SF3580
HW 3

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2018/11/29

Task 2

Task 4

Task 5

(a)

The matrix A is diagonalizable with the eigendecomposition $A = QDQ^{-1}$, where D is a diagonal matrix. For such structures it holds that $\sin(A) = Q\sin(D)Q^{-1}$. Thus we can validate the result for the Schur-Parlett method, which is

$$\sin(A) = \sin\left(\begin{bmatrix} 1 & 4 & 4 \\ 3 & -1 & 3 \\ -1 & 4 & 4 \end{bmatrix}\right) \approx \begin{bmatrix} 0.846192 & 0.0655435 & -0.187806 \\ 0.33476 & 0.385017 & -0.141244 \\ -0.190921 & 0.192478 & 0.848269 \end{bmatrix}. \tag{1}$$

which in norm differs $4.28e-16$ from $Q\sin(D)Q^{-1}$.

```
using LinearAlgebra
function schur_parlett(A,f)
    T,Q,ev=schur(A)
    n = size(A,1)
    F = zeros(n,n)
    for i=1:n
        F[i,i]=f(T[i,i])
    end
    for p=1:n-1
        for i=1:n-p
            j=i+p
            s=T[i,j]*(F[j,j]-F[i,i])
            for k=i+1:j-1
                s = s + T[i,k]*F[k,j]-F[i,k]*T[k,j];
            end
            F[i,j]=s/(T[j,j]-T[i,i])
        end
    end
    F=Q*F*Q';
    return F
end
```

(b) & (c)

It is clear from Figure 1 that the number of flops required for Schur–Parlett is not discernibly affected by N, at leat for $N \in 10, 50, 100, 150, 200, 250, 300$. This is not suprsining, as often the most computationally

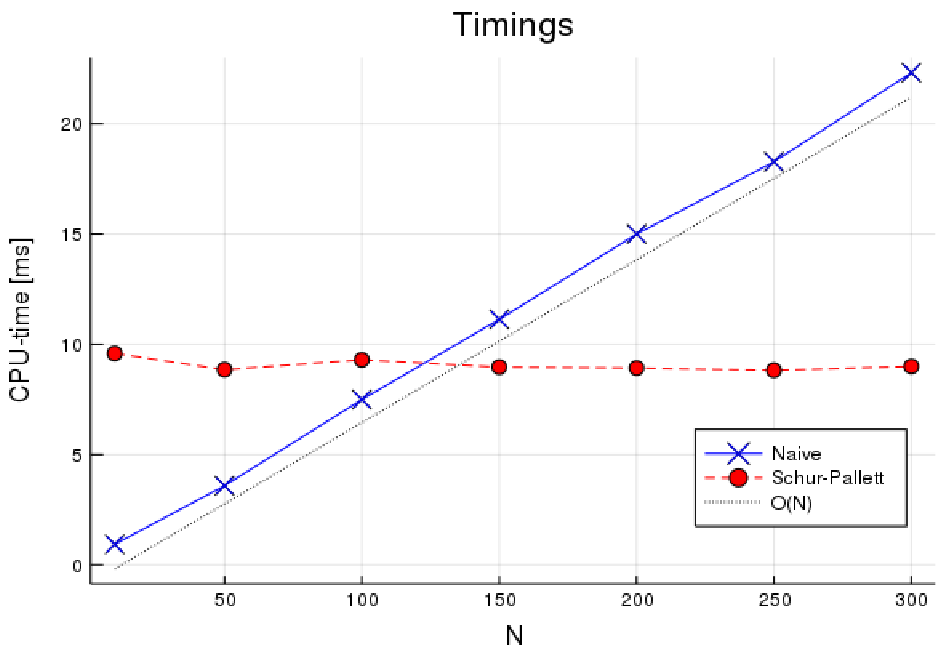

Figure 1: Task 5, (b) & (c): CPU–time in milliseconds, as a function of N.

demanding part of the Schur–Parlett method, in is performing the Schur decomposition, which scales like $\mathcal{O}(n^3)$. Once obtained, the function f is only applied to the diagonal elements, which are scalars.

For the naive appraoch the number of flops is proportional to N. A matrix multiplication is of $\mathcal{O}(n^3)$, thus performing N matrix gives $\mathcal{O}(Nn^3)$, which we read from Figure 1. The black line corresponds to the line $0.08 + 0.07\,N$.

Task 6

Task 7