SF3580
HW 3

Anna Broms & Fredrik Fryklund

December 8, 2018

Task 2

(a)

The QR-algorithm is implemented and applied to the given matrix, see Julia code and Figure 1.

(b) and (c)

The eigenvalues of the matrix A are (by construction) $[2^{[0:7]}, \lambda_9, 2^9]$, where

$$\lambda^9 = 2^9 \left(0.99 - \frac{1}{5\alpha} \right). \tag{1}$$

After n iterations, the elements below the diagonal in the QR-iterates will be *proportional* to $|\lambda_i/\lambda_j|^n$, where $i < j$. The large α, the dominating eigenvalues are λ_9 and $\lambda_1 0$. Computing $|\lambda_i/\lambda_j|^n$, we obtain

$$|\lambda_i/\lambda_j|^n = \left(0.99 - \frac{1}{5\alpha} \right)^n. \tag{2}$$

Taking this as a meassure of the error and setting a tolerence of 10^{-10}, we obtain

$$\left(0.99 - \frac{1}{5\alpha} \right)^n \leq 10^{-10}$$
$$\Leftrightarrow n \cdot \log(0.99 - \frac{1}{5\alpha}) \leq -10 \log(10) \tag{3}$$
$$n \geq \frac{-10 \log(10}{\log(0.99 - \frac{1}{5\alpha})}.$$

The predicted number of iterations is plotted together with the obtained number of iterations in Figure 1. The predicted number of iterations is proportional to the true number of iterations, as expected.
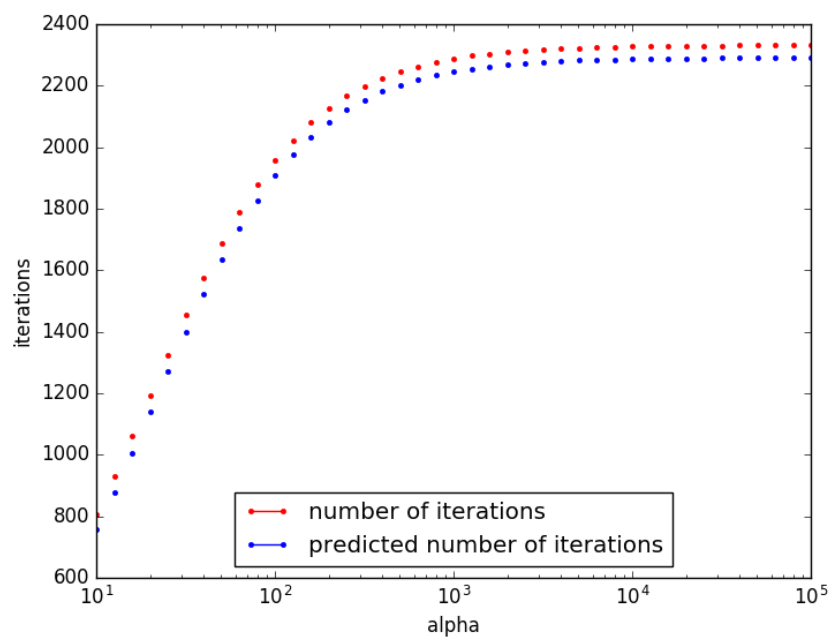
Figure 1: Task 2: obtained and predicted number of iterations for the QR-method applied to the given matrix.