SF3580
HW 4

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Task 1

We implement the naive approach and compare it to the Bartels-Stewart method on random matrices. The dependence of the simulation time on n is illustrated in Figure ??. We observe the complexity

Task 3

We show that $\text{vec}(\mathbf{u}\mathbf{v}^T) = \mathbf{v} \otimes \mathbf{u}$, where $\mathbf{u}$ and $\mathbf{v}$ are two vectors of length n. Starting with the left hand side, we have

$$uv^T = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \begin{bmatrix} v_1, & v_2, & \dots, & v_n \end{bmatrix} = \begin{bmatrix} u_1v_1, & u_1v_2, & \dots, & u_1v_n \\ u_2v_1 & u_2v_2, & \dots, & u_2v_n \\ \vdots & \vdots & \ddots & \vdots \\ u_nv_1 & u_nv_2 & \dots & u_nv_n \end{bmatrix}. \tag{1}$$

Thus,

$$\text{vec}(\mathbf{u}\mathbf{v}^T) = \begin{bmatrix} u_1v_1, \dots, u_nv_1, u_1v_2, \dots, u_nv_2, \dots, u_1v_n, \dots, u_nv_n \end{bmatrix}. \tag{2}$$

The right hand side expression can be written as

$$\mathbf{v} \otimes \mathbf{u} = \begin{bmatrix} v_1\mathbf{u} \\ v_2\mathbf{u} \\ \vdots \\ v_n\mathbf{u} \end{bmatrix}, \tag{3}$$

which is the same as the expression for the left hand side in (2).

Task 4

We consider

$$A = \begin{bmatrix} 0 & a \\ 1 & 0, \end{bmatrix} W = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \tag{4}$$

and determine for which values of a that the Lyapunov equation has a unique solution.

The answer is given using theorem 4.1.1 from the lecture notes. The eigenvalues of the real matrix A is $\lambda = \pm\sqrt{a}$. The theorem states that the Lyapunov equation has a unique solution if and only if $\lambda_1 \neq -\lambda_2$. The only possibility for this to hold is if $a = 0$.

Task 5

(a)

Task 12