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October 31, 2017

1 4.1

$$\hat{y} = \underset{y}{\mathrm{argmax}} P(X|y)$$

$$P(X|y) = \prod_{k=1}^{n} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x^{(k)} - \mu_{yk})^2}{2\sigma^2}) \right) \propto \exp(-\sum_{k=1}^{n} (x^{(k)} - \mu_{yk})^2)$$

$\sum_{k=1}^{n} (x^{(k)} - \mu_{yk})^2$. , $\hat{y}$ μ_y . .

2 4.2

ROC (FPR, TPR). S.

$$S = \frac{TPR + 1 - FPR}{2}$$

2 .

E(S). N_1 - 1, N_0 - 0.

$$E(TPR) = \frac{TP}{TP + FN} = \frac{E(TP)}{N_1} = \frac{E(\sum_{x:y_x=1} I\{a(x) = 1\})}{N_1} = \frac{N_1 \cdot p}{N_1} = p$$

$$E(FPR) = \frac{FP}{FP + TN} = \frac{E(FP)}{N_0} = \frac{E(\sum_{x:y_x=0} I\{a(x) = 1\})}{N_0} = \frac{N_0 \cdot p}{N_0} = p$$

,

$$E(S) = \frac{p + 1 - p}{2} = \frac{1}{2}$$

.

3 4.3

$P(y \neq y_n)$.

$1NN$, 1 x 0 x_n . ..

$$E_N = P(y \neq y_n) = P(1|x)P(0|x_n) + P(1|x_n)P(0|x)$$

n ,

$$E_N = 2P(1|x)P(0|x) = 2E_B(1 - E_B) = 2E_B - 2E_B^2 \leq 2E_B$$