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Decision Support

A framework for conceptualising hybrid system dynamics and agent-based simulation models

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ABSTRACT

The growing complexity of systems and problems that stakeholders from the private and public sectors have sought advice on has led systems modellers to increasingly use multimethodology and to combine multiple OR/MS methods. This includes hybrid simulation that combines two or more of the following methods: system dynamics (SD), discrete-event simulation, and agent-based models (ABM). Although a significant number of studies describe the application of hybrid simulation across different domains, research on the theoretical and practical aspects of combining simulation modelling methods, particularly the combining of SD and ABM, is still limited. Existing frameworks for combining simulation methods are high-level and lack methodological clarity and practical guidance on modelling decisions and elements specific to hybrid simulation that modellers need to consider. This paper proposes a practical framework for developing a conceptual hybrid simulation model that is built on reviews and reflections of theoretical and application literature on combining methods. The framework is then used to inform and guide the process of conceptual model building for a case study in controlling the spread of COVID-19 in care homes. In addition, reflection on the use of the framework for the case study led to refining the framework itself. This case study is also used to demonstrate how the framework informs the structural design of a hybrid simulation model and relevant modelling decisions during the conceptualisation phase.

1. Introduction

Researchers/practitioners using single simulation modelling approaches (i.e., system dynamics (SD), discrete-event simulation (DES), and agent-based models (ABM)) face challenges representing the multi-dimensional nature of complex systems composed of interactive and interconnected constituents with dynamic behaviours (Brailsford et al., 2013; Eldabi et al., 2016). Hybrid simulation that combines different simulation methods continues to have a strong appeal to the Operational Research (OR)/Management Science community as it offers an opportunity to overcome these challenges and capture important characteristics and behaviours of such systems (Mustafee et al., 2017; Nguyen et al., 2020a, 2020b, 2020c). The divergent philosophical approaches of SD and ABM possess strong and unique explanatory capabilities to address various dimensions of system complexity. Hybrid simulation helps encompass micro, meso, and macro perspectives, strategic, tactical, and operational levels, as well as detail and dynamic complexity (Bobashev et al., 2007; Morel & Ramanujam, 1999; Phelan, 1999). SD, a top-down continuous simulation method, represents complex system structures using stocks, flows, feedback, and time delays to study their behaviour over time (Sterman, 2000). While SD does not directly address emergence due to the fixed nature of system structures, it captures learning through changes in loop dominance caused by non-linearities. In contrast, ABM, a bottom-up simulation method, models autonomous, dynamic, and adaptive systems by simulating the behaviour of individual-level entities known as agents and the interaction between agents and their environment (Wilensky & Rand, 2015). These interactions give rise to emergent system-level outcomes, which can, in turn, influence subsequent behaviours and interactions and lead to self-organization. The aggregate-level behaviour and individual-level interactions captured by SD and ABM respectively mean that combining them in a hybrid model provides an opportunity to gain a more comprehensive understanding of complex systems. Kazakov et al. (2021) provide support for this complementarity through proposing a problem-structuring approach that combines resource-feedback and agent-based perspectives and incorporates several theories that align with SD and ABM.

Various works have outlined the key activities involved in the

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modelling process for single simulation methods, including conceptual modelling for SD (Sterman, 2000), ABM (Railsback & Grimm, 2019), DES (Banks et al., 2005; Hoover & Perry, 1989; Robinson, 2014), and generic modelling described in MS/OR (Hillier & Lieberman, 1995; Law et al., 2007). However, the modelling processes for single simulation methods do not offer sufficient support for dealing with the increased complexity of hybrid simulation models and additional decisions on when, why, and how to combine the methods. In addition, despite the growing interest and popularity in this approach (Brailsford et al., 2019; Howick & Ackermann, 2011), guidance for designing and utilising hybrid models, especially for those combining SD and ABM, is still at a high level and therefore, not sufficiently practical. The application of hybrid simulation described in the existing literature is also ad-hoc without explicit rationalisation on why and how the methods are combined. To address the methodological gap of combining SD and ABM, this paper proposes a practical framework for developing a conceptual hybrid SD-ABM model.

The framework aims to assist modellers in making decisions on when, why, and how to combine simulation methods in future studies, which will facilitate the process of developing hybrid models in addition to improving the models' usefulness. It also suggests an approach for describing the conceptual presentation of the overall hybrid model to i) communicate the model design effectively to stakeholders and the wider research communities for confidence building and ii) facilitate verification and the development of computerized simulation models.

The next section provides a literature review of existing methodological guidance for combining simulation methods. Section 3 presents a proposed hybrid simulation modelling framework for combining SD and ABM. Section 4 describes its application to a case study that investigates the transmission dynamics of COVID-19 across networks of care homes by sharing bank/agency staff. The framework in Section 3 makes relevant reference to different parts of the case study in Section 4. The paper concludes by discussing the contributions and implications of this work in practice, its limitations, and future research.

2. Literature review

A literature review was undertaken to identify practical or methodological guidance for combining SD and ABM. This sought to explore why, when, and how modellers should combine SD and ABM and identify the limitations of existing guidance in informing practice. Appendix 1 details the search terms used for this review. We included two types of publications in this review: i) theoretical papers (including those providing case studies to support the theoretical discussion) which describe frameworks to aid the combining of SD and ABM models or multi-method approaches more generally (Appendix 1) and ii) case studies that discuss the application of hybrid SD-ABM models in various application contexts (Appendix 2). Theoretical papers provided an overview of existing methodology for combining simulation methods and both types of studies revealed gaps in the methodological clarity and practice of combining simulation methods. The methodology overview and gaps are discussed in the remainder of this section. Case studies also served as a basis to collate, elaborate, and extend the discussion on combining SD and ABM in Section 3.

While the early works on hybrid simulation focus on using one simulation method to validate outputs generated by another and triangulating outputs (Phelan, 2004; Rahmandad & Sterman, 2008; Scholl, 2001), hybrid simulation applications have evolved to diverse approaches for combining methods. Akkermans (2001) and Schieritz and GroBler (2003) employed SD to model the internal decision logic or cognitive structure of individual agents within ABM. However, these studies did not specify how the agents interact with each other nor did they explicitly explain the role of SD in facilitating such interactions. Borshchev and Filippov (2004) suggested two ways to combine SD and ABM. First, SD is used to represent the internal processes inside discretely communicating agents, a design frequently adopted in hybrid simulation models within supply network studies (Schieritz & GroBler, 2003). Second, agents, such as people and households, live in an environment, such as housing, jobs and infrastructure, whose dynamics is modelled using SD. Lorenz and Jost (2006) highlighted the use of SD in creating an active, dynamic environment for agents of an ABM. In addition, Martinez-Moyano et al. (2007) and Swinerd and McNaught (2012 and 2014) focused on elucidating input-output interactions and running sequences between SD and ABM sub-models in hybrid models. While most studies delineate static designs of hybrid SD-ABM models, a number of studies in epidemiology and ecology modelling propose a dynamically adaptive design. In this design, a hybrid model switches between SD and ABM representations based on a predetermined threshold (Bobashev et al., 2007; Vincenot et al., 2011; Wallentin & Neuwirth, 2017). The concept of this design originates from the application of the law of large numbers and the central limit theorem when the number of active agents in an ABM is large and reaches a threshold. In this case, aggregating the behaviour of similar agents and modelling their behaviour through mean-field approximations should be possible. Conversely, when the number of agents becomes small, the SD model is switched back to the ABM to avoid artifacts possibly caused by the SD. Appendix 1 includes further details on the theoretical underpinnings of combining SD and ABM.

From the papers included in Appendix 1, six high-level designs were identified that summarise the existing theoretical guidance on approaches to combining simulation methods as follows (Borshchev & Filippov, 2004; Chahal & Eldabi, 2008; Morgan et al., 2017; Mustafee et al., 2018; Swinerd & McNaught, 2012; Wallentin & Neuwirth, 2017):

i) *Parallel:* Independent single-method models are developed either to address different aspects of the same problem or to represent the same problem for direct comparison.
ii) *Sequential:* Two or more sub-models embedded in different simulation modelling methods run sequentially once and the output of one sub-model becomes the input to another sub-model.
iii) *Interaction:* Distinct single-method sub-models interact cyclically multiple times during run time. A sequential design can be considered as a special case of the interaction design.
iv) *Enrichment:* Single simulation methods are combined to form one unified hybrid model in which one method dominates and is enhanced by elements of another.
v) *Integration:* One seamless hybrid model in which it is impossible to identify where one simulation approach ends and the other begins. Swinerd and McNaught (2012) describe three different integrated designs of combining SD and ABM, including agents with rich internal structure, stocked agents, and parameters with emergent behaviour.
vi) *Dynamically switching:* The model dynamically alters among different SD-ABM configurations. The switching point is informed by a threshold at which the size of the population of interest is small or large enough, making the impact of heterogeneity among individuals of each entity type on the model's outcomes become more or less significant, respectively. The rationalised use of this design is to optimize the trade-off between the predictive and computational modelling performance.

Existing literature on hybrid simulation reveals four major limitations. Firstly, the literature lacks clarity on the processes and aspects that modellers should take into account when reaching a decision on the design of a hybrid model. This lack of clarity obfuscates the reasons for and benefits of applying hybrid simulation in the existing literature and the choice of a specific hybrid simulation design. Eldabi et al. (2016) asserted that existing hybrid simulations tend to be "ad hoc and pragmatic with no clear methodology" (p1389). Secondly, the guidance reviewed describes hybrid simulation designs at a high level and emphasises their differences based on the direction of interaction and frequency of interaction over a time window. Although enrichment, interaction, and integration designs share many similarities, they differ in the separability of single-method sub-models, which refers to the extent to which the sub-models can function independently without significant interdependence. They also differ in method dominance, which is the emphasis placed on each method within the hybrid model. The relative nature of these characteristics leads to the difficulty in selecting an appropriate design for a hybrid model. Thirdly, although there is some evidence of conceptual models in the existing literature (Jo et al., 2015; Kolominsky-Rabas et al., 2015; Páez-Pérez & Sánchez-Silva, 2016), it is far from comprehensive. Whilst sub-models are often described, the linkages between them are not explained. Conceptual modelling, especially for hybrid simulation, has been the most under-researched area in the simulation modelling lifecycle (Brailsford et al., 2019; Tako et al., 2019). This gap leads to difficulty in communicating the modelling design and its rationale, and poor communication is a challenge for confidence-building, reproducibility, and the development of generic lessons about when, why, and how to develop hybrid models. Finally, most of the existing hybrid simulation modelling studies focus on dealing with issues in particular domains rather than improving the methodological clarity of combining SD and ABM (i.e., application studies in Appendix 2 outnumbered theoretical studies in Appendix 1).

The remainder of this paper will focus on the above key gaps revealed by the literature review, which highlight the lack of comprehensive and practical guidance for designing and developing a hybrid SD-AB model, including the lack of a detailed presentation of a conceptual hybrid simulation model. In particular, the focus of the framework discussed in this paper is the technical design aspects of the conceptual modelling of a hybrid simulation.

3. A hybrid simulation modelling framework for combining SD and ABM in a hybrid model

This section presents a theoretical framework that guides the conceptual modelling phase of a hybrid SD-AB simulation study. The framework provides practical instructions that specify steps modellers should take to build a conceptual hybrid SD-AB simulation model. It also suggests the elements that modellers should describe to provide an overarching representation of a conceptual hybrid model for other modellers and stakeholders. These elements include i) modules – self-contained single-simulation-method components constituting a hybrid model, ii) abstraction level, modelling method rationale, and content for each module, and iii) their linkages (i.e., flows of information, interfaces, and updating rules). Each of these elements is defined in this section. The framework includes stages and steps that align with key activities from existing conceptual modelling processes for single simulation methods. These are as follows: i) define the problem of interest (stage 1) and ii) formulate the conceptual model to represent the problem (Steps 3.2 and 3.3 of stage 3) (Banks et al., 2005; Hillier & Lieberman, 1995; Hoover & Perry, 1989; Law et al., 2007; Railsback & Grimm, 2019; Robinson, 2014; Sterman, 2000). Stage 2, step 3.1 of stage 3, and stage 4 are specific to hybrid simulation to assist the decision on the appropriateness of combining methods, determining single-method modules with hybrid models, and designing links between the modules, respectively. Therefore, they do not correspond with activities from any existing conceptual modelling processes for single simulation methods. Stages 3 and 4 provide detailed activities for hybrid simulation, referring to relevant frameworks from the literature when necessary.

The proposed framework offers clearer and more detailed guidance on developing a conceptual hybrid simulation model, emphasising practical aspects compared to existing guidance. It also includes new insights into modelling practice beyond what is currently discussed in the literature. The framework builds on an earlier version presented in a conference paper by Nguyen et al., (2020a, 2020b, 2020c). The earlier version was solely based on the synthesis of existing guidance on combining different simulation modelling methods and only described the conceptual modelling process for hybrid simulation at a high level. It touched on the definition of different elements of a hybrid simulation model but did not provide sufficient detail to inform practice. For example, it defined interfaces but did not explain different interface designs and how modules exchange information at their interfaces. Also, the clarity of the conceptual model design steps and iterations between them were improved from this version, rendering it more practical and user-friendly. The existing guidance typically discusses the high-level benefits of hybrid simulation to justify its use, such as complementarity and the ability to capture various system levels. By contrast, the updated version of the framework, which is based on further analysis and synthesis of hybrid SD-ABM studies across different application domains (Appendix 2), provides greater insights into different application contexts for hybrid simulation, the rationale of hybrid simulation uses for each context, and designs of linkages between SD and ABM modules. This nuanced perspective facilitates the ease with which modellers can relate these application contexts to the specific problems they aim to address. The framework was also enhanced and refined through reflections on the process of building the case study hybrid model described in Section 4. For example, we recognised that selecting the high-level hybrid simulation modelling design (e.g., interaction, enrichment, and integration) described in existing guidance before defining the linkages between modules was not practical, and the former did not inform the latter; thus, we removed this step from the process. Our framework incorporates confidence-building approaches to specific stages and steps, an aspect absent from existing guidance. We iterated between developing the model and developing the framework.

Fig. 1 shows the framework for conceptualising a hybrid simulation model, consisting of four main stages. In stage 1, modellers explore the problem of interest by defining the modelling objectives, scoping the problem, and specifying its characteristics as would normally be done in simulation modelling studies using single methods (for example, this is referred to as understanding the problem situation and determining the modelling objectives in Robinson (2008) and problem formulation in Balci (2012)). In stage 2, modellers determine whether an individual simulation method or a hybrid simulation method is most appropriate to model the problem of interest based on the exploration in stage 1. We considered only SD and ABM in this framework, but it is extendable to include DES. Modellers would follow and adhere to the same principles described within the stages and steps of the framework to build a hybrid simulation model that contains DES modules. Pertinent guidance related to DES will be integrated at each relevant stage and step. Stage 3 consists of activities to design the modules comprising the hybrid model, and stage 4 comprises activities to link the modules. The stages are explained in more detail in the following sub-sections. It is noted that modellers may have to iterate between these stages and steps several times to reach a design that fits their modelling purposes. Throughout the description of the framework, the confidence-building activities that modellers should implement are also discussed. This includes both black-box and white-box validation. Although black-box validation, which considers whether the overall behaviour of the model represents the behaviour of the real system with sufficient accuracy for its purpose (Kleijnen & Wan, 2007; Robinson, 1997), requires a completed simulation model, modellers should plan for this activity at the model conceptualisation phase. Therefore, planning for black-box validation is discussed within the framework.

3.1. Stage 1: exploring the problem

The first stage is to explicitly define the nature of a problem under investigation and the modelling objectives, similar to what is normally done in a single modelling method study. This stage is vital to help identify the model scope and the level of detail required and, thus, the selection of appropriate simulation modelling methods (Randers, 1980; Roberts et al., 2012; Robinson, 2014; Wilensky & Rand, 2015).

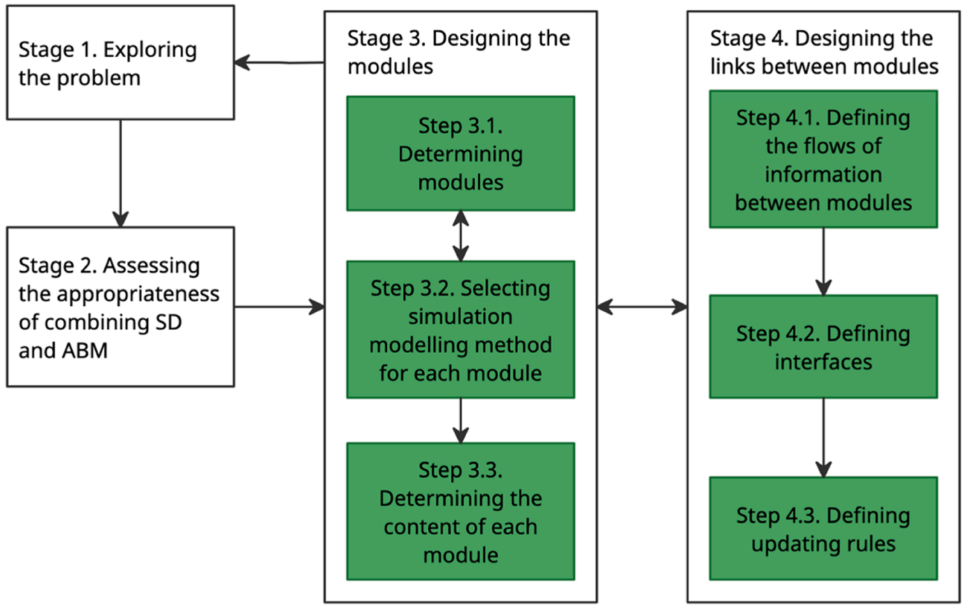

Fig. 1. The conceptualisation stages of the hybrid SD-ABM simulation modelling framework.

Identifying the model outcomes is also useful to inform the content of the model.

In this stage, modellers need to gain a detailed understanding of the problem as it guides modelling decisions. Identifying the problem characteristics that are important to include in the modelling study is domain-specific and requires input from the problem owners and other relevant stakeholders. For example, building a simulation model to explore a public health policy or a health promotion program usually involves defining problem characteristics such as the target population (e.g., age, gender, and risk factors), the target settings, potential interventions, the health outcomes, and the time horizon adequate to capture differences in outcomes across interventions. Gaining an understanding of the problem characteristics builds on a combination of expert knowledge, literature review, and data from sources such as interviews with stakeholders, surveys, focus group discussions, participatory processes, observations, laboratory experiments, or network analyses.

In addition, modellers consulting with the problem owners and relevant stakeholders need to identify and justify the modelling assumptions and simplifications arising from scoping the model. They also need to discuss constraints such as the timeline of the modelling study, difficulties associated with data collection, access to applicable data, and cost constraints (Mykoniatis & Angelopoulou, 2020). If these constraints cannot be reasonably addressed, the objectives and the scope of the model need to be revised.

3.2. Stage 2: assessing the appropriateness of combining SD and ABM

Once the problem is articulated, modellers need to establish that simulation is appropriate to tackle the problem and then whether a hybrid simulation modelling approach is more appropriate compared to a single simulation method. Each simulation modelling method has strengths and limitations, making it more or less suited for specific problems at different levels of decision-making (Nguyen et al., 2020a, 2020b, 2020c; Scholl, 2001). Therefore, the problem characteristics explored in stage 1 and stakeholders' decision-making perspectives (e.g., operational, tactical, and strategic) should drive the selection of simulation modelling methods. Modellers will choose a hybrid simulation modelling approach that combines the strengths of SD and ABM if one simulation paradigm has difficulty capturing the complexity of the problem on its own. Table 1 presents contexts that motivate the application of SD-ABM combinations based on a literature review of hybrid simulation modelling studies (Appendix 2). This context-based approach was inspired from the classification of problem situations in Lättilä et al. (2010). Examples are selected from the studies included in Appendix 2 to demonstrate each contextual application.

It is worth noting that, as Fig. 1 indicates, assessing the appropriateness of combining SD and ABM is likely to be iterative. For example, while modellers implement their choice for a hybrid SD-ABM simulation model in stage 3, they may learn new characteristics that lead them to redefine the problem addressed in stage 1 and reassess the method(s) selected in stage 2. Another example is that if a modeller initially selects SD in stage 2 and then finds it difficult to formulate the casual relationships for a highly uncertain variable, they may decide to return to step 1 to explore the variable and then reassess the method selected in stage 2.

In selecting the appropriate simulation modelling approach, modellers also need to consider practical issues that may constrain hybrid simulation modelling work. Building a hybrid simulation model is likely to take more time, especially if modellers lack knowledge and experience in both SD and ABM (Mingers, 2001). Furthermore, modellers may lack access to potentially high-cost software that combines different simulation method modules. Modellers may also lack the skills to code from scratch and combine these modules.

Modellers should plan for black-box validation to facilitate its implementation when a completed simulation model is built. Modellers can utilise the pattern-oriented modelling approach which is used to uncover the internal organisation of entities and mechanisms in agent-based complex systems. This approach provides a framework for comparing models with qualitative and quantitative empirical patterns observed at various system levels (Grimm et al., 2005). It enables the identification of essential system elements necessary to capture the processes aligned with the model's purpose. Part of the aim is to strike a balance between simplicity and inclusiveness, avoiding a narrow set of assumptions and parameter sets that only coincidentally reproduce specific empirical outputs. These patterns can also help rationalise critical design decisions of the model (see black box validation in Section 4.5). Since pattern-oriented modelling occurs at different system levels, it is essential to consider the patterns reproduced by the overall hybrid models at this stage and those reproduced by the ABM components within the hybrid models, as described in stage 3.

Table 1
Description of context that motivates the application of hybrid SD-ABM models.

Typical context of application of hybrid simulation (When?)	Example	Module design	Method often used in previous studies[a]	Benefits of hybrid simulation models compared with single modelling methods (Why?)
Strategic policy decisions with consideration of a wide range of operational/local circumstances	Capacity planning of solar energy resources by modelling the electricity system with a flexible structure that captures energy demand in a region characterized by different households (e.g., singles, couples, families with children) and their electricity consumption during weekdays, weekends, different times of the day and different seasons (Mazhari et al., 2009)	Using ABM to zoom in on one part of the system modelled in SD	SD	Provide richer insight into the interdependences between the behaviours of a system at a macro level and the behaviours of multiple agents involved at the micro level Contribute to explaining why a strategic policy may fail to improve operational performance Offer flexibility to model different operational circumstances or intervention scenarios explicitly
Focusing on causal relationships in a system with stochastic and/or highly uncertain elements	Causal relationships between risk metrics and variables, including risk perception, value perception, and risk preference, in modelling technological innovation risks that involve uncertainties caused by multiple agents with conflicting information and objectives and that are subject to limited perception and behavioural capacity (Wu et al., 2010)	Using ABM to model uncertain/stochastic elements in the system modelled in SD	Stochastic SD	Help model stochastic/uncertain elements in casual relationships explicitly by entering variation into the appropriate sources/ decision levels of the model Provide richer insight by capturing parameters with emergent behaviours
Involving interdisciplinary processes, several organisational factors (e.g., social, economic, epidemiological, and political)	The assessment of innovative health technologies prior to their launch involving interdisciplinary processes: population dynamics, disease dynamics, healthcare financing, and healthcare (Kolominsky-Rabas et al., 2015)	SD and ABM modules represent different disciplines	Both SD and ABM	Model the system in a more natural way Harmonize interdisciplinary expertise of experts whose views may be rooted in either SD or ABM Optimize trade-off between the computational and the predictive performance of the model
Comprising multiple interconnected subsystems	The problem of large infrastructure systems development comprising interconnected sub-systems and involving the partnership of public and private entities with conflicting goals and information asymmetry (Glock et al., 2016; Páez-Pérez & Sánchez-Silva, 2016)	SD and ABM modules represent different sub-systems	Both SD and ABM	Model the system in a more natural way Address and satisfy different views of stakeholders on the system
Social and/or spatial interactions between entities affecting and/ or affected by the dynamic global environment	Social-spatial fragmentation and segregation affected by cause-effect chains of urban shrinkage (Haase et al., 2012)	Agents live in an environment represented by an SD module	ABM	The active, dynamic urban environment, where a spatial, social structure of agents live, is characterized by casual relationships and, thus, difficult to model using state variables representing the environment Provide richer insights into relationships between agents' behaviours and external environment
Social and/or spatial interactions between entities affecting and/ or affected by their internal dynamics	Modelling the complex safety behaviours (e.g., resting decision) of truck drivers in interaction with co-workers (Goh & Askar Ali, 2016)	Embedding an SD module in each agent to represent its internal structure	ABM	Internal dynamics are complex and, thus, difficult to model using state variables Provide richer insights into relationships between agents' internal dynamics and their behaviours

[a] This column shows the simulation methods that are often used in previous studies to address similar questions. This is based on the literature review provided in the example studies.

3.3. Stage 3: designing the modules

A model can consist of several components called “modules”. A module should principally be self-contained and bounded with pre-defined interfaces (input and output) to the external world, including other modules. In a hybrid simulation model, we find it useful to consider a module as one logical component of a hybrid model developed using one of the simulation modelling methods (Onggo, 2014). It is noted that the term ‘module’ as used in the context of hybrid simulation does not refer to the ‘module’ within the software design technique of modular programming. In an integrated hybrid model, the boundary between modules is not explicit because the interfaces between modules are intertwined. In this case, it is more challenging to define the modules but still doable. For example, we can identify the single-method modules and their interfaces in four hybrid models that Brailsford et al. (2019) suggested as truly integrated (Alzraiee et al., 2015; Bergman et al., 2008; Chatfield & Pritchard, 2013; Varol & Gunal, 2015). Appendix 3 details these models' designs.

In this stage, modellers will determine and describe constituent modules of the model, levels of abstraction, and the simulation modelling method used to build each module. Stages 3 and 4 assist in white-box validation, which determines that the constituent parts of the model represent the corresponding parts of the real system with adequate accuracy for its purpose (Kleijnen & Wan, 2007). These stages offer a plausible design of the hybrid model's structure for presenting to stakeholders and experts for face validation and interface validation (see white-box validation in Section 4.5).

3.3.1. Step 3.1: determining modules

Several approaches are appropriate for determining modules. Djanatliev and German (2015) suggested defining independent problem areas within a specific domain scope and modelling each area using one of the simulation methods. For example, Kolominsky-Rabas et al. (2015) developed a model for assessing innovative health technologies prior to their launch that involves interdisciplinary processes and is divided into modules by these disciplines, including population dynamics, disease dynamics, healthcare financing, and healthcare operation. Alternatively, modellers can use a hierarchical breakdown. For example, in the context of healthcare, modules can describe global, national, regional, institutional, individual person, and internal levels (e.g., internal body processes and disease progression). Modellers can also define modules based on the application contexts described in Table 1 (for an example, see the case study in Section 4.1).

3.3.2. Step 3.2: selecting simulation modelling methods for each module

After identifying the modules of a problem, modellers need to determine the level of abstraction and justify the selection of a particular simulation modelling method used for each module (Brailsford et al., 2013) (for an example, see the case study in Section 4.2). Modellers may need to iterate between step 3.1 and step 3.2, splitting or merging modules. While rationalizing the choice of a simulation modelling method for each module, modellers may decide to partition a module if they cannot build it using one single modelling method. They may also consider combining modules with the same modelling method to simplify the model structural design while maintaining the "module" principle definition.

3.3.3. Step 3.3: determine the content of each module

Having identified the simulation method for each module, modellers determine the content of each module. SD modules will contain key variables, influencing factors, and feedback interrelations. For ABM modules, modellers identify key agents, their characteristics and behavioural rules, and their interactions. Modellers can adopt Section 2 (Entities, state variables, and scales) and Section 4 (Design concepts) of the Overview, Design concepts, and Details (ODD) protocol to develop the content for each ABM module (Grimm et al., 2017) (e.g., the ODD protocol in Nguyen et al. (2022a, 2022b)). These sections cover types of agents and their characteristics, and they provide an overview of their interactions and behavioural rules and what the model's time steps represent in reality. SD and ABM modules can be described using their own conceptual modelling tools, such as Stock-and-flow Diagrams and Causal Loop Diagrams (Coyle, 1997; Maani & Cavana, 2000; Richardson, 1991; Richardson & Pugh, 1997; Roberts et al., 1997; Sterman, 2001) for SD and State-chart diagrams, Agent-Object-Relationship diagrams for ABM (Scheidegger et al., 2018; Wagner, 2003). In the STRESS guideline for strengthening the reporting of empirical simulation studies, Monks et al. (2019) suggested three checklists for describing the basic conceptual building blocks of SD, ABM, and DES models. In designing the content for each module, modellers must also keep in mind the modelling objectives to justify why they include or exclude particular elements. Additionally, modellers should record any assumptions and simplifications made during this step and present them to the problem owner and any relevant stakeholders to ensure the validity and credibility of the model. At this step, modellers should plan confidence-building approaches for individual modules using the existing standard approaches for single-method models.

3.4. Stage 4: designing the links between modules

In this stage, modellers need to define elements to link the modules comprising the hybrid model, including information flows, interfaces, and updating rules. Performing this stage also provides learning about stage 3. The modules' scope and content determined in stage 3 must be sufficient to provide the links between the modules and define their interfaces. This approach of defining modules, interfaces, and flows shares similarities with the Discrete EVent System specification (DEVS) formalism, where DES models are broken down into atomic or coupled components, each having well-defined input and output ports, state transitions, and time advancements (Vangheluwe, 2000; Zeigler & Vahie, 1993).

3.4.1. Step 4.1: defining the flows of information between modules

In this step, modellers decide what information modules should exchange and the direction of this information's flow between modules (see Fig. 3 of the case study in Section 4 for an example of information flows between modules). Modellers explicitly define whether information flows between two modules in one or both directions. This will inform the design of interfaces between modules in step 4.2. Modellers also need to describe the frequency of information flows that inform the detailed design of updating rules in step 4.3 (e.g., once a day, twice a week).

3.4.2. Step 4.2: defining interfaces

In this step, modellers need to define clear and logical interfaces for each pair of modules. An interface between the two modules defines how the information is passed from the generating module to the

Table 2
Interfaces between SD and ABM modules in a hybrid model (A detailed description, visual presentations, and examples for each design are provided in Nguyen et al. (2022a, 2022b)).

Interface	Description
Information flows from SD module to ABM module	
(1a) Stock levels define agent-specific state variables	The level of a stock in an SD module embedded in each agent of an ABM module can determine a characteristic (i.e., state variable) of that agent.
(1b) Generating agents from stocks	Small crowds of individual agents with specific characteristics can be generated from stocks representing large population numbers. Individual agents can be generated using distribution functions based on existing empirical data or theories to represent the necessary heterogeneity of these agents.
(2) Stock levels define behaviours of individual agents	Stock levels in an SD module determine the corresponding behaviours that individual agents in an ABM module will execute.
(3) Stock levels bound aggregate measures of agents	A stock level in an SD module bounds an aggregated measure of agents in an ABM module. The aggregated measure of agents must not exceed the level of a particular stock. Aggregate measures of agents can be the sum of values for an agent-specific state variable or the size of the agent population with a specific characteristic. While a stock level directly affects the behaviour of individual agents in interface design (2), in this design, it indirectly affects behaviour based on the collective measure of agents, summing up their state variables.
(4) Stock levels define agents' network topologies	The levels of stocks in the SD module determine the corresponding spatial relationship and/or interacting network topology among agents in the ABM module.
Information flows from ABM module to SD module	
(5) Agents' state variables affect flows	Agents' state variables may evolve during a simulation as they execute a behaviour or interact with other agents and/or the environment. Changes in agents' state variables can affect flows in an SD module.
(6) Behaviours of agents affect flows	Behaviours of agents in an ABM module can influence flows in an SD module by increasing/decreasing parameters used in equations for flows.
(7) Aggregated measures of agents affect flows	An aggregated measure of agents in an ABM can influence a flow in an SD module. When SD and ABM modules represent different parts of a system and agents physically move from the ABM module to the SD module, they are removed from the ABM module and aggregated into a stock in the SD module. This movement is represented as an inflow of the stock.
(8) Network topologies affect flows	The spatial/social relationship and/or network topologies of agents in an ABM module can affect the flows in an SD module.

receiving module during the running time of the hybrid model. Table 2 provides an overview of information flows between components of an SD module (i.e., stocks and flows) and an ABM module (i.e., agent-specific state variables, agents' behaviours, aggregated measures of agents, and network topology). A detailed discussion of categories of information flows can be found in Nguyen et al. (2022a, 2022b). These categorisations emerged from a literature review of hybrid SD-ABM models across various domains and were based on reflection from the modelling process in the case study described in Section 4. The complete description of hybrid modelling studies and information flows can be found in Appendix 2.

While the interface designs 1–3 and 6–8 have been used in hybrid SD-ABM models discussed in the literature, interface designs 4 and 5 emerged during the modelling process of the case study discussed in Section 4. The interfaces described in the reviewed studies were analysed and grouped into categories. Simultaneously, potential interface designs for the case study model and the key elements of SD and ABM that can interact were considered. This iterative process led to the emergence of designs 4 and 5. Below are hypothetical examples of these two newly proposed interface designs for demonstration purposes.

Stock levels define agents' network topology: This interface design would be useful to capture the coordinated actions of agents in response to changes in the internal/external environment. A hypothetical example is a hybrid model for a pandemic that consists of an SD module that simulates the transmission dynamics in the community. The ABM module simulates healthcare facility agents interconnected by the patient-transfer pathways defined by a network topology. When the infected population (a stock in the SD module) increases to a certain threshold that the current network topology could no longer cope with, such transferring pathways may need to reform to handle this increasing demand (network topology changes).

Agents' state variables affect flows: An example of this interface design is a hybrid model that comprises an ABM module simulating transmissions between individual staff and resident agents in a care home and an SD module representing its connected hospitals. Resident agents have a state variable characterising their state of infection (e.g., susceptible and infected). Infected residents are assumed to require acute medical care and are, therefore, admitted to hospitals. This implies that a change in resident agents' state of infection will affect the admission inflow to a patient stock in the hospital SD module.

3.4.3. Step 4.3: defining updating rules

Updating rules define when information is sent from one module to another and how new information is handled by the receiving module to maintain the logical consistency of the hybrid model (Onggo, 2014). Modellers specifically need to address the following issues when defining updating rules: i) SD and ABM modules in a hybrid simulation model may use different time advancement methods. SD is compatible with both the continuous and the discrete concept of time (Sterman, 2000); the latter allows SD to advance using fixed-time increments. ABM typically advances using fixed-time increments but can adopt variable-time increments; ii) although the modules in a hybrid SD-ABM model may use the same time advancement method, they may use different time units; iii) modellers need to consider how updating rules would impact the modelling results and what implications there are for interpreting the model findings; iv) it is crucial to determine the logical order of several updates occurring at a pre-defined point in time as such order could affect the modelling results; v) modules use different simulation modelling software which has its own internal time management; and vi) modellers need to consider the run-time of a model when defining updating rules. We will discuss the first two issues concerning the synchronisation of time advancement methods and time units of modules in the next two paragraphs. This discussion implicitly assumes that the modules are run on a multi-method simulation software, such as AnyLogic. If the modules are executed using different software, synchronisation needs to employ the synchronisation algorithms from parallel and distributed simulation to avoid causality errors. The last two paragraphs of this section will explore the third and fourth issues. The fifth and sixth issues are out of the scope of this framework as the framework focuses on building a conceptual model.

Red triangle: Fixed-time increments advancement; Cyan triangle: Variable-time increment advancement.

If the modules in a hybrid SD-ABM model use fixed-time increment advancements with the same unit of time, updates can be easily done when the hybrid model advances its simulation time. If the modules use fixed-time increments but different units of time, updates can occur synchronously or asynchronously. Synchronously, all modules in a hybrid model will pass their information to other recipient modules at predefined simulation points, which can be, for example, the time step of one of the modules (Fig. 2A). Asynchronously, every time a module advances its simulation time, the module's status may alter and it will send new information to recipient modules which the interfaces define (Fig. 2B).

Additionally, information exchanges and updates can occur at variable-time increments in one module or both modules. The updating points can be triggered by stock levels or rates of an SD module reaching particular thresholds, agents of an ABM module executing specific behaviours, or specific properties of an ABM module emerging. Updates can occur synchronously when all modules in a hybrid model pass their information to other recipient modules at triggered variable-time increments in one module (Fig. 2C). Asynchronously, one module can send its information to the recipient module at its predefined simulation points (e.g., end of its time step) whilst the other module passes its information at triggered variable-time increments (Fig. 2D). All modules can also send their information to other recipient modules at their own triggered variable-time increments (Fig. 2E).

As a model is an abstraction of reality used for a specific objective, updating is unlikely to occur at the same frequency as in a real system. Therefore, modellers need to assess how the timing of updating rules would affect the modelling results and their interpretations. For example, components of a system may exchange information and update their status every second in reality; however, the modules of a hybrid model representing these components may be defined to update every hour or every day. Less frequent updates, in this case, are chosen as they are sufficient to meet the modelling objective and reduce the runtime. For another example, infectious disease models that update too infrequently (e.g., weekly rather than daily) could affect the transmission dynamics and thus potentially levels of infection.

At a pre-defined point of time, several modules of the hybrid model may need to exchange information and update their status. Determining the order of these updates is important to support the logic underlying the model and, thus, has implications on the modelling results. A tabular summary of updating rules is helpful for communicating the hybrid model design and facilitates face validation and computerised model building. Such a table can include when updates occur, the order of the updates at each updating point of time, the modules that send and receive information, and what information is exchanged between the modules (e.g., Table 3 in Section 4.4).

4. The hybrid SD-ABM model case study: the impact of sharing temporary bank/agency staff on COVID-19 transmission across care homes

In this section, the applicability of the framework presented in Section 3 will be demonstrated through a case study.

Modellers have used both SD and ABM to study the transmission dynamics of infectious diseases and interventions, which have contributed to informing policy decisions on infection prevention and control. On the one hand, SD models of infection control have simulated the population as aggregates of sub-populations representing different states of infection rather than individuals with distinct characteristics and behaviours (Keeling & Rohani, 2008). Therefore, these SD models

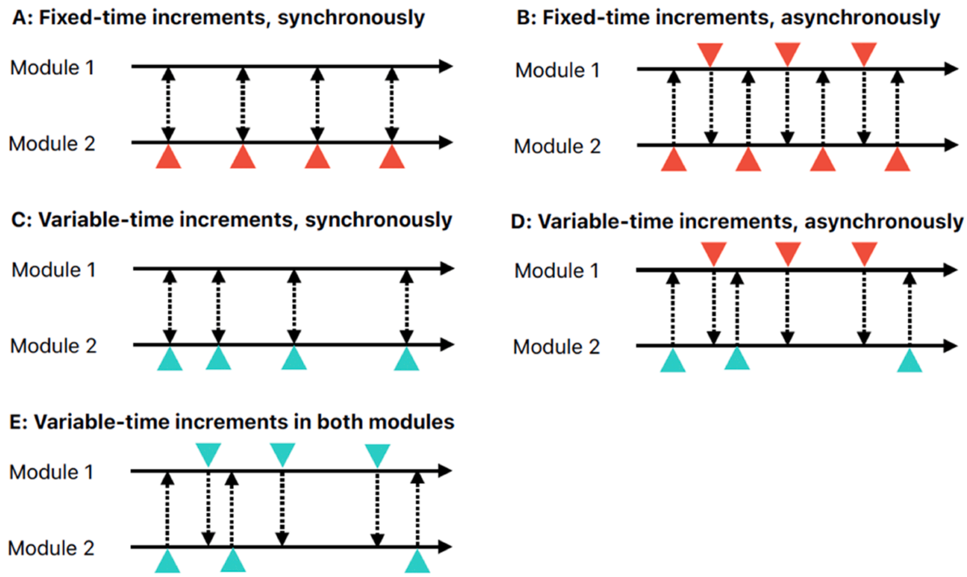

Fig. 2. Synchronisation between modules in a hybrid SD-ABM model.

Table 3
Updating rules at each time step (daily).

Execution order	Sending module	Receiving module	Information
At the beginning of each time step			
1	Network (ABM)	Temporary Staff (ABM)	Request bank/agency staff based on the daily demand
2	Temporary Staff (ABM)	Network (ABM)	Schedule job – Allocate bank/ agency staff into care home agents
3	Network (ABM)	Intra-facility (SD)	Daily staffing level affects the number of contacts with residents per staff member.
	Temporary Staff (ABM)	Intra-facility (SD)	Ingress of virus – The number of infectious bank/agency staff members (an aggregated measure) allocated to a care home agent affect the force of infection for susceptible residents and staff (flows) in that facility.
At the end of each time step			
4	Intra-facility (SD)	Temporary Staff (ABM)	Bank/agency staff acquire infection from infectious residents and other staff members.
	Intra-facility (SD)	Network (ABM)	The number of permanent staff members self-isolating due to COVID-19 determine the need of additional bank/agency staff in the next time step.

provide a holistic, cross-sectional view, patterns, and trends of the system over time to support policymakers' strategic decisions that impact a large population rather than longitudinally tracking specific individuals. On the other hand, ABM has expanded on the infectious disease epidemiological research primarily established by SD or compartmental models (Miksch et al., 2019; Willem et al., 2017). By simulating complex individual interactions and behaviours and spatial heterogeneity in the healthcare system, ABM has provided more details about the underlying mechanism and nature of the pathogen transmission (Megiddo et al., 2018; Miksch et al., 2019; Perez & Dragicevic, 2009; Stephenson et al., 2020). The method has also enabled the explicit testing of various infection control strategies such as cohorting, contact tracing, and social distancing (Enanoria et al., 2016; Farthing & Lanzas, 2021; Ferguson et al., 2005; Hotchkiss et al., 2005). In contrast to SD, capturing stochastic effects resulting from heterogeneous populations is a key feature of ABM. Accordingly, it has significantly enhanced our understanding of epidemics. As each simulation method has pros and cons, combining them in a hybrid simulation model can help address interconnected and intricate problems relating to infection control and generate richer insight (Nguyen et al., 2020a, 2020b, 2020c).

In this section, we demonstrate how the framework presented in Section 3 has informed the design of a hybrid SD-ABM model developed to investigate the impacts of bank/agency staff working across different care homes on COVID-19 spread patterns as well as targeted interventions to mitigate these impacts. It is noted that bank/agency staff in the UK are temporary or flexible workers hired through specialised external agencies or staff banks to provide care and support in healthcare settings. The detailed description of the modelling case study is beyond the scope of this paper and can be found in Nguyen et al. (2022a, 2022b).

4.1. Stage 1: exploring the problem

The model has been used to assist policymakers from the UK Department of Health and Social Care who considered effective interventions targeting bank/agency staff. According to evidence, staff working across different care homes are at a greater risk of COVID-19 infection than those working in a single care home, and using these staff significantly increases the risk of outbreaks among residents (Ladhani et al., 2020; Shallcross et al., 2021). However, knowledge is limited on the extent to which staff work in multiple care homes and contribute to spreading infection, which interventions effectively target this group, and how interventions interact to undermine or enhance each other when implemented concurrently.

The modellers worked closely with the stakeholders (care homes, health and social care partnerships, and policymakers) throughout the modelling process. In the problem articulation phase, discussions with the stakeholders were critical to gaining insights, scoping the problem, and determining the modelling objectives. Issues such as the study timeline, availability and access to data, and challenges in data collection were also addressed in this phase. The primary data collected through interviews and discussions with the stakeholders and secondary data available at public databases and provided by care homes, Lanarkshire Health and Social Care Partnerships contributed to informing the conceptual model's structure. Regular consultation with stakeholders helped justify the modelling assumptions and simplifications emerging from the first three phases of the modelling process, contributing to building confidence in the model. Engaging stakeholders in the modelling process increased the model's credibility and their buy-in, which ensured the implementation of the model's recommendations.

During the development of the model amidst the COVID-19 pandemic, stakeholder engagement was primarily conducted through virtual meetings and interviews due to practical constraints. The adoption of a more comprehensive stakeholder participation approach (i.e., facilitated modelling) (Franco & Montibeller, 2010), presented challenges in this context. Ideally, under different circumstances, stakeholder-oriented workshops for model building could have been facilitated using established OR methods, such as soft systems methodology (SSM) (Checkland & Scholes, 1999), group model building (GMB) (Richardson & Andersen, 1995), strategic options development and analysis (SODA) (Ackermann & Eden, 2010), multi-criteria decision analysis (MCDA) (Köksalan et al., 2011), and PartiSim (Tako & Kotiadis, 2015).

4.2. Stage 2: assessing the appropriateness of combining SD and ABM

The complexity and multi-scale characteristics of the problem, which involves interactions between intra-facility transmissions and inter-facility transmissions, pose a challenge to the use of SD or ABM alone, the two commonly used simulation methods in infectious disease modelling. However, a benefit of combining SD and ABM was that a hybrid model could achieve a comprehensive representation of such complexity by modelling multiple levels of aggregation. This makes it well-suited to stakeholders operating across these different levels (i.e., governments, health and social care partnerships, care homes). Hybrid simulation offers the flexibility to direct stakeholder's attention to the appropriate level of detail in different parts of the system, allowing stakeholders to gain a deeper understanding of the system and, therefore, leading to increased confidence in the model. Also, by combining SD and ABM, the model can demonstrate how interactions between agents and the overall system impacts outcomes, providing stakeholders with a more lucid understanding of the system's behaviours and the underlying mechanisms driving those behaviours. This enhanced understanding empowers stakeholders to grasp the dynamics of the system, leading to more effective communication and informed decision-making processes.

4.3. Stage 3: designing the modules

4.3.1. Step 3.1: determining modules

Fig. 3 shows the architectural design of the integrated hybrid SD-ABM model. Each of the modules and their linkages are described in stages 3 and 4 respectively. The hybrid model contained three modules built using either SD or ABM: Network (of Care Homes) (ABM), Temporary Staff (ABM), and Intra-facility (transmission in individual care homes) (stochastic SD). The concept of a network consisting of several agents representing sub-populations/healthcare facilities with a rich internal structure built using SD is similar to Vincenot and Moriya (2011) and Barnes et al. (2011). In these models, persons/patients move between sub-population/facility agents and spread the epidemics across a network. Their movement was modelled implicitly via behavioural rules of agents (i.e., how sub-population/facility agents exchange their persons/patients) in the network. These exchanged persons/patients were still considered homogeneous. However, the nature of such movement is different from the movement of bank/agency staff across care homes. Therefore, it was essential to consider bank/agency staff in a separate module (i.e., Temporary Staff Module). The choice of an appropriate simulation modelling method for each module is explained in the following subsections.

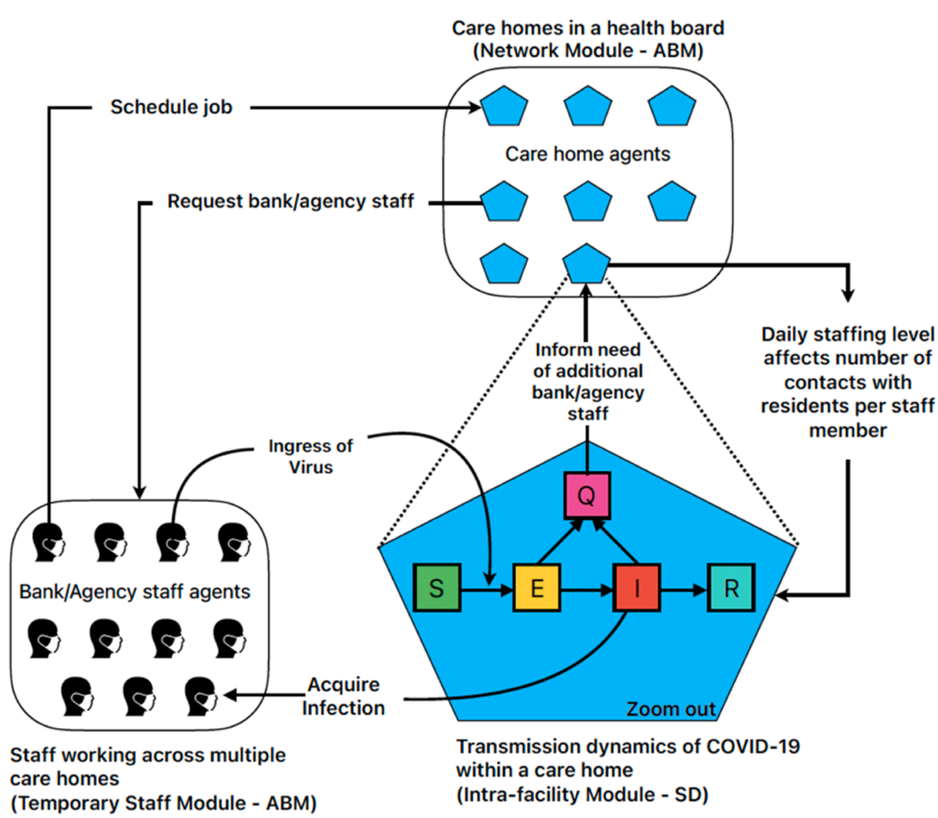

Fig. 3. Architectural design of the integrated hybrid SD-AB model comprising three modules.

4.3.2. Step 3.2: selecting simulation methods for each module

Network module: The Network module comprises care home agents connected via shared bank/agency staff. The state variables, including staffing levels, resident population sizes, levels of bank/agency staff use, and intra-facility transmission rates, characterise the care home agents. We selected ABM for this module as it is important for our research questions to capture the heterogeneity of care homes in terms of ingress risk and intra-facility transmission dynamics, which impact the facilities' risk of experiencing outbreaks. Shared bank/agency staff can spread SARS-CoV-2 from one care home with a current outbreak to other care homes with no infected case, triggering new outbreaks. The flexibility of ABM compared with SD allows for easy and quick modifications in network composition and modelling certain interventions explicitly, such as grouping care homes into 'bubbles'. Network composition and interventions that alter such composition can affect how the virus spreads across constituent care homes.

Network Module (ABM) shows a network of heterogeneous care home agents, each containing an SD Intra-facility Module representing the dynamics of within-home transmission (Note: S: Susceptible, E: Exposed, I: Infectious, R: Recovered, Q: Staff in isolation). Bank/Agency staff agents (Temporary Staff Module) shared between the care homes can spread the virus across the network. Arrows show the information flows between modules with the information content they exchange. Reproduced from "Hybrid simulation modelling of networks of heterogeneous care homes and the inter-facility spread of COVID-19 by sharing staff", by Nguyen et al. (2022a, 2022b), PLoS Computational Biology, 18 (1): e1009780. Copyright 2022 by Elsevier B.V. Reprinted with permission.)

Temporary Staff module: The Temporary Staff module models bank/agency staff as agents whose state variables characterise their infection state, testing and isolation status, and record of movement across care homes. The working schedules of bank/agency staff change daily, depending on their decisions and care homes' demands and preferences. This feature results in the stochasticity of their movement across care homes which is important to account for at the individual level. The collective movement actions of bank/agency staff agents can also lead to the emergence of events, such as several concurrent outbreaks across care homes with low community infection prevalence. While previous models discussed in Section 4.1 did not study interventions targeting persons/patients moving between sub-populations/facilities, the flexibility of ABM enables the explicit modelling of interventions that restrict the movement of bank/agency staff within a bubble of care homes.

Intra-facility module: Each care home agent of the Network module comprises a stochastic SD module that represents its intra-facility transmission of COVID-19. The aggregation of individuals was based on their roles (i.e., staff members or residents), state of infection, and testing and isolation status. SD is used as it is difficult to represent the intra-facility transmission dynamics using agents' state variables without making several further modelling assumptions about the behaviours of these variables over time. Stakeholders and modellers would have needed persuading about the validity and credibility of such assumptions and this level of abstraction. The stakeholders included teams from the Scottish Government and the UK Government, which consisted of infectious disease modellers external to the project and decision-makers, and they were familiar with and accepted the well-established epidemiological SD model (Anderson, 1991; Daley & Gani, 2001). In addition, as the impact of heterogeneous individual characteristics and behaviours and micro operational structures within care homes on intra-facility transmission is not the objective of this hybrid model and has been investigated in previous ABM models (Nguyen et al., 2020a, 2020b, 2020c; Nguyen et al., 2021), SD is preferable for this module to keep the model simple and with lower computational intensity. Additionally, as the studied problem focuses on inter-facility transmissions in a network mediated by bank/agency staff, each care home is viewed as a sub-system from a holistic perspective. The macro characteristics and behaviours of care homes, rather than the micro characteristics and behaviours of individuals within each care home, are of concern to decision-makers at a regional/national level whose decisions this model aims to support. Employing ABM to simulate the interactions within care homes could have been counterproductive, potentially diverting the attention of decision-makers from the core problem by delving into unnecessary levels of detail. We also used stochastic SD instead of traditional deterministic SD to capture the stochasticity of intra-facility transmission dynamics and the extinction of the virus which affect the risk of outbreaks in each care home. As part of the confidence-building process, we compared the stochastic SD Intra-Facility module with parallel deterministic SD and ABM models offering complementary representations of the same system at a different level of abstraction (details in Nguyen et al. (2022a, 2022b)). This approach helped gain insights into any differences in outcomes generated by different simulation modelling methods, from that obtaining plausible explanations of the system behaviour.

4.3.3. Step 3.3: determining the content of each module

As developing the content of single-method modules is not the focus of this paper, readers can refer to Nguyen et al. (2022a, 2022b) for details.

4.4. Stage 4: designing the links between modules

4.4.1. Step 4.1: defining the flows of information between modules

Fig. 3 describes the information flows between the modules.

4.4.2. Step 4.2: defining interfaces

- Interface between Network module (ABM) and Temporary Staff module (ABM)

Care homes seek to recruit a specific number of bank/agency staff daily, based on their current bank/agency staff usage and shortages due to COVID-related isolation. The allocation of bank/agency staff to care homes follows rules established through stakeholder discussions with care homes, supplying agencies, and bank/agency staff. For each care home agent, unallocated and non-isolating bank/agency staff members are randomly assigned or chosen based on their work history. These rules encourage the consistent utilisation of bank/agency staff within the same care home. In the event of an insufficient number of available bank/agency staff, the care homes may face understaffing for that day.

- Interface between Network module (ABM) and Intra-facility module (SD)

Agents' state variables affect flows: Care home agents' daily staffing level determined by their state variables, including the desired number of staff members on duty, the daily number of unfilled staff positions, and the number of residents, affect the transmission rates (in the SD module) within care homes. As it is implicitly assumed that the overall care home workload does not change regardless of daily staffing level, staff on duty will have to carry out extra workloads to maintain the quality of care delivered to residents if there is a staff shortage. Therefore, the daily number of contacts with residents per staff member at work used in the transmission rates of the SD Intra-facility module is dependent on daily staffing levels.

Stock levels affect agents' state variables: The number of permanent staff members self-isolating due to COVID-19 in a care home (i.e., a stock in Intra-facility module) affect the demand for bank/agency staff to cover absent staff on a given day during the pandemic (i.e., a state variable of care home agents in Network module).

- Interface between Temporary Staff module (ABM) and Intra-facility module (SD)

Aggregate measures of agents affect flows: The daily number of infectious bank/agency staff members increase the forces of infection for susceptible residents and susceptible permanent staff in the Intra-facility module.

Stock levels affect agents' state variables: The levels of stocks representing the number of infectious staff and residents in the Intra-facility module affect bank/agency staff agents' state variable representing their state of infection. Susceptible bank/agency staff acquire infection via interactions with infectious residents and other staff members at a rate that is equal to the force of infection in staff in the care home where they have worked.

4.4.3. Step 4.3: defining updating rules

Table 3 shows the updating rules between the modules. As the time unit commonly used to report epidemiological data and describe clinical characteristics of COVID-19 in the literature is daily, the ABM modules Network and Temporary Staff use a daily time step. The stochastic SD module Intra-facility is theoretically in continuous time, with the time step dt representing an infinitesimally small interval (Allen, 2008; Ossimitz & Mrotzek, 2008). In practical implementation, the module operates with a finite time step dt of $\frac{1}{2}^7$ days to ensure that numerical outcomes are as close as possible to those of a continuous model without the burden of carrying out too many calculations. The modules also exchange daily data as bank/agency staff are scheduled daily, and their infection states affect the states of SD modules in the care homes where they work on this time scale. The execution order was specified when multiple updates occur at the same point in time as described in Table 3 since this order would affect the results. For example, if update 2 occurs before update 1, bank/agency staff would be allocated based on the demand of care homes in the previous day. The model ran for 90-day and 180-day time steps as the former covered the period for which the client planned response strategies to contain the spread of COVID-19, and the latter was to examine the robustness of the findings in a longer term.

4.5. Model confidence-building relevant to the conceptual modelling phase

As part of the white-box validation, the model as described above was presented explicitly to and challenged by care home stakeholders, including representatives from Health and Social Care Partnership, Public Health, and staff and managers of care homes in Lanarkshire, Department of Health and Social Care, the Scottish Government Data Analysis Research Group, and the UK Social Care Working Group. The clear presentation of the model structure, including its constituent modules and interfaces between modules, helped to facilitate discussions with stakeholders and experts with various degrees of knowledge and experience in simulation modelling. This supported gaining stakeholders' trust in the model's validity, by demonstrating that it sufficiently represents the investigated system and that model assumptions are appropriate for the model's purposes. Harper et al. (2021) suggest that trust between the stakeholder and the model is one of the trust facets essential in model acceptability and confidence to implement results.

In addition to the confidence-building purpose, the pattern-oriented modelling approach (Grimm et al., 2005) that was adopted in the black-box validation helped inform some of the conceptual model designs. Patterns of the risk of infection for staff and residents and the risk of outbreak occurrence observed in care homes in the UK were identified at the problem exploring stage (i.e., stage 1 in the framework) and these were taken into account when designing the model. For example, as the risk of outbreak occurrence in care homes is dependent on their size and staff-to-resident ratio (Burton et al., 2020; Green et al., 2021; Scottish Government, 2020), it implies that care homes need to be considered at the individual level and that their resident population size and staffing level are important characteristics for capturing transmission dynamics.

5. Discussion

This paper contributes to the field of modelling and simulation from a methodological perspective by proposing a stepwise, detailed, and practical framework for developing a conceptual hybrid simulation model. The framework addresses a lack of methodological clarity on combining simulation methods, specifically combinations of SD and ABM, that was revealed in the literature review. It is developed based on a review of existing guidance for combining SD and ABM and reflection upon the process of building a hybrid simulation model. It focuses on the conceptual modelling process for hybrid models, which has been identified as "the least developed stage in the modelling cycle, despite its importance" (Brailsford et al., 2019). Conceptual modelling also helps the structural modelling and validation processes and is considered an important tool for model confidence building in healthcare (Roberts et al., 2012). In particular, the contribution of this paper to hybrid simulation is through addressing the following issues: i) a description of when SD and ABM should be combined, ii) an explanation of why SD and ABM combinations are required, iii) an explanation of how information is exchanged between SD and ABM modules at their interfaces, iv) a description of the elements including modules, their interfaces, and updating rules that are essential for reporting a conceptual hybrid model, and v) a description of how modellers can plan the confidence-building process for the individual modules and the overarching hybrid model at different stages of the framework. The modules constituting a hybrid model, justification for the selected simulation method for each module, interfaces, and updating rules are characteristics of the hybrid model. Reporting these characteristics provides a comprehensive overarching presentation of a conceptual hybrid model that facilitates communications of the model design and enables other modellers to take forward general lessons.

The paper also discusses new practices for modelling interfaces between SD and ABM modules in a hybrid simulation model. Previous frameworks for hybrid simulation have described different modes of interaction between simulation methods, focusing on the system view, method dominance, and direction and frequency of interaction. Examples include the Hierarchical, Process Environment, and Integrated modes in Chahal and Eldabi (2008) and the Sequential, Enriching, Interaction, and Integration models in Morgan et al. (2017), Swinerd and McNaught (2012), and Martinez-Moyano et al. (2007). Swinerd and McNaught (2012) expand the concept of the Integration mode into three generic designs of combining SD and ABM, namely agents with rich internal structure, stocked agents, and parameters with emerging behaviours. However, the description of these interaction modes is still abstract and has not explicitly explained how the information is passed between different simulations. This paper addresses this issue by categorizing the designs of an interface between SD and ABM modules and defining how SD/ABM modules generate the information and how the receiving ABM/SD modules handle such information for each design. These interface designs also explain other forms of feedback that go beyond what has been generally discussed in previous hybrid models: i) the SD module generates information that shapes the agents' environment or affects their decision-making and ii) the aggregation of the agents' characteristics or actions represents a stock or parameter in the SD module. The research also proposes two new interface designs: i) a stock level defines the agents' network topology and ii) the agents' state variables affect flows.

The framework is intended to guide modellers in thinking through different aspects and issues critical for developing a hybrid model. Whilst it presents choices for linking modules, the framework does not provide an exhaustive list of options. This is particularly due to the 'art' of modelling. Different modellers may choose to represent a situation in different ways. However, the framework provides a guiding structure which future research can further develop as the literature using hybrid models expands. At this point, the framework requires further use in practice beyond the application in Section 4 to build confidence in its validity and practicality.

Following the extensive review and synthesis of hybrid models in various application areas that contribute to the development of the hybrid simulation modelling framework, the remaining contribution is from the reflection of a single modeller/researcher bound by a healthcare context, and its application is only demonstrated in a single case study. This impacts the generalizability of the decisions on whether to choose SD or ABM and the practicability of the framework. Further testing of the framework is necessary through applications in other contexts.

An extension of this research could explore the values of the proposed interface designs between SD and ABM modules which have not been applied in the existing hybrid models. Future research can also consider how to combine three simulation methods (i.e., SD, ABM, and DES) in a hybrid simulation model and whether the framework can be extended to support the development of hybrid models that include other types of Operational Research/Management Science methods. This may include exploring further designs of interfaces between modules depending on the methods selected for combination.

This research has discussed different scenarios where hybrid simulation models are preferred compared to using single simulation methods and explained the benefits of using hybrid simulation for each application scenario based on reviewing and analysing existing models. However, it remains unclear how individual modellers or modelling teams decide on the use of hybrid simulation and what key factors affect their decision, as there is little discussion on the decision-making process in the literature. Therefore, analysing existing models is not sufficient to address these issues, and other research methods, such as in-depth interviews, can be used to explore the practice of combining multiple methods (Ackermann & Howick, 2022). This can help provide richer insights into the decision-making process of selecting hybrid simulations. Understanding this decision-making process would be helpful to draw generic lessons to aid the selection of appropriate methods.

Declaration of competing interest

None

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2024.01.027.

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