

SF3580

HW 2

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1 Task 2

2 Task 4

The linear system of equations

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

is considered.

2.1 (a)

We determine the constants α , β and γ such that for the iterates x_0, \dots, x_3 of the conjugate gradient method we obtain $\text{span}(x_0, x_1, x_2, x_3) = \text{span}(c_0, c_1, c_2, c_3)$, where

$$C = [c_0, c_1, c_2, c_3] = \begin{bmatrix} 1 & \alpha & 0 & \gamma \\ 1 & 0 & \beta & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2)$$

For this purpose, we use Lemma 2.2.4 from the lecture notes, stating that

$$\text{span}(b, Ab, \dots, A^{m-1}b) = \text{span}(x_0, x_1, \dots, x_m). \quad (3)$$

Thus, we want to compute α , β and γ such that

$$\text{span}(b, Ab, A^2b, A^3b) = \text{span}(c_0, c_1, c_2, c_3). \quad (4)$$

We can directly conclude that $\text{span}(b) = \text{span}(c_0)$. Next, we want to make sure that $\text{span}(b, Ab) = \text{span}(c_0, c_1)$. By column reduction we thus find that $\alpha = 0$. Using that $\text{span}(b, Ab, A^2b) = \text{span}(c_0, c_1, c_2)$, we can similarly find that $\beta = -1$ and finally, using (4), and again reduce columns, we identify that $\gamma = 6$.

2.2 (b)

See the implemented Julia code. We have replaced ??? with $\|Ax - b\|_{A^{-1}}$. Comparing \mathbf{x}_{opt} and \mathbf{x}_{cg} , we obtain a difference of $1.51 \cdot 10^{-11}$.

2.3 (c)

For GMRES, ??? is replaced by $\|Ax - b\|_2$. Now, comparing \mathbf{x}_{opt} with $\mathbf{x}_{\text{gmres}}$, we obtain the difference $1.06 \cdot 10^{-11}$.

3 Task 5

4 Task 6

4.0.1 (a)

We compare GMRES and CGN for a given matrix B and right hand side b . The result of the comparison is visualised in Figure ??.

4.0.2 (b)

The result can be related to the convergence theory for CGN and GMRES.

5 Task 7