ASTRONOMICAL ALGORITHMS

SECOND EDITION

Jean Meeus

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Foreword (to the first edition)

People who write their own computer programs often wonder why the machine gives inaccurate planet positions, an unreal eclipse track, or a faulty Moon phase. Sometimes they insist, bewildered, "and I used double precision, too." Even commercial software is sometimes afflicted with gremlins, which comes as quite a shock to anyone caught up in the mystique and presumed infallibility of computers. Good techniques can help us avoid erroneous results from a flawed program or a simplistic procedure — and that's what this book is all about.

In the field of celestial calculations, Jean Meeus has enjoyed wide acclaim and respect since long before microcomputers and pocket calculators appeared on the market. When he brought out his *Astronomical Formulae for Calculators* in 1979, it was practically the only book of its genre. It quickly became the "source among sources", even for other writers in the field. Many of them have warmly acknowledged their debt (or should have), citing the unparalleled clarity of his instructions and the rigor of his methods.

And now this Belgian astronomer has outdone himself yet again! Virtually every previous handbook on celestial calculations (including his own earlier work) was forced to rely on formulae for the Sun, Moon, and planets that were developed in the last century — or at least before 1920. The past 10 years, however, have seen a stunning revolution in how the world's major observatories produce their almanacs. The Jet Propulsion Laboratory in California and the U.S. Naval Observatory in Washington, D.C., have perfected powerful new machine methods for modeling the motions and interactions of bodies within the solar system. At the same time in Paris, the Bureau des Longitudes has been a beehive of activity aimed at describing these motions analytically, in the form of explicit equations.

Yet until now the fruits of this exciting work have remained mostly out of reach of ordinary people. The details have existed mainly on reels of magnetic tape in a form comprehensible only to the largest brains, human or electronic. But *Astronomical Algorithms* changes all that. With his special knack for computations of all sorts, the author has made the essentials of these modern techniques available to us all.

In some cases there is even no need to write a program. For instance, the old TRS-80 Model I contained a built-in function which sorted 1000 numbers in 9 seconds, and 8000 numbers in 83 seconds. It appears that the sorting time is approximately proportional to N here, not to N^2, so probably the QUICKSORT method was used.

To conclude, we can recommend the "straight insertion" (SIMPLE SORT) if the set of data to be sorted is not too large, for instance for $N < 200$. For larger sets it is well worth while to use QUICKSORT.

Besides numerical data, often strings (names) are to be sorted, such as X$(1) = "Ceres", X$(2) = "Pallas", etc. Each character has its own value. The complete list with all signs constitutes the so-called ASCII table, a part of which is given in Table 6.D. [ASCII = "American Standard Code for Information Interchange".]

TABLE 6.D

Visible ASCII Characters

After each character its decimal code is given

space	32	8	56	P	80	h	104
!	33	9	57	Q	81	i	105
"	34	:	58	R	82	j	106
#	35	;	59	S	83	k	107
$	36	<	60	T	84	l	108
%	37	=	61	U	85	m	109
&	38	>	62	V	86	n	110
'	39	?	63	W	87	o	111
(	40	@	64	X	88	p	112
)	41	A	65	Y	89	q	113
*	42	B	66	Z	90	r	114
+	43	C	67	[	91	s	115
,	44	D	68	\	92	t	116
-	45	E	69	]	93	u	117
.	46	F	70	^	94	v	118
/	47	G	71	_	95	w	119
0	48	H	72	`	96	x	120
1	49	I	73	a	97	y	121
2	50	J	74	b	98	z	122
3	51	K	75	c	99	{	123
4	52	L	76	d	100	|	124
5	53	M	77	e	101	}	125
6	54	N	78	f	102	~	126
7	55	O	79	g	103		

Chapter 7

Julian Day

In this Chapter we give a method for converting a date, given in the Julian or in the Gregorian calendar, into the corresponding Julian Day number (JD), or vice versa.

General remarks

The Julian Day number or, more simply, the *Julian Day* (*) (JD) is a continuous count of days and fractions thereof from the beginning of the year −4712. By tradition, the Julian Day begins at Greenwich mean *noon*, that is, at 12^h Universal Time. If the JD corresponds to an instant measured in the uniform scale of Dynamical Time, the expression *Julian Ephemeris Day* (JDE) (**) is often used. For example,

1977 April 26.4 UT = JD 2443 259.9
1977 April 26.4 TD = JDE 2443 259.9

In the methods described below, the Gregorian calendar reform is taken into account. Thus, the day following 1582 October 4 (Julian calendar) is 1582 October 15 (Gregorian calendar).

(*) In many books we read "Julian *Date*" instead of "Julian Day". A *date* consists of a year number, a month, and a day of the month, in any calendar. For me, a Julian date is a date in the Julian calendar, just as a Gregorian date refers to the Gregorian calendar. The JD has nothing to do with the Julian calendar.

(**) Not JED as it is sometimes written. The "E" is a sort of index appended to "JD": JDE = (Julian Day)$_{\text{Ephemeris}}$. The name *Ephemeris* comes from "Ephemeris Time", the old name for the uniform Dynamical Time. The abbreviation JDE has been used in the *Minor Planet Circulars* until 1991 inclusively, when it was changed to JDT. Here the "T" means Terrestrial Dynamical Time (see Chapter 10). But what if we want to refer to the Barycentric Dynamical Time, or in cases where the very small difference between TDT and TDB does not matter? For this reason, I prefer to continue to use the abbreviation JDE.

The Gregorian calendar was not at once officially adopted by all countries. This should be kept in mind when making historical research. In Great Britain, for instance, the change was made as late as in 1752, and in Turkey not before 1927.

The Julian calendar was established in the Roman Empire by Julius Caesar in the year −45 and reached its final form about the year +8. Nevertheless, we shall follow the astronomers' practice consisting of extrapolating the Julian calendar indefinitely to the past. In this system we can speak, for instance, of the solar eclipse of August 28 of the year −1203, although at that remote time the Roman Empire was not yet founded and the month of August was still to be conceived!

There is a disagreement between astronomers and historians about how to count the years preceding the year 1. In this book, the "B.C." years are counted astronomically. Thus, the year before the year +1 is the year zero, and the year preceding the latter is the year −1. The year which the historians call 585 B.C. is actually the year −584. (Do *not* use the mention "B.C." when using negative years! "−584 B.C.", for instance, is incorrect.)

The astronomical counting of the negative years is the only one suitable for arithmetical purposes. For example, in the historical practice of counting, the rule of divisibility by 4 revealing the Julian leap years no longer exists; these years are, indeed, 1, 5, 9, 13, ... B.C. In the astronomical sequence, however, these leap years are called 0, −4, −8, −12 ..., and the rule of divisibility by 4 subsists.

We will indicate by INT(x) the greatest integer less than or equal to x. For example:

$$\mathrm{INT}(7/4) = 1 \qquad \mathrm{INT}(5.02) = 5$$
$$\mathrm{INT}(8/4) = 2 \qquad \mathrm{INT}(5.9999) = 5$$

There may be a problem with negative numbers. In most programming languages, INT(x) has the definition given above. In that case we have, for instance, INT(−7.83) = −8, because −7 is indeed larger than −7.83.

But in other languages, such as FORTRAN 77, INT is the integer part of the *written* number, that is, the part of the number that precedes the decimal point. In that case, INT(−7.83) is −7. This is called *truncation*, and some programming languages have both functions: INT(x) having the first of the above-mentioned meanings, and TRUNC(x) or FIX(x).

Hence, take care when using the INT function for negative numbers. (For positive numbers, both meanings yield the same result). In the formulae given in this Chapter, however, the argument of the INT function is always positive.

Calculation of the JD

The following method is valid for positive as well as for negative years, but not for negative JD.

Let Y be the year, M the month number (1 for January, 2 for February, etc., to 12 for December), and D the day of the month (with decimals, if any) of the given calendar date.

- If $M > 2$, leave Y and M unchanged.
  If $M = 1$ or 2, replace Y by $Y-1$, and M by $M+12$.
  In other words, if the date is in January or February, it is considered to be in the 13th or 14th month of the preceding year.
- In the *Gregorian* calendar, calculate

$$A = \mathrm{INT}\left(\frac{Y}{100}\right) \qquad B = 2 - A + \mathrm{INT}\left(\frac{A}{4}\right)$$

In the *Julian* calendar, take $B = 0$.

- The required Julian Day is then

$$\mathrm{JD} = \mathrm{INT}(365.25\,(Y+4716)) + \mathrm{INT}(30.6001\,(M+1)) + D + B - 1524.5 \qquad (7.1)$$

The number 30.6 (instead of 30.6001) will give the correct result, but 30.6001 is used so that the proper integer will always be obtained. [In fact, instead of 30.6001, one may use 30.601, or even 30.61.] For instance, 5 times 30.6 gives 153 exactly. However, most computer languages would not represent 30.6 exactly — see in Chapter 2 what we said about BCD — and hence might give a result of 152.9999998 instead, whose integer part is 152. The calculated JD would then be incorrect.

In formula (7.1), the constant 4716 has been added to the argument of the first INT function, in order to avoid trouble for negative years.

Example 7.a — Calculate the JD corresponding to 1957 October 4.81, the time of launch of Sputnik 1.

Here we have $Y = 1957$, $M = 10$, $D = 4.81$.
Because $M > 2$, we leave Y and M unchanged.
The date is in the Gregorian calendar, so we calculate

$$A = \mathrm{INT}(1957/100) = \mathrm{INT}(19.57) = 19$$
$$B = 2 - 19 + \mathrm{INT}(19/4) = 2 - 19 + 4 = -13$$
$$\mathrm{JD} = \mathrm{INT}(365.25 \times 6673) + \mathrm{INT}(30.6001 \times 11) + 4.81 - 13 - 1524.5$$
$$\mathrm{JD} = 2436\,116.31$$

Example 7.b — Calculate the JD corresponding to January 27 at 12^h of the year 333.

Because $M = 1$, we have $Y = 333 - 1 = 332$ and $M = 1 + 12 = 13$.

Because the date is in the Julian calendar, we have $B = 0$.

$$\mathrm{JD} = \mathrm{INT}(365.25 \times 5048) + \mathrm{INT}(30.6001 \times 14) + 27.5 + 0 - 1524.5$$
$$\mathrm{JD} = 1842\,713.0$$

The following list gives the JD corresponding to some calendar dates. These data may be useful for testing a program.

2000 Jan. 1.5	2451 545.0	1600 Dec. 31.0	2305 812.5
1999 Jan. 1.0	2451 179.5	837 Apr. 10.3	2026 871.8
1987 Jan. 27.0	2446 822.5	−123 Dec. 31.0	1676 496.5
1987 June 19.5	2446 966.0	−122 Jan. 1.0	1676 497.5
1988 Jan. 27.0	2447 187.5	−1000 July 12.5	1356 001.0
1988 June 19.5	2447 332.0	−1000 Feb. 29.0	1355 866.5
1900 Jan. 1.0	2415 020.5	−1001 Aug. 17.9	1355 671.4
1600 Jan. 1.0	2305 447.5	−4712 Jan. 1.5	0.0

If one is interested only in dates between 1900 March 1 and 2100 February 28, then in formula (7.1) we have $B = -13$.

In some applications it is needed to know the Julian Day JD_0 corresponding to January 0.0 of a given year. This is the same as December 31.0 of the preceding year. For a year in the *Gregorian* calendar, this can be calculated as follows.

$$Y = \text{year} - 1 \qquad A = \text{INT}\left(\frac{Y}{100}\right)$$

$$JD_0 = \text{INT}(365.25\,Y) - A + \text{INT}\left(\frac{A}{4}\right) + 1721\,424.5$$

For the years 1901 to 2099 inclusively, this reduces to

$$JD_0 = 1721\,409.5 + \text{INT}(365.25 \times (\text{year} - 1))$$

When is a given year a leap year ?

In the ***Julian calendar***, a year is a leap (or bissextile) year of 366 days if its numerical designation is divisible by 4.
All other years are common years (365 days).
For instance, the years 900 and 1236 were bissextile years, while 750 and 1429 were common years.

The same rule holds in the ***Gregorian calendar***, with the following exception: the centurial years that are *not* divisible by 400, such as 1700, 1800, 1900, 2100, are common years. The other century years, which *are* divisible by 400, are leap years, for instance 1600, 2000, and 2400.

The *Modified Julian Day* (MJD) sometimes appears in modern work, for instance when mentioning orbital elements of artificial satellites. Contrary to the JD, the Modified Julian Day begins at Greenwich mean *midnight*. It is equal to

$$\text{MJD} = \text{JD} - 2400\,000.5$$

and therefore MJD = 0.0 corresponds to 1858 November 17 at 0^h UT.

Calculation of the Calendar Date from the JD

The following method is valid for positive as well as for negative years, but not for negative Julian Day numbers.

Add 0.5 to the JD, and let Z be the integer part, and F the fractional (decimal) part of the result.

If $Z < 2299\,161$, take $A = Z$.

If Z is equal to or larger than 2299 161, calculate

$$\alpha = \text{INT}\left(\frac{Z - 1867\,216.25}{36524.25}\right)$$

$$A = Z + 1 + \alpha - \text{INT}\left(\frac{\alpha}{4}\right)$$

Then calculate

$$B = A + 1524$$

$$C = \text{INT}\left(\frac{B - 122.1}{365.25}\right)$$

$$D = \text{INT}(365.25\,C)$$

$$E = \text{INT}\left(\frac{B - D}{30.6001}\right)$$

The day of the month (with decimals, if any) is then

$$B - D - \text{INT}(30.6001\,E) + F$$

The month number m is $E - 1$ if $E < 14$; $E - 13$ if $E = 14$ or 15

The year is $C - 4716$ if $m > 2$; $C - 4715$ if $m = 1$ or 2

Contrary to what has been said about formula (7.1), in the formula for E the number 30.6001 may *not* be replaced by 30.6, even if the computer calculates exactly. Otherwise, one would obtain February 0 instead of January 31, or April 0 instead of March 31.

Example 7.c — Calculate the calendar date corresponding to JD 2436 116.31.

2436 116.31 + 0.5 = 2436 116.81

$Z = 2436\,116$ and $F = 0.81$

Because $Z > 2299\,161$, we have

$$\alpha = \text{INT}\left(\frac{2436\,116 - 1867\,216.25}{36524.25}\right) = 15$$

$$A = 2436\,116 + 1 + 15 - \text{INT}\left(\frac{15}{4}\right) = 2436\,129$$

Then we find

$B = 2437\,653$ $\quad C = 6673$ $\quad D = 2437\,313$ $\quad E = 11$

day of month = 4.81

month $m = E - 1 = 10$ (because $E < 14$)

year $= C - 4716 = 1957$ (because $m > 2$)

Hence, the required date is 1957 October 4.81.

Exercise : Calculate the calendar dates corresponding to

JD = 1842 713.0 and JD = 1507 900.13.

Answers: 333 January 27.5 and −584 May 28.63.

Time interval in days

The number of days between two calendar dates can be found by calculating the difference between their corresponding Julian Days.

Example 7.d — The periodic comet Halley passed through the perihelion of its orbit on 1910 April 20 and on 1986 February 9. What is the time interval between these two passages?

1910 April 20.0 corresponds to JD 2418 781.5
1986 Febr. 9.0 corresponds to JD 2446 470.5

The difference is 27 689 days.

Exercise : Find the date exactly 10 000 days after 1991 July 11.

Answer: 2018 November 26.

Day of the week

The day of the week corresponding to a given date can be obtained as follows. Compute the JD for that date at 0^h UT, add 1.5, and divide the result by 7. The remainder of this division will indicate the weekday, as follows: if the remainder is 0, it is a Sunday, 1 a Monday, 2 a Tuesday, 3 a Wednesday, 4 a Thursday, 5 a Friday, and 6 a Saturday.

The week was not modified in any way by the Gregorian reform of the Julian calendar. Thus, in 1582, Thursday October 4 was followed by Friday October 15.

Example 7.e — Find the weekday of 1954 June 30.

1954 June 30.0 corresponds to JD 2434 923.5

2434 923.5 + 1.5 = 2434 925

The remainder of the division of 2434 925 by 7 is 3. Hence it was a Wednesday.

Day of the Year

The number N of a day in the year can be computed by means of the following formula [1].

$$N = \text{INT}\left(\frac{275\,M}{9}\right) - K \times \text{INT}\left(\frac{M+9}{12}\right) + D - 30$$

where M is the month number, D the day of the month, and

$K = 1$ for a leap (bissextile) year,
$K = 2$ for a common year.

N takes integer values, from 1 on January 1, to 365 (or 366 in leap years) on December 31.

Example 7.f — 1978 November 14.

Common year, $M = 11$, $D = 14$, $K = 2$.
One finds $N = 318$.

Example 7.g — 1988 April 22.

Leap year, $M = 4$, $D = 22$, $K = 1$.
One finds $N = 113$.

Let us now consider the reverse problem: the day number N in the year is known, and the corresponding date is required, namely the month number M and the day D of that month. The following algorithm was found by A. Pouplier, of the Société Astronomique de Liège, Belgium [2].

As above, take

$K = 1$ in the case of a leap year,
$K = 2$ in the case of a common year.

$$M = \mathrm{INT}\left(\frac{9\,(K+N)}{275} + 0.98\right)$$

If $N < 32$, then $M = 1$

$$D = N - \mathrm{INT}\left(\frac{275\,M}{9}\right) + K \times \mathrm{INT}\left(\frac{M+9}{12}\right) + 30$$

REFERENCES

1. Nautical Almanac Office, U.S. Naval Observatory, Washington, D.C., *Almanac for Computers for the Year 1978*, page B2.
2. A. Pouplier, letter to Jean Meeus, 1987 April 10.

Chapter 8

Date of Easter

In this Chapter we give a method for calculating the date of the Christian Easter Sunday of a given year. For the Jewish Pesach, see next Chapter.

Gregorian Easter

The following method has been given by Spencer Jones in his book *General Astronomy* (pages 73–74 of the edition of 1922). It has been published again in the *Journal of the British Astronomical Association*, Vol. 88, page 91 (December 1977) where it is said that it was devised in 1876 and appeared in Butcher's *Ecclesiastical Calendar*.

Unlike the formula given by Gauss, this method has no exception and is valid for all years in the Gregorian calendar, hence from the year 1583 on. The procedure for finding the date of Easter is as follows:

Divide	by	Quotient	Remainder
the year x	19	—	a
the year x	100	b	c
b	4	d	e
$b + 8$	25	f	—
$b - f + 1$	3	g	—
$19a + b - d - g + 15$	30	—	h
c	4	i	k
$32 + 2e + 2i - h - k$	7	—	l
$a + 11h + 22l$	451	m	—
$h + l - 7m + 114$	31	n	p