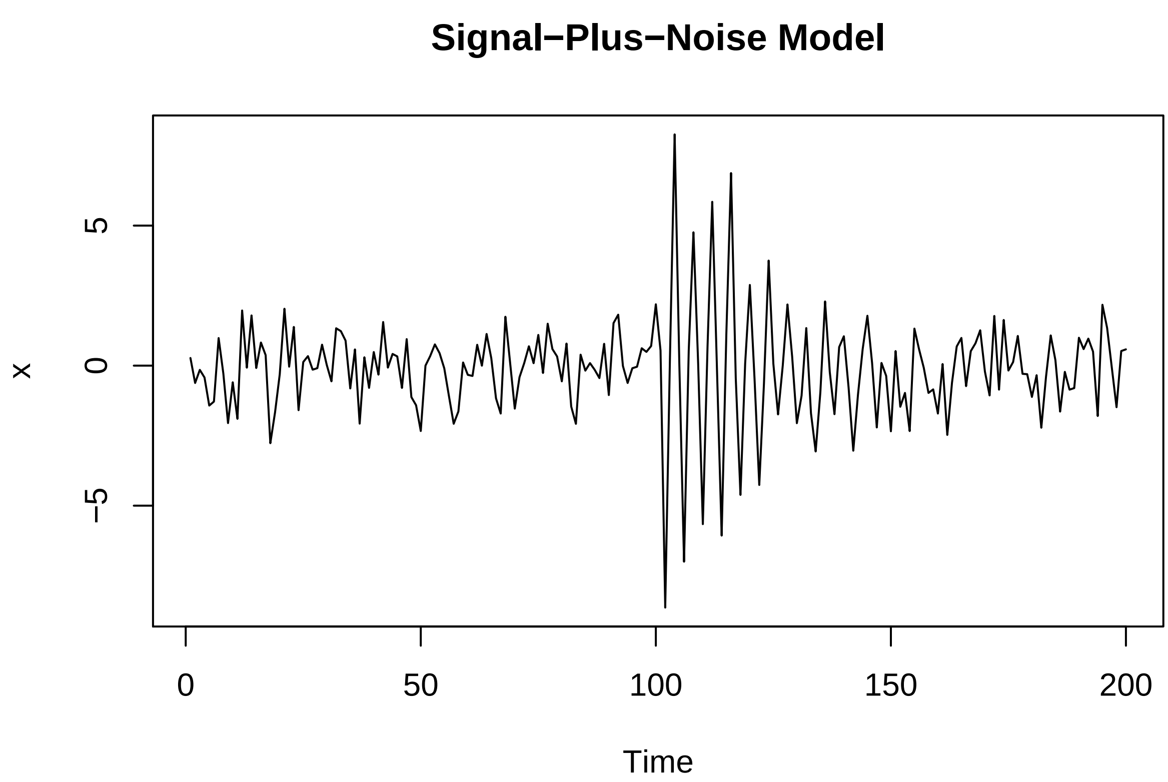
3) Simulate the data of a signal plus noise model as desccribed in problem 1.2a (of the book)

```
s = c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*101:200/4))
x = ts(s + rnorm(200, 0, 1))
```

b) Draw the time series plot

```
par(mfrow=c(1,1))
plot.ts(x, main="Signal-Plus-Noise Model")
```



And comment on the plot by comparing it with the time series plot in figure 1.7

The data appears to be pure noise before t=100 because that is where the second function kicks in. It looks like the explosion without the warning at the beginning. The earthquake on the other hand appears most consistant after t=1000 because it doesn't fade out like out model.

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4) **Do all parts of problem 1.3**

a) **Generate 100 observations from the autoregression** $x_t = -0.9x_{t-2} + w_t$

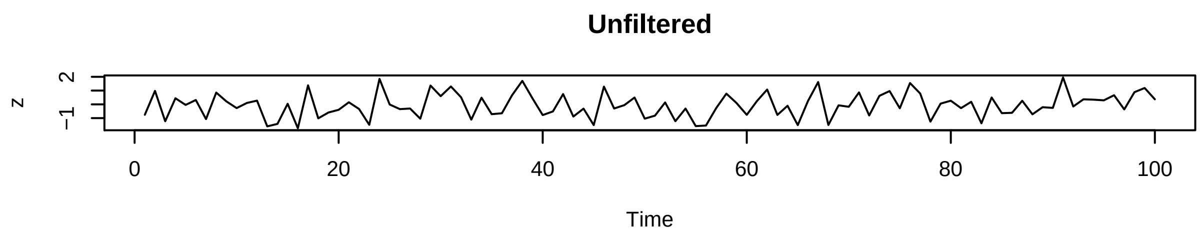
```
z = rnorm(100, 0, 1)
x = filter(z, filter=c(0, 0.9), method="recursive")
```

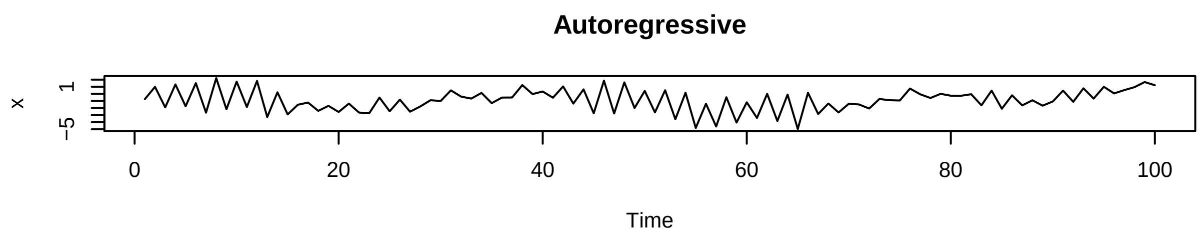
Then apply the moving average filter $v_t = \frac{x_t + x_{t-1} + x_{t-2} + x_{t-3}}{4}$

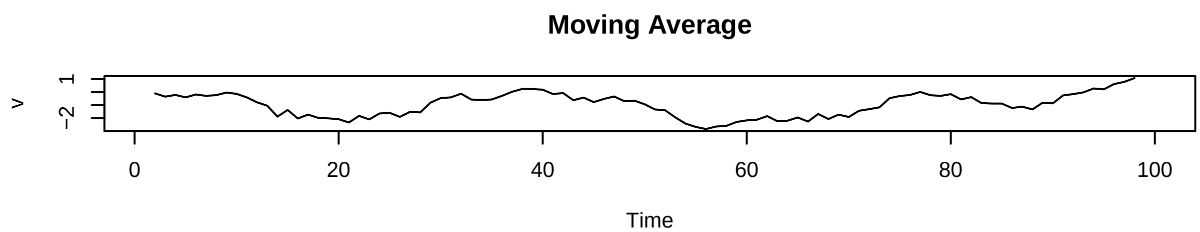
```
v = filter(x, filter=rep(1/4, 4), method="convolution")
```

Then plot

```
par(mfrow=c(3, 1))
plot.ts(z, main="Unfiltered")
plot.ts(x, main="Autoregressive")
plot.ts(v, main="Moving Average")
```



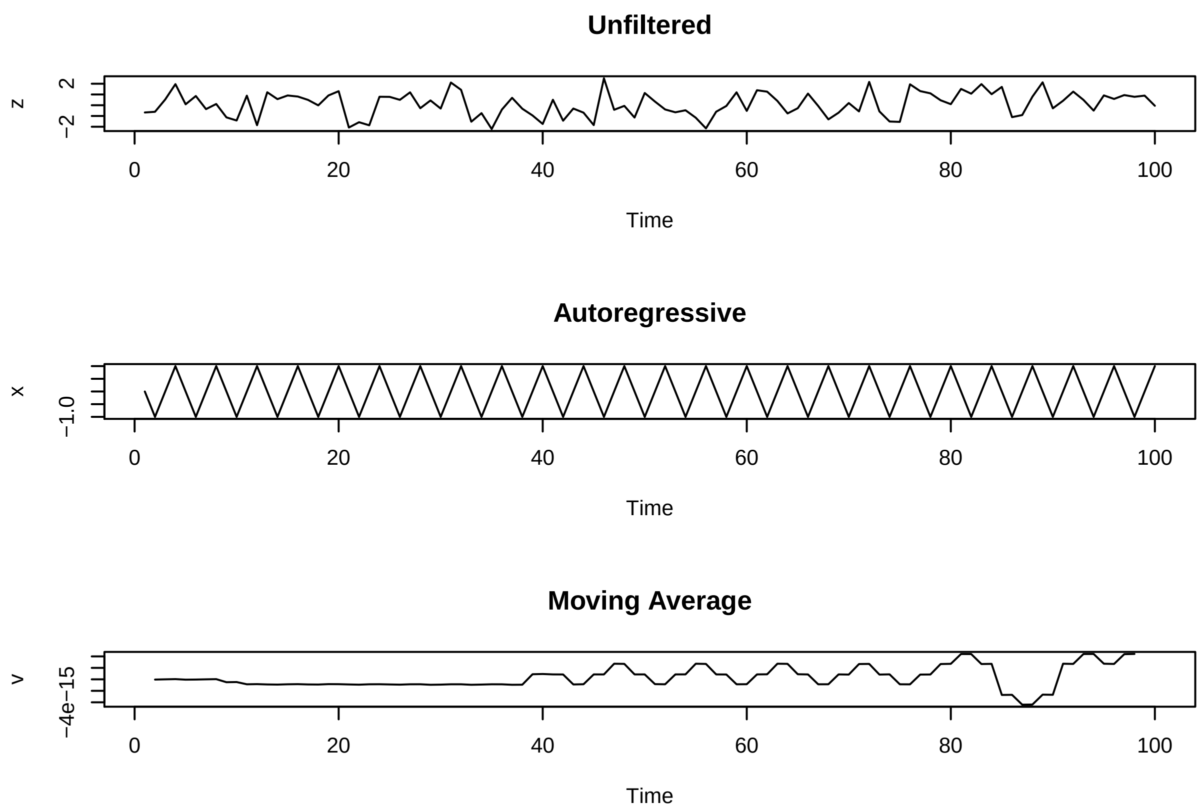




b) **Repeat for** $x_t = cos(\frac{2\pi t}{4})$

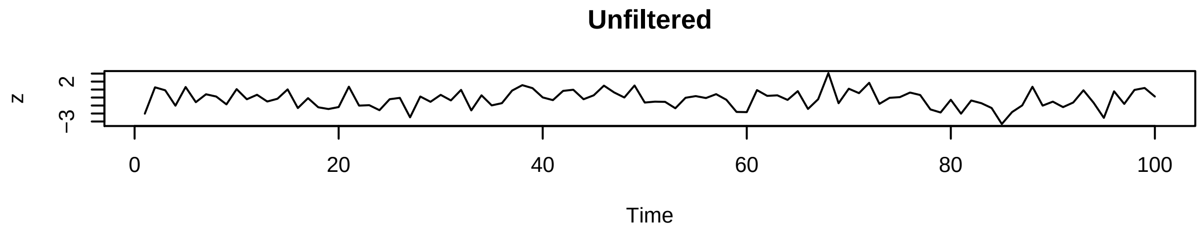
```
z = rnorm(100, 0, 1)
x = cos(2*pi*1:100/4)
v = filter(x, filter=rep(1/4, 4), method="convolution")
par(mfrow=c(3, 1))
plot.ts(z, main="Unfiltered")
```

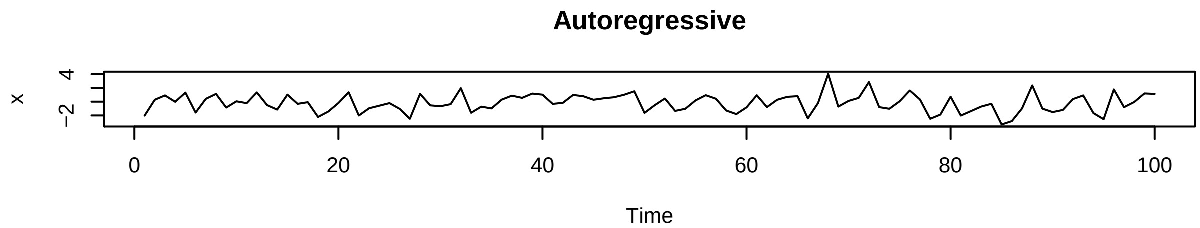
```
plot.ts(x, main="Autoregressive")
plot.ts(v, main="Moving Average")
```

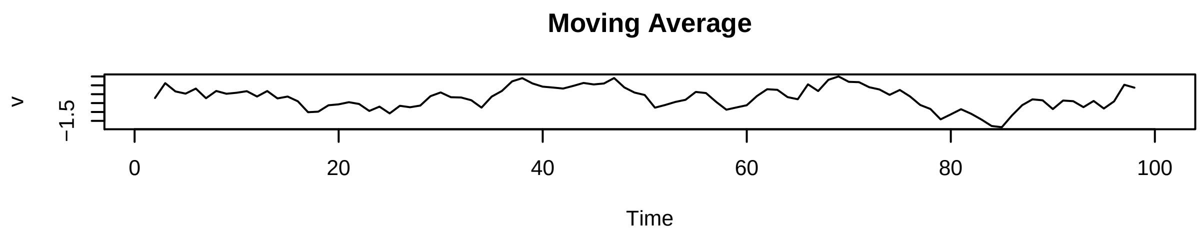


c) Repeat for $x_t = cos(\frac{2\pi t}{4}) + w_t$

```
z = rnorm(100, 0, 1)
x = cos(2*pi*1:100/4) + z
v = filter(x, filter=rep(1/4, 4), method="convolution")
par(mfrow=c(3, 1))
plot.ts(z, main="Unfiltered")
plot.ts(x, main="Autoregressive")
plot.ts(v, main="Moving Average")
```







d) Compare and contrast all

For part a the autoregressive function follows the trend of the unfiltered input but it is less noisey. The moving average on top of that removes a lot more noise making the graph look relatively smooth. Part b on the other hand is not affected by any noise so it is just a sharp sine wave. The moving average produces a weird effect where there is almost no noise but it increases toward the end. Part c being part b with added noise smooths out over time like in part a, but has a slight resemblance to a sine wave.