SF3580

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1 Task 2

1.1 (a)

Insert figure

1.2 (b)

Insert figure

1.3 (c)

Insert figure

The Rayleigh quotient only uses the symmetric part of A in

$$r(\mathbf{x}) = \mathbf{x}^H A \mathbf{x}$$

assuming $\mathbf{x}$ is normalized. The matrix A is no longer symmetric, i.e. $A \neq A^H$, but any square matrix can be decomposed into a symmetric part A_s and a nonsymmetric part A_{ns} by

$$A = \underbrace{\frac{1}{2}\left(A + A^H\right)}_{=A_s} + \underbrace{\frac{1}{2}\left(A - A^H\right)}_{=A_{ns}}.$$

Thus

$$r(\mathbf{x}) = \mathbf{x}^H A_s \mathbf{x} + \mathbf{x}^H A_{ns} \mathbf{x} = \mathbf{x}^H A_s \mathbf{x}$$

since

$$\mathbf{x}^H A_{ns} \mathbf{x} = \mathbf{x}^H A \mathbf{x} - \mathbf{x}^H A^H \mathbf{x} = 0.$$

For a nonsymmetric matrix all avaliable information is not used.

2 Task 3

The performance of different versions of Gram-Schmidt orthogonalizations is investigated when combined with the Arnoldi method. We consider the matrix A constructed by

```
Random.seed!(0); A=matrixdepot("wathen",nn,nn)
```
(1)

where we choose $nn = 500$ and use m number of iterations in the Arnoldi method.

Results for CPU time and orthogonality of the basis in Q is given in Table 1.

Table 1: Comparison for different types of Gram-Schmidt (GS) orthogonalisation in the Arnoldi method: SGS (single GS), MDS (modified GS), DGS (double GS), TGS (triple GS), where *time* is the measured CPU time and *orth* is the orthogonality of the basis in terms of $\|Q_m^T Q_m - I\|$.

m	SGS time	SGS orth	MGS time	MGS orth	DGS time	DGS orth	TGS time	TGS orth
5	188 ms	$4.82 \cdot 10^{-13}$	241 ms	$3.28 \cdot 10^{-15}$	246 ms	$3.81 \cdot 10^{-15}$	298 ms	$3.84 \cdot 10^{-15}$
10	496 ms	$2.15 \cdot 10^{-12}$	705 ms	$1.32 \cdot 10^{-14}$	665 ms	$4.19 \cdot 10^{-15}$	820 ms	$4.23 \cdot 10^{-15}$
20	1.10 s	$1.04 \cdot 10^{-11}$	2.03 s	$6.41 \cdot 10^{-14}$	1.682 s	$5.66 \cdot 10^{-15}$	2.16 s	$5.72 \cdot 10^{-15}$
50	4.05 s	$1.06 \cdot 10^{-10}$	9.95 s	$6.04 \cdot 10^{-13}$	6.69 s	$6.18 \cdot 10^{-15}$	9.49 s	$6.38 \cdot 10^{-15}$
100	12.0 s	$4.77 \cdot 10^{-10}$	37.8 s	$3.29 \cdot 10^{-12}$	21.7 s	$7.82 \cdot 10^{-15}$	31.2 s	$7.87 \cdot 10^{-15}$

It can be concluded that double Gram-Schmidt performs the best in terms of orthogonalization error, while single Gram-Scmidt is the fastest among the algorithms. Triple Gram-Schmidt performs almost exactly as well as double Gram-Schmidth in terms of error, which is not surprising. It was stated during the lecture that the best result possible to achieve using multiple Gram-Schmidt is indeed obtained for double Gram-Schmidt. Note however that the CPU time required for triple Gram-Schmidt is of course considerably larger than for double Gram-Schmidt. The orthogonalization errors for the modified Gram-Schmidt is smaller than for single Gram-Schmidt but the algorithm is in this example seen to be worse than double Gram-Schmidt both in terms of CPU time and orthogonalization error.

3 Task 4

We investigate a primitive version of the Arnoldi method. Let

4 Task 6

test

5 Task 8

test