1

November 15, 2017

1 1.1

y , $y_i, i \in \{1, .., l\}$ - , y.
:

$$E(MSE_1) = E\Big(y - \frac{1}{l}\sum_{i=1}^{l} y_i\Big)^2 = Ey^2 - 2 \cdot Ey \cdot E\Big(\frac{1}{l}\sum_{i=1}^{l} y_i\Big) + E\Big(\frac{1}{l}\sum_{i=1}^{l} y_i\Big)^2$$

, $\forall i \in \{1, .., l\} Ey_i = Ey_1$.

$$E(MSE_1) = Ey^2 - 2 \cdot Ey \cdot Ey_1 + \frac{1}{l^2} E\Big(\sum_{i=1}^{l} y_i\Big)^2 = Ey^2 - 2 \cdot Ey \cdot Ey_1 + \frac{1}{l} Ey_1^2 + \frac{1}{l^2} l(l-1)(Ey_1)^2 = Ey^2 - 2 \cdot Ey \cdot Ey_1 + \frac{1}{l}$$

$$E(MSE_2) = E(y - y_i)^2 = Ey^2 - 2 \cdot Ey \cdot Ey_1 + Ey_1^2$$

:

$$E(MSE_1) - E(MSE_2) = Ey^2 - 2 \cdot Ey \cdot Ey_1 + \frac{1}{l} Ey_1^2 + (1 - \frac{1}{l})(Ey_1)^2 - Ey^2 + 2 \cdot Ey \cdot Ey_1 - Ey_1^2 = (\frac{1}{l} - 1)Dy_1 < 0$$

, .

2 1.3

$$p(x) = \frac{1}{(2 \cdot \pi)^{\frac{n}{2}} \cdot \sqrt[2]{|\Sigma|}} \cdot \exp\Big(-\frac{1}{2} \cdot (x - \mu)^T \cdot \Sigma^{-1} \cdot (x - \mu)\Big),$$

μ - , Σ - .

$$H(p) = -\underset{R^n}{\int \cdots \int} p(x) \cdot \ln p(x) dx =$$

$$= \underset{R^n}{\int \cdots \int} p(x) \cdot \left(\frac{1}{2} \cdot (x - \mu)^T \cdot \Sigma^{-1} \cdot (x - \mu) + \ln((2 \cdot \pi)^{\frac{n}{2}} \cdot \sqrt[2]{|\Sigma|})\right) dx =$$

$$= \frac{1}{2} \cdot E\left(\sum_{i,j} (x_i - \mu_i) \cdot (\Sigma^{-1})_{i,j} \cdot (x_j - \mu_j)\right) + \frac{1}{2} \cdot \ln((2 \cdot \pi)^n \cdot |\Sigma|) =$$

$$
\begin{aligned}
&= \frac{1}{2} \cdot \sum_{i,j} \Big(E((x_i - \mu_i) \cdot (\Sigma^{-1})_{i,j} \cdot (x_j - \mu_j)) \Big) + \frac{1}{2} \cdot \ln((2 \cdot \pi)^n \cdot |\Sigma|) = \\
&= \frac{1}{2} \cdot \sum_i \sum_j (\Sigma)_{i,j} \cdot (\Sigma^{-1})_{i,j} + \frac{1}{2} \cdot \ln((2 \cdot \pi)^n \cdot |\Sigma|) = \\
&= \frac{1}{2} \cdot \sum_i (\Sigma \cdot \Sigma^{-1})_{i,i} + \frac{1}{2} \cdot \ln((2 \cdot \pi)^n \cdot |\Sigma|) = \\
&= \frac{1}{2} \cdot \sum_i (E)_{i,i} + \frac{1}{2} \cdot \ln((2 \cdot \pi)^n \cdot |\Sigma|) = \\
&= \frac{n}{2} + \frac{1}{2} \cdot \ln((2 \cdot \pi)^n \cdot |\Sigma|) = \\
&= \frac{1}{2} \cdot \ln((2 \cdot \pi \cdot e)^n \cdot |\Sigma|)
\end{aligned}
$$

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