## Phys 129 lectures

25Apr22

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## Note to students

These are my raw, unedited lecture notes. They are intended to remind me what to say during lecture, so you are reading a very terse conversation between me and myself. You will not find good explanations here, though you will find some useful examples. I am providing the notes to you as an index to the course content, so that you can, for example, find where a particular topic is discussed in the recorded lectures. This will work most of the time, but there are places where the order of the material in the recordings is somewhat different than what you see here.

At the beginning of each lecture, you will see the names of the video files in which the corresponding content is covered. You can see the video file names on the course web page by moving the mouse cursor over the lecture links. The time at which a particular topic appears in a recorded lecture is sometimes shown in square brackets, for example like this: [12:46].

WARNING: The recorded lectures are from spring 2020, and some of these notes are from before then. While the content they contain is still useful, you must refer to the course web page for the current versions of due dates, assignment guidelines, and course rules and procedures. In addition, some of the programs and hardware discussed in the recordings have been replaced with newer versions.

If you print this file and bring it to class, you can take your own notes on the page next to mine.

\_\_\_\_\_\_

1.

Online:

unpack\_rpi.mp4
rpi\_install.mp4
passwords.mp4
homework\_overview.mp4

Introduction textbooks

```
attendance required
   lab times, 5223 hours
Install Raspbian
   assemble RPi
     use cover with opening for jumper wires
     insert SD card
   run through rpi_install.txt - stop before update script
shutting down safely
password security
  hash functions (md5, sha1, sha256)
  password hashing and shadow file (salting)
Preview web page
Homework guidelines on course web page
Homework #1 on course web page
  document password
  preview HW #1
keyboards, mice, monitors
  UCSB surplus
_____
Online:
  course_info.mp4
  course_intro.mp4
  numbers_and_files.mp4
_____
Turn on networking
Run update script
Intro to 129L presentation
Late hw -10% 24 h, 0 after
grading
                            summer with final exam
  attendance 10
                            10
  homework 65
                            45
  final project 25
                            45
Office hours: Tuesday 5:00-6:00
fixres
vm
<Ctrl-Alt> F1
Introduction to files
  xxd
     binary numbers
     hexadecimal numbers
     octal numbers
```

**ASCII** 

```
cat
      less
   Python
      python3
      ipython3
      spyder3
      1 + 1
      0b11110000
      0xf0
      bin(240)
      hex (240)
      oct (240)
      0010
3.
   Online:
      theshell.mp4
   _____
   ls (with options: -a, -l, -F)
   type -a
   man
      searching in man page
   help
   Directories
   Filesystem
      pwd
      cd
      mkdir
      rmdir
   Typing tricks
      tab completion
      <ESC>-.
      history, up and down arrows, !N
      .bash_history
   Shell aliases
      alias foo='echo "Hi there!"'
      .bashrc
         # begins shell comments
      type -a
   Shell variables, environment variables
      var='hello there'
      echo $var
      export FOO='this is foo'
      python3
        import os
        a = os.getenv('FOO')
        print(a)
      export var
      env
```

Control characters - demonstrate with vi

```
^D - End of transmission 0x04
   ^C - Interrupt
   ^Z - Stop process
       jobs
       fg
cat
  > (file redirection)
  >> (append)
   concatenate using multiple files on command line
   cat file.txt | cat | cat | rev
Files
  file
  ср
  mv
   rm
  less
_____
Online:
  more_shell.mp4
  flashdrive.mp4
_____
grep
WC
Pipes
   1s | grep
Shell wildcards
     ls *.txt
     ls /etc/*.conf
      ls .??*
File extensions not so important in UNIX
command substitution
   $()
   cat 'ls *.txt'
   cat $(ls *.txt)
Files, processes, kernel
  htop
  ps
   echo "Hi there!" > /dev/pts/0
Standard input, output, error (file descriptors vs. files)
cat
   stdin -> stdout
  takes stdin from command line arguments, more than one possible
   concatenate
grep foo * 2>/dev/null
```

```
id -a
      chmod
      sudo
      /etc/passwd
      /etc/shadow
      /etc/group
      /dev/snd/*
   PATH
      run proctemp script from ~
   CDPATH
   Text editor
      vi
         .vimrc
         :r in vi
         :r! in vi
   Format flash drives
      rsync
   Intro to shell scripts
      #!/bin/sh
      clear
      echo "Hello, world!"
5.
   _____
   Online:
     python1.mp4
   _____
   Machine code, assembly language, compiler, interpreter
   Python scripts
      #!/usr/bin/env python3
      # hello.py - Say hello
      # 29Apr19 Everett Lipman
      print('Hello, world!')
   Comments in code
   6 ways to execute Python:
      interpreter, #!, command line, IDE,
      ipython, jupyter
   Implicit and explicit typecasting
      type()
      a = 5
      type(a)
      b = 5.5
      type(b)
      c = 'foo'
      type(c)
```

Permissions

```
d = '5.5'
   d
   x = float(d)
   x = float(c) -> error
print
   introduce print formatting
   a = 5.5
   print('%d' % a)
   print('%f' % a)
   print('%.3f' % a)
   scientific notation
   b = 5.5e10
   print('%.3f' % b)
   print('first: %f second: %e' % (a,b))
   tuples
   c = (a,b)
   print('first: %f second: %e' % c)
   print(..., end='')
   \n in strings
      print('Hello\n\nThere\n')
formatting need not be inside a print call
   s = 'The number is %.3f' % 3.1415
man 3 printf
dir()
   a = 5.5
   dir(a)
   dir(float)
   b = 5 + 2j
   dir(b)
   b.conjugate()
import
   sin(3)
   import math
   dir(math)
   math.sin(3)
   from math import *
   sin(3)
   PYTHONSTARTUP
      startup3.py
   np.sin(3) - will operate on arrays
   np.sin(np.arange(100))
Python help()
   help(np.arange)
   help(print)
   import math
   help(math.sin)
   help('modules')
PYTHONPATH
```

```
_____
Online:
  programming1.mp4
_____
range(), lists, tuples
  iterables vs. lists
  list()
  a = list(range(10))
  b = tuple(range(10)
  a[3] = 100
  b[3] = 100
     tuples immutable
  range(1, 6, 1)
  a = np.arange(10)
  b = np.arange(1,100,2)
   len()
for loops
while loops
   import time
  while True:
     print('still going...')
     time.sleep(0.5)
  i = 0
  while i < 5:
      i = i + 1
     print(i)
input
  instr = input('string, please: ')
  print(instr)
  type(instr)
  instr = input('number, please: ')
  conversion int(), float()
  a = int(instr)
  type(a)
Exit codes
  ls foo.txt
  echo $?
subprocess
   subproc_ls.py
   demonstrate in Python shell
Functions and methods [33:25]
  def add(a,b):
     c = a + b
     return(c)
  add strings
     operator overloading
      __add__() method
   docstring
     """, '''
      a = """This is a
```

```
multi-line string."""
      print(a)
Reading and writing files
   file_readlines.py
   help(open)
   careful_write.py
   open files for append
   with/as
      with open('lines.txt', 'r') as infile:
         lines = infile.readlines()
         print(lines)
      with open('out.txt', 'w') as outfile:
         outfile.write('Hello, file!\n')
Clarify files vs. file descriptors
   file is a place to send data to or get data from
   disk vs. virtual (in kernel)
Conditionals
   1 < 2
   1 > 2
   i == 6 \text{ vs } i = 6
   for i in range(10):
      print(i)
      if i > 5:
         print('i > 5 !!!')
         break
      else:
         print('i <= 5')</pre>
i = -1
while i <= 5:
   i = i + 1
   print(i)
if 1 < 2 and True:
   print('hi')
if 1 < 2 or True:
   print('hi')
5 // 2 and 5 / 2
5 % 2
Python objects [1:14:25]
   strings
      string methods
      help(str)
      split() with and without argument
         a = 'Hello, there, this is a string.'
         a.split()
         a.split(',')
         a.split('ll')
   list methods
      a = list(range(6))
      dir(a)
```

```
7.
   Online:
      programming2.mp4
   Integer overflow
      r = range(1, 10000, 2)
      list(r)
      b = 0
      for a in r:
        b = b + a*a
      b
      a = np.arange(1, 10000, 2)
      b = a*a
      b.sum()
             # -837059544 on RPi
   a = np.arange(1, 100000, 2)
   type(a[0]) # <class 'numpy.int32'> on RPi
   b = np.arange(1, 100000, 2, dtype='int64')
   type(b[0]) # <class 'numpy.int64'> on RPi
   a.sum()
   b.sum()
   Integer calculation with proper size [12:46]
      a = np.arange(1, 10000, 2, dtype='int64')
      b = a*a
             # 16666665000 on RPi
      b.sum()
   Parallel calculation with numpy
      a = np.arange(1000000)
      b = np.sqrt(a)
      never use loops if numpy function can do the job
   Arbitrary precision integers
      21580943287502943875934 * 42549872943879274
         -> 918266394892314543987627518927329991916
      918266394892314543987627518927329991916 / 42549872943879274
      918266394892314543987627518927329991916 // 42549872943879274
   Floating point numbers
      binary representation
      mantissa, exponent
      IEEE 754
      float.hex()
         def md(x):
           print('%.55f' % x)
         a = 2.0
         a = 4.0
         a = 3.5
           md(a)
           a.hex()
```

a.reverse()
a.append()

```
a = 0.7
         md(a)
         a.hex()
            '0x1.66666666666p-1'
           = (1 + 6/16 + 6/256 + 6/4096 + ...)*2^{-1}
          ^{\sim} = (1 + 0.375 + 0.0234375 + 0.00146484375) * 0.5
           = 0.69951171875
         a.as_integer_ratio()
   sys.float_info
  print (np.finfo(np.float))
  Representation error and roundoff error
      a = [0.1] * 10
      sum(a)
      sum(a) == 1.0
List comprehensions
   [ i + i for i in range(10) ]
Bytes and character strings
   chr(65), chr(0x41), ord('A'), hex(ord('A'))
   a = b'Abytestring', list(a)
  b = [102, 111, 111]
   ''.join([chr(x) for x in b])
Names and objects
   immutable types
      integers
      tuples
  a = list(range(10))
  b = list(range(10))
   a
  b
   is keyword
      a is b
   a.reverse()
   a
  b
  c = b
   c is b
  b
   c.reverse()
  b
  d = c.copy()
   d.reverse()
Scoping and local variables
   a = 1
   def add(x, y):
      a = x + y
      return(a)
   def add(x, y):
      global a
      a = x + y
      return(a)
   a
```

```
Exceptions and error handling
   error_handling.py
  errprint.py
  try:
     1/0
  except Exception as foo:
     print(foo)
_____
Online:
  programming3.mp4
_____
Slicing, zero first
  a = list(range(100))
  b = list(range(10, 110, 1))
  a[10:20]
  b[10:20]
  a[:50]
  a[50:]
  a[-10]
  a[-10:]
  a[10:20:2]
  a[::-1]
      [i:j:k] - if i omitted, 0 for k > 0, n-1 for k<0.
              - if j omitted, n for k > 0, -n-1 for k<0.
              - if k omitted, 1
   assignment with slicing
     a[30:40] = range(10)
     a[40:60] = 5 -> error
        a[40:60] = [5]*20
         [5]*20
        a[40:60] = [5]*10
        len a
     a = np.arange(100)
     a[40:60] = 5
Dictionaries [11:19]
  person = {}
  person['color'] = 'blue'
  person['age'] = 47
  person['height'] = 1.776
  person['food'] = 'pizza'
  person['car'] = 'Toyota'
keys() method
sorted(iterable)
   iterable may be dict
   a = list('hi there')
   sorted(a)
List sort() method
```

```
a = list('hi there')
      a.sort()
   None
      months list
      months dictionary keyed on numbers
   Debugging with print()
      reportloop.py
   ps, htop, kill
      kill cat in another terminal
      kill -9 bash
      kill -STOP cat
      man 7 signal
      echo $$
   Symbolic links
      demonstrate ln -s
      mv, rm a symlink
      /usr/bin/python3
      absolute and relative symbolic links
   Optimization [55:05]
      don't do more computation than necessary
         polynomial evaluation
            a + bx + cx^2 + dx^3 : 3 additions, 6 multiplications
            a + x(b + x(c + xd)): 3 additions, 3 multiplications
      optimize inner loops
      use more memory
         swapping two elements in a list vs. creating a swapped list
         lookup tables
      avoid function calls
      recursion - not good in Python
   time shell builtin
      time -p sleep 2
   time module [1:12:42]
      time.perf_counter()
      timeloop.py
      pass
_____
   Online:
      graphics1.mp4
   ===========
   prime.c and prime.py
   prime.py optimization
      roundoff - // vs. / with 86795643218674325
         a = 86795643218674325.0
         a.as_integer_ratio()
      optimize inner loop by removing else: - 10% improvement
   Pythonic factorization
      a = 1232324123
```

```
np.sqrt(a)
  b = np.arange(35105) + 2
  b
   c = a % b
  b[c == 0]
   a/_
   23957*51439
apt-get and aptitude
   sudo aptitude install sysvbanner
argc, argv
   arguments.py
Operator precedence
  1+1*3
   6/2*3
  https://docs.python.org/3.9/reference/expressions.html
Numpy arrays
  np.arange()
  np.linspace()
  a = np.loadtxt('wind.dat')
  type(a)
  dir(a)
  help(a)
  a.dtype
  a.shape
  type(a[0])
  a[0].shape
   a[0][0]
   a[0][2]
  type(a[0][0])
   a.T
      different view of a, not a new object
      b = a.T
      c = a.copy().T
      a[1][1] = 99
      b
   a = np.zeros(100, dtype='complex')
Plotting
   drows = np.loadtxt('wind.dat')
  wdat = drows.T
   f1, ax1 = plt.subplots()
   ax1.plot(wdat[0], wdat[1], 'o')
   ax1.set_xlim(-5,25)
  ax1.set_ylim(0,12)
   ax1.errorbar(wdat[0], wdat[1], yerr=wdat[2], fmt='o', capsize=3)
   # ax1.plot(wdat[0], wdat[1], 'o')
   f1.show()
   f2, ax2 = plt.subplots()
   ax2.plot( np.sin(np.linspace(0,np.pi,1000)) )
   f2.show()
   ax2.plot( np.cos(np.linspace(0,np.pi,1000)) )
   f2.show()
```

```
10.
    _____
    Online:
      graphics2.mp4
    ==========
    Summer only:
    Project
       guidelines
      https://www.adafruit.com/category/105
      Use of packaged equation solvers, ray tracers, etc. in projects
    Online: project.mp4
    Graphics
      pixels, screen memory, 24-bit color
      xmag demonstration
      X = 100
      Y = 100
       f1, ax1 = plt.subplots()
      pvals = np.zeros((X, Y, 3), dtype='uint8')
      pvals[50,50,:] = (255,0,0)
      ax1.imshow(pvals, interpolation='none')
      f1.show()
       for i in range (16, 64, 1):
          for j in range (16, 64, 1):
             pvals[i,j,:] = (0,0xee,0)
       ax1.imshow(pvals, interpolation='none')
      f1.show()
       set color block with slicing instead of loop
          pvals[20:40, 20:40, :] = (0,0,0xff)
          ax1.imshow(pvals, interpolation='none')
          f1.show()
    img.py and cmapimg.py
    Complex numbers
      a = 5+2j
      type(a)
      a = 5+j does not work
      b = complex(5)
      a = 5+1j
      a*a
      a+a
       5+1j * 5+1j (unexpected result)
       (5+1j)*(5+1j)
   Mandelbrot and Julia sets
    Fifos (queues)
```

simple\_plot.py

demonstrate saving .eps files

```
from collections import deque
      a = [1, 2, 3]
      q = deque(a)
      q.append()
      q.popleft()
   Stacks
      append(), pop()
      dc
   PostScript
      qs
      hello, world
        stack-based syntax
        coordinate system
      gv
      rt345.eps
      Blue Book and PLRM
      vector graphics
        curveto.ps
        bigletter.ps
      EPS
        rt345.eps
        PS translate
        gv coordinate display
      petal.ps
11.
   _____
   Online:
      networking1.mp4
   _____
   School year only:
   Project
      guidelines
      https://www.adafruit.com/category/105
      Use of packaged equation solvers, ray tracers, etc. in projects
   Online: project.mp4
   Network protocol stack
      application email, http
                 TCP, UDP
      transport
      network
                IP
      link
                 ethernet
                 cable, radio wave, optical fiber, etc.
      physical
      ifconfig
        ip command Linux-specific
        netstat -ie
      ΙP
        IP numbers
        port numbers
        /etc/services
      UDP
```

```
TCP
   Basics of DNS
      host
    traceroute
   ping
    URLs
    nc -Cv web.physics.ucsb.edu 80
      GET /~phys129/lipman/ HTTP/1.0
    CRLF in HTTP and SMTP
    wget
      update script function
12.
    _____
    Online:
      networking2.mp4
    _____
    nc -Cv elo.physics.ucsb.edu 25
      HELO ealrpi1
      MAIL FROM: mrsmtp@example.com
      RCPT TO:lipm1@elo.physics.ucsb.edu
      DATA
      Subject: fake mail
       This is fake mail
      QUIT
    RFCs
      http://www.rfc-editor.org/rfc-index.html
      SMTP 821
      HTTP 2068
         RFC 7230, section 3.1.1 "Request Line"
    nc -lv 1024 < file
    nc 127.0.0.1 1024
    netstat --inet
    lsof -i
   Buffer overflows and stack smashing
    client.py
      also can write to socket
    server.py
       also can read from socket
    Python Requests library and Beautiful Soup
```

http://docs.python-requests.org/en/master/

import requests

https://www.crummy.com/software/BeautifulSoup/bs4/doc/

```
type(r)
     dir(r)
     r.content
     r.encoding
     r.text
     print(r.text)
   BRING HARDWARE TO NEXT LECTURE
13.
   -----
   Online:
     data_acquisition.mp4
   --> Air Can
   School year only:
     Detailed project scope due to me
   generators
     def gen():
        yield 1
        yield 2
        yield 3
     a = gen()
     next(a)
     next(a)
     next(a)
     next(a)
   Object-oriented programming vs. imperative programming
   stripchart.py
     classes
   I2C bus
   MCP9808
     https://www.adafruit.com/product/1782
     Wiring diagram on course web page
     tempdemo.py
   ADS1015 and solar cell
     adcdemo.py
     fastadc.py
        Real-time kernels
14.
   _____
   Online:
     fourier.mp4
   _____
   Sampling
     Nyquist theorem
     aliasing
```

r = requests.get('http://web.physics.ucsb.edu/~phys129/lipman/')

```
convolution and signal recovery from samples
         (f*g)(t) = \int_{-\infty}^{\infty} f(tau)g(t - tau) dtau
   /etc/init.d, /etc/init
      man service
   FFT
      evaluate polynomial at n complex nth roots of 1
      O(n^2) \rightarrow O[nlog(n)]
   matplotlib.mlab.psd() and numpy.fft
      fft_spectrum.py
      psd_spectrum.py
15.
   -----
   Online:
      random.mp4
   ==========
   man random
      RNG period
   dir(np.random)
   help(np.random)
   np.random.random()
      np.random.random(100)
      np.random.random((10,10))
   np.random.seed()
   np.random.randint()
   np.random.uniform()
   Monte Carlo
      area/value of pi
      simple integration
      For N evaluations, error goes as 1/sqrt(N)
      for any number of dimensions. For grid
      (trapezoidal rule), error goes as 1/N^{2/d}
      in d dimensions.
   LaTeX
      latex texample.tex
      latex texample.tex
      dvips texample.dvi
      ps2pdf texample.ps
16.
   _____
   Online:
      fork_exec_tracing.mp4
   _____
   fork() and threading
      fork_example.py
      starting a new process
        help exec
        sleep 3
```

exec sleep 3

```
man exec, execve
         shell forks, then calls wait(2)
         import os
         help(os.execv)
         -> in second terminal, ps -uxw | tail -5
         os.execv('/bin/sleep', ['/bin/sleep', '10'])
         -> ps -uxw | tail -5; show that process name changed
      import threading
      dir (threading)
      thread_example.py
   Homework problems
      fork_ls.py demo
      threadchart.py demo
   System call tracing
      man strace
      strace 1s
         trace output goes to stderr
      strace 1s nosuchfile 2> foo
         run on date
         run on errprint.py
         run on fork_example.py
17.
   ______
   Online:
      distributions1.mp4
   _____
   Coin toss/binomial mean and SD
      P(N,k) = \frac{N!}{k!(N-k)!} p^k q^{(N-k)}
      mean = Np
      \sigma = \sqrt(Npq)
   Binomial sampling example
      Dice game: win $5 on 6, lose $1 on any other roll
      you play:
      N = 300
      w = 50
      p = 1/6, uncertainty?
      Npq = 300*(1/6)*(5/6) = 41.667
      \sigma = 6.45
      w = 50 +/- 6.45
      p = 0.167 +/- 0.022
      cousin plays:
      N = 100
      w = 31, fair?
      Np = 16.67, sqrt(Npq) = 3.73
      (31 - 16.67)/3.73 = 3.8 \text{ sd}
      Both tails: 1/6900
      Probability of w = 31 or greater is 1/13,800
      Best estimate for cousin:
      N = 100, w = 31
```

```
p = w/N = 0.31
      sqrt(Npq) = 4.6, 4.6/100 = 0.046
      p = 0.31 + / - 0.046
        -----
      Other examples
         election polling
         count fish in lake
         all error bars touch line
    Poisson distribution
      Binomial as Np \rightarrow 0, q \rightarrow 1
      P(n) = (\mu^n/n!)e^{-\mu}
       \sigma = \sqrt(Np) (q \approx 1)
      histogram fluctuation for uniform random numbers
          a = np.random.random(10000)
          f1, ax1 = plt.subplots()
         ahist = ax1.hist(a, 10)
         f1.show()
         compare np.sqrt(Npq) to np.sqrt(Np)
          f2, ax2 = plt.subplots()
         ahist = ax2.hist(a, 100)
          f2.show()
          compare np.sqrt(Npq) to np.sqrt(Np)
    Nonuniform random numbers
      Gaussian distribution
       1/\text{sigma*sqrt}(2\pi) \exp(-(x-mu)^2/2\pi)
      a = np.random.random(10000)
      f1, ax1 = plt.subplots()
       ahist = ax1.hist(a, 50)
      f1.show()
      b = np.random.normal(1.0, 0.1, 10000)
      f2, ax2 = plt.subplots()
      bhist = ax2.hist(b, 50, (0.6, 1.4))
      f2.show()
      from scipy import stats
      x = np.linspace(0.6, 1.4, 1000)
       ax2.plot(x, 160*stats.norm.pdf(x, 1.0, 0.1))
      f2.show()
18.
    _____
    Online:
       integration.mp4
      distributions2.mp4
    _____
    Discrete integration
       careful with dx
       find the integral of y = x from 0 to 1:
         N = 10000
         dx = 1.0/N
         x = np.linspace(0, 1.0-dx, N)
         y = x
         ydx = y*dx
```

```
y.sum()*dx
      trapezoidal rule
      simpson's rule (parabola)
      other methods
      numint.py
   For the error function, \sigma = 1/\sqrt{2}.
   So we integrate one tail by giving the number
   of standard deviations divided by \sqrt(2)
   as the argument to erfc(). With N = 100,
   p = 1/6, w = 31, we are 3.846 \setminus sigma from
   the mean, so:
   from scipy import special
   0.5*special.erfc( 3.846/np.sqrt(2) )
      6.0030881107545723e-05 = 1 in 16658.
   from scipy import stats
   x = np.arange(101)
   a = stats.binom.pmf(x, 100, 1.0/6.0)
   a[31:].sum()
      0.00029578488275368661 = 1 in 3381.
   f, ax = plt.subplots()
   ax.plot(a, drawstyle='steps-post')
   f.show()
   Integer histograms
      a = np.random.randint(1,7,1000)
      f, ax = plt.subplots()
      ax.hist(a)
      f.show()
      inthist.py
   Arbitrary nonuniform random numbers demo
      g(y) = 2y, f(x)dx = dx = g(y)dy = 2ydy (2y is normalized on 0-1)
      dx = 2ydy
      x = y^2, y = sqrt(x)
      x = np.random.random(10000)
      f1, ax1 = plt.subplots()
      xhist = ax1.hist(x, 50)
      f1.show()
      y = np.sqrt(x)
      f2, ax2 = plt.subplots()
      yhist = ax2.hist(y, 50)
      f2.show()
19.
   _____
   Online:
      diffeq1.mp4
   =========
   Finite differencing
      draw curve with straight line approximation to tangent
```

ydx.sum()

```
[x(t + dt) - x(t)]/dt
   Q/C - iR = 0
      i = -dQ/dt
   dQ/dt + Q/RC = 0
  C = 10 \text{ uF}, R = 20k, RC = 0.2
  Q(t + dt) - Q(t) = -Q(t)dt/RC
  Q(t + dt) = Q(t)*(1 - dt/RC)
   capfd.py
Second derivative finite difference
   draw curve with straight line approximation to tangent
      three points, two slopes
      { [x(t+2dt) - x(t+dt)]/dt - [x(t+dt) - x(t)]/dt }/dt
      {x(t+2dt) - 2x(t+dt) + x(t)}/{dt^2}
  m d^2x/dt^2 + kx = 0
  d^2x/dt^2 + w^2x = 0
  x(t+2dt) = 2x(t+dt) - x(t)(1 + w^2*dt^2)
   shmfd.py
Senior thesis overview (Laplace's equation in einzel lens)
_____
Online:
  diffeq2.mp4
_____
2-d finite differencing
  D = del, p = phi
  Laplace eqn
  E = -Dp
  D.E = D.(-Dp) = d^2p/dx^2 + d^2p/dy^2 = \rho\langle p \rangle
       = 0 in free space
   {p(x+dx,y) - 2p(x,y) + p(x-dx)}/dx^2} +
   {p(x,y+dy) - 2p(x,y) + p(x,y-dy)}/dy^2 = 0
   4 p(x,y) = p(x+dx,y) + p(x-dx,y) + p(x,y+dy) + p(x,y-dy)
  p(x,y) = [p(x+dx,y) + p(x-dx,y) + p(x,y+dy) + p(x,y-dy)]/4
Relaxation
   laplace_cap.py
a = np.arange(9).reshape((3,3)) + 1
np.roll(a,-1,axis=0)
np.roll(a,1,axis=0)
np.roll(a,-1,axis=1)
lapcap_roll.py
lapcap_live.py
Sparse matricies
 -4p(x,y) + p(x+dx,y) + p(x-dx,y) + p(x,y+dy) + p(x,y-dy) - b = 0
```

Ax = b; b from boundary points  $x = A^{-1}b$ 

ESCI evaluations