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SECTION: O

ASSIGNMENT 04

VECTOR SPACES

Solution 01:

Property 1:

$$\text{Let } \vec{u} = (x, y, z)$$
$$\vec{v} = (x', y', z')$$
$$\vec{u} + \vec{v} = (xx', yy', zz') \in R$$

V is closed under addition.

Property 2:

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$
$$\vec{u} + \vec{v} = (x, y, z) + (x', y', z')$$
$$\vec{u} + \vec{v} = (xx', yy', zz')$$
$$\vec{u} + \vec{v} = (x'x^*, y'y^*, z'z^*)$$
$$= (x', y', z') + (x, y+z)$$
$$\vec{u} + \vec{v} = \vec{v} + \vec{u}$$

Property 3:

$$\text{Let } \vec{u} = (x, y, z)$$
$$\vec{v} = (x', y', z')$$
$$\vec{w} = (w_1, w_2, w_3)$$

$$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$$

$$\vec{u} + (\vec{v} + \vec{w}) = (x, y, z) + (x', y', z' + w_1, w_2, w_3)$$
$$= (x, y, z) + (x'w_1, y'w_2, z'w_3)$$
$$= (xx'w_1, yy'w_2, zz'w_3)$$
$$= ((xx') + w_1, (yy') + w_2, (zz') + w_3))$$
$$= (xx', yy', zz') + (w_1, w_2, w_3)$$
$$= (x, y, z + x', y', z') + (w_1, w_2, w_3)$$
$$= (\vec{u} + \vec{v}) + \vec{w}$$

Property 4:

$$\text{Let } \vec{u} = (x, y, z)$$
$$\vec{0} = (0, 0, 0)$$
$$\vec{0} + \vec{u} = (x, y, z) + (0, 0, 0)$$
$$= (x0, y0, z0)$$

That is not satisfied.

- Hence, V is not a vector space because 4^{th} property is not satisfied.

Solution 02:

$$\text{Let } \vec{u} = (x_1, y_1)$$
$$\vec{u} = (x_1, -3x_2)$$
$$\vec{v} = (x_2, -3x_2)$$
$$\vec{u} + \vec{v} = (x_1 + x_2, -3x_1 - 3x_2)$$
$$= (x_1 + x_2, -3(x_1 + x_2))$$

Closed under addition

$$\text{Let } \vec{u} = (x_1, -3x_1)$$
$$K\vec{u} = K(x_1, -3x_1)$$
$$= Kx_1, -3K_1x_1$$

Closed under scaler multiplication.

- So, Set $\{(x, y); y = -3x, x \in \mathbb{R}\}$ is sub-space of $\mathbb{R}^2$.

Solution 03:

$$\text{Let } X = (7, 8, 9)$$

Then,

$$X = K_1 u + K_2 v + K_3 w$$
$$(7,8,9) = K_1(2,1,4) + K_2(1,-1,3) + K_3(3,2,5)$$
$$(7,8,9) = (2K_1, K_1, 4K_1) + (K_2, -K_2, 3K_2) + (3K_3, 2K_3, 5K_3)$$
$$= (2K_1, K_1, 4K_1 + K_2, -K_2, -3K_2 + 3K_3, 2K_3, 5K_3)$$
$$= (2K_1 + K_2 + 3K_3, K_1 - K_2 + 2K_3, 4K_1 + 3K_2 + 5K_3)$$

$$2K_1 + K_2 + 3K_3 = 7$$
$$K_1 - K_2 + 2K_3 = 8$$
$$4K_1 + 3K_2 + 5K_3 = 9$$

By using Gauss elimination method,

$$= \left[\begin{array}{ccc|c} 2 & 1 & 3 & 7 \\ 1 & -1 & 2 & 8 \\ 4 & 3 & 5 & 9 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 1 & -1 & 2 & 8 \\ 4 & 3 & 5 & 9 \end{array}\right] R_1 - R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & -3 & 1 & 9 \\ 4 & 3 & 5 & 9 \end{array}\right] R_2 - R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 3 & -1 & -9 \\ 0 & -5 & 1 & 13 \end{array}\right] -1R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/3 & -3 \\ 0 & -5 & 1 & 13 \end{array}\right] R_2/3$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/3 & -3 \\ 0 & 0 & -2/3 & -2 \end{array}\right] R_3-(-5)R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/3 & -3 \\ 0 & 0 & -1 & -3 \end{array}\right] 3R_3/2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & -1/3 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right] -1R_3$$

$$K_1 + 2K_2 + K_3 = -1 \quad \text{--- (i)}$$
$$K_2 - \tfrac{1}{3}K_3 = -3 \quad \text{--- (ii)}$$
$$\boxed{K_3 = 3}$$

By putting $K_3 = 3$ in eq. (ii),

$$K_2 - \tfrac{1}{3}(3) = -3$$
$$K_2 - 1 = -3$$
$$K_2 = -3 + 1$$
$$\boxed{K_2 = -2}$$

By putting $K_2 = -2$ and $K_3 = 3$ in eq. (i),

$$K_1 + 2(-2) + 3 = -1$$
$$K_1 - 4 + 3 = -1$$
$$K_1 - 1 = -1$$
$$K_1 = -1 + 1$$
$$\boxed{K_1 = 0}$$

Thus,

$$(7,8,9) = 0(2,1,4) - 2(1,-1,3) + 3(3,2,5)$$

expressed in linear combination as:

$$X = 0\vec{u} - 2\vec{v} + 3\vec{w}$$