Homework #9

Physics 129 Spring 2022

Problems due **Saturday, May 28, at 11:55 P.M.**

Please read the homework guidelines handout on the course web page.

Before attempting this assignment, ensure your RPi is connected to the Internet, then run the `update_physrpi` script.

Better answers and code will get better grades.

Reading

None this week.

Problems

1. Coin Toss Simulation. Write a program that

a. contains a function simulating 100 coin tosses, returning the number of heads thrown

b. plots a histogram displaying the result of calling the function 1000 times

c. overlays on the plot, in a different color, a graph of the Gaussian distribution with the same mean, standard deviation, and maximum value as the binomial distribution that corresponds to your coin toss simulation.

For this problem only, instead of a text file, turn in a pdf file produced with LaTeX that contains a title, your name, your final plot, a figure caption, and any information you would have put in the text file.

Hints: $G(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$ $\quad \mu = Np \quad \sigma = \sqrt{Npq} \quad q = 1 - p.$

```
latex texample.tex
latex texample.tex
dvips texample.dvi
ps2pdf texample.ps
```

2. Counting Simulation. Write a program that

a. has a function which simulates photon counting for 1000 one-millisecond intervals, with the probability of a detection in each interval being 0.002. The function should return the total number of detections.

b. plots a histogram displaying the result of calling the function 1000 times

c. overlays on the plot, in a different color, a graph of the Poisson distribution with mean, standard deviation, and maximum value corresponding to this simulation.

Hints: $P(n) = \frac{\mu^n e^{-\mu}}{n!}$ $\quad \mu = Np \quad \sigma = \sqrt{\mu}$.

3. Gradschool Admissions. A physics department gets $N = 787$ applicants for its Ph.D. program, of whom it admits $N_A = 146$.

a. The best estimate p of the admission probability is N_A/N, so $N_A = Np$. A repeated admissions process with N as given above and actual admission probability equal to p will produce values for N_A drawn from a distribution. What is the standard deviation σ_A of this distribution?

b. What is the 1-standard deviation uncertainty in p corresponding to σ_A?

c. Assume p is the exact admission probability. In other words, neglect the uncertainty you calculated above. Using `binom.pmf()`, compute the probability that from a randomly selected group of 154 applicants, 48 or more are admitted.

   Hint:
   ```
   from scipy import stats
   stats.binom.pmf(x_array, N, p)
   ```

d. Usually (normally?), a Gaussian is a good approximation to a binomial distribution. However, sometimes you can run into trouble if you are looking at the tails of the distributions. Use `erfc()` to calculate the probability from the previous part using a Gaussian approximation. By what factor is the resulting answer too small?

   Hint:
   ```
   from scipy import special
   0.5*special.erfc(...)
   ```

e. From a group of 154 applicants, 48 are admitted. Find the best estimate p_G of the admission probability for this group, and the 1-standard deviation uncertainty in p_G.

f. Find the best estimate p_N of the admission probability for applicants not in the group of 154, and the corresponding 1-standard deviation uncertainty.

g. Turn in a plot showing the three Gaussian approximations to the distributions of p_N, p, and p_G in different colors. Make the areas of the Gaussians proportional to the populations of the corresponding groups.

4. Numerical Integration. Write a program that computes $\int_{-\infty}^{\infty} e^{-x^2}\,dx$

a. by adding the areas of a lot of small rectangles

b. using a Monte Carlo simulation.

You may not use Python libraries to do the integration (write your own code). Produce a plot for each method of the fractional error (compared with the known value) as a function of N, where N is the number of rectangles or random points.