Capital Replacement and the Demand for Clean Technology

Felix Samy Soliman

University of Zurich*

27th October 2025

Abstract

I study why clean technologies are adopted slowly and how slow adoption undermines clean innovation. Using an event study around large energy price swings, I provide evidence that industries with short-lived assets see greater increases in energy efficiency and green patenting, consistent with lock-in among users of long-lived assets. To assess policy implications for the green transition, I embed the feedback between irreversible investment and energy saving innovation in an integrated assessment model. Slow adoption delays the pass-through of clean innovation to energy demand relative to benchmark models. The sluggish uptake of innovation justifies higher carbon taxes if the social cost of carbon rises with cumulative emissions. These higher taxes decrease investment, which negatively affects R&D incentives, further limiting the power of green innovation in facilitating emissions reductions in the short to medium run. Replacement subsidies can partially substitute for carbon taxes. Uniform subsidies improve fuel efficiency but raise emissions via scale effects. Redirecting these subsidies to electrification is a more effective second best when the electricity mix is sufficiently clean.

*E-mail address: felix.samysoliman@econ.uzh.ch. Many thanks to my supervisors David Hémous and Alessandro Ferrari. Adrien Auclert, Elizabeth Baldwin, Matias Bayas-Erazo, Luigi Bocola, Maya Eden, Claudia Gentile, Elie Gerschel, Diego Känzig, Pete Klenow, Simon Lepot, Michael Peters, Juan Carlos Suarez Serrato, Christopher Tonetti, Florian Trouvain and seminar participants at LSE, Munich (CESifo), Oxford, Stanford and Zurich (UZH) provided helpful comments.

1 Introduction

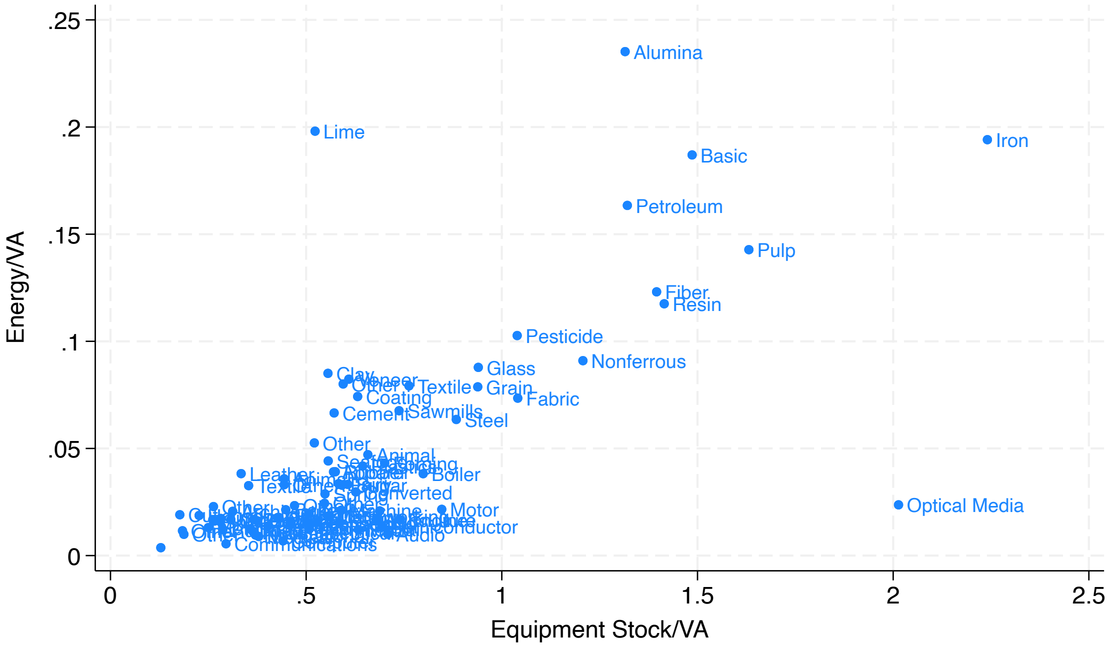
Reducing CO2 emissions to limit damage from global warming is an important policy priority. Given that nearly 90% of global CO2 emissions stem from fossil fuel combustion, abatement must come either from retrofitting the existing capital stock or through replacement with more efficient vintages. Empirical evidence suggests the second channel is more important: Over short horizons, capital and energy are close to perfect complements (Hassler, Krusell, and Olovsson, 2021), implying existing combustion processes are not easily modifiable. By contrast, the role of technology adoption through capital replacement is well documented. Entrants with new capital are approximately 30% more energy-efficient than incumbents across the US manufacturing and power-sector (Clay, Jha, Lewis, and Severnini, 2021; Linn, 2008).

The importance of capital replacement for reducing emissions is in tension with current goals of a rapid transition. Infrequent replacement, driven by the highly durable nature of fossil fuel consuming capital, creates significant path dependence in fuel use, despite ongoing breakthroughs in clean technology. Low adoption across many industrial processes also weakens incentives to develop these technologies, reinforcing the shortage of low-emission alternatives (IEA, 2020). With 1.5-degree warming already out of reach, this lack of adoption and innovation presents a roadblock to meet warming thresholds set in the 2015 Paris agreement.

In this paper, I combine new reduced-form evidence with a structural model to quantify how the feedback between adoption and innovation matters for fossil fuel use dynamics and resulting climate damages. To illustrate the role of durability, I compare industries with high and low durability during periods of persistently elevated energy prices. Industries with high durability exhibit weaker shifts toward clean innovation and smaller improvements in energy intensity in response to energy price shocks. Motivated by this evidence, I develop a multisector integrated assessment model (IAM) with vintage capital and endogenous energy-saving technical change. Existing climate-economy models assume either that energy use can be freely adjusted after capital is installed or new technologies are adopted immediately (Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Golosov, Hassler, Krusell, and Tsyvinski, 2014). The quantitative analysis yields two main findings. First, optimal carbon taxes reduce short to medium run emissions significantly less than in standard models. Emissions decline slowly because new technologies are only slowly adopted and carbon tax induced investment declines limit R&D incentives. This weaker response also implies higher optimal taxes when damages rise with cumulative emissions. Achieving a 45 percent cut within 25 years, as in the Paris goals, would require a tax about four times higher compared to the benchmark in Golosov et al. (2014). If marginal damages rise linearly, the optimal tax is about 10 to 15 percent higher. Second, replacement subsidies that boost demand for new machines improve equilibrium energy efficiency via market size effects. However, scale effects dominate, so these policies raise overall emissions. If firms can electrify, replacement subsidies to switch from fuel to electricity powered machines can be a more effective second-best policy, provided the electricity mix is sufficiently clean.

To motivate the analysis, I first empirically study how capital adjustment shapes the dynamics of

Figure 1: Capital versus Energy Intensity Across Manufacturing Industries



The figure plots the ratio of equipment capital to value added against the ratio of energy use to value added across manufacturing industries in the US. Data are from the NBER CES Manufacturing database.

energy efficiency. Figure 1 shows a tight link between capital and energy intensity across U.S. manufacturing industries.[1] I test whether this relationship also matters for how quickly industries adopt vintages with higher energy efficiency. Because capital is replaced less frequently in industries with low depreciation rates, these industries may experience smaller improvements in energy efficiency after energy price increases. I exploit two long upswings in aggregate energy prices, starting in 1973 and post-2000, both preceded by periods of relative price stability. I use an event-study design around these increases to test for differential responses as a function of industry depreciation rates. In line with the sluggish demand hypothesis, high-depreciation industries see larger gains in energy efficiency and green patenting. The innovation effects are consistent with greater replacement increasing firms' incentives to innovate. Quantitatively, industries at the 75th percentile of the depreciation-rate distribution experience a 14% decline in energy intensity and a 31% increase in the share of green patents, relative to the 25th percentile.

To interpret the implications of these estimates for the dynamics of the green transition, I develop a theoretical model of adoption and innovation in which demand for new vintages arises from firms replacing old capital with new. The model quantifies (i) long- and short-run elasticities of energy demand, (ii) the interaction between adoption and technological development, and (iii) transition paths under carbon taxes, replacement subsidies, and industrial electrification. Long-run general-equilibrium adjustments are key to addressing these questions because transitional dynamics extend well beyond the time horizon that

[1] Manufacturing accounts for 23% of all carbon emissions in the U.S. economy, even before accounting for indirect emissions through the use of electricity generated by electric utilities (EPA, 2021). The majority of these emissions come through energy use.

event study approaches can cover. Differential effects across industries mask aggregate adjustments, such as declines in investment induced by carbon taxes, which in turn feed back into adoption and innovation. Finally, the model establishes the quantitative importance of slow adoption relative to existing integrated assessment models.

The key novelty of the model is the two-way relationship between replacement of old machines and R&D in new vintages. On the demand side, users of capital goods operate machines with fixed energy requirements that scrap to replace it with a new machine. The energy efficiency of this new machine depends on the R&D decision of machine producers supplying the capital. Because R&D to improve energy efficiency entails a fixed overhead cost, the model features a market size effect: higher replacement rates increase innovation incentives. Thus, the model is the first to combine vintage capital with demand-driven, factor-augmenting technical change. Carbon taxes or higher energy prices raise the demand for energy-efficient equipment among machine users. However, higher energy prices also reduce the investment rate. This contraction occurs because, as energy prices rise, profitability per unit of capital declines, making the fixed cost of investing in a new machine less attractive.

I embed this model of replacement-driven technological progress into a dynamic general equilibrium model with climate–economy interactions, following Golosov et al. (2014). Final output is produced using manufacturing, services, and electricity in a nested CES production function. Manufacturing employs capital with higher and lower durability, mirroring the reduced-form analysis, and capital in electricity generation is either fossil-fuel-consuming or clean. In both manufacturing and fossil-fuel-based electricity, capital embodies energy efficiency, and replacement drives innovation incentives.[2] I cast the model in continuous time, which allows me to leverage tools from Achdou, Han, Lasry, Lions, and Moll (2022), facilitating rapid computation.[3] I handle transition dynamics using a collocation approach instead of the commonly used finite-difference method in the time dimension (Hémous, Lepot, Sampson, and Schärer, 2023; Schesch, 2024). Together, these methods allows me to use the simulated method of moments to match model to data.

The calibration combines parameter estimates from external sources with moment matching, targeting sectoral data on investment, depreciation, energy use, and prices for the U.S. economy. The elasticity of aggregate energy efficiency with respect to energy prices is governed by the dispersion of idiosyncratic replacement cost shocks. I calibrate this elasticity to match the reduced-form evidence on differential changes in energy efficiency across industries. The calibrated model matches the full time path of the coefficients from the event study. Matching these empirical transition speeds provides credibility to the central goal of my quantitative exercises: How much does slow uptake of new vintages matter for short- and medium-run energy dynamics?

In the first set of counterfactuals, I study how the economy responds to a permanent 250% increase

[2]In clean energy, investment also works through replacement, but I abstract from innovation in that sector. Extensions to exogenously falling investment prices or endogenous innovation would be straightforward.

[3]As firms are subject to idiosyncratic shocks to the value of replacement, the model formally belongs to the family of heterogeneous-agent continuous-time models analyzed by Achdou et al. (2022) with a continuous and time-varying distribution of vintages in each sector.

in energy prices.[4] Such an increase is on the same order of magnitude as the mid-1970s run-up in energy prices, as well as projections of what is required for the European Union to meet its decarbonization goals (Hochmuth, Krusell, and Mitman, 2025). The simulations indicate a prolonged transition to an equilibrium with higher energy efficiency. For example, in durable manufacturing the transition half-life is 25 years for the average technology embodied in firms' capital. These slow improvements in energy efficiency are accompanied by an initial sharp decline in investment, as firms delay upgrading until the technology has matured. To further establish the role of slow depreciation, I consider a counterfactual economy with depreciation calibrated to match a half-life of 5 years for capital goods. While both economies show similar long-run gains in energy efficiency and declines in energy use, cumulative energy use is 8 percent higher over the first 25 years of the transition. This comparison provides a lower bound on the importance of slow depreciation, as most models of endogenous technical change assume that capital fully depreciates over five or ten years. I also investigate short- and long-run elasticities of substitution between capital and energy. In the short run, energy requirements are fixed, so the factor share of energy moves one-for-one with the energy price, as in models with Leontief aggregate production function. Over time, energy efficiency improves, and electricity production substitutes toward clean energy, generating a long-run elasticity of 0.3-0.4.[5]

Second, I evaluate the effectiveness of the globally optimal climate policy in curbing fossil fuel emissions. To solve for the optimal policy, I build on insights from Nuño and Moll (2018) to derive a social marginal value function that governs the social planner's replacement decisions. I show that capital replacement choices are efficient absent environmental externalities. I calibrate damage parameters to generate carbon taxes on the order of $150 per ton of carbon, motivated by recent evidence of substantial temperature induced GDP losses (Bilal and Känzig, 2024; Nath, Ramey, and Klenow, 2025). From a global perspective, these taxes generate substantial welfare gains, especially in the long run once cleaner vintages are adopted. Relative to workhorse IAMs such as Golosov et al. (2014), these gains are smaller because energy is much less substitutable in the main model than in the Cobb-Douglas economy considered in their paper. In contrast, the Cobb-Douglas case displays essentially no transitional dynamics with energy use shifting down immediately. These differences in the elasticity of emissions with respect to carbon taxes raise the optimal carbon tax if the social cost of carbon is increasing in cumulative emissions. For a sharply increasing social cost of carbon, or an equivalently strict carbon budget, the sluggish response of energy demand justifies substantially higher taxes. To lower emissions by 45% within 25 years requires nearly four times higher taxes in my model than in the Cobb-Douglas case. I also contrast with a case of no substitutability between capital and energy. In that case, decarbonization occurs without technological adjustment, as investment must fall significantly. Without technological adjustment, energy use remains persistently higher than in my model.

[4] I treat the price of fossil fuels as exogenous to the U.S. economy.

[5] This long-run elasticity is lower than implied by the Cobb-Douglas assumption used in many macro-climate models. I also consider an calibration with larger intertemporal R&D spillovers, which doubles the long-run elasticity. Despite the larger long-run elasticity, simulations show that energy use dynamics over the first 25 years of the transition are very close to the baseline case of smaller spillovers.

Carbon taxes have proved politically infeasible in many countries, leading policy makers to adopt alternative policy tools to lower emissions. Examples include investment tax credits to stimulate investment into more energy-efficient machines or clean energy subsidies. The framework developed in this paper provides a natural laboratory to assess the effectiveness of such policies. In the model, unconditional replacement subsidies can stimulate innovation, thereby potentially lowering emissions. Quantitatively, these increases in productivity are outweighed by a scale effects: more frequent replacement raises the economy's capital stock. The model-predicted scale effects match empirical estimates of the elasticity of investment to the cost of capital (Chodorow-Reich, 2025). This scale effect outweighs the productivity effect, so emissions increase on net.

More targeted subsidies can perform better. Such policies appear in the U.S. Inflation Reduction Act and the Federal Climate Action bill Germany passed in 2024. I extend the model with a second margin of adjustment: electrification. When replacing equipment, firms choose between a new fossil fuel machine and an electrified machine. The benefit is that firms can avoid paying carbon taxes. I calibrate the cost of an electrified machine to match a rate of electrification of 12%, consistent with the share of energy use coming through electricity in U.S. manufacturing.[6] Model simulations suggest that subsidies towards electrification are a powerful second-best policy towards reducing fossil fuel use in manufacturing. A 5% subsidy to manufacturing investment raises fossil fuel use by 1%, whereas redirecting the same subsidy to electrification cuts it by 10%. For such policies to reduce overall emissions, the power sector must be sufficiently decarbonized so additional demand is met with clean energy.

Related Literature. This paper contributes to several strands of the literature on environmental macroeconomics and growth.[7] First, I contribute to the literature using integrated assessment models to study optimal climate policy (Barrage and Nordhaus, 2024; Golosov et al., 2014). The integrated assessment framework in these papers builds on the neoclassical growth model with exogenous technology. Acemoglu et al. (2012) extend these models by studying how policy should engineer a green transition when innovation can be redirected towards non-polluting energy sources. While they show that endogenous technical change can drive green growth, how quickly such innovations are adopted remains an open question. My paper fills this gap by developing a model of demand-driven innovation and adoption in which take-up of new technologies is constrained by slow depreciation. Hassler et al. (2021) and Casey (2023) attribute higher long-run substitutability between energy and non-energy inputs to gradual technological development, even as short-run energy demand may be inflexible. Relative to these papers, my model incorporates feedback between adoption and innovation and captures the gradual response of energy efficiency implied by imperfect depreciation across vintages. My model also accounts for the fact that high carbon taxes may lower R&D incentives through their negative effects on investment, a channel absent from existing work in the directed technical change literature. A second contribution is to study pathways for industrial decarbonization. Over the last 40 years, improvements in energy efficiency were

[6]To validate this model of fuel switching I compare model responses to well-identified evidence on fuel switching in response to carbon prices from Kaartinen and Prane (2024). The model replicates the empirical fuel-switching response well.

[7]Bilal and Stock (2025) and Hassler and Krusell (2018) survey this literature.

the most significant contributor to the decline in the U.S. economy's carbon intensity (Casey, 2023), but green electricity opens the possibility of electrification as an alternative for reducing emissions. In an extension, I introduce electrification as an alternative to fossil fuel saving technical change.

A smaller literature studies the role of capital replacement for growth and innovation, emphasizing market size effects linked to the replacement rate (Bertolotti and Lanteri, 2024; Hsieh, 2001; Krusell, 1998). However, these papers do not consider factor-specific technological change, which is crucial for studying the green transition and directed technological change more broadly. I also show how differences in depreciation rates across industries affect how quickly innovation improves measured productivity. These differences are important even across manufacturing industries, where depreciation varies only from 5 to 10 percent. The adoption channel is likely even more significant when comparing manufacturing with services, where differences in depreciation are larger.

An important idea of this paper is that technology is embodied in the capital stock, so benefiting from technology requires investment. This idea is central to "putty-clay" models of capital accumulation. Atkeson and Kehoe (1999) were the first to apply the putty-clay approach of Solow (1962) to the capital–energy nexus. Recent work asks whether sharp increases in carbon taxes or energy prices strand assets by making inefficient vintages unprofitable to operate (Campiglio, Dietz, and Venmans, 2022; Gilchrist, Martinez, and Rickard, 2024; Wei, 2003). My framework abstracts from the short-run costs of stranded assets, and instead emphasizes the feedback between innovation and adoption along the transition. This feedback matters quantitatively, as energy prices or carbon taxes encourage innovation, which allows the economy to transition away from fossil fuels. My model of demand driven innovation also accounts for innovation spillovers driven by market size effects. These spillovers are absent in putty-clay models where producing firms choose their own capital energy mix when building a new machine. Market size effects provide an important rationale for replacement subsidies, which I analyze in this paper.

On the empirical side, this paper connects to a rich literature emphasizing the importance of innovation and capital adjustment costs in mediating the effects of energy price increases. Hawkins-Pierot and Wagner (2023) show that the energy efficiency of entering firms is significantly more sensitive to rising energy prices than that of incumbent firms. Rates of establishment churn from Business Dynamics Statistics (BDS) data are positively correlated with the sectoral depreciation rates I construct. Thus, higher depreciation and more entry are related margins that make the capital stock more flexible and allow more short-run substitution away from energy. Relatedly, Capelle, Kirti, Pierri, and Bauer (2023) link the vast heterogeneity in energy intensity across firms in the same industry to the age of the capital stock. While such age-driven, within-industry dispersion also arises in my model, I emphasize sectoral characteristics shaping speed at which industries adjust. Finally, my estimates of the differential cross-industry effects of energy prices on green innovation contribute to the literature on innovation responses to energy price shocks and environmental regulation (Aghion, Dechezleprêtre, Hemous, Martin, and Van Reenen, 2016; Dugoua and Gerarden, 2025; Popp, 2002). I add to this literature by showing that feedback from adoption to innovation can undermine innovation incentives in low-depreciation sectors.

Roadmap. This paper is structured as follows. Section 2 presents the empirical results. Section 3 outlines the model. Section 4 describes the calibration and counterfactual analysis. Section 5 concludes.

2 Empirics

The main aim of the empirical analysis is study the role of slow depreciation and durability as factors mediating the pass-through of energy prices to energy efficiency. This section describes construction and sources for the key inputs into this investigation as well as summary statistics. Then I present the results for the main empirical exercise comparing energy efficiency and rates of green patenting across two major episodes of energy price increases.

2.1 Data, Methodology and Descriptives

This section introduces the three main data sources for the empirical analysis are introduced in this section. Additional data used for robustness checks is discussed at the end of the results section.

Manufacturing Data. Data on value added, energy expenditure and energy price indices comes from the NBER CES Manufacturing database (Becker, Gray, and Marvakov, 2021). I deflate energy expenditures and value added using industry-specific output deflators and industry-specific energy-price indices. The data on equipment stock shown in Figure 1 uses the real value of the equipment capital stock reported in this database.

Patent data. I start from the universe of USPTO granted patents available from Patstat. In robustness checks, I estimate my regressions on the subset of biadic patents, i.e. patents that were granted by one other patent office in addition to the USPTO. Aggregating this data to the industry level requires a concordance since patent data is reported at the level of technology classes. Let $p_{jk} = 1$ if patent k lists CPC4 technology code j and let N_k be the total number of codes of patent k. A single patent may have several technology codes associated with it. The patent count for industry i is then constructed as

$$p_i = \sum_j \sum_k \omega_{ij} \frac{p_{jk}}{N_k}$$

where ω_{ij} is concordance weight to aggregate patents from CPC4 to NAICS4 from Lybbert and Zolas (2014). The number of green patents is constructed analogously from all USPTO granted patents that list among their technology code the classification "Y02". This classification has been used widely in academic research (Calel and Dechezleprêtre, 2016; Känzig, 2023).

Depreciation data. I construct industry-level depreciation rates from equipment capital-stock and investment data at the 4-digit NAICS level available from the Federal Reserve. With this data, I estimate depreciation rates by inverting the following perpetual-inventory equation

$$K_{i,t+1} = (1 - \delta_{i,t})K_{i,t} + I_{i,t}. \tag{1}$$

This is the same perpetual inventory equation that the Federal Reserve uses to construct industry-level capital stocks (Gilbert and Mohr, 1996). $\delta_{i,t}$ is computed from equipment capital stocks and investment expenditures. These depreciation rates obtained in this way naturally vary across years because of shifts in the composition of investment across assets within industries, base-year effects (Whelan, 2002), as well as the cyclical behavior of investment. To account for this uncertainty in measurement, I average the depreciation rate across 1990-2020, a period during which depreciation rates were relatively constant in the aggregate (Dalgaard and Olsen, 2025). I discuss these issues in more detail in Appendix A.1, and show robustness of my results to using a measure of capital stock durability based on an average of service lives of individual assets using asset by industry investment shares from the BEA capital flow table as weights.

Empirical approach. My analysis aims to test whether pass through of energy price shocks to energy intensity and the share of green patents depends on capital durability. The empirical strategy leverages two sources of energy price variation. First, I evaluate the differential response of industries to aggregate swings in the price of energy using an event study framework. Second, I use an IV approach that relies on industries' differential exposure to fuel price shocks to generate plausibly exogenous variation in energy prices across industries.

Event-Study Approach. Figure 2 plots the evolution of the average price of energy paid by U.S. manufacturing industries.[8] The time series exhibits two distinct upswings. The top panel shows that the real price of energy rises 2.5-fold within 15 years after the 1973 oil-price shock. The first oil price shock does not fully explain the persistent increase. Historical accounts of U.S. energy prices emphasize additional domestic and global shocks to energy markets during this time period (Holdren, 1990; Lifset, 2014). An important question is whether industries anticipated such a persistent response to the initial shocks. Kilian and Murphy (2014) argue that inventory behavior indicates producers expected prolonged periods of higher prices after the initial shock. The Panel on the right shows a smaller but significant run-up in energy prices from 2000 to 2005, again led by a dramatic post-2000 increase in oil prices. I use these two episodes of rapid, persistent increases to evaluate how industries respond to such shocks.[9]

Concretely, I estimate the differential response to the shock following the local-projections approach to difference-in-differences estimation suggested in Dube, Girardi, Jorda, and Taylor (2023)

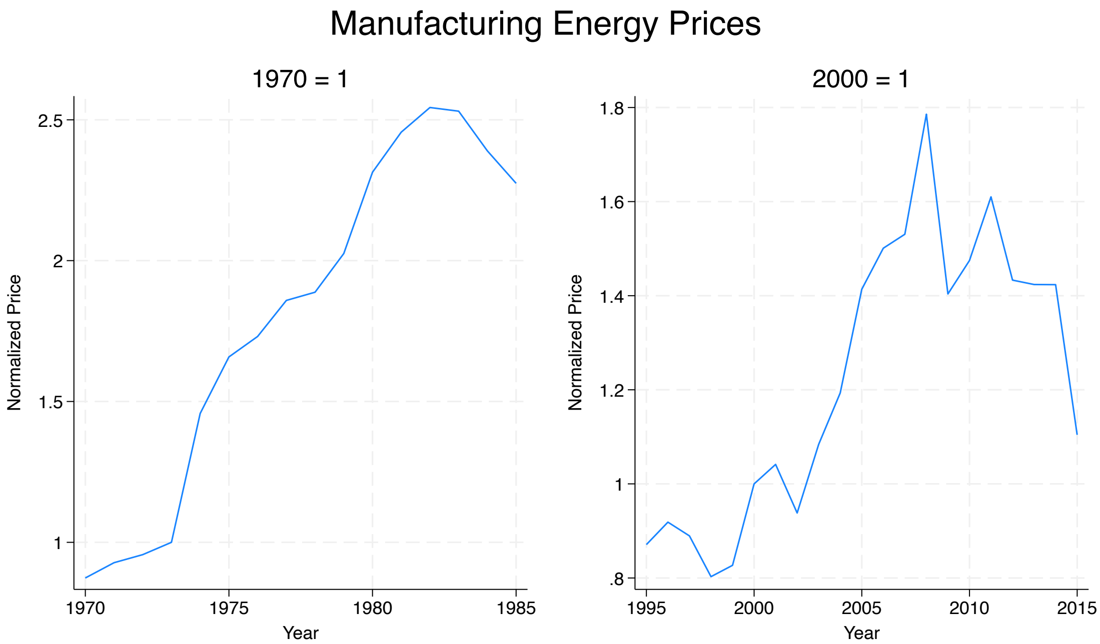
$$\log(y_{it}) - \log(y_{i-1}) = \alpha_t + \beta_t \delta_i + \boldsymbol{\gamma_t}'\boldsymbol{x_i} + \epsilon_{it} \text{ for } t \in \{-5, \dots, 15\} \tag{2}$$

This regression compares how energy intensity and the share of green patents evolve around the shock, relative to the pre-shock year $t = -1$. Coefficients β_t $t < 0$ reveal whether industries trended differentially prior to the increase in energy prices. Because depreciation may correlate with industry characteristics, I control for capital and energy intensity, the 1990–2010 industry-level increase in imports from China,

[8] This data is provided by the State Energy Data System (SEDS) as collected by U.S. Energy Information Administration (EIA).

[9] While oil is not a major fuel in manufacturing, U.S. fossil fuel prices co-move strongly (Serletis and Herbert, 1999), which explains the large energy-price increase following the oil shock in Figure 2.

Figure 2



Note: Both plots show the annual manufacturing energy price calculated as a fuel share weighted average across energy sources from the U.S. Energy Information Administration, deflated by the CPI. Underlying fuel prices are averages for the industrial sector.

and fuel shares to capture secular fuel-switching trends. Standard errors are clustered at the four-digit NAICS level.

Descriptive Patterns. Table 1 provides summary statistics for the key variables. Panels A and B show these variables for the 5 lowest and highest depreciation rate industries in the sample. These panels illustrate the type of comparisons the empirical analysis relies on. Industries such as steel or industrial machinery feature low depreciation rates around 5 percent, while capital stocks in the apparel industry feature much higher depreciation rates. The variation in depreciation rates closely tracks differences in mean service lives. Capital in low depreciation industries has on average much longer service lives, with a correlation of -0.82 shown in Panel C. By contrast, depreciation is only weakly related to capital intensity, the energy cost share, or the share of green patents. These low correlations indicate that durability and energy intensity are distinct at the industry level. Capital stocks for the higher depreciation industries in Panel B still require significant energy expenditures to operate. Among low-depreciation industries (e.g., alumina, steel, machinery manufacturing), energy use varies widely. Likewise, high-depreciation industries (e.g., apparel) differ substantially in energy intensity.

Overall, variation in depreciation rates is sufficiently distinct from plausibly related characteristics identified ex ante, motivating an empirical comparison of high- versus low-depreciation industries. To ensure robustness to differential pass-through by energy or capital intensity, all regressions control for baseline characteristics; I also report estimates without controls.

Table 1: Comparison of High and Low Depreciation Industries

Industry	Depreciation	Cap. intensity	Energy cost share	Green share	Durability
Panel A: Lowest depreciation (Top 5 by rank)					
Steel Product from Purchased Steel	0.05	1.43	0.08	0.06	27.16
Machine Shops; Turned Product; and Screw, Nut, and Bolt	0.05	0.93	0.03	0.02	24.26
Industrial Machinery	0.05	0.86	0.02	0.01	25.21
Coating, Engraving, Heat Treating, and Allied Activities	0.05	0.97	0.07	0.13	24.26
Nonferrous Metal (except Aluminum) Production and Processing	0.06	1.74	0.09	0.08	25.69
Panel B: Highest depreciation (Bottom 5 by rank)					
Veneer, Plywood, and Engineered Wood Product	0.10	0.99	0.09	0.12	15.33
Fiber, Yarn, and Thread Mills	0.10	1.83	0.12	0.02	19.69
Sawmills and Wood Preservation	0.10	0.99	0.07	0.04	15.33
Cut and Sew Apparel	0.10	0.73	0.02	0.01	18.57
Footwear	0.11	0.81	0.02	0.00	18.65
Panel C: Correlations					
Depreciation	1.00				
Cap. intensity	-0.10	1.00			
Energy cost share	-0.07	0.55***	1.00		
Green share	-0.20*	0.16	0.17	1.00	
Durability	-0.82***	0.35***	0.15	0.12	1.00

This table presents summaary statistics on measures of depreciation, capital intensity, energy intensity, share of green patents and durability across four-digit manufacturing industries. All figures present averages using post 1990 data. Industry-level depreciation is constructed from inverting the perpetual inventory equation based on industry-level capital stocks and investment flows. *Cap. intensity* refers to the ratio of equipment capital to value added. *Energy cost share* is calculated as the ratio of energy costs to value added. *Green share* is the ratio of green to total parents. *Durability* is the weighted average service life of assets using asset-industry specific investment as weights. Panels A and B contrast low and high depreciation industries. Panel C shows the correlations between these variables across all 81 manufacturing industries in the sample. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

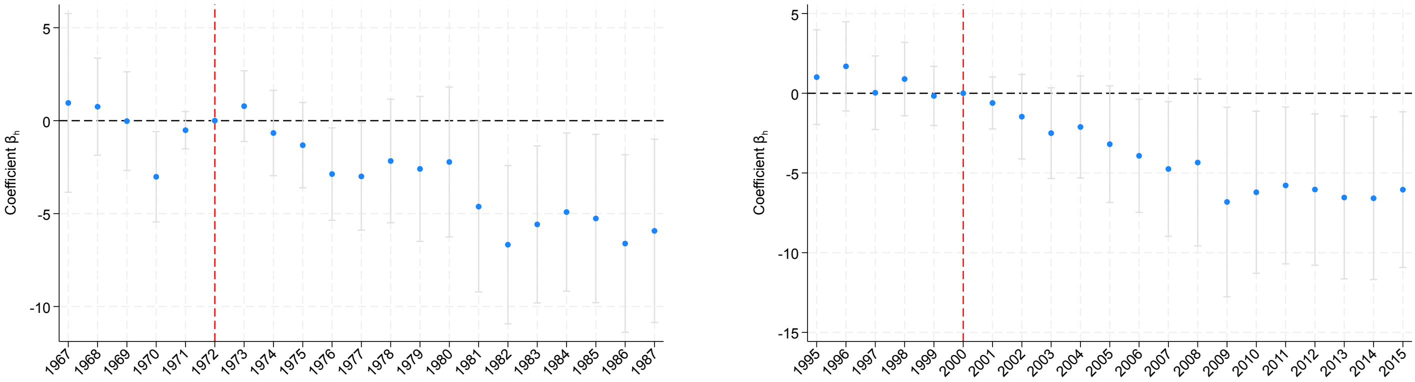
2.2 Results

Figure 3 shows the results from estimating equation (2) on data pre and post energy price increase. The left-hand panel (Figure 3a) shows that after the 1973 oil-price shock, energy intensity declines more in high-depreciation industries. The effects are economically sizable: an industry at the 75th percentile of depreciation sees a 14% decline in energy intensity relative to the 25th percentile. The dynamics post energy price shock differ from those observed in the pre-period. In times of stable energy prices, high and low depreciation rate industries evolve similarly in terms of energy intensity.

Next, I apply the same approach to the log share of green patents. Since data on USPTO granted patents is available only after 1976, I can only evaluate the effect on the post 2000s run up in energy prices. Figure 4 presents the results. Prior to the energy price shock, there is no differential trend in green patenting. Starting around 2003, however, when energy prices started to increase green patenting rates increase significantly in high versus low depreciation industries. Multiplied by the interquartile range of the depreciation rate distribution, these effects amount to a 34 percent relative increase the share of green patents.

An important question is whether these increases in the share of green patents reflect increases in green patents versus differential declines in overall patents. Results for the (log) level of green and overall patents are presented in Figure A2. Estimates indicate a long-run increase in the level of green patents, with a transitory decline in overall patenting rates.

Figure 3: Effect on energy intensity

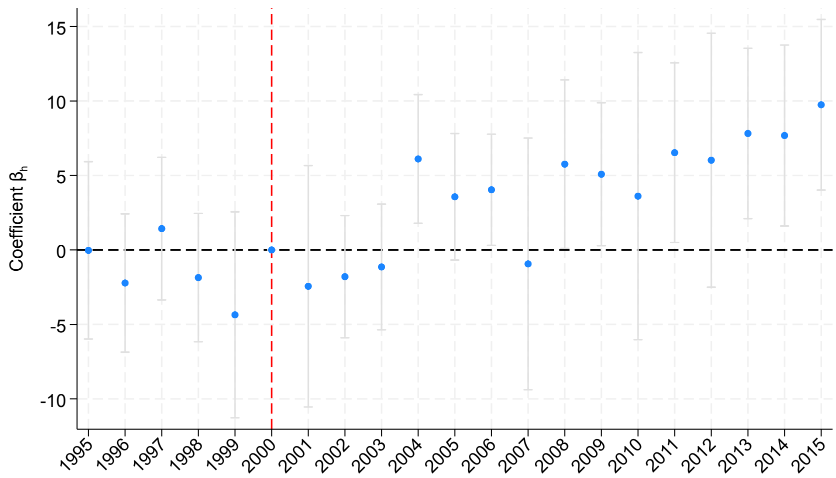


(a) First energy price shock: $t = 0$ in 1973

(b) Second energy price shock: $t = 0$ in 2000

Note: This Figure plots coefficients$\{\beta_t\}_{t=-5}^{15}$ around the two increases in the real price of energy paid by industrial users. The exposure variable to the increase in energy prices is the industry-level depreciation rate. The dependent variable is the (log) ratio of real energy use divided by real value added. Panel (a) shows effects around the first energy price increase starting in 1973. Panel (b) repeats shows effects for the second energy price shock starting in 2000. Standard errors are clustered at the four-digit NAICS level.

Figure 4: Effect on (log) share of green patents. Second energy price shock: $t = 0$ in 2000



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$around the two increases in the real price of energy paid by industrial users. The exposure variable to the increase in energy prices is the industry-level depreciation rate. The dependent variable is (log) ratio of green to total patents. Patent data is only available from 1976, so results are only presented for the second energy price increase.

IV estimates. One shortcoming of the event-study approach is that major increases in the aggregate price of energy were accompanied by broader macroeconomic downturns. Notably, the response of energy intensity and green patenting does not arise at business cycle frequency. However, short-run event study estimates could conflate the reaction to broader macroeconomic events with the reaction to energy prices. To move beyond aggregate increases in energy prices, I rely on industry-level energy price indices to test for differential pass through. The regression model is

$$\log y_{it} = \alpha_i + \alpha_t + \rho \log y_{it-1} + \beta_0 \log p^e_{it-1} + \beta_1 \log p^e_{it-1} \times \delta_i + \epsilon_{it}.$$

In this specification, α_i and α_t are unit and time fixed effects. The coefficient of interest is β_1. I include one lag of the dependent variable to allow the effect of energy prices to accumulate over time.[10] Standard errors are clustered at the four-digit NAICS level. To address endogeneity concerns about industry-level energy prices, I apply an instrumental variables approach. These prices differ because industries differ in the types of fuels they consume (gas, coal, oil, electricity). I construct a fixed-weight energy price index:

$$\tilde{p}^e_{i,t} = \sum_f \gamma_{if,1990} p_{f,t}$$

where $\gamma_{if,1990}$ is the share of total energy consumption industry i derived from fuel $f \in \{Coal, Gas, Oil, Electricity\}$ in 1990. $p_{f,t}$ is price observed for those fuels over the period 1990-2018. The data on fuel shares is taken from the Manufacturing Energy Cost Survey (MECS). The benefit of this IV approach relative to OLS is it avoids reverse causality from reduced energy demand to lower energy prices as well as fuel switching effects. The first stage regression of this IV approach has a Kleibergen-Paap F-Statistic of 35.6 (cf. Table **??**).[11] In addition to a strong first stage, this research design relies on exogeneity of the fuel shares with respect to unobserved shocks to energy intensity and the share of green patents (Goldsmith-Pinkham, Sorkin, and Swift, 2020). In support of this assumption, Figures A3 and A4 show that none of the fuel shares predict changes in the outcomes of interest prior to 1990.

Given these validity checks, Table 2 presents the second stage results. Columns (1)-(3) present results for (log) energy intensity. Column (1) shows that on average energy prices have only a small effect on energy intensity. Column (2) confirms industries with higher depreciation rates see larger reductions in response to increases in energy prices. Column (3) adds control variables by interacting the price of energy with aforementioned industry characteristics. Results remain robust to adding these control variables. The results for the share of green patents paint a similar picture. Column (4) shows a statistically insignificant, positive effect of energy prices on the share of green patents. However the effect on innovation is again larger for high depreciation industries as can been seen in columns (5)-(6). I obtain broadly similar results using OLS instead of the IV approach. Table A2 reports these results.

Robustness Checks. The results are robust along a number of dimensions:

[10] Results are in not sensitive to the inclusion of lagged dependent variables.

[11] I have verified that my results also hold in the reduced form, i.e. if I directly plug in $\tilde{p}^e_{it}$ in place of p^e_{it}.

Table 2: IV estimates: Second stage results

	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
log(p)	-0.0260	-0.0122	-0.0894	0.346	0.318	-0.0582
	(-0.20)	(-0.09)	(-0.92)	(1.41)	(1.35)	(-0.24)
Lagged Outcome	0.859***	0.855***	0.862***	0.586***	0.570***	0.538***
	(31.60)	(30.14)	(33.43)	(6.45)	(6.26)	(5.52)
log(p) x delta		-2.318**	-2.450***		10.15***	8.585**
		(-2.21)	(-2.70)		(2.79)	(2.33)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	81	81	81	81	81	81
Kleibergen-Paap F-Stat	34.1	18.6	31.6	36.6	19.1	33.8
Observations	2187	2187	2106	2187	2187	2106

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This table shows IV estimates of the differential effects of industry-level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the instrumented interaction of baseline energy intensity and capital intensity with industry-level energy prices. First stages estimates are presented in Table A1. Standard errors are clustered at the four-digit NAICS level.

1. Heterogeneity in short-run substitution elasticity: Estimates in Figure 3 suggest high depreciation industries display a higher elasticity of substitution between energy and non-energy inputs. One potential threat to interpreting these effects as driven by depreciation is they may also exhibit a higher short-run elasticity. To investigate this possibility, I run regressions of (log) energy cost share in value added on (log) energy prices

$$\log(\text{energy share})_t = \alpha + \beta \log p_{e,t} + \epsilon_t.$$

I estimate this equation separately for each industry, resulting in a vector of short-run pass through rates. The resulting coefficients have a mean of 1.05 across industries, consistent with evidence of perfect complements in Hassler et al. (2021). Figure A1 shows that short-run pass-through displays no significant correlation with depreciation. These results strengthen the interpretation of depreciation as a source of long-run, but not short-run, substitution, as would be implied by faster adoption of more energy-efficient capital.

2. Discrete treatment variable: An alternative way of estimating the differential effect of energy prices is to define a dummy variable equal to one if industry i is in the top quartile of the empirical distribution of depreciation rates. Figures A5, A6 and A7 show the results. For the second energy price shock, effects are highly consistent across the two approaches. For the first energy price shock and outcome (log) energy intensity, estimates are qualitatively similar but not statistically significant

when using the discrete measure. The depreciation rates are close to uniformly distributed across manufacturing industries such that there is no natural cut-off. Therefore, results based on the continuous measure are arguably preferable.

3. Biadic patents: Patents differ widely in quality, so I repeat the estimation shown in Table A2 including only biadic patents. I define a patent as biadic if it has been granted by USPTO and one other patent agency. Columns (4)-(6) in Table A3 show the results. This delivers very similar results to the OLS estimates of Table A2.
4. Alternative durability measure: To assess the robustness of my results to alternative measures of durability, I construct a measure of average asset life of equipment capital for each industry T_i. This measure is based on BLS estimates of asset lives for disaggregated asset classes and industry by asset class level investment from the 1997 BEA capital flow table. This variable is not always available at the four digit level since BEA industries are often more aggregated than four digit NAICS. The resulting durability measure has a negative correlation with depreciation of -0.82 as shown in Table 1. Table A4 shows the results. While results are slightly noisier, they remain statistically significant.
5. The effects in Figures A6 represent differential effects across industries and do not capture potentially important general equilibrium adjustments. These cannot be credibly identified with the empirical approach taken so far, as time fixed effects are contaminated by simultaneously occurring macroeconomic events. Identified macroeconomic shocks provide an alternative source of variation allowing for identification of general equilibrium responses. The shocks I draw on are oil supply shocks provided by Baumeister and Hamilton (2019). I use these shocks in a local projection framework similar to Känzig (2021). Further details on the estimation approach are provided in Appendix A.1. Figure A8 shows that these oil price shocks lead to a persistent increase in the real price of energy paid by industrial consumers, where the energy price series is the same series as in Figure 2. Figure A9 shows results of the shock on innovation and energy intensity across high and low depreciation industries. Effects on innovation are consistent with the cross-industry analysis. High depreciation sector see a roughly 10 percent increase in the share of green patents, with a smaller response for low depreciation industries. This difference is statistically significant at the 90% confidence level at the 1 year horizon. To interpret the magnitude note that the shock is scaled to increase energy prices by 10 percent on impact. For energy intensity, the response is zero for high depreciation industries and even positive at some horizons for low depreciating industries. The pattern for low depreciation industries stems from value added declining faster than energy demand. However the difference is not statistically significant. Broadly, these results are in line with the cross-industry analysis in terms of the differential effect. The less persistent nature of energy price shocks defined at the business cycle frequency makes it more challenging to identify effects on measures of productivity such as energy intensity.

Summary. These results provide the first evidence that pass-through of energy prices to energy intensity and innovative activity depends on industry-level depreciation rates. The evidence supports the hypothesis that high depreciation industries display an effectively higher elasticity of substitution between energy and non-energy inputs. Why would this elasticity depend on depreciation? High depreciation industries naturally experience a higher churn of their equipment capital. When energy prices rise, these industries then invest into equipment with higher energy efficiency. While the technology embodied in investment goods is inherently difficult to measure directly, the effect on green patenting rates shows a clear increase in the demand for clean technology among high depreciation industries.

3 The Model

Model overview. The model in this section aims to describe the feedback between incentives of machine users to replace capital and suppliers of the capital to improve the energy efficiency of those machines. Machine users are heterogeneous in two dimensions: Across sectors, machines depreciate more slowly in the high than in the lower durability sector. This sectoral heterogeneity mirrors the reduced form cross-industry regressions. Within sectors, firms are subject to idiosyncratic shocks to the value of replacing their capital. Because firms only update their technology when they replace the capital through a lumpy investment decision, these shocks generate a distribution of vintages in each sector.

Faced with this demand side, innovating firms choose price and energy efficiency embodied in these machines. The model features a market size effect whereby higher investment rates increase innovation because it allows firms to spread the overhead cost of R&D over more units. Higher energy prices or more stringent carbon policy makes energy efficient machines more valuable. At the same time, these forces lower aggregate investment, reducing R&D incentives.

I embed this two-way relationship between adoption and technological development in a standard macro-climate general equilibrium model a la Golosov et al. (2014). The model quantifies how slow machine replacement shapes the effects of energy-price shocks, optimal carbon policy, and the benefits of sector-specific interventions.

3.1 Growth model

I consider a continuous time, infinite horizon economy. The preferences of the infinitely lived, representative households are represented by the utility function

$$U = \int_0^\infty e^{-\rho t} \log C(t) dt \tag{3}$$

where $C(t)$ is consumption of the final good and ρ is the discount rate. The representative household saves in bonds at interest rate $r(t)$ leading to a standard Euler equation for consumption growth

$$\frac{\partial C(t)}{\partial t} = C(t)(r(t) - \rho). \tag{4}$$

The final output good is produced by combining manufacturing output, X_m, services X_s and electricity

X_s

$$Y(t) = \left(\left(X_m(t)^{\kappa} X_s(t)^{1-\kappa} \right)^{\frac{\varrho-1}{\varrho}} + \nu X_e(t)^{\frac{\varrho-1}{\varrho}} \right)^{\frac{\varrho}{\varrho-1}}. \tag{5}$$

and denote output net of climate damages by $\widetilde{Y}(t) = \exp\big(-\psi\left(S(t) - \bar{S}\right)\big)\, Y(t)$. The term $\exp\big(-\psi\left(S(t) - \bar{S}\right)\big)$ expresses aggregate productivity as a function of climate damages as in Golosov et al. (2014). $S(t)$ denotes the amount of carbon in the atmosphere in period t, equal to $\bar{S}$ in the base-period concentration. The dynamics of the climate system are represented as in Golosov et al. (2014):

$$S(t) = S_P(t) + S_D(t), \tag{6}$$

$$\dot{S}_P(t) = \varphi_L\, \mathcal{E}(t), \tag{7}$$

$$\dot{S}_D(t) = -\varphi\, S_D(t) + \big(1 - \varphi_L\big)\, \varphi_0\, \mathcal{E}(t). \tag{8}$$

Atmospheric concentration of carbon $S(t)$ is the sum of carbon that remains in the atmosphere permanently $S_P(t)$ and carbon that is partially absorbed by the climate system over time $S_D(t)$. A share φ_L of current emissions $\mathcal{E}(t)$ stays in the atmosphere forever, increasing $S_P(t)$. φ_0 is the share that is not immediately absorbed, while φ determines the rate at which non-permanent carbon is absorbed over time.[12] Emissions result from fossil fuel use decision of capital goods producers as I specify below, unlike the DICE model where emissions are modelled in direct proportion to final output (Barrage and Nordhaus, 2024).

The three inputs X_i into this aggregate are produced as follows. Services are produced linearly from labor $X_s = A_L L_s(t)$. Manufacturing combines capital and labor using a Cobb-Douglas production function

$$X_m(t) = K_m(t)^{\alpha} (A_L L_m(t))^{1-\alpha} \tag{9}$$

where manufacturing capital is in turn a composite of durable capital $K_m(d,t)$ and less durable capital $K_m(nd,t)$ that are combined in CES-fashion with elasticity of substitution η.

$$K_m(t) = \left(\beta^{1/\eta}\, K_m(d,t)^{\frac{\eta-1}{\eta}} + (1-\beta)^{1/\eta}\, K_m(nd,t)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{10}$$

The two sectors divide manufacturing into one producing with more versus less durable capital mirroring the reduced form analysis. By modelling manufacturing output as combining sector specific capital with an aggregate endowment of labor, I abstract from sector specific differences in labor shares.[13] Finally, electricity production combines electricity capital and labor

[12] The Golosov et al. (2014) climate model provides a strongly simplified representation of the climate system relative to more full-fledged models (Folini, Friedl, Kübler, and Scheidegger, 2025). Importantly, Dietz, Van Der Ploeg, Rezai, and Venmans (2021) show that this simple representation acccurately captures the immediate link between cumulative emissions and surface warming. The accurate representation of near term climate dynamics makes it a useful benchmark for examining how short-run technological lock in alters carbon-policy conclusions. A shortcoming of this climate model is that it abstracts from state dependence in terms of the climate system's ability to absorb carbon over time (Dietz et al., 2021).

[13] Empirically, I find that both sectors are similar in terms of their ratio of the aggregate wage bill to value added as measured in the NBER CES Manufacturing database.

$$X_e(t) = K_e(t)^{\varphi}(A_L L_e(t))^{1-\varphi}. \tag{11}$$

Electricity capital $K_e(t)$ is a composite of fossil fuel capital $K_e(f,t)$ and green energy capital $K_e(g,t)$

$$K_e(t) = \left(K_e(f,t)^{\frac{\vartheta-1}{\vartheta}} + A_g^{\frac{1}{\vartheta}} K_e(g,t)^{\frac{\vartheta-1}{\vartheta}}\right)^{\frac{\vartheta}{\vartheta-1}}. \tag{12}$$

3.1.1 Investment dynamics.

I begin by describing the investment problem of firms producing capital goods $K_e(f,m), K_m(d,t), K_{nd}(t)$. Each of these sectors i consists of a mass of ex-ante identical firms N_i. I omit sectoral indices since firms in each sector are symmetric except for the depreciation rate of their capital.

Static Profit Maximization. I first describe the static profit maximization problem for firms in each sector and then determine their investment policy.

Firms produce output $k(t)$ using a CES-vintage capital production function

$$k(t) = \left(\int_0^1 \min\{A_e(\nu,t-a)e(\nu,t), k(\nu,t)\}^{\frac{\sigma-1}{\sigma}} d\nu\right)^{\frac{\sigma}{\sigma-1}} \tag{13}$$

Each machine ν combines predetermined capital $k(\nu,t)$ and machine-specific energy efficiency $A_e(\nu,t-a)$ with a freely variable energy input $e(\nu,t)$. A continuum of machines $\nu \in [0,1]$ ensures tractability when deriving demand for new machines and thus innovation incentives of machine producers. This production function imposes two important assumptions.

First, firms produce using vintage capital. Denoting by a the age of the machine and by $t-a$ the time since purchase, energy efficiency remains constant across the life of a machine. I further assume that firms can only replace all machines ν at once, implying that $t-a$ does not vary within the firm. All machines are purchased at the same time, and the dynamic optimization problem underlying this decision is described below.

Second, capital and energy are modeled as perfect complements. Given predetermined capital and technology per machine, profits depend only on utilization, which is determined by the energy input. Faced with an exogenous energy price $p_e(t)$ and taking the output price $P^K(t)$ as given, firms solve:

$$\max_{\{e(\nu,t)\leq \frac{k(\nu,t)}{A(\nu,t-a)}\}_{\nu\in[0,1]}} P^K(t)\,k(t) - p_e(t)\int_0^1 e(\nu,t)\,d\nu \quad \text{s.t.} \quad (13), \tag{14}$$

and the resulting energy use satisfies

$$e(\nu,t) = \min\left\{\left(\frac{P^K(t)\,k(t)^{1/\sigma}\,A_e(\nu,t-a)}{p_e(t)}\right)^{\sigma},\ \frac{k(\nu,t)}{A(\nu,t-a)}\right\}. \tag{15}$$

In equilibrium, the allocation of capital across machines, as well as their energy efficiency will be symmetric - $k(\nu,t) = k(t)$ and $A_e(\nu,t-a) = A_e(t-a)$. In that case, $A_e(t-a)$ and $k(t)$ become sufficient statistics for firm profits. We can use the constant returns to scale assumption to write profits as

$$\pi(k(t), A_e(t-a), t) = k(t)\mathbf{1}_{P^K(t) - \frac{p_e(t)}{A_e(t-a)} \geq 0} \left(P^K(t) - \frac{p_e(t)}{A_e(t-a)} \right) \tag{16}$$

Profits are equal to the level of capital $k(t)$ multiplied by its marginal revenue product $\bar{\pi}(t) = \left(P^K(t) - \frac{p_e(t)}{A_e(t-a)} \right)$ which is increasing in the output price $P^K(t)$ and the level of technology $A_e(t-a)$ and declining in the energy price $p_e(t)$. Whenever marginal revenue is negative, the firm does not produce and profits are zero. Profits do not differ across varieties within a firm, but they do differ across firms that replaced capital at different times $t-a$. If $A_e(t-a)$ grows over time, firms with more recent vintages earn higher profits.

Discussion of assumptions. Before describing the role of this profit function for investment dynamics, I briefly discuss the assumptions underlying it. Prior research documents the empirical relevance of technological lock-in for understanding energy efficiency dynamics in manufacturing. Manufacturing plants are most sensitive to energy prices at entry (Hawkins-Pierot and Wagner, 2023), consistent with technological lock-in after major investments. Linn (2008) documents higher energy efficiency among entrants than incumbents, suggesting that firms benefit from technology growth when they invest in new capital. Second, I assume capital and energy are perfect complements within each machine. At the firm-level, several papers estimate a very low elasticity of substitution between energy and non-energy inputs at the firm level (Chen, Chen, Liu, Suárez Serrato, and Xu, 2025; Hawkins-Pierot and Wagner, 2023; Ryan, 2018). Hassler et al. (2021) document a near perfect short-term correlation between the price of energy and the energy factor share of GDP.

Dynamic Replacement Problem. Firm capital $k(t)$ is determined through irreversible investment in new machines. The investment decision is based on the Hamilton-Jacobi-Bellman equation

$$r(t)V(k,t) = \underbrace{\pi(k,t)}_{\text{Profits}} - \underbrace{\delta k V_k + V_t}_{\text{Drift}} + \underbrace{\lambda(\mathcal{V}(k,t) - V(k,t))}_{\text{Expected Replacement Gain}} \quad . \tag{17}$$

Time t and capital k are the state variables of the firm's problem. Firms also differ in technology, but using $\frac{\partial k(t)}{\partial t} = -\delta k(t)$ we can infer age $a = \frac{1}{\delta} \log! \left(\frac{\bar{k}(t)}{k} \right)$ and thus $A_e(t-a) = A_e(t,k)$. $\bar{k}$ denotes the level of capital of a new machine.

The instantaneous return to capital $r(t)V(k,t)$ has three components. First, the flow of profits the firm makes from its capital stock $\pi(k,t)$. The drift component captures deterministic capital gain dynamics due to depreciation, $\delta k V_k$, and potential transition dynamics V_t. Third, the firm receives replacement opportunities at Poisson rate λ. Upon a replacement opportunity, firms draw an additive shock ϵ to the value of a new machine $V(\bar{k},t) - P(t)\bar{k}$, yielding an expected gain $\mathcal{V}(k,t) - V(k,t)$. The overall size of a new machine $\bar{k}$ is exogenous.[14] Replacement takes place whenever

$$V(\bar{k}, t) - P(t)\bar{k} + \epsilon > V(k, t). \tag{18}$$

Conditional on the shock ϵ, firms are more likely to replace when new capital is cheap, or relatively more profitable. The shocks can represent idiosyncratic differences in firms to switch from old to new machines, or a random scrap value independent of the remaining capital. The endogenous cost of new capital $P(t)\bar{k}$ is derived below from innovating firms supplying the capital. Relative profitability of new versus old capital depends on how far the old machine has depreciated, as well as the gap in marginal products. Using the optimal policy and integrating over shocks, the expected gross gain is $\mathcal{V}(k, t) = \mathbb{E}_\epsilon\big[\max V(\bar{k}, t) - P(t)\bar{k} + \epsilon, V(k, t)\big]$.

Law of motion for distribution. Idiosyncratic shocks across firms imply that firms replace their capital at different times, giving rise to a distribution of capital $\mu(k, t)$. The dynamics of this distribution are given by

$$\frac{\partial \mu(k, t)}{\partial t} = \frac{\partial}{\partial k}(\delta k \mu(k, t)) - \lambda P^R(k, t)\mu(k, t) \tag{19}$$

with $\delta \bar{k} \mu(\bar{k}, t) = \lambda \int_0^{\bar{k}} P^R(k, t)\mu(k, t)dk$ and in each sector we have $\int_0^{\bar{k}} \mu(k, t) = N_i$.[15]

The first term reflects inflow of firms into capital level k, while the second term reflects outflow of all firms with capital k that receive a replacement opportunity. The replacement probability $P^R(k, t) = \mathbb{P}(V(\bar{k}, t) - P(t)\bar{k} - V(k, t) > -\epsilon)$ is obtained by integrating the investment policy (18) across the density of ϵ.

Demand for new technologies. When a firm receives a replacement opportunity, it can scrap its existing capital and purchase a new machine composed of varieties $k(\nu, t)$. The overall size of this machine is constrained to $\bar{k}$ in the presence of constant returns and perfect competition. Firms selling the varieties $k(\nu, t)$ also choose their energy efficiency $A_e(\nu, t)$. Thus, the demand for varieties $k(\nu, t)$ determines innovation incentives.

Abstracting from variable utilization, the firm maximization problem determining demand for each variety can be stated as

[14] The assumption of exogenous arrival rate of replacement opportunities with shocks to each option drawn upon arrival follows Arcidiacono, Bayer, Blevins, and Ellickson (2016). An alternative approach would be to continuously allow firms to replace their capital. Introducing idiosyncratic shocks to the resulting HJB-Variational Inequality requires adding a second state variable. For details see Achdou et al. (2022) and related notes available on Ben Moll's website `https://benjaminmoll.com/codes/`. The current approach avoids this issue because the HJB equation holds exactly - not as an inequality - allowing me to represent the value of capital in expected value terms, i.e. after integrating out the shocks.

[15] To see the intuition for the capital drift in this equation, abstract from replacement and write the share of firms with capital in the interval $[k,\, k + dk]$ at time $t + dt$ as

$$\mu(k, t + dt)\, dk \;=\; \mu\big(k(1 + \delta dt), t\big)\,(1 + \delta dt)\, dk.$$

The additional factor $(1 + \delta dt)$ reflects the fact that these firms come from an interval of width $dk(1 + \delta dt)$. Taking a first order approximation of $\mu\big(k(1 + \delta dt), t\big) \approx \mu(k, t) + \delta k \mu_k(k, t)dt$ and letting $dt \to 0$ gives the result. Equivalently, we can derive the law of motion starting from the discrete time law of motion for the cumulative distribution function. I include this derivation in Appendix B.1.

$$
\begin{aligned}
\max_{k(\nu,t)} \mathcal{L} \;=\; & \underbrace{\int_t^\infty e^{-H(t,\tau)}\Big[P^K(\tau)\big(\int_0^1 k(\nu,t)^{\frac{\sigma-1}{\sigma}}\,d\nu\big)^{\frac{\sigma}{\sigma-1}} - \int_0^1 \frac{p_e(\tau)}{A_e(\nu,t)}\,k(\nu,t)\,d\nu\Big]d\tau}_{\text{Operating Profits}} - \underbrace{\int_0^1 p(\nu,t)\,k(\nu,t)\,d\nu}_{\text{Investment Cost}} \\
& +\; \xi(t)\underbrace{\Big[\bar{k} - \big(\int_0^1 k(\nu,t)^{\frac{\sigma-1}{\sigma}}\,d\nu\big)^{\frac{\sigma}{\sigma-1}}\Big]}_{\text{Capacity Constraint}}.
\end{aligned}
\tag{20}
$$

Profits stem from operating profits over the replacement cycle net of investment cost. In the presence of perfect competition and constant returns to scale in production, the capacity constraint $\bar{k}$ pins down the overall level of capital.

The length of the replacement cycle is stochastic as reflected by the discount factor

$$
H(t,\tau) = \delta(\tau - t) + \int_t^\tau r(u) + \lambda P^R(\bar{k}e^{-\delta(u-t)}, u)du,
$$

summarizing depreciation, standard discounting of future profits at the interest rate and the evolution of the replacement probability due to capital depreciation and transition dynamics. By taking into account how this probability changes over time, firms' sensitivity to energy prices crucially depends on future replacement dynamics. Suppose the firm anticipates likely replacement in the future, for example because of higher energy efficiency in the future. The firm then discounts current and future energy prices when buying a machine today. Intuitively, current and future energy prices matter less for today's purchase if the firm anticipates facing those prices with a different machine than the one it is buying today.

As shown in Appendix B.2, this profit maximization problem gives rise to isoelastic demand for machines

$$
k(\nu,t) = \bar{k}\left(\frac{\tilde{p}(\nu,t)}{\tilde{P}(t)}\right)^{-\sigma} \tag{21}
$$

where $\tilde{p}(\nu,t) = p(\nu,t) + \tilde{p}_e(\nu,t)$ is the sum of the up-front purchasing price $p(\nu,t)$ and life time energy costs given by

$$
\tilde{p}_e(\nu,t) = \frac{\tilde{p}_e(t)}{A_e(\nu,t)} = \frac{\int_t^\infty e^{-H(t,\tau)} p_e(\tau)\mathbf{1}_{P^K(\tau)\geq \int \frac{p_e(\tau)}{A_e(\nu,t)}d\nu} d\tau}{A_e(\nu,t)}.
$$

$\tilde{P}(t) = (\int_0^1 \tilde{p}(\nu,t)^{1-\sigma}d\nu)^{\frac{1}{1-\sigma}}$ is the associated CES price index. Demand for machines is influenced both by the purchase price $p(\nu,t)$, as well as the downstream energy cost of the machine $\tilde{p}_e(\nu,t)$.

3.1.2 Supply of Technology.

Each machine $k(\nu,t)$ is produced by a monopolistically competitive machine producer setting price $p(\nu,t)$ and energy efficiency $A_e(\nu,t)$ to maximize profits

$$
\pi(\nu,t) = \max_{p(\nu,t),\,A_e(\nu,t)} M(t)k(\nu,t)(p(\nu,t) - c) - w(t)\frac{A_e(\nu,t)^{\frac{1}{\theta}}}{\gamma\phi(t)} \;\text{ s.t. (21).} \tag{22}
$$

Firms' unit margin $p(\nu,t)-c$ is the difference between the purchase price of a machine and the marginal cost of capital c (denoted in units of the final good). Quantity sold is the product of the mass of firms that replace $M(t)=\int_0^1 \lambda P^R(k,t)\mu(k,t)dk$ times unit sold to each firm that replaces. R&D into energy efficiency represents an overhead cost and is subject to decreasing returns $\theta<1$. Total R&D cost are $w(t)\frac{A_e(\nu,t)^{\frac{1}{\theta}}}{\gamma\phi(t)}$ where $\gamma\phi(t)$ shifts the productivity of resources put into R&D. The optimal price equals

$$p(\nu,t)=\frac{\sigma}{\sigma-1}c+\frac{1}{\sigma-1}\frac{\tilde{p}_e(t)}{A_e(\nu,t)}. \tag{23}$$

A simple intuition for the effect of life time energy costs on the purchase price of a machine comes from the derived elasticity of demand

$$-\frac{d\log k(\nu,t)}{d\log p(\nu,t)}=\sigma\frac{p(\nu,t)}{\tilde{p}(\nu,t)}<\sigma.$$

With higher energy prices, firms become less sensitive to the purchase price of a machine $p(\nu,t)$, flattening the demand curve in $k(\nu,t)-p(\nu,t)$ space, leading to an increase in markups. In a symmetric equilibrium (cf. Appendix B.2), the first order condition for energy efficiency can be rearranged to

$$A_e(\nu,t)=A(t)=(\theta\gamma\phi(t)M(t)\bar{k}\tilde{p}_e(t)/w(t))^{\frac{\theta}{1+\theta}}. \tag{24}$$

Higher energy prices have direct and indirect effects on the supply of energy efficiency. The direct effect stems from higher energy prices increasing capital users' willingness to pay for energy efficiency. Faced with this higher willingness to pay, firms supplying the capital increase energy efficiency as machine users are now more easily induced to buy from them. Indirect effects arise from energy prices lowering the replacement rate $M(t)$. The mechanism is as follows: The relative benefit of having a new machine versus an old machine shrinks with higher energy prices. Intuitively, investing in a new machine is worthwhile if the machine has high returns. Faced with relatively lower profitability, firms delay replacement, shrinking the market for new machines and hence equilibrium energy efficiency. Combining (23) and (24), we obtain the investment cost of a new machine

$$P(t)=\frac{\sigma}{\sigma-1}c+\frac{1}{\sigma-1}\frac{\tilde{p}_e(t)}{A_e(t)}. \tag{25}$$

To capture spillovers from innovation, I assume that $\phi(t)$ represents an endogenous component of R&D productivity and follows a simple law of motion

$$\frac{\partial\phi(t)}{\partial t}=-\delta_A\phi(t)+A_e(t). \tag{26}$$

The exogenous component γ is assumed constant. Importantly, these spillovers are not internalized by firms because they solve a static profit maximization problem.[16] In other endogenous growth models,

[16] The particular law of motion for research productivity implies my model shares similarities with spillovers in semi-endogenous growth models. Along the transition path, spillovers can deliver growth in technology. In steady state, productivity is constant,

firms benefit from R&D investment over multiple periods. In my model, firms do benefit from their own investment because it raises future R&D productivity. Conditional on productivity $\phi(t)$ firms need to invest into R&D every period to maintain a given level of technology.

Equilibrium. To close the model, I add the resource constraint

$$\widetilde{Y}(t) = C(t) + \sum_i (c\bar{k}M_i(t) + p_e(t)\mathcal{E}_i(t)) + c\bar{k}M_e(g,t) \tag{27}$$

where the sum across sectors i includes the three sectors operating with fossil fuel powered capital and $M_e(g,t)$ denotes the replacement rate into renewable using capitals. $\mathcal{E}_i(t)$ denotes sectoral fossil fuel use. The remaining output is used for consumption. The labor market clearing condition states that exogenous endowment of labor equals labor demand into R&D $L_r(t)$, manufacturing $L_m(t)$, services $L_s(t)$ and electricity production $L_e(t)$:

$$L = L_r(t) + L_m(t) + L_s(t) + L_e(t). \tag{28}$$

3.2 Model solution and Calibration.

Solution method. The steady state of the model is solved following Achdou et al. (2022). I use a finite difference method to discretize the capital grid and solve the Hamilton-Jacobi-Bellman in equation (17) numerically, given a guess of the endogenous variables. To solve for the distribution (19), I apply this discretization to approximate the derivative $\frac{\partial \mu(k,t)}{\partial k}$ and calculate the model implied replacement probability based on the solution to the HJB. The steady state distribution (19) can then be solved as a linear system.

Transitional dynamics are handled similarly: I discretize the time dimension following a collocation approach similar to Hémous et al. (2023) and Schesch (2024). The endogenous variables are approximated as Chebyshev polynomials in the time dimension.[17] Time derivatives are then computed based on the Chebyshev differentiation matrix associated with the polynomial, instead of the usual finite difference approximation involving equally-spaced, neighbouring points (Trefethen, 2000). Using these approximations, I follow very similar steps as for the steady state to solve for the value function $V(k,t)$ and the distribution $\mu(k,t)$. Appendix C contains further details on the numerical methods used to solve the model.

Calibration. I calibrate the model to cross sectional moments from the U.S. manufacturing sector for the year 2010 as well as the difference-in-difference estimates presented in Section 2. Several key parameters are internally calibrated using the simulated method of moments. The remaining parameters

so only exogenous growth in other factors, can deliver permanent increases in energy efficiency $A_e(t)$. The particular specification for spillovers is motivated by the fact that it allows me to derive an analytical expression for steady-state productivity, $\phi = A\frac{\gamma}{\delta_A}$, which is typically not possible in semi-endogenous models.

[17] I use 19 grid points in the time dimension. I assume the system has converged to a new steady state in 500 years. To deal with non-stationarity induced by the climate system, I hold climate damages constant at the value reached after 500 years (Cai and Lontzek, 2019).

are set based on external sources. Table 3 lists all parameters, their values and sources used in the calibration.

Dividing manufacturing into two sectors. The model features durable and non-durable capital using industries. I sort the 86 four-digit industries in the NBER CES Manufacturing database based on the depreciation rates used in the empirical analysis. Industries in the lower quartile of the distribution of depreciation rates are sorted into the durable using sector. While the exact split is arbitrary, the distribution of depreciation rates does not feature major spikes, justifying this procedure. I use a 25%-75% split so that sector specific policies for durable capital using industries are sufficiently narrowly targeted.

Externally calibrated parameters - Climate Block. The calibration of the law of motion for the evolution of atmospheric concentration (6) involves three parameters. I follow Dietz et al. (2021) and set the share of emissions that remains in the atmosphere permanently at $\phi_L = 0.2$. $\phi_0 = 0.402$ implies roughly 40% of non-permanent emissions are absorbed immediately. Non-permanent emissions depreciate at rate $1 - \phi = 0.00231$. The elasticity of damages to additional increases in atmospheric concentration is set to $\psi = 0.00011$, following the higher value proposed by Acemoglu, Aghion, Barrage, and Hémous (2023). This higher value is motivated by recent literature estimating large GDP losses from increases in temperature (Bilal and Stock, 2025; Nath et al., 2025). Initial values for the carbon stock are a stock of permanent emissions of $S_P = 684GtC$ and a stock of emissions $S_D = 159GtC$ subject to depreciation over time.

Externally calibrated parameters - Aggregate Production Function. I calibrate the aggregate production function (5) by setting the labor share $\alpha = 0.5$, consistent with recent evidence in Kehrig and Vincent (2021) for U.S. manufacturing. BLS data indicates a labor share of 0.25 for electric utilities, leading me to set $\varphi = 0.75$. Next, I determine the parameters of equation (10) mapping sector specific capital levels into aggregate manufacturing capital. Oberfield and Raval (2021) and Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2025) estimate cross-industry elasticities of substitution around between two-digit manufacturing sectors. Based on their estimates, I set $\eta = 1.2$. The elasticity of substitution $\vartheta = 1.85$ between clean and dirty energy is set following estimates by Papageorgiou, Saam, and Schulte (2017) on the substitutability between clean and dirty capital. I set $\beta = 0.286$ based on the share of industry value added accounted for by the durable sector.

Externally calibrated parameters - Supply of Technology. I set the elasticity of substitution σ across capital varieties equal to 6. Since energy prices provide an additional force driving up markups (cf. 23), this leads to markups of 1.25. I assume strongly decreasing returns to R&D at the firm-level with $\theta = 0.35$.[18] To calibrate the law of motion for research productivity $\phi(t)$ in (26), I assume productivity depreciates at rate $\delta_A = 0.05$. This value is consistent with estimates of organizational forgetting for durable goods manufacturing by Benkard (2000) and on the lower end of R&D depreciation values in Hall

[18]This value is in line with estimates of the elasticity of patenting with respect to R&D spending in Bloom, Schankerman, and Van Reenen (2013) of around 0.3 I discuss below that this value of θ is line with independent evidence on the elasticity of innovation with respect to energy prices.

(2007) for Compustat firms as a whole.

Normalized Parameters. I normalize the size of machines in both sectors to $\bar{k} = 1$ and set the arrival rate of replacement opportunities $\lambda = 1$ as in Arcidiacono et al. (2016). With $\lambda = 1$, firms receive an average of one replacement opportunity per year. Aggregate labor is normalized to ten, meaning the labor productivity parameter should be interpreted in per capita terms. I also set $c = 1$, i.e. a unitary conversion rate of final output to investment as is standard in neoclassical growth.

Table 3: Base-Year Model Calibration Summary

Parameter	**Value**	**Source / Target**
Internally calibrated parameters		
Aggregate productivity A_L	3610.3	World GDP (World Bank)
Mass of firms in each sector N_m, N_e	2961.9, 8883.7	Sectoral investment share (BEA)
Depreciation rates manufacturing $\delta_e(d),\ \delta_e(nd)$	0.03, 0.05	Perpetual-inventory–method implied δ (NBER Manufacturing)
Taste-shock dispersion σ_ε	5	DiD event study estimate
Energy efficiency R&D productivity $\gamma_m,\ \gamma_e$	0.602, 8.16	Sectoral fossil fuel shares
Electricity productivity ν	297.157	Electricity share of GDP
Green energy productivity A_g	0.145	Share of green electricity
Externally imposed parameters		
Manufacturing share κ	0.11	Manufacturing value added (NBER-CES) as share of GDP
Share of durable capital β	0.286	Share of durables in manufacturing value added
Capital share (manufacturing) α	0.50	Kehrig and Vincent (2021)
Capital share (electricity) φ	0.75	BLS data
Elasticity of substitution between electricity and service–manufacturing composite ϱ	0.25	—
Elasticity of substitution η (between $K_m(d,t)$ and $K_m(nd,nt)$)	1.2	Oberfield and Raval (2021)
Elasticity of substitution ϑ (between $K_e(f,t)$ and $K_e(g,nt)$)	1.85	Papageorgiou et al. (2017)
Within sector elasticity of substitution σ	6.0	Markup of 1.25
Knowledge depreciation δ_A	0.05	Benkard (2000)
Discount rate ρ	0.02	Real rate of 2 percent
Decreasing returns in innovation θ	0.35	Imposed
Industrial energy price $p_e(t)$	\$12 per million BTU	MECS data
Depreciation rates electricity $\delta_e(f), \delta_e(g)$	0.03, 0.05	-
Arrival rate of replacement opportunity λ	1.0	Normalized
Size of machine $\bar{k}$	1.0	Normalized

Internally calibrated parameters. Internally calibrated parameters. I calibrate labor productivity A_L to match the value of world GDP reported by the World Bank (\$66 trillion).[19]

The mass of firms N_e, N_m in electricity and manufacturing is chosen to match sectoral private invest-

[19] While all other parameters are taken from U.S. data, I match world rather than U.S. GDP so that the social planner internalizes global damages from CO_2 emissions.

ment into fixed assets reported in BEA data (2.4% and 0.6% of GDP for manufacturing and the power sector, respectively). I assume an equal number of firms in each subsector. Physical depreciation rates are chosen to match the perpetual-inventory implied rates used in the reduced-form analysis based on NBER manufacturing data. I match the observed sectoral depreciation rates of 5.8% and 8.3%. Since empirical depreciation rates are inflated by firms discarding capital before full depreciation (obsolescence), physical depreciation rates are lower. I apply the same depreciation rates to clean and dirty capital in the electricity sector, close to the calibration in Gilchrist et al. (2024) who set them to 5 and 10 percent, respectively.

I assume shocks to the cost of replacing a machine follow a logistic distribution. I identify the scale parameter of the distribution by matching the cross-sector DiD effect of a 1973-sized permanent energy price shock on equilibrium energy efficiency. Specifically, I treat the DiD effect in Figure 3a as reflecting the differential impact of energy on average energy efficiency 7 years into the transition after a permanent 250% increase in energy prices. The DiD identifies the dispersion within this model because with low variance, the decline in investment is amplified which lowers the energy efficiency response.

The exogenous component of R&D productivity γ is set to match sectoral data on energy expenditures relative to value added in the energy and manufacturing sectors. With lower R&D productivity, energy efficiency falls, raising the energy share. Electricity productivity ν is chosen to match the GDP share of electricity (1.5%). Clean energy productivity A_G is calibrated so that the model matches a clean energy share of 10% based on 2010 data for the U.S.

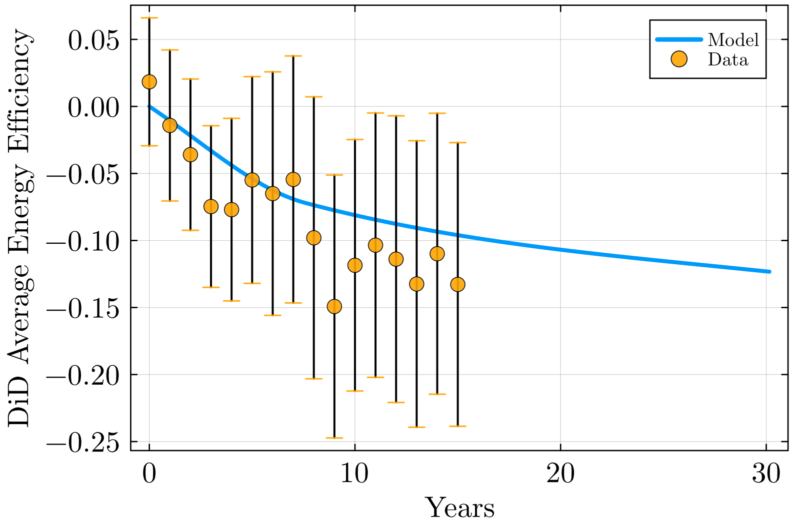
3.3 Validation

The model also matches key untargeted moments and transitional dynamics. I test the model with a permanent 250% energy-price increase, mirroring the sustained increase of the early 1970s. I then run the same difference-in-differences event study as in Figure 3 on the simulated data and compare model predictions to data.[20] Figure 5 reports the result. By construction, model and data line up 7 years into the transition. More important, the model also reproduces the differential adjustment path between low and high depreciation industries.

The model's aggregate innovation response also fits external evidence. Popp (2002) estimates a long-run elasticity of clean patents to energy prices of about 0.35, similar to Acemoglu et al. (2023). Dugoua and Gerarden (2025) find larger elasticities, 0.5-0.6. I map these facts to the model using two shocks. With a 30% price increase, R&D labor rises 16.3% (elasticity 0.54). With a 250% increase, R&D labor rises 68.4% (elasticity 0.27). The lower elasticity induced by large shocks reflects stronger declines in investment, muting innovation incentives. This non-constant elasticity matches the reduced-form literature. Popp (2002) exploits the large energy price swings of the 1970s–1980s and omits time fixed effects, so his estimates also capture the downturn-driven drop in patenting. By contrast, Dugoua and Gerarden (2025)

[20] I multiply event study coefficients in Figure 3 by the difference in depreciation rates between low and high depreciation sectors implied by my calibration.

Figure 5: Differential Response Of Energy efficiency: Model Versus Data



Notes. This figure compares the response of average energy efficiency in the model to the event study estimates in Figure 3. The model counterfactual corresponds to a permanent 250 percent increase in the exogenous price of energy $p_e(t)$. Empirical event study coefficients are scaled by the difference in depreciation rates between low and high depreciation sectors. The y-axis corresponds to the percent increase in average energy efficiency in low relative to high depreciation industries. The comparison extends to 15 years after the energy price increase.

relies on cross-country variation and smaller swings, yielding larger elasticity estimates.

Taking stock, my model assumes that energy and capital are perfect complements at the level of individual machines. Short-run energy use responds little, consistent with macro data. Over time, adoption of new machinery raises energy efficiency gradually. The model quantifies differences in transition speed driven by depreciation and fits well identified evidence on the response of clean technology to energy prices. By matching these important moments, I can credibly evaluate how slow adoption shapes the clean transition.

4 Quantitative Analysis

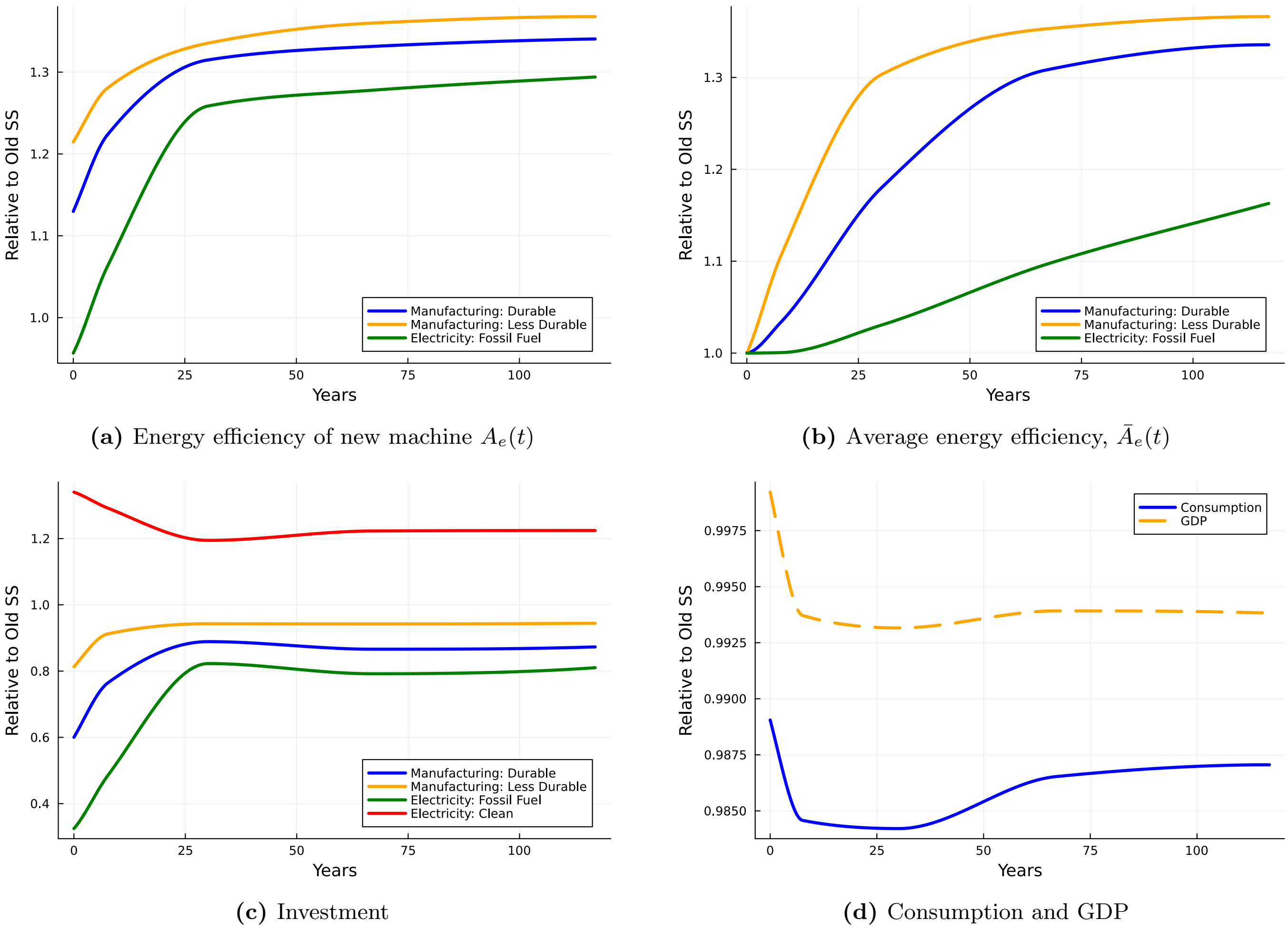
4.1 Aggregate Effects of Energy Price Shocks

I start by analyzing the effects of a permanent, 250% increase in the price of energy. Energy price swings of this magnitude were observed in the 1970s (cf. Figure 2) or among European countries after the Ukraine war.[21] Figure 6 presents the transitional dynamics resulting from this shock. All variables are expressed relative to their pre-shock steady state value. Results on the effects of energy price shocks abstract from climate-economy feedback.[22]

Panels D11a and D11b show the evolution of $A_e(t)$, the energy efficiency embodied in new machines, and the average energy efficiency embodied in machines active in the market $\bar{A}_e(t) = \int_0^{\bar{k}} A(k)\mu(k,t)dk$.

[21] See data on industrial energy prices available from `https://www.gov.uk/government/statistical-data-sets/international-industrial-energy-prices`.

[22] This assumption matters for longer-run outcomes because I calibrate the model to the world economy, so the feedback from low emissions to damages is non-trivial.



(a) Energy efficiency of new machine $A_e(t)$

(b) Average energy efficiency, $\bar{A}_e(t)$

(c) Investment

(d) Consumption and GDP

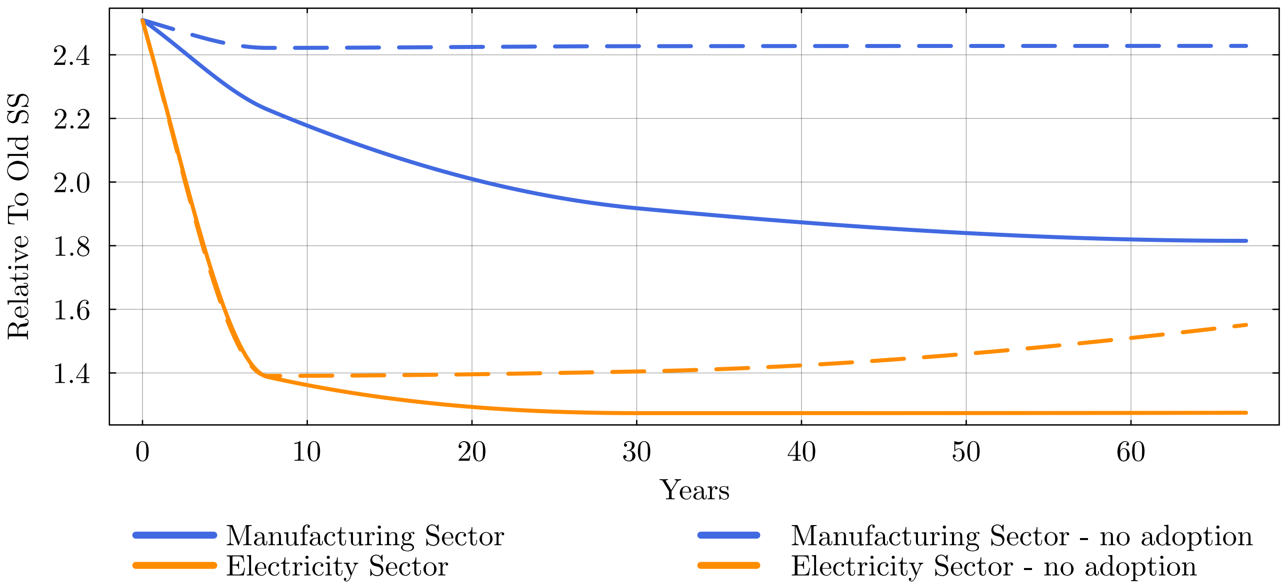
Figure 6: Transition Dynamics After Permanent 250 Percent Increase In The Price Of Energy.

Notes. This figure presents simulated responses of sectoral and macroeconomic aggregates in response to a permanent 250 percent increase in the exogenous price of energy $p_e(t)$. The y-axis measures responses relative to the baseline value of the variable prior to the shock. The simulations do not account for feedback from emissions to climate damages.

Energy efficiency of new machines increases relatively quickly for both manufacturing industries. The response of fuel efficiency in electricity generation is smaller quantitatively and takes more time to materialize. These dynamics are driven by gradually rising R&D productivity via the law of motion for $\phi(t)$ in equation (26). Additionally, Figure D11c shows sharp initial declines in investment in fossil-fuel-using sectors. The initially low investment rates depress R&D incentives and even lead to a small decline in energy efficiency in the fossil fuel sector. The collapse in fossil fuel electricity is particularly pronounced because clean-energy producers not directly affected can gain market share after the shock.[23] Improvements in $A_e(t)$ translate into a drawn-out increase in average efficiency $\bar{A}_e(t)$ (Figure D11b). The differential effect between durable and less durable manufacturing mirrors the event study results shown in Figure 5. The high depreciation sector has higher investment rates and exhibits faster convergence rates. The prolonged collapse in investment into fossil fuel using electricity leads to a very slow transition of energy efficiency

[23]Further contributing to the dynamics of investment is the low initial profitability of investment. As the economy transitions to a lower capital stock, capital goods prices increase along the transition. With profitability increasing, firms delay their investment.

Figure 7: Fuel Shares After Permanent Increase In Energy Price



Notes. The figure plots the simulated dynamics of the fuel cost share in manufacturing and electricity production after a 250 percent permanent increase in the price of energy. The y-axis shows the value of the fuel cost share relative to its pre-shock value. The cost share increases by 2.5 on impact. The dotted lines show fuel cost shares in the respective sectors if energy efficiency is held fixed at its pre-shock value.

in that sector. Turning to macroeconomic aggregates, Panel D11d shows that both consumption and output decline after the shock. The decline in consumption is more pronounced initially as higher energy prices force consumers to cut back immediately.[24] As investment rises again, consumption further drops, and then recovers slightly as the response of technology partially undoes the effect on energy spending. Output is initially almost unaffected because capital is fixed in the short term and there is no effect on utilization in my model. The fall in investment then leads to a fall in output over time. Effects on output and consumption are relatively moderate given that the fuel cost share in my calibrated model is only about 1.7%. The model omits energy spending on transport and buildings, which makes up a significant share of aggregate energy spending.

While capital and energy are perfect complements at the level of individual machines, adoption of more energy efficient varieties makes capital and energy substitutable over time. To illustrate this effect, Figure 7 plots fuel cost shares for the manufacturing and electricity sector over time. As energy prices increase by a factor of 2.5, the energy share increases by the same factor initially, reflecting perfect complementarity between capital and energy in the short run. In the manufacturing sector, the fuel share then slowly declines over time as firms adopt clean capital. Eventually, the fuel cost share converges to around 1.8 times its pre-shock level. The implied long-run elasticity of substitution is thus about 0.3.[25] In the power generation sector, the transition is more rapid because renewables expand quickly and fossil fuel generation

[24]The energy price driven decline in consumption can provide substantial amplification in demand determined economies (Auclert, Monnery, Rognlie, and Straub, 2023; Känzig, 2023).

[25]Suppose, in the long-run, sectoral production is given by $F(K,E) = \left(K^{\frac{\sigma-1}{\sigma}} + E^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$. Normalizing the price of K to

collapses. The implied long-run elasticity of substitution is 0.35. The dotted lines show what happens if technology is held fixed. In that case, the manufacturing energy cost share barely declines. The small decline is driven by substitution between sectors based on initial differences in energy efficiency. For the electricity sector, the cost share behaves very similar without adoption. Since technology adoption is very slow in that sector, fuel efficiency improvements only become more important in the long run. Without them, the long run fuel cost share would be 22 percent higher. These results show the importance of innovation in generating substitutability between capital and energy.

Since these long-run elasticities are significantly below the Cobb-Douglas benchmark frequently used in the literature, I consider robustness of my results to larger long-run elasticities. To do so, I modify the law of motion for $\phi(t)$ in equation (26) to generate a partial equilibrium, steady state elasticity of energy efficiency A_e to energy prices that is twice as large as in the baseline case. Appendix D contains details on the recalibration and Figure D11 repeats the exercise from Figure 6. The results are intuitive: Energy efficiency increases significantly more in the long-run, which translates into smaller declines in GDP, investment and consumption in the long-run. Short- to medium-run effects effects are very similar, however. Figure D12 considers how the larger effects on technology translate to energy use. The larger long-run improvements primarily manifest after around 25 years of the transition, with almost identical declines in aggregate energy use for the first 25 years. The similar short-term behavior of average energy efficiency and energy use reflects the fact that independent of the long-run elasticity, the short-run decline in investment constrains short-term energy efficiency improvements. Overall, these findings suggest that short-to-medium run transition dynamics are robust to different calibrations of the long-run elasticity.

Next, I show that changing the speed of adoption by increasing depreciation leads to faster improvements in energy efficiency and more rapid decarbonization. To establish this, I recalibrate my model so that the average implied depreciation (accounting for obsolescence) across sectors equals 0.138 or, equivalently, a half-life of 5 years.[26] I then compare energy demand and technology adoption in response to the same 250 percent increase in energy prices. Results from this exercise are shown in Figure 8.

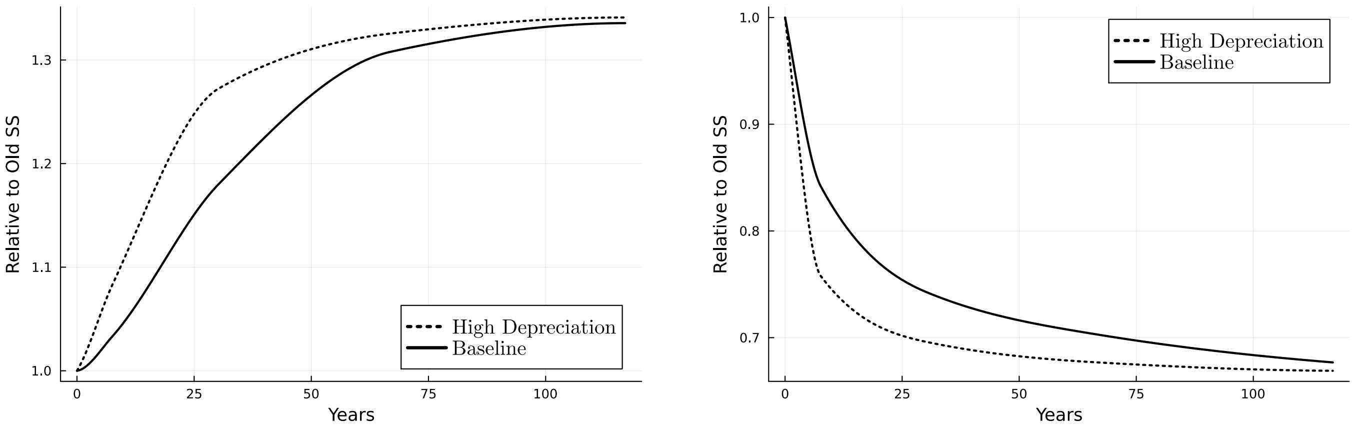
Panel 8a compares the response of average energy efficiency for the durable manufacturing sub-sector across models. The dotted line shows much faster convergence, similar to the cross-sectoral differences in convergence speed across sectors. These differences then also translate to a differential response of aggregate energy demand. In the long-run, both models respond similarly to the energy price shock. However, there are significant differences in the short to medium. Since improvements in energy efficiency only manifest themselves in delayed fashion, cumulative energy demand in the main model is 7.7% higher over the first 25 years of the transition. This finding also shows that abstracting from slow depreciation

1, the first order condition for cost minimizations can be expressed in therms of the cost share of energy s_e:

$$\frac{s_e}{1 - s_e} = p_e^{1-\sigma}.$$

The implied σ can be computed based on solving this equation in changes $\widehat{\frac{s_e}{1-s_e}} = \widehat{p_e}^{1-\sigma}$, where $\widehat{x}$ denotes the gross change.

[26]The model is recalibrated so that the higher depreciation model still matches the same data on aggregate energy demand and capital.



(a) $\bar{A}_e(t)$ in durable manufacturing subsector

(b) Aggregate Energy Demand

Figure 8: Technology and Energy Demand Comparisons

Notes. This figure compares the response of average energy efficiency and aggregate energy demand across models. The dotted line corresponds to a model with average depreciations across sectors set to 0.138, implying a half-life of 5 years. The straight line corresponds to the main model. The model counterfactual corresponds to a permanent 250 percent increase in the exogenous price of energy $p_e(t)$. The y-axis measures responses relative to the baseline value of the variable prior to the shock.

is not innocuous. Many climate-economy endogenous growth models assume full depreciation between periods by arguing that most capital is depreciated after 5 or 10 years (Acemoglu et al., 2012; Casey, 2023). In doing so, these papers drastically overstate the speed of technology adoption, with quantitatively important implications for the speed of the energy transition.

4.2 Policy Analysis

4.2.1 Optimal Policy

After illustrating the main mechanics of the model, I study the optimal climate policy. Following Nuño and Moll (2018), I set up the social planner problem as a constrained maximization problem treating the law of motion for the distribution $\mu(k,t)$ as a constraint. To economize on notation, I state the problem for a single energy-using sector. All qualitative results extend to the multisector case, and quantitative results use the full model.

The social planner maximizes consumption plus utility from replacement shocks. Since these are expressed in dollar terms in competitive equilibrium, I multiply them by the marginal utility of consumption to express them in utility terms. The utility-maximization problem is subject to four constraints: the law of motion for the climate block, the law of motion for the distribution over capital levels, a resource constraint, and a labor-market-clearing condition. I solve for the welfare-maximizing allocation which can be decentralized by using a carbon tax, output subsidy to innovating firms and an innovation subsidy. In this formulation, the planner does not internalize effects of replacement or innovation on future research productivity via the law of motion for $\phi(t)$ in equation (26). While this is done for computational reasons as the dynamic innovation subsidy is not available in closed form, this is unlikely to matter quantitatively.[27]

$$\max_{\{C(t),S(t),\mu(t),h(k,t,\epsilon),A_e(t)\}} \quad \int_0^\infty e^{-\rho t}\big[\log C(t)\big]\,dt + \lambda^C(t)\lambda\int_0^{\bar{k}}\int \mu(k,t)\,h(k,t,\epsilon)\,\epsilon\,f(\epsilon)\,d\epsilon\,dk$$

$$\text{s.t.} \qquad \lambda^C(t) : \underbrace{C(t) = \exp\!\big(-\psi[S(t)-\bar{S}]\big)Y(t) - p_e(t)\,\mathcal{E}(t) - I(t)}_{\text{Resource constraint}}, \quad \forall t$$

$$\lambda^L(t) : \underbrace{\bar{L} = L^R(t) + L^P(t)}_{\text{Labor market clearing}}, \ \forall t$$

$$\eta(t) : \underbrace{\dot{S}(t) = \big[\varphi_L + (1-\varphi_L)\varphi_0\big]\mathcal{E}(t) - \varphi(1-\varphi_L)\varphi_0\int_0^\infty e^{-\varphi u}\,\mathcal{E}(t-u)\,du = 0}_{\text{Climate dynamics}}, \quad \forall t$$

$$j(k,t) : \underbrace{\begin{aligned}\dot{\mu}(k,t) &= \frac{\partial}{\partial k}\big(\delta k\,\mu(k,t)\big) - \lambda\mu(k,t)\int h(k,t,\epsilon)\,f(\epsilon)\,d\epsilon \\ &\quad + \lambda\mathbf{1}_{k=\bar{k}}\int_0^{\bar{k}}\mu(x,t)\int h(x,t,\epsilon)\,f(\epsilon)\,d\epsilon\,dx\end{aligned}}_{\text{KFE}}, \quad \forall (k,t)\in[0,\bar{k}]\times[0,\infty)$$

$$\text{where} \ \ \mathcal{E}(t) = \int_0^{\bar{k}}\mu(k,t)\frac{k}{A(k)}\,dk \ \ \text{and} \ \ I(t) = \bar{k}\int_0^{\bar{k}}\mu(k,t)\int h(k,t,\epsilon)\,f(\epsilon)\,d\epsilon\,dk$$

$$\text{and} \ \ L^R(t) = \frac{A_e(t)^{\frac{1}{\theta}}}{\gamma\phi(t)} \ \ \text{and} \ \ Y(t) \ \ \text{as in} \ \ (5).$$

After observing replacement shocks ϵ to all firms receiving a replacement draw, the planner chooses to replace $h(\cdot) = 1$ whenever

$$h(k,\epsilon,t) = \begin{cases} 1 & \text{if } j(\bar{k},t) - j(k,t) - \lambda^C(t)\bar{k}) + \lambda^C(t)\epsilon > 0, \\ 0 & \text{otherwise.}\end{cases}$$

This replacement policy mirrors the replacement behavior of firms in the competitive equilibrium: Firms replace their capital if the private value of replacement exceeds the value of continuing with the existing machine. The planners replacement rule is characterized in terms of the social value of marginally more firms with capital k, $j(k,t)$, and the social cost of investing, $\lambda^C(t)\bar{k}$. The planner replaces whenever the marginal social value of new capital, $j(\bar{k},t) - \lambda^C(t)\bar{k}$, exceeds the marginal social value of a firm of continuing to operate with current capital $j(k,t)$. I assume the planner treats the shocks to the value of replacement $\tilde{\epsilon} = \lambda^C(t)\epsilon$ as increasing utility, but rearranging the planner problem to include them in the resource constraint shows that an interpretation in terms of shocks to the cost of replacement is equivalent.

To derive a HJB equation characterizing the value of $j(k,t)$, I algebraically manipulate the constraint on the law of motion for $\mu(k,t)$. First, we can integrate out the replacement policy to rewrite terms

[27] In particular, I compare the allocation with carbon taxes to a business as usual scenario where only the remaining inefficiencies are shut down via the appropriate policy tools. I have also quantified the effects of carbon taxes without any knowledge spillovers, whether internalized or not. While the innovation response is smaller, the effects are very similar.

involving $h(k,t)$ as $\lambda(\mathcal{J}(k,t)-j(k,t))$ where $\mathcal{J}=\mathbb{E}_\epsilon \max\{j(\bar{k},t)-\lambda^C(t)(\bar{k}-\epsilon), j(k,t)\}$. Second, we can integrate $e^{-\rho t} j(k,t)(\dot{\mu}(k,t)-\frac{\partial}{\partial k}(\delta k \mu(k,t)))$ by parts as in Nuño and Moll (2018). With these steps in place, shown in more detail in Appendix B.3, we can take the (Gateaux) derivative with respect to $\mu(k,t)$, noting the dependency of aggregate output, investment and emissions/energy on the distribution. Doing so allows us to derive the HJB equation that characterizes the the social marginal value function $j(k,t)$.

$$\rho\, j(k,t) = \underbrace{\lambda^C(t)\, k \left[e^{-\psi\,(S(t)-\bar{S})} \frac{\partial Y(t)}{\partial K(t)} - \frac{p_e(t)+\Lambda(t)}{A(k)} \right]}_{\text{Profits}} \tag{29}$$

$$+\underbrace{\partial_t j(k,t) \;-\; \delta\, k\, \partial_k j(k,t)}_{\text{Drift}} \tag{30}$$

$$+\underbrace{\lambda \left(\mathcal{J}(k,t) - j(k,t) \right)}_{\text{Replacement gain}}\;. \tag{31}$$

Analogous to the firm-level HJB for $V(k,t)$, the instantaneous social return $\rho j(k,t)$ comprises "profits" proportional to machine capital k, a drift component, and the expected gain from a replacement opportunity. The main difference from the firm HJB is that the profit component is adjusted for the social cost of carbon (SCC), $\Lambda(t)$. Additionally, the cost of replacing a machine is $\lambda^C(t)\bar{k}$ which contrasts to the competitive equilibrium where firms pay a markup over marginal cost.[28] The social cost of carbon is given by the continuous-time analogue of the formula in Golosov et al. (2014):

$$\Lambda(t) = \frac{\eta(t)}{\lambda^C(t)} = \psi \int_t^\infty e^{-\rho(\tau-t)}\, \frac{\lambda^C(\tau)}{\lambda^C(t)}\, e^{-\psi\,[S(\tau)-\bar{S}]}\, Y(\tau) \left[\varphi_L + (1-\varphi_L)\varphi_0\, e^{-\varphi(\tau-t)} \right] d\tau. \tag{32}$$

Intuitively, $\Lambda(t)$ captures present and future damages of increasing emissions today. The model assumes these damages come in the form of current and future losses in productivity. Future output $Y(\tau)$ is valued at the marginal utility of consumption $\lambda^C(\tau)$. The term in square brackets accounts for the link between emissions and future carbon concentrations. As emissions are slowly absorbed into the atmosphere, their impact on future damages falls. One important assumption in the Golosov et al. (2014) formulation of damages is that, for a constant income to consumption ratio $\lambda^C(t)Y(t)$, the SCC is is constant with respect to cumulative emissions. This means the cost of an additional ton of CO2 does not rise with atmospheric concentration. Thus, slowly declining emissions do not affect optimal tax rates in this formulation because the SCC remains constant even as atmospheric concentration $S(t)$ rises. If damages are more convex, as would be required to rationalize carbon budgets or warming thresholds, a lower elasticity of emissions to carbon taxes on the transition would also justify higher taxes.

The optimal level of energy efficiency is given by the derivative with respect to $A_e(t)$

$$-\left(\int_t^\infty \lambda^C(\tau) e^{-\rho(\tau-t)} \big(p_e(\tau)+\Lambda(\tau)\big) \frac{\partial \mathcal{E}(\tau)}{\partial A_e(t)}\, d\tau \right) = \lambda^C(t) \frac{\lambda^L(t)}{\gamma\theta\phi(t)} A_e(t)^{\frac{1}{\theta}-1}. \tag{33}$$

[28] If $\Lambda(t)=0$, the replacement decision of firms is efficient. This can be seen from setting $j(k,t)=\lambda^C(t)V(k,t)=\frac{V(k,t)}{C(t)}$, i.e. after converting the social marginal value function from utils to dollars, the HJB equations are equal.

The left hand side measures the discounted benefit of energy savings $\frac{\partial \mathcal{E}(\tau)}{\partial A_e(t)}$ in all future periods.[29] These benefits reflect lower energy imports and lower climate damages. Discounted future energy savings are equal to

$$\int_t^\infty e^{-\rho(\tau-t)} \frac{\partial \mathcal{E}(\tau)}{\partial A_e(t)} \, d\tau = -\frac{\int_t^\infty \bar{k} e^{-(\delta+\rho)(\tau-t)} \mu(\bar{k} e^{-\delta(\tau-t)}, \tau) \, d\tau}{A_e(t)^2}. \tag{34}$$

Energy savings accrue over the replacement cycle of firms that adopt the machine $\mu(\bar{k}, t)$. As capital depreciates, and firms receive new replacement draws, the share of initial adopters still using $A_e(t)$ falls. Additionally, the initial capital $\bar{k}$ depreciates physically, lowering potential energy savings as energy demand from a machine falls. Putting these together and rearranging yields

$$A_e(t) = \left[\frac{\theta \gamma \phi(t)}{\lambda^L(t)} \int_t^\infty e^{-(\rho+\delta)(\tau-t)} \ \frac{\lambda^C(\tau)}{\lambda^C(t)} \, \bar{k} \, \mu\big(e^{-\delta(\tau-t)}, \tau\big) \, \big(p_e(\tau) + \Lambda(\tau)\big) \, d\tau \right]^{\frac{\theta}{1+\theta}}. \tag{35}$$

This expression is conceptually equivalent to the energy efficiency choice of innovating firms in competitive equilibrium. In particular, Appendix B.3 shows that, in both cases, sensitivity to current and future energy prices depends on the share of firms adopting at time t and on how many firms still use the technology in future periods, as determined by the evolution of the replacement hazard.

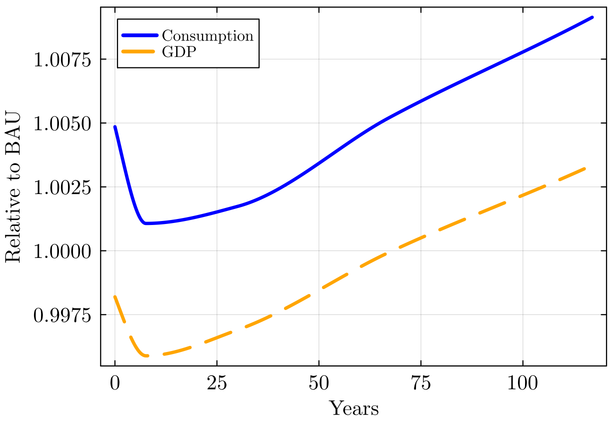
Optimal Policies Quantified. To study the quantitative effects of the optimal climate policy, I compare the economy without carbon taxes to one where I impose the optimal carbon tax from equation (32). The business-as-usual scenario is an economy in which I correct the markup distortion but impose no further tax on energy. Figure 9 presents the results.

Panel 9a shows the dynamics of GDP and consumption. Consumption initially increases as investment falls in response to a higher cost of capital, followed by a slight dip as lower investment reduces output. Eventually, abatement benefits dominate: lower climate damages raise GDP and consumption above the business-as-usual scenario without the carbon tax. The technology response in Panel 9b is similar in magnitude to that in Figure D11a following a large energy-price shock.

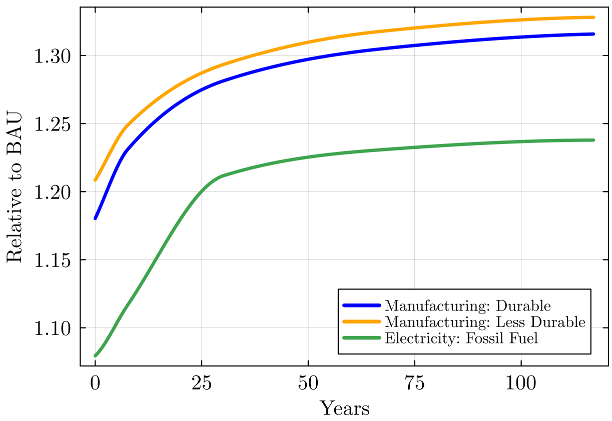
The lower two panels of Figure 9 compare carbon taxes and energy demand in the main model to models with higher and lower elasticities of substitution between capital and energy. In both the Cobb-Douglas and Leontief case, I abstract from innovation - energy efficiency remains fixed at its pre-shock value. Additionally, households own capital, and investment is reversible, as in the neoclassical growth model. These models can be seen as extending the Golosov et al. (2014) model to multiple forms of capital. All three models are calibrated to match the same level of capital and energy demand in each sector. I provide analytical and calibration details in Appendix B.4.

Across models, I find similar paths for carbon taxes, with slight growth along the transition as consumption rises. While the tax rate is similar, there are significant differences in the response of energy demand (Figure 9d). The Cobb-Douglas case predicts a drastic, immediate reduction in energy use. Transitional dynamics are relatively unimportant in that case, as energy can fall without significant re-

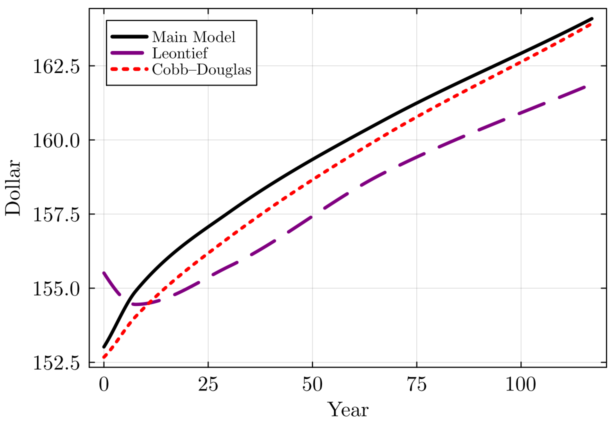
[29] I omit the energy efficiency choice of each variety ν. Given symmetry across varieties ν, the planner chooses a common level $A_e(\nu, t) = A_e(t)$.



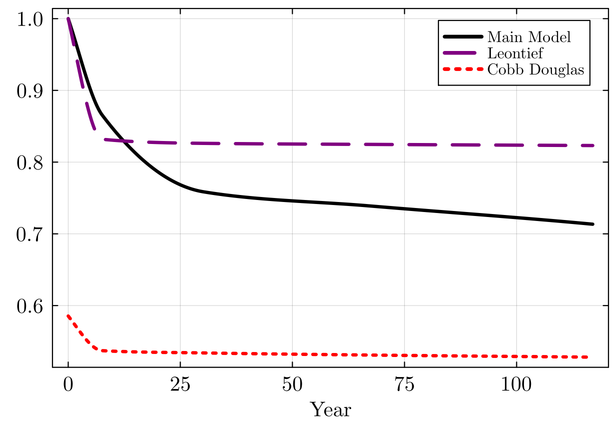
(a) Consumption & GDP



(b) $A_e(t)$



(c) Tax across models



(d) Energy demand across models

Figure 9: Optimal Climate Policy Versus Business as Usual

Notes. This figure compares the responses of sectoral and macroeconomic aggregates under the optimal carbon tax to a business-as-usual scenario. The business-as-usual scenario implements a set of production subsidies that undo the markup distortion in machinery production. The dashed and dotted lines in the lower panels refer to neoclassical growth models with fixed energy efficiency. The dotted line assumes capital and energy are combined in Cobb-Douglas fashion, while the dashed line assumes they are perfect complements.

ductions in the aggregate capital stock. The Leontief economy with zero substitutability between capital and energy has similar short-run energy dynamics as my main model. However, as technology improves, energy demand continues to fall in the model with endogenous technology, generating substantial further energy savings. The short-lived transitions in both models show that it is the interaction of adoption and incomplete depreciation that leads to drawn out transitional dynamics.[30]

If marginal damages are increasing in cumulative emissions rather than constant as in Golosov et al. (2014), optimal tax rates can be much higher. The Paris agreement goal of limiting warming to 2 degrees would for example require cutting emissions by around 60 percent by 2050. To illustrate the difference in tax required to meet such targets, I ask how high the carbon tax must be to achieve the same emissions reduction implied by the Cobb-Douglas model in Figure 9d 25 years into the transition. I find the tax required to achieve this must be nearly four times higher at around 600 dollars. Figure D13

[30]The swift transitional dynamics in the neoclassical growth model match the findings in King and Rebelo (1993).

shows energy use dynamics implied by these tax scenarios. Notably, the constant tax only achieves this emissions reductions after 25 years, with 38 percent higher cumulative energy use within the first 30 years compared to the reduction in the Cobb-Douglas model with a much lower tax. In Figure D14, I compare the investment response across the two tax scenario. Investment declines significantly more in response to the higher tax rate, illustrating the cost of large carbon taxes. These findings illustrate that if the SCC is highly convex, optimal policy requires drastically higher taxes, especially if emissions reductions are to occur soon. Figure D15 illustrates the same idea by using a damage function $\exp\left(-\psi(S(t)-\bar{S})^2\right)$ as in Campiglio et al. (2022). I recalibrate ψ to match the SCC at $t=0$ as in the main model. This calibration implies 10-15 percent higher taxes in the baseline model relative to Cobb-Douglas, and 10 percent lower relative to the zero substitutability case implied by the Leontief aggregate production function.

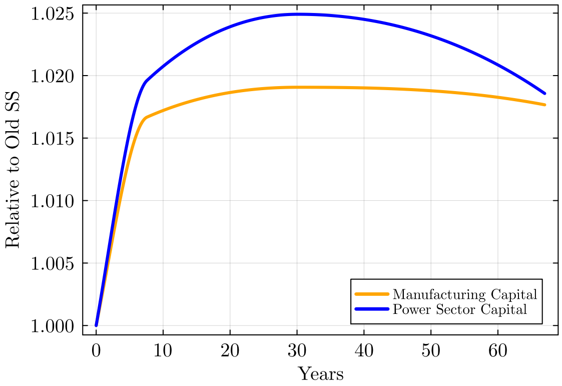
4.2.2 Replacement Subsidies

Governments regularly subsidize investment in high-durability assets. For example, bonus depreciation policies primarily lower investment costs for long-duration assets (Curtis, Garrett, Ohrn, Roberts, and Suarez-Serrato, 2023; Zwick and Mahon, 2017). By stimulating investment, market size effects may increase energy efficiency. However, such policies also have scale effects by investment into machinery. There has also been a proliferation of policies aiming to stimulate investment in machinery to lower plant-level environmental footprints. The Inflation Reduction Act set aside \$10 billion to subsidize manufacturing firm investments capable of reducing facility level carbon emissions by more than 20 percent.[31] In the EU, the German government plans to spend \$4.6 billion to support adoption of low-emissions machinery in durable manufacturing industries. To investigate the effects of these types of subsidies, I proceed as follows. First, I analyze replacement subsidies that do not differentiate between vintages, similar to bonus depreciation policies. Then, I extend the model to allow firms to choose a machine that operates on electricity and analyze subsidies for lower-emissions machinery.

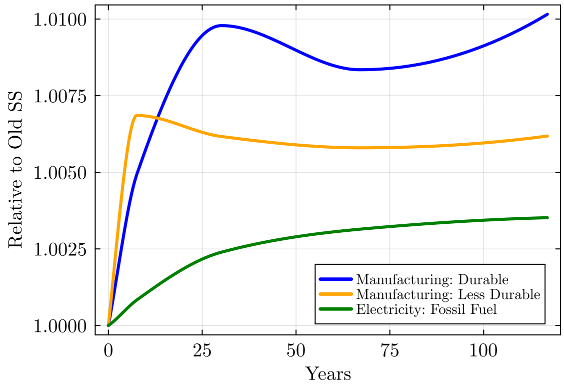
To analyze bonus depreciation policies, I assume firms receive a 5 percent subsidy to the cost of investing into a new machine, broadly in line with the fiscal cost of bonus depreciation policy passed under the Tax Cuts and Jobs Act in 2017.[32] Figure 10 displays the results. In Panel 10a, I plot the responses of sectoral capital stocks, with both manufacturing and power sector capital rising by around 2 percent. This long-run response implies a long-run elasticity of capital with respect to the user cost of capital of about 40%, in line with estimates surveyed in Chodorow-Reich (2025). Panel (10b) shows that this increase in investment leads to a small increase in average energy efficiency through a market size effect in equation (24). The net effect of the increase in scale and the improvement in efficiency is the increase in aggregate energy demand of slightly less than 1 percent shown in Panel (10c). Thus,

[31] See `https://www.irs.gov/credits-deductions/businesses/advanced-energy-project-credit` for policy details.

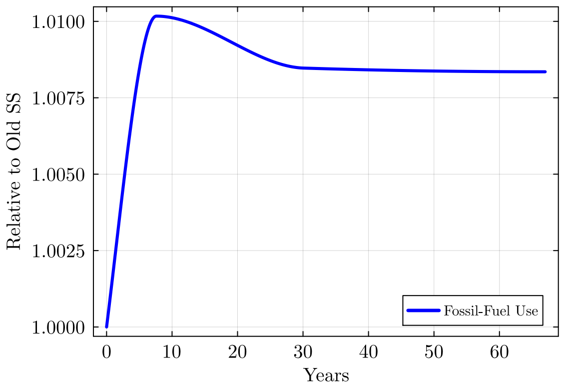
[32] Bonus depreciation has a fiscal cost over 10 years on the order of \$30 billion per year (Curtis et al., 2023). This cost estimate can be compared to annual manufacturing investment spending into equipment of around \$200 billion. Taking into account that not all bonus depreciation tax cuts go to manufacturing plants, I approximate the policy with a 5 percent subsidy that I also apply to the clean and fossil fuel power plants.



(a) $K_i(t)$



(b) $\bar{A}_e(t)$



(c) Aggregate Energy Demand

Figure 10: Technology and Energy Demand Response To Replacement Subsidies.

Notes. This figure shows the response of average energy efficiency, sectoral capital and aggregate energy demand to a permanent 5% subsidy to the cost of investing.

unconditional investment subsidies do not raise energy efficiency enough to reduce overall energy demand and emissions.

4.2.3 Electrification

Next, I study the effectiveness of carbon taxes and replacement subsidies when firms can electrify production. Electrification is central to the green transition, since improvements in renewable energy may enable advanced countries to decarbonize the power sector. However, electrification also requirees turnover of the capital stock. This makes my model a suitable laboratory to investigate whether carbon taxes and subsidies can steer firms to electrify. The baseline model abstracted from electrificaiton because U.S. manufacturing firms rely primarily on fossil fuels to meet their energy demand. MECS data indicate that about 87% of U.S. manufacturing energy demand is met by fossil fuels, with only 13% from electricity. Electrification rates are much higher in other advanced countries (Kaartinen and Prane, 2024). This discrepancy indicates the availability of alternative technologies allowing manufacturing firms to produce with electricity.[33]

To study these issues, I extend the firm-investment model to allow firms to switch between fuel types at replacement opportunities. Let V_F denote the value of a fossil fuel machine and V_E the value of an electrified machine; the system of Bellman equations becomes:

$$r(t)\,V_E(k,t) = \pi_E(k,t) \;-\; \delta k\,\partial_k V_E(k,t) \;+\; \partial_t V_E(k,t) \;+\; \lambda\big(\mathcal{V}_E(k,t) - V_E(k,t)\big), \tag{36}$$

$$r(t)\,V_F(k,t) = \pi_F(k,t) \;-\; \delta k\,\partial_k V_F(k,t) \;+\; \partial_t V_F(k,t) \;+\; \lambda\big(\mathcal{V}_F(k,t) - V_F(k,t)\big). \tag{37}$$

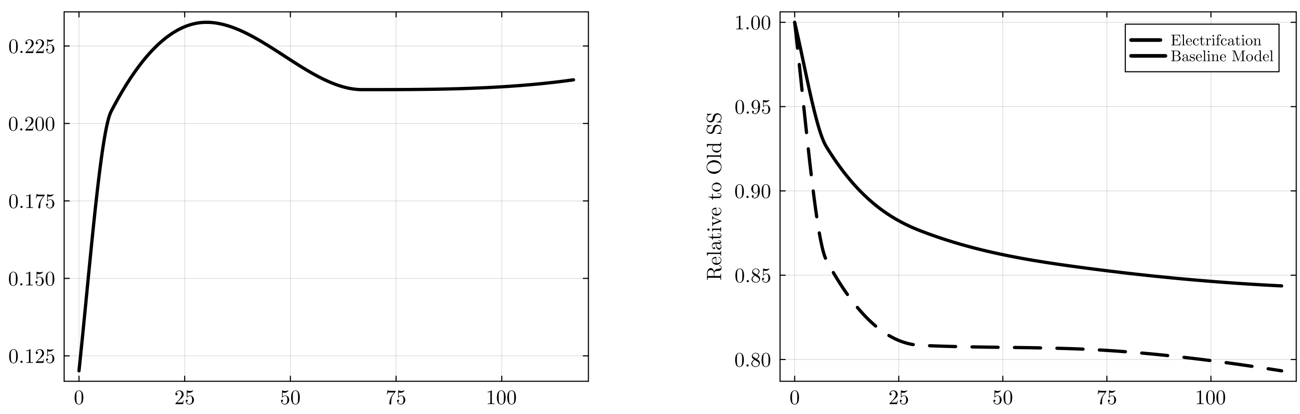
Replacement opportunities arrive at rate λ and when faced with a replacement opportunity a firm chooses between replacing with a machine using the same fuel, a machine using a different fuel or continuing to operate with the current machine. Machines differ in their purchase price, but I abstract from switching costs. Each option comes with a choice-specific shock and I assume these shocks are drawn from common distribution. The fuel-specific expected values of a replacement opportunity are

$$\mathcal{V}_F(k,t) = \mathbb{E}_\varepsilon\Big[\max\big\{V_F(\bar{k},t) - \bar{k}\,P_f(t) + \varepsilon^F_{\text{rep}},\; V_E(\bar{k},t) - \bar{k}\,P_e(t) + \varepsilon^F_{\text{sw}},\; V_F(k,t) + \varepsilon^F_{\text{keep}}\big\}\Big], \tag{38}$$

$$\mathcal{V}_E(k,t) = \mathbb{E}_\varepsilon\Big[\max\big\{V_E(\bar{k},t) - \bar{k}\,P_e(t) + \varepsilon^E_{\text{rep}},\; V_F(\bar{k},t) - \bar{k}\,P_f(t) + \varepsilon^E_{\text{sw}},\; V_E(k,t) + \varepsilon^E_{\text{keep}}\big\}\Big]. \tag{39}$$

Using the fuel-specific replacement and switching rates, we can write the law of motion for the distribution

[33] These differences in electrification rates are also apparent at the sector level. Figure A10 plot the share of energy demand coming from electricity across three-digit NAICS sectors in U.S. manufacturing. Comparing to Figure 1 in Kaartinen and Prane (2024), rates of electrification are much lower in U.S. compared to Swedish manufacturing, even within these broad sectors.



(a) Electricity Share in Manufacturing

(b) Aggregate Energy Demand

Figure 11: Electrification In Response to Carbon Taxes

Notes. This figure shows the response of the share of energy demand coming from electricity and the aggregate fossil fuel demand. The right hand panel compares fossil fuel demand in the baseline model (straight line) to the extension allowing for electrification (dashed line).

of firms as

$$\partial_t \mu_F(k,t) = \partial_k\big(\delta\, k\, \mu_F(k,t)\big) \;-\; \lambda\Big(P_F^{\text{rep}}(k,t) + P_F^{\text{sw}}(k,t)\Big)\,\mu_F(k,t), \tag{40}$$

$$\partial_t \mu_E(k,t) = \partial_k\big(\delta\, k\, \mu_E(k,t)\big) \;-\; \lambda\Big(P_E^{\text{rep}}(k,t) + P_E^{\text{sw}}(k,t)\Big)\,\mu_E(k,t). \tag{41}$$

Electrified machines combine predetermined capital with electricity. To determine profits, I assume a constant electricity price set so that electricity costs equal fossil fuel costs.[34] For simplicity, these electrified machines are assumed to be supplied by perfectly competitive producers that do not innovate. Thus, I calibrate the cost $P_e(t)$ of an electrified machine to match the share of manufacturing energy demand coming through electricity. To validate this model of fuel- switching, I rely on reduced form estimates of fuel switching in response to carbon prices from Kaartinen and Prane (2024). They find a 10 percent increase in the share of energy coming from electricity among manufacturing firms. My model predicts a long-run response of 12%, close to their event study estimates at longer horizons. To illustrate how fuel switching changes the response to carbon prices, I consider transitional dynamics in response to an ad-hoc carbon price of \$60. The results are shown in Figure 11.

Panel 11a shows that the electricity share nearly doubles. The mild overshoot is explained by the gradual improvement of energy efficiency which makes fossil fuel machines relatively more attractive later in the transition. The fuel switching response has a significant effect on the elasticity of manufacturing fossil fuel demand in response to the carbon tax increase. The straight line in Panel 11b shows fossil fuel demand over time in the baseline model. Since fuel switching occurs rapidly, fossil fuel demand falls much faster in the model with electrification. Transition dynamics are much faster with electrification because

[34] The model could accommodate richer electricity price calibrations or a general equilibrium extension where electricity is purchased from the power sector. I opt for a simpler version because manufacturing firms differ widely in the prices they pay for electricity. Capturing these complexities accurately is beyond the scope of this paper.

it allows firms to avoid the carbon tax entirely. The baseline model features a slower transition, as energy efficiency improves gradually in response to carbon taxes (Figure 9b). This gradual improvement makes immediate replacement unattractive for firms.

Finally, I consider replacement subsidies to incentivize electrification. In particular, I assume the government makes the subsidy available only to firms purchasing an electrified machine, holding the total policy cost constant. Similar to the $60 carbon tax, the subsidy doubles the share of electricity in manufacturing energy demand. Figure 12 shows how the two subsidies compare in terms of their effect on manufacturing energy demand.

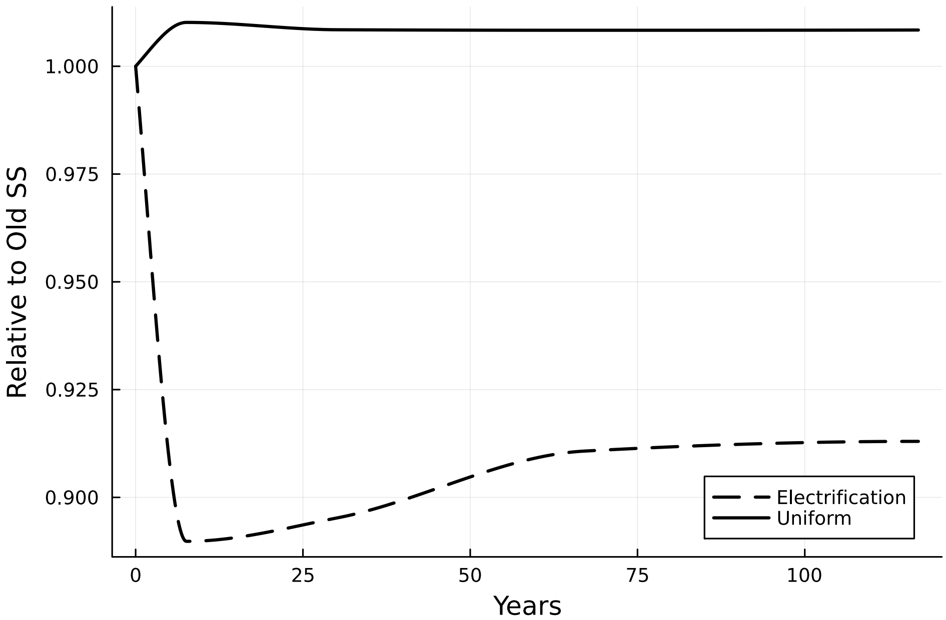


Figure 12: Fossil Fuel Demand in Response to Uniform and Electrification Subsidies

The straight line shows the increase in fossil fuel use in the baseline model as in Panel 10c. The dashed line shows that electrification lowers fossil fuel demand by around 10%. Notably, this transition occurs rapidly. The rapid transition is driven by firms anticipating growth in the capital stock. This growing capital stock leads to declining profits along the transition, making early adoption beneficial. These differential effects of subsidies amount to a 13 percent reduction in cumulative energy use over the first 25 years of the transition. These results illustrate the promise of electrification in delivering a more rapid reduction in fossil fuel use. The overall environmental implications hinge on whether the fuel mix used to supply the additional electricity is sufficiently clean. The results presented are indicative of the potential of electrification if it allows firms to fully avoid carbon taxes. If a significant fraction of electricity is produced using fossil fuels, electrification is likely to be less attractive because carbon taxes would pass through to electricity prices.

5 Conclusion

The transition towards less fossil fuel intensive production requires replacing slowly depreciating capital with lower emissions vintages. I show empirically that capital durability shapes these transition dynamics. Industries with less durable capital adjust to energy price shocks, with larger improvements in energy efficiency among high depreciation industries. These improvements in energy efficiency are accompanied by increases in green innovation, consistent with feedback between adoption and innovation. To study how this feedback affects the green transition, I calibrate a multisector IAM with vintage capital and endogenous improvements in energy augmenting technology. The complementarity between capital and energy combined with the slow adoption of new vintages make energy demand highly inelastic in the short-run. Assuming counterfactually high depreciation rates leads to much faster adoption and a greater elasticity of fossil fuel use with respect to carbon taxes. These findings illustrate that slow adoption limits the power of technology to lower emissions in the short-run. To spur adoption, policy makers frequently provide manufacturing firms with generous investment subsidies. My model captures an important benefit of such subsidies: By raising investment, these subsidies incentivize innovation. Model simulations show that such efficiency improvements are outweighed by scale effects, raising overall energy use. While these findings caution against the use of uniform subsidies, I also consider subsidies targeted at electrifying manufacturing firms. My model predicts a doubling of the share of energy demand from electricity if existing subsidies were conditioned on electrification.

Overall, this paper is part of an emerging literature emphasizing the role of capital adjustment costs for climate dynamics (Dietz et al., 2021). An open question is whether capital replacement is always efficient once carbon taxes are in place. There may be spillover effects when firms switch towards cleaner vintages. My model considers spillovers to innovation, but other forms of spillovers may render firm replacement decisions socially inefficient. An example would be lower switching costs if many firms adopt simultaneously. The existence of such spillovers would further justify policies that coordinate and accelerate the transition to clean technology.

References

Acemoglu, Daron, Philippe Aghion, Lint Barrage, and David Hémous (**2023**). Climate Change, Directed Innovation, and Energy Transition: The Long-Run Consequences of the Shale Gas Revolution. *Working Paper*

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous (**2012**). The Environment and Directed Technical Change. *American Economic Review* 102(1), pp. 131–166

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll (**2022**). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. *The Review of Economic Studies* 89(1), pp. 45–86

Aghion, Philippe, Antoine Dechezleprêtre, David Hemous, Ralf Martin, and John Van Reenen (**2016**). Carbon Taxes, Path Dependency, and Directed Technical Change: Evidence From the Auto Industry. *Journal of Political Economy* 124(1), pp. 1–51

Arcidiacono, Peter, Patrick Bayer, Jason R. Blevins, and Paul B. Ellickson (**2016**). Estimation of Dynamic Discrete Choice Models in Continuous Time With an Application to Retail Competition. *The Review of Economic Studies* 83(3), pp. 889–931

Atkeson, Andrew and Patrick J. Kehoe (**1999**). Models of Energy Use: Putty-Putty Versus Putty-Clay. *American Economic Review* 89(4), pp. 1028–1043

Auclert, Adrien, Hugo Monnery, Matthew Rognlie, and Ludwig Straub (**2023**). Managing an Energy Shock: Fiscal and Monetary Policy. *Working Paper*

Barrage, Lint and William Nordhaus (**2024**). Policies, Projections, and the Social Cost of Carbon: Results From the DICE-2023 Model. *Proceedings of the National Academy of Sciences* 121(13), p. e2312030121

Bartelme, Dominick, Arnaud Costinot, Dave Donaldson, and Andres Rodriguez-Clare (**2025**). The Textbook Case for Industrial Policy: Theory Meets Data. *Journal of Political Economy* 133(5), pp. 1527–1573

Baumeister, Christiane and James D. Hamilton (**2019**). Structural Interpretation of Vector Autoregressions With Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks. *American Economic Review* 109(5), pp. 1873–1910

Becker, Randy A., Wayne B. Gray, and Jordan Marvakov (**2021**). NBER-CES Manufacturing Industry Database (1958–2018, Version 2021a). *Database*

Benkard, C. Lanier (**2000**). Learning and Forgetting: The Dynamics of Aircraft Production.

American Economic Review 90(4), pp. 1034–1054

Bertolotti, Fabio and Andrea Lanteri (**2024**). Capital Replacement and Innovation Dynamics. *Working Paper*

Bilal, Adrien and Diego R. Känzig (**2024**). The Macroeconomic Impact of Climate Change: Global vs. Local Temperature. *Working Paper*

Bilal, Adrien and James H. Stock (**2025**). Macroeconomics and Climate Change. *Working Paper*

Bloom, Nicholas, Mark Schankerman, and John Van Reenen (**2013**). Identifying technology spillovers and product market rivalry. *Econometrica* 81(4), pp. 1347–1393

Cai, Yongyang and Thomas S Lontzek (**2019**). The social cost of carbon with economic and climate risks. *Journal of Political Economy* 127(6), pp. 2684–2734

Calel, Raphael and Antoine Dechezleprêtre (**2016**). Environmental policy and directed technological change: evidence from the European carbon market. *Review of economics and statistics* 98(1), pp. 173–191

Campiglio, Emanuele, Simon Dietz, and Frank Venmans (**2022**). Optimal Climate Policy As If the Transition Matters. *Working Paper*

Capelle, Damien, Divya Kirti, Nicola Pierri, and German Villegas Bauer (**2023**). Mitigating Climate Change at the Firm Level: Mind the Laggards. *Working Paper*

Casey, Gregory (**2023**). Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation. *Review of Economic Studies*

Chen, Qiaoyi, Zhao Chen, Zhikuo Liu, Juan Carlos Suárez Serrato, and Daniel Y. Xu (**2025**). Regulating Conglomerates: Evidence From an Energy Conservation Program in China. *American Economic Review* 115(2), pp. 408–447

Chodorow-Reich, Gabriel (**2025**). The Neoclassical Theory of Firm Investment and Taxes: A Reassessment. *Working Paper*

Clay, Karen, Akshaya Jha, Joshua A Lewis, and Edson R Severnini (**2021**). Impacts of the clean air act on the power sector from 1938-1994: Anticipation and adaptation. Technical report, National Bureau of Economic Research

Curtis, E. Mark, Daniel G. Garrett, Eric Ohrn, Kevin A Roberts, and Juan-Carlos Suarez-Serrato (**2023**). Capital Investment and Labor Demand: Evidence from 21st Century Stimulus Policy. *Working Paper*

Dalgaard, Carl-Johan and Morten Olsen (**2025**). Has the Real Rate of Return "Depreciated"? *Journal of Economic Growth* 30(1), pp. 49–85

Dietz, Simon, Frederick Van Der Ploeg, Armon Rezai, and Frank Venmans (2021). Are Economists Getting Climate Dynamics Right and Does It Matter? *Journal of the Association of Environmental and Resource Economists* 8(5), pp. 895–921

Dube, Arindrajit, Daniele Girardi, Oscar Jorda, and Alan M. Taylor (2023). A Local Projections Approach to Difference-in-Differences Event Studies. Technical report, National Bureau of Economic Research

Dugoua, Eugenie and Todd D. Gerarden (2025). Induced Innovation, Inventors, and the Energy Transition. *American Economic Review: Insights* 7(1), pp. 90–106

Folini, Doris, Aleksandra Friedl, Felix Kübler, and Simon Scheidegger (2025). The Climate in Climate Economics. *Review of Economic Studies* 92(1), pp. 299–338

Gilbert, Charles E. and Michael F. Mohr (1996). Capital Stock Estimates for Manufacturing Industries: Methods and Data. *Working Paper*

Gilchrist, Simon, Joseba Martinez, and Natalie Rickard (2024). The Green Energy Transition in a Putty-Clay Model of Capital. *Working Paper*

Goldsmith-Pinkham, Paul, Isaac Sorkin, and Henry Swift (2020). Bartik Instruments: What, When, Why, and How. *American Economic Review* 110(8), pp. 2586–2624

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014). Optimal Taxes on Fossil Fuel in General Equilibrium. *Econometrica* 82(1), pp. 41–88

Hall, Bronwyn H. (2007). Measuring the Returns to R&D: The Depreciation Problem. *Working Paper*

Hassler, John and Per Krusell (2018). Environmental Macroeconomics: The Case of Climate Change. *Handbook of Environmental Economics* 4, pp. 333–394

Hassler, John, Per Krusell, and Conny Olovsson (2021). Directed Technical Change as a Response to Natural Resource Scarcity. *Journal of Political Economy* 129(11), pp. 3039–3072

Hawkins-Pierot, Jonathan and Katherine Wagner (2023). Technology Lock-In and Costs of Delayed Climate Policy. *Working Paper*

Hémous, David, Simon Lepot, Thomas Sampson, and Julian Schärer (2023). Trade, Innovation and Optimal Patent Protection. *Working Paper*

Hochmuth, Philipp, Per Krusell, and Kurt Mitman (2025). Distributional Consequences of Becoming Climate-Neutral. *Working Paper*

Holdren, John P. (1990). Energy in Transition. *Scientific American* 263(3), pp. 156–163

Hsieh, Chang-Tai (2001). Endogenous Growth and Obsolescence. *Journal of Development*

Economics 66(1), pp. 153–171

Hulten, Charles R and Frank C Wykoff (**1981**). The estimation of economic depreciation using vintage asset prices: An application of the Box-Cox power transformation. *Journal of Econometrics* 15(3), pp. 367–396

IEA (**2020**). Aligning investment and innovation in heavy industries to accelerate the transition to net-zero emissions. *IEA Commentary*

Kaartinen, Maija and Ossian Prane (**2024**). Carbon Pricing and Fuel Switching by Firms: Theory and Evidence. *Working Paper*

Känzig, Diego R. (**2021**). The Macroeconomic Effects of Oil Supply News: Evidence From OPEC Announcements. *American Economic Review* 111(4), pp. 1092–1125

——— (**2023**). The Unequal Economic Consequences of Carbon Pricing. Technical report, National Bureau of Economic Research

Kehrig, Matthias and Nicolas Vincent (**2021**). The Micro-Level Anatomy of the Labor Share Decline. *The Quarterly Journal of Economics* 136(2), pp. 1031–1087

Kilian, Lutz and Daniel P. Murphy (**2014**). The Role of Inventories and Speculative Trading in the Global Market for Crude Oil. *Journal of Applied Econometrics* 29(3), pp. 454–478

King, Robert G. and Sergio T. Rebelo (**1993**). Transitional Dynamics and Economic Growth in the Neoclassical Model. *The American Economic Review* 83(4), pp. 908–931. URL `http://www.jstor.org/stable/2117585`

Krusell, Per (**1998**). Investment-Specific R&D and the Decline in the Relative Price of Capital. *Journal of Economic Growth* 3, pp. 131–141

Lifset, Robert (**2014**). *American Energy Policy in the 1970s*. University of Oklahoma Press

Linn, Joshua (**2008**). Energy Prices and the Adoption of Energy-Saving Technology. *The Economic Journal* 118(533), pp. 1986–2012

Lybbert, Travis J. and Nikolas J. Zolas (**2014**). Getting Patents and Economic Data to Speak to Each Other: An 'Algorithmic Links With Probabilities' Approach for Joint Analyses of Patenting and Economic Activity. *Research Policy* 43(3), pp. 530–542

Montiel Olea, José Luis and Mikkel Plagborg-Møller (**2021**). Local Projection Inference Is Simpler and More Robust Than You Think. *Econometrica* 89(4), pp. 1789–1823

Nath, Ishan B, Valerie A Ramey, and Peter J Klenow (**2025**). How much will global warming cool global growth? *Working Paper*

Nuño, Galo and Benjamin Moll (**2018**). Social Optima in Economies With Heterogeneous Agents. *Review of Economic Dynamics* 28, pp. 150–180

Oberfield, Ezra and Devesh Raval (**2021**). Micro Data and Macro Technology. *Econometrica* 89(2), pp. 703–732

Papageorgiou, Chris, Marianne Saam, and Patrick Schulte (**2017**). Substitution Between Clean and Dirty Energy Inputs: A Macroeconomic Perspective. *Review of Economics and Statistics* 99(2), pp. 281–290

Popp, David (**2002**). Induced Innovation and Energy Prices. *American Economic Review* 92(1), pp. 160–180

Ryan, Nicholas (**2018**). Energy Productivity and Energy Demand: Experimental Evidence From Indian Manufacturing Plants. *Working Paper*

Schesch, Constantin (**2024**). Pseudospectral Methods for Continuous-Time Heterogeneous-Agent Models. *Journal of Economic Dynamics and Control* 163, p. 104856

Serletis, Apostolos and John Herbert (**1999**). The message in North American energy prices. *Energy Economics* 21(5), pp. 471–483

Solow, Robert M. (**1962**). Substitution and Fixed Proportions in the Theory of Capital. *The Review of Economic Studies* 29(3), pp. 207–218

Trefethen, Lloyd N. (**2000**). *Spectral Methods in MATLAB*. SIAM

Wei, Chao (**2003**). Energy, the Stock Market, and the Putty-Clay Investment Model. *American Economic Review* 93(1), pp. 311–323

Whelan, Karl (**2002**). A Guide to US Chain Aggregated NIPA Data. *Review of Income and Wealth* 48(2), pp. 217–233

Zwick, Eric and James Mahon (**2017**). Tax policy and heterogeneous investment behavior. *American Economic Review* 107(1), pp. 217–248

A Data and Robustness Checks

A.1 Methods.

Depreciation Rates. National and industry-level economic accounts construct asset j capital stocks based on the formula $K_{j,t} = \sum_j^{S_j} \phi_{j,i} I_{j,t-1}$ where $i \in \{1, S_j\}$ denotes how many years ago the investment took place with S_j being the maximum possible service life (Gilbert and Mohr, 1996). The parameters ϕ summarize the underlying age-efficiency function: How much does an investment from year i contribute to the current capital stock j in year t? To calibrate this extended perpetual inventory method, the Federal Reserve Bank combines BLS data on mean service lifes of individual assets with probabilistic model of discard and decay of capital goods. This calibration goes back to pioneering work by Hulten and Wykoff (1981) that used prices of used equipment to determine rates of decay.

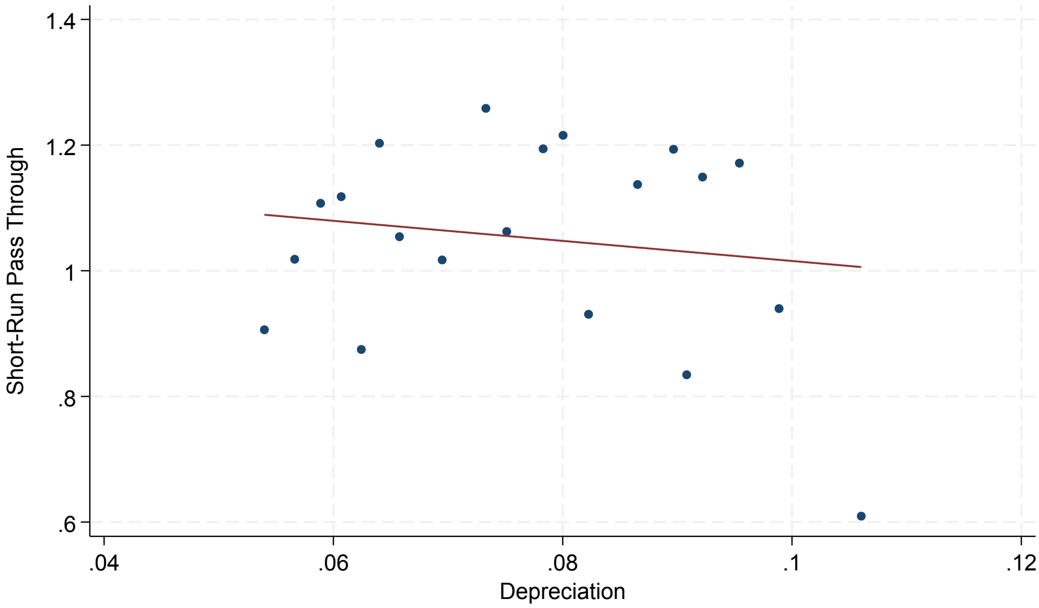
The perpetual inventory method in equation (1) simplifies this model to by assuming a constant rate of decay as capital ages. Depreciation rates obtained from inverting (1) thus partially capture changes in relative share of vintages. A boom in investment may lower depreciation since the Fed model assumes that new assets depreciate less than older assets. Since investment is highly cyclical, I average the depreciation rates across the post-1990 period to smooth out such effects.

Local Projection Implementation. To estimate the effects of oil-price shocks on industry-level outcomes I the following local projections framework

$$y_{i,t+h} - y_{i,t} = \alpha + \beta_{t+h} shock_t + \gamma_t x_t' + \epsilon_t \text{ for } h \in \{1, 2, 3, 4\}. \tag{42}$$

The outcome presents the long difference between outcome y h years after the shock relative to the year of the shock. I cumulate the oil price shocks from Baumeister and Hamilton (2019) to the annual level, giving me the shock measure $shock_t$. The vector of controls x_t includes lags of the shock and baseline values of the dependent variable to account for mean reversion. I additionally control for the lag of the dependent variable, in line with the lag-augmentation approach suggested by Montiel Olea and Plagborg-Møller (2021). Panels (a)-(d) in Figure A9 use outcomes aggregated to the the sector level, where I define a high and a low depreciation sector based on the industries being above or below the 75th percentile of depreciation rate distribution. To estimate the interaction effect, I estimate equation (42) on a panel (with an observation each the for high and low depreciation sector) and add an interaction between $shock_t$ and a dummy equal to one for the high depreciation sector. Outcomes are measured in logs.

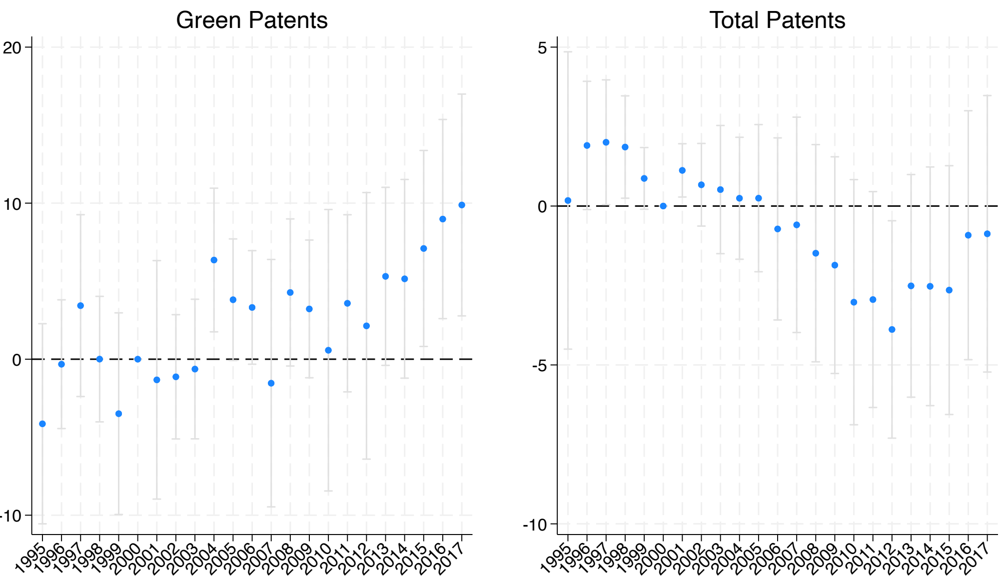
Figure A1: Short-run pass through and depreciation



Note: This Figure shows a binscatter plot of the short-run, industry-level elasticity of the cost share of energy with respect to energy prices against industry-level depreciation rates. Pass through rates are obtained by regressing the (log) energy cost share in industry value added on the (log) real price of energy. The plot shows that short-run pass-through is uncorrelated to depreciation.

A.2 Robustness Checks

Figure A2: Second energy price shock - (Log) Green and Total Patents



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the two increases in the real price of energy paid by industrial users. The dependent variable is the (log) ratio of real energy use divided by real value added. Both panels use the continuous depreciation rate measure as the exposure variable. The left hand side shows results for (log) of green patents. The right hand side shows effects for (log) total patents. Standard errors are clustered at the four-digit NAICS level.

Table A1: First Stage

	(1)
Fixed Weight log(p)	0.461***
	(5.97)
Industry F.E.	yes
Year F.E.	yes
Clusters	81
F-Statistic	35.6
Observations	2268

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This Table shows the first stage regressions results from regressing industry-level energy prices on the fixed weight energy price index used as an instrumental variable in Table 2.

Table A2: OLS results

	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Outcome	0.859***	0.855***	0.879***	0.577***	0.561***	0.537***
	(32.69)	(31.27)	(26.15)	(6.20)	(6.07)	(5.51)
log(p)	-0.0188	-0.0201	0.160*	-0.127	-0.131	-0.391**
	(-0.36)	(-0.38)	(1.97)	(-1.41)	(-1.40)	(-2.27)
log(p) x delta		-2.063**	-2.223***		10.41***	9.143***
		(-2.13)	(-2.84)		(3.31)	(2.82)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	81	81	81	81	81	81
Observations	2187	2187	2106	2187	2187	2106

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This Table shows OLS estimates of the differential effects of industry-level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the interaction of baseline energy intensity and capital intensity with industry-level energy prices. Standard errors are clustered at the four-digit NAICS level.

Table A3: OLS results: Biadic Patents

	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Outcome	0.859***	0.855***	0.879***	0.529***	0.516***	0.495***
	(32.63)	(31.28)	(26.13)	(5.90)	(5.85)	(5.35)
log(p)	-0.0160	-0.0172	0.162**	-0.127	-0.130	-0.404**
	(-0.30)	(-0.32)	(1.99)	(-1.24)	(-1.21)	(-2.08)
log(p) x delta		-2.033**	-2.173***		10.73***	9.282***
		(-2.09)	(-2.77)		(3.20)	(2.68)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	80	80	80	80	80	80
Observations	2160	2160	2080	2160	2160	2080

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: This Table shows OLS estimates of the differential effects of industry-level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of biadic green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the interaction of baseline energy intensity and capital intensity with industry-level energy prices. Effects in columns (1)-(3) differ from A2 because the sample is restricted to industries with non-zero biadic patents in all years. Standard errors are clustered at the four-digit NAICS level.

Table A4: OLS results: Alternative duration measure

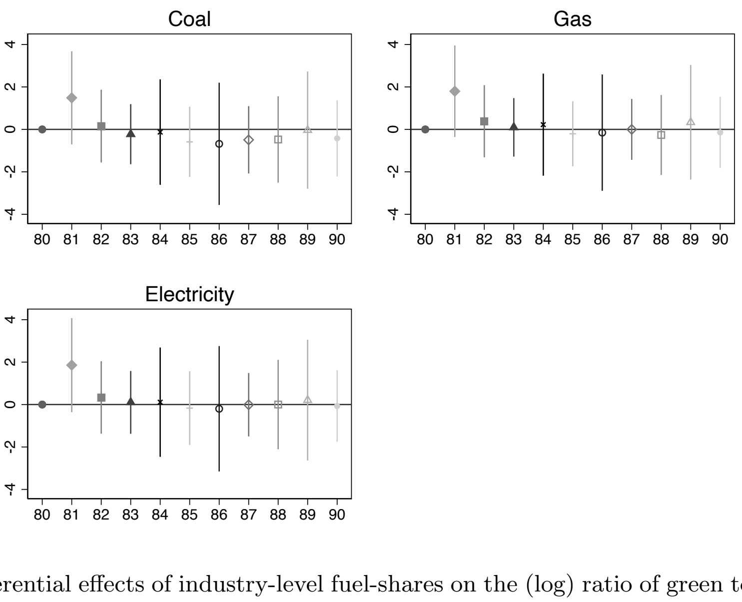
	E/VA			Share Green Patents		
	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Outcome	0.859***	0.861***	0.876***	0.579***	0.568***	0.542***
	(33.69)	(33.52)	(33.48)	(6.23)	(5.71)	(5.12)
log(p)	0.00319	-0.00736	0.156	-0.0521	-0.0554	-0.337*
	(0.07)	(-0.15)	(1.63)	(-0.65)	(-0.64)	(-1.76)
log(p) x T		0.00404	0.00950**		-0.0319**	-0.0285*
		(1.01)	(2.39)		(-2.36)	(-1.94)
Industry F.E.	yes	yes	yes	yes	yes	yes
Year F.E.	yes	yes	yes	yes	yes	yes
Controls	no	no	yes	no	no	yes
Clusters	81	72	72	81	72	72
Observations	2187	1944	1872	2187	1944	1872

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

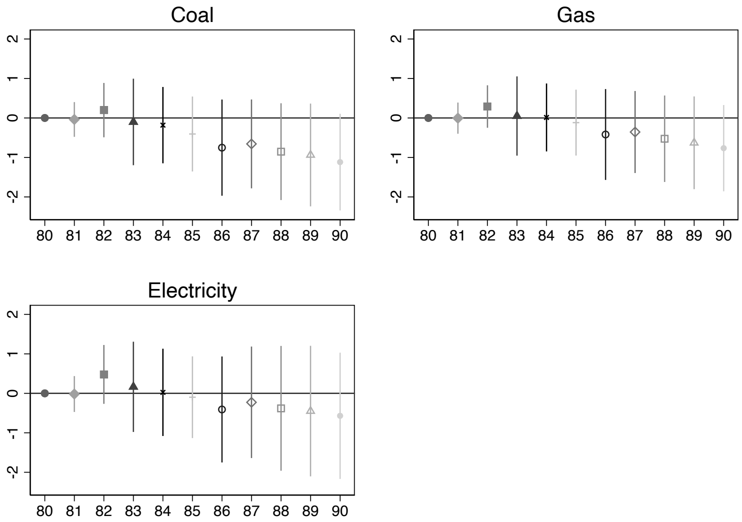
Note: This table shows OLS estimates of the differential effects by durability of industry-level energy prices on the (log) ratio of real energy use divided by real value added (Columns (1)-(3)) and the (log) share of green patents (Columns (4)-(6)). Columns (4) and (6) additionally control for the interaction of baseline energy intensity and capital intensity with industry-level energy prices. Durability is defined as the weighted average of industry-asset level service lifes reported by the BLS. Standard errors are clustered at the four-digit NAICS level.

Figure A3: Pretrends of fuel shares: patents



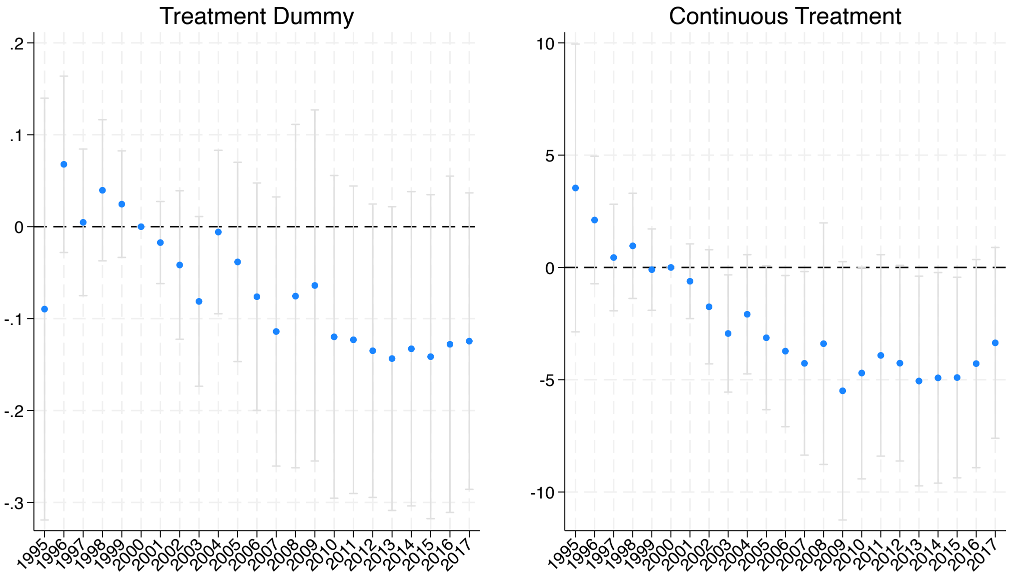
Note: This Figure shows differential effects of industry-level fuel-shares on the (log) ratio of green to total patents prior to 1990.

Figure A4: Pretrends of fuel shares: patents



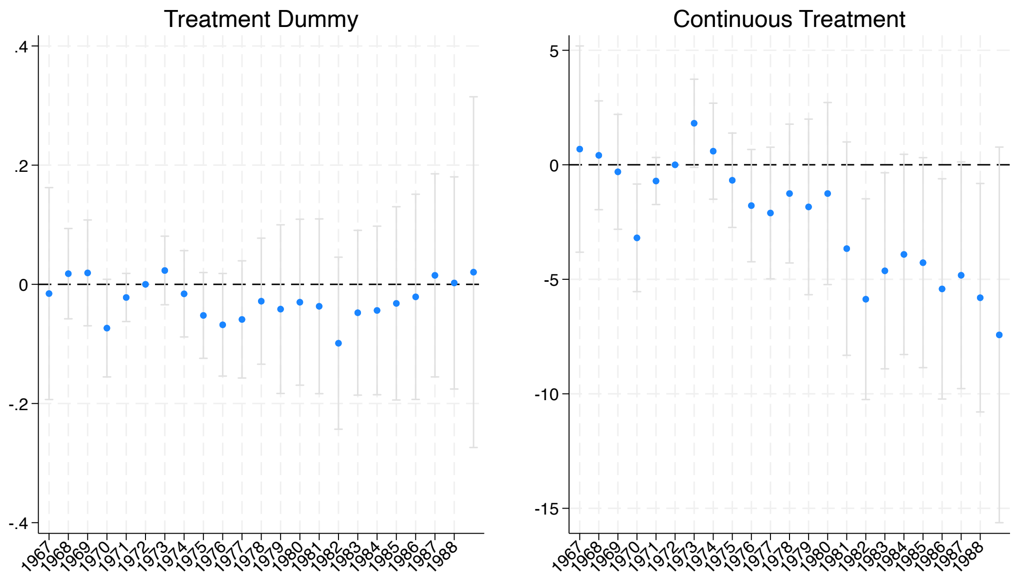
Note: This Figure shows differential effects of industry-level fuel-shares on the (log) ratio of energy to value added prior to 1990.

Figure A5: Discrete versus continuous exposure: Second energy price shock - energy intensity



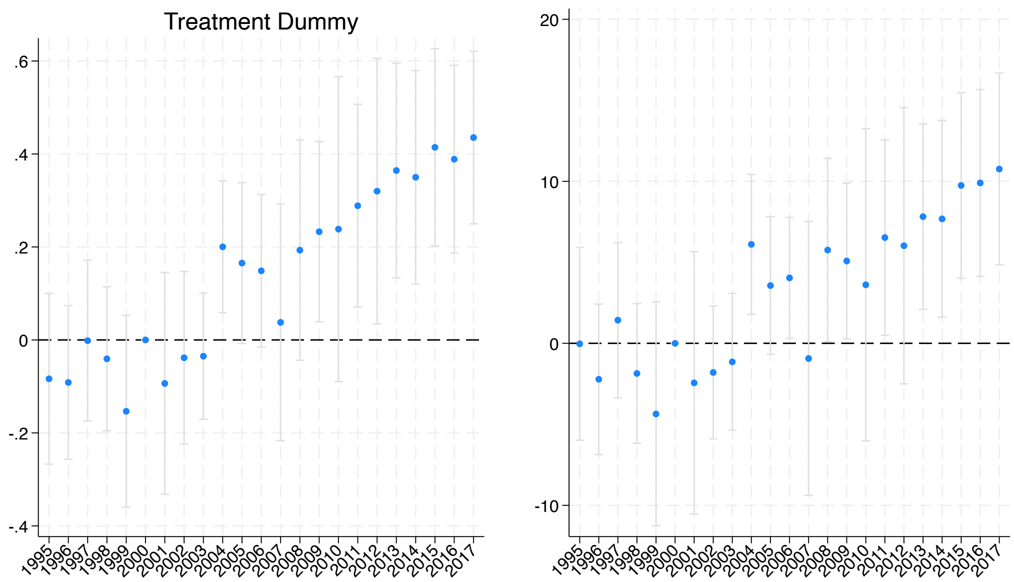
Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the two increases in the real price of energy paid by industrial users. The dependent variable is the (log) ratio of real energy use divided by real value added. The right hand side panel shows effects using the continuous depreciation rate variable as the measure of exposure. The left hand side panel uses a dummy equal to one for industries with depreciation rate above the 75th percentile of the depreciation rate distribution across industries. Standard errors are clustered at the four-digit NAICS level.

Figure A6: Discrete versus continuous exposure: First energy price shock - energy intensity



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the two increases in the real price of energy paid by industrial users. The dependent variable is the (log) ratio of real energy use divided by real value added. The right hand side shows effects using the continuous depreciation rate variable as the measure of exposure. The left hand side uses a dummy equal to one for industries with depreciation rate above the 75th percentile of the depreciation rate distribution across industries. Standard errors are clustered at the four-digit NAICS level.

Figure A7: Discrete versus continuous exposure: Second energy price shock - share of green patents



Note: This Figure plots coefficients $\{\beta_t\}_{t=-5}^{15}$ around the two increases in the real price of energy paid by industrial users. The dependent variable is the (log) ratio of green to total patents. The right hand side shows effects using the continuous depreciation rate variable as the measure of exposure. The left hand side uses a dummy equal to one for industries with depreciation rate above the 75th percentile of the depreciation rate distributiona across industries. Standard errors are clustered at the four-digit NAICS level.

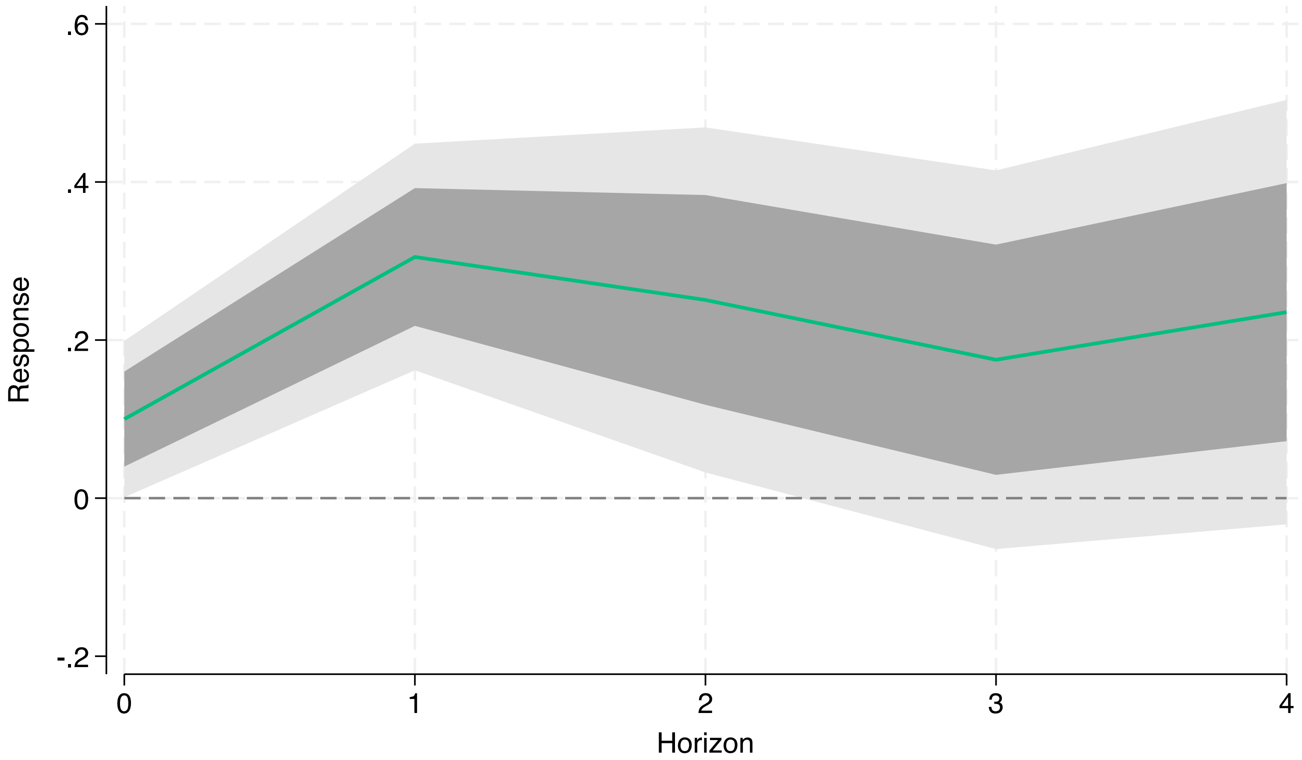
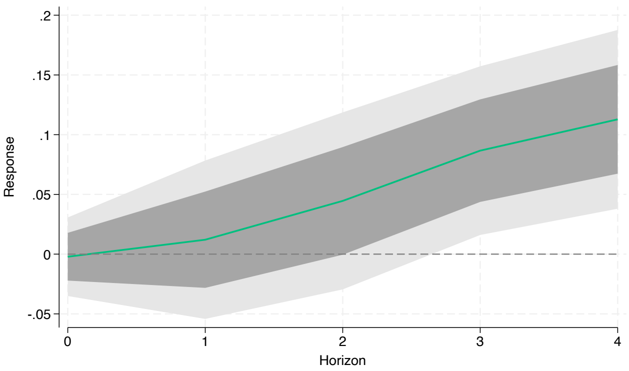
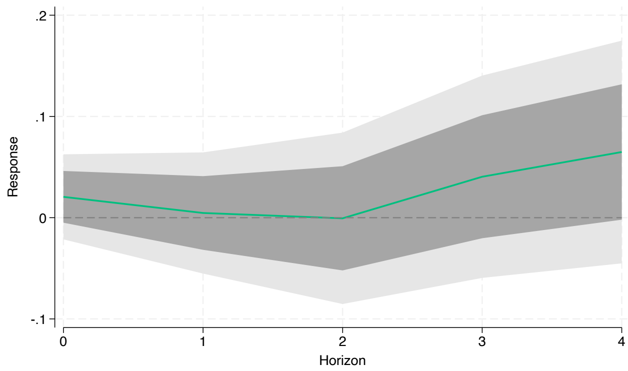


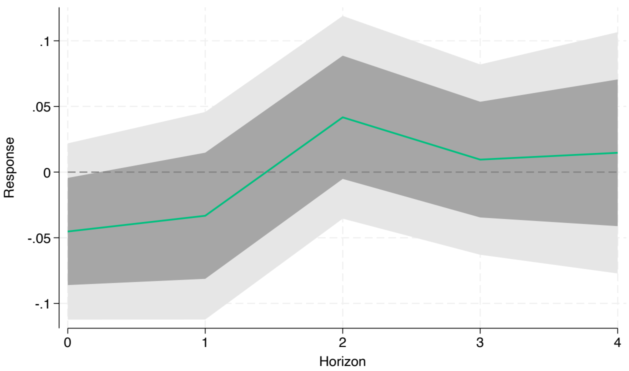
Figure A8: Impulse–response estimates from regressing industrial energy prices on oil-price shocks. Shaded areas denote 90% and 68% confidence bands. Confidence bands are based on the lag augmentation approach by Montiel Olea and Plagborg-Møller (2021). Estimates represent growth rates relative to the year of the shock. Shocks are constructed based on high-frequency identified oil supply shocks from Baumeister and Hamilton (2019), aggregated to annual frequency. Shocks are scaled scaled to increase the outcome (energy prices) by 10% on impact.



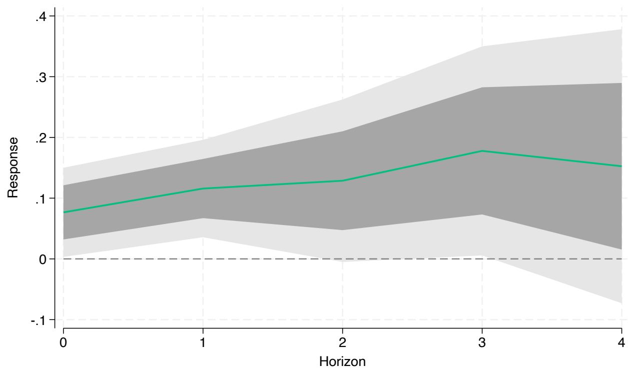
(a) Low Depreciation – Energy Intensity



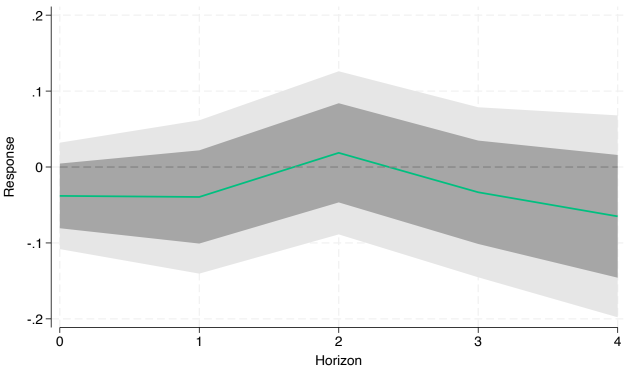
(b) Low Depreciation – Share Green



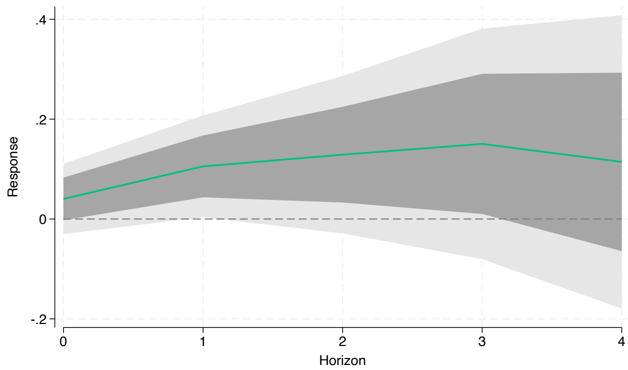
(c) High Depreciation – Energy Intensity



(d) High Depreciation – Share Green



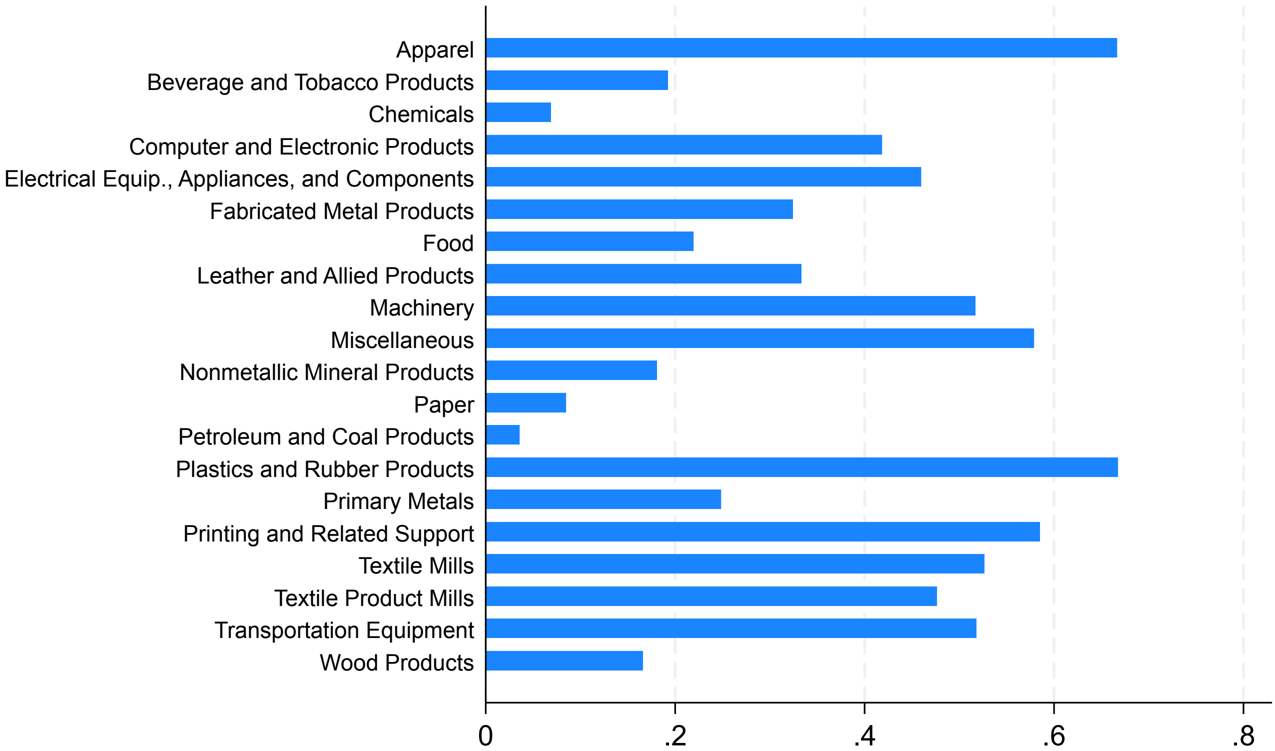
(e) Interaction – Energy Intensity



(f) Interaction – Share Green

Figure A9: Impulse–response estimates by outcome and depreciation rate. Shaded areas denote 90% and 68% confidence bands. Confidence bands are based on the lag augmentation approach by Montiel Olea and Plagborg-Møller (2021). All outcomes represent growth rates relative to the year of the shock. Shocks are constructed based on high-frequency identified oil supply shocks from Baumeister and Hamilton (2019), aggregated to annual frequency. Shocks are scaled scaled to increase the real price of energy paid by industrial users by 10% on impact.

Figure A10: Sectoral Share of Electricity in Total Energy Consumption



Note: This figure plots the distribution of the share of electricity in total energy consumption across 3-digit NAICS manufacturing sectors in the U.S. Data is from the Manufacturing Energy Cost Survey.

B Additional Derivations

B.1 Derivations HJB and KFE

This appendix provides derivations of the HJB characterizing $V(k,t)$ in equation (17) and an alternative derivation of the law of motion for the density $\mu(k,t)$ in (19). Both derivations start from discrete time analogues.

Derivation HJB. Allowing for arbitrary period length dt, the discrete time analogue of (17) is the Bellman equation

$$V(k,t) = \pi(k,t)\,dt + \big(1 - r(t)\,dt\big)\Big[(1-\lambda\,dt)\,V\big(k(1-\delta\,dt),\,t+dt\big) + \lambda\,dt\,\mathcal{V}\big(k(1-\delta\,dt),\,t+dt\big)\Big]. \tag{43}$$

Rearranging and dividing by dt yields

$$0 = \pi(k,t) + \frac{(1-r\,dt)(1-\lambda\,dt)\,V\big(k(1-\delta\,dt),t+dt\big) - V(k,t)}{dt} + (1-r\,dt)\,\lambda\,\mathcal{V}\big(k(1-\delta\,dt),t+dt\big). \tag{44}$$

Multiplying the second term out, we get

$$\begin{aligned}\frac{(1-r\,dt)(1-\lambda\,dt)\,V\big(k(1-\delta\,dt),t+dt\big) - V(k,t)}{dt} = {} & \frac{V(k,t+dt)-V(k,t)}{dt} \\ & + \frac{\big[(1-r\,dt)(1-\lambda\,dt)-1\big]\,V(k,t+dt)}{dt} \\ & + (1-r\,dt)(1-\lambda\,dt)\,\frac{V\big(k(1-\delta\,dt),t+dt\big)-V(k,t+dt)}{dt}.\end{aligned} \tag{45}$$

Taking limits of all three terms

$$\lim_{dt\to 0}\frac{V(k,t+dt)-V(k,t)}{dt} = V_t(k,t). \tag{46}$$

$$\lim_{dt\to 0}\frac{\big[(1-r\,dt)(1-\lambda\,dt)-1\big]\,V(k,t+dt)}{dt} = -(r(t)+\lambda)\,V(k,t). \tag{47}$$

$$\lim_{dt\to 0}(1-r\,dt)(1-\lambda\,dt)\,\frac{V\big(k(1-\delta\,dt),t+dt\big)-V(k,t+dt)}{dt} = -\delta k\,V_k(k,t). \tag{48}$$

Combining (46)–(48) yields

$$0 = \pi(k,t) + V_t(k,t) - (r(t)+\lambda)\,V(k,t) - \delta k\,V_k(k,t) + \lambda\,\mathcal{V}(k,t) \tag{49}$$

which can be rearranged to (17).

Derivation of distribution. Denote by $M(k,t)$ the cumulative distribution function associated with $\mu(k,t)$and write its law of motion as

$$M(k,t+dt) = \int_0^{k(1+\delta dt)} (1 - dtP^R(\tilde{k},t))\mu(\tilde{k},t)d\tilde{k}.$$

Split the integral in two and notice there are no firms between $k(1+\delta dt)$ and k so that

$$M(k,dt) = \int_0^k (1 - dtP^R(\tilde{k},t)\mu(\tilde{k},t)dk + [M(k(1+\delta dt),t) - M(k,t)]\,(1 - dtP^R(k(1+\delta dt,t)).$$

Subtract $M(k,t)$ from both sides, divide by dt and and let $dt \to 0$. Using

$$\lim_{dt\to 0} \frac{M(k(1+\delta dt,t) - M(k,t)}{dt} = \delta k\mu(k,t),$$

we obtain

$$\frac{\partial M(k,t)}{\partial t} = \delta k\mu(k,t) - \int_0^k P^R(\tilde{k},t)\mu(\tilde{k},t)d\tilde{k}.$$

Differentiating with respect to k gives equation (19).

B.2 Derivations Demand and Supply of Machines

Production Function for $E(t)$:

$$E(t) = \left[\left(K_{ef}^{\varphi} L_{ef}^{1-\varphi}\right)^{\frac{\vartheta-1}{\vartheta}} \; + \; A_G \left(K_{ef}^{\varphi} L_{ef}^{1-\varphi}\right)^{\frac{\vartheta-1}{\vartheta}}\right]^{\frac{\vartheta-1}{\vartheta}} \tag{50}$$

Define $\rho_e = \frac{1}{1-\vartheta}$, and write the optimal labor ratio as

$$r = \frac{L_{eg}}{L_{ef}} = A_G^{\frac{1}{1-\rho_e(1-\varphi)}} \left(\frac{K_{eg}(t)}{K_{ef(t)}}\right)^{\eta_e}$$

where $\eta_e = \frac{\varphi\rho_e}{1-(1-\varphi)\rho_e}$.

Defining $L_e(t) = L_{ef}(t) + L_{eg}(t)$, and using the ratio, we can write the production function as

$$E(t)^{\rho_e} \;=\; L^{\rho_e(1-\varphi)}\,(1+r)^{-\rho_e(1-\varphi)}\left(K_{ef}^{\rho_e\varphi} + r^{\rho_e(1-\varphi)}\,A_G\,K_{eg}^{\varphi\rho_e}\right) \tag{51}$$

Using the definition of the labor ratio, we can write this as

$$(1+r)^{-\rho_e(1-\varphi)} = K_{ef}^{\eta_e\rho_e(1-\varphi)}\left(K_{ef}^{\eta_e} + A_G^{\frac{1}{1-\rho_e(1-\varphi)}}\,K_{eg}^{\eta_e}\right)^{-\rho_e(1-\varphi)}$$

and

$$\left(K_{ef}^{\rho_e\varphi} + r^{\rho_e(1-\varphi)}\,A_G\,K_{eg}^{\varphi\rho_e}\right) = K_{ef}^{-\eta(1-\varphi)\rho_e}\left(K_{ef}^{\eta_e} + A_G^{\frac{1}{1-\rho_e(1-\varphi)}}\,K_{eg}^{\eta_e}\right)$$

we simplify this expression to get

$E(t) = L_e(t)^{1-\varphi}K_e(t)^{\varphi}$ where

$$K_e(t) = \left(K_{ef}^{\eta_e} + A_G^{\frac{1}{1-(1-\varphi)\rho_e}}\,K_{eg}(t)^{\eta_e}\right)^{\frac{1}{\eta_e}}. \tag{52}$$

Demand function (21). Defining $\tilde{p}(\nu,t) \;\equiv\; p(\nu,t) \;+\; \int_t^{\infty} e^{-H(t,\tau)}\frac{p_e(\tau)}{A_e(\nu,t)}\,d\tau$, and $k(t) \;\equiv$

$\left(\int_0^1 k(\nu,t)^{\frac{\sigma-1}{\sigma}}\,d\nu\right)^{\frac{\sigma}{\sigma-1}}$, maximizing (20)

$$\mathcal{L} \;=\; \int_t^\infty e^{-H(t,\tau)}\left[P^K(\tau)\,k(t) - \int_0^1 \frac{p_e(\tau)}{A_e(\nu,t)}\,k(\nu,t)\,d\nu\right]d\tau \;-\; \int_0^1 p(\nu,t)\,k(\nu,t)\,d\nu \;+\; \xi(t)\,[\,\bar{k} - k(t)\,].$$

with respect to $k(\nu,t)$ gives

$$\left(\int_t^\infty e^{-H(t,\tau)}P^K(\tau)\,d\tau - \xi(t)\right)\,k(t)^{1/\sigma}\,k(\nu,t)^{-1/\sigma} \;=\; \tilde{p}(\nu,t).$$

which can be rearranged to

$$k(\nu,t) \;=\; k(t)\left(\frac{\int_t^\infty e^{-H(t,\tau)}P^K(\tau)\,d\tau - \xi(t)}{\tilde{p}(\nu,t)}\right)^{\sigma}. \qquad (*)$$

Defining the CES price index

$$\tilde{P}(t) \;\equiv\; \left(\int_0^1 \tilde{p}(\nu,t)^{\,1-\sigma}\,d\nu\right)^{\frac{1}{1-\sigma}},$$

the lagrange multiplier on the capacity constraint $\xi(t)$ is equal to the discounted marginal product of capital net of the energy inclusive cost of new machine $\tilde{P}(\nu,t)$

$$\int_t^\infty e^{-H(t,\tau)}P^K(\tau)\,d\tau - \tilde{P}(t) = \xi(t) \;=$$

. Substituting this into the first order condition, we obtain

$$k(\nu,t) \;=\; \bar{k}\left(\frac{\tilde{p}(t)}{\tilde{P}(\nu,t)}\right)^{-\sigma}.$$

Price (23). Maximizing (22) with respect to $p(\nu,t)$ gives

$$-\sigma(p(\nu,t)-c)\tilde{p}(\nu,t)^{-\sigma-1} + \tilde{p}(\nu,t)^{-\sigma} = 0$$

Using the definition of $\tilde{p}(\nu,t)$ and solving gives (23).

Technology (24). Plugging back the optimal price (23), the firm solves

$$\max_{A(\nu,t)} \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\tilde{P}(t)^{\sigma}\left(c+\frac{\tilde{p}_e(t)}{A(\nu,t)}\right)^{1-\sigma}M(t)\bar{k} - \frac{A(\nu,t)^{\frac{1}{\theta}}}{\phi(t)}$$

The first order condition is $M(t)\bar{k}\tilde{P}(t)^{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\left(c+\frac{\tilde{p}_e(t)}{A(\nu,t)}\right)^{-\sigma}\frac{\tilde{p}_e(t)}{A(\nu,t)^2} - \frac{1}{\theta\phi(t)}A(\nu,t)^{\frac{1}{\theta}-1} = 0$. In a symmetric equilibrium $\tilde{P}(t) = \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(c+\frac{\tilde{p}_e(t)}{A_e(t)}\right)^{\sigma}$ and rearranging yields (24).

B.3 Derivations Planner Problem

Derivation of social marginal value function $j(k,t)$**:** I first collect all terms featuring $h(k,t,\epsilon)$ in the social planner problem

$$\lambda\mu(k,t)h(k,t,\epsilon)\left(j(\bar{k},t)-j(k,t)-\lambda^C(t)(\bar{k}-\epsilon)\right).$$

Substituting in the optimal replacement policy, and integrating over ϵ, we get

$$\int \lambda\mu(k,t)h(k,t,\epsilon)\left(j(\bar{k},t)-j(k,t)-\lambda^C(t)(\bar{k}-\epsilon)\right)f(\epsilon)d\epsilon = \lambda\mu(k,t)(\mathcal{J}(k,t)-j(k,t)).$$

Next, we integrate $\int_0^\infty \int_0^{\bar{k}} e^{-\rho t} j(k,t)\left(\frac{\partial\mu(k,t)}{\partial t}-\partial_k(\delta\mu(k,t))\right)dkdt$ by parts with respect to k and t. First, with respect to t, applying the transversality condition $\lim_{t\to\infty} e^{-\rho t}j(k,t)$ and dropping the initial value $\mu(k,0)$, we have

$$\int e^{-\rho t}j(k,t)\frac{\partial\mu(k,t)}{\partial t}dt = \int e^{-\rho t}\mu(k,t)(\rho j(k,t)-\partial_t j(k,t))dt.$$

Second, with respect to k, we have

$$-\int_0^\infty\int_0^{\bar{k}} e^{-\rho t}j(k,t)\partial_k(\delta\mu(k,t))dkdt = \int_0^\infty e^{-\rho t}\left[\,j(k,t)\,\delta k\,\mu(k,t)\,\right]_0^{\bar{k}}dt - \int_0^\infty e^{-\rho t}\int_0^{\bar{k}} \mu(k,t)\,\delta k\,\partial_k j(k,t)\,dk\,dt.$$

The first term drops out because $\mu(0,t)=0$ and $\mu(\bar{k},t)$ is already pinned down by the replacement policy $h(k,t)$. Thus, we can collect the terms involving $\mu(k,t)$ in the constraint on the law of motion as

$$-\mu(k,t)(\rho j(k,t)-\partial_t j_t(k,t)+\delta k\partial_k j(k,t)).$$

Since the Gateaux derivative of Y with respect $\mu(k,t)$ is equal to $\frac{\delta Y(t)}{\delta\mu}(k,t)=k\frac{\partial Y(t)}{\partial k}$, we get

$$\frac{\delta\mathcal{L}}{\delta\mu}(k,t)=\lambda^C(t)k\left(e^{-\gamma(S(t)-\bar{S})}\frac{\partial Y(t)}{\partial K(t)}-\frac{p_e(t)+\Lambda(t)}{A(k)}\right)-\rho j(k,t)+\partial_t j(k,t)-\delta k\partial_k j(k,t)+\lambda(\mathcal{J}(k,t)-j(k,t))=0,$$

which yields (29). See Definition 3 in Nuño and Moll (2018) for further details on the Gateaux derivative $\delta\mathcal{L}$.

Comparing planner choice of $A_e(t)$ **(35) to competitive outcome (24)**

Recall that energy efficiency in the competitive equilibrium outcome equals $A_e(t)=(\frac{\theta\phi(t)M(t)\bar{k}\tilde{p}_e(t)}{w(t)})^{\frac{\theta}{1+\theta}}$ where

$$M(t)\tilde{p}_e(t)=M(t)\bar{k}\int_t^\infty e^{-\delta(\tau-t)-\int_t^\tau r(u)+\lambda P^R(\bar{k}e^{-\delta(u-t)},u)du}p_e(\tau)d\tau.$$

The social planner sets $A_e(t)=A_e(t)=\left[\frac{\theta}{\lambda^L(t)}\int_t^\infty e^{-(\rho+\delta)(\tau-t)}\,\phi(t)\,\frac{\lambda^C(\tau)}{\lambda^C(t)}\,\bar{k}\,\mu\big(e^{-\delta\tau},\tau\big)\,\big(p_e(\tau)+\Lambda(\tau)\big)\,d\tau\right]^{\frac{\theta}{1+\theta}}$

To see that these are equivalent, first note

$$M(t)\,e^{-\int_t^\tau \lambda P^R(\bar{k}e^{-\delta(s-t)},s)ds}=\delta\bar{k}e^{-\delta(\tau-t)}\,\mu(\bar{k}e^{-\delta(\tau-t)},\tau),$$

i.e. the mass of firms that replaced in period t, times the probability of survival until period τ is equal to the cross-sectional share of firms with capital $\bar{k}e^{-\delta(\tau-t)}$ in period τ. Second, we can integrate the consumption Euler equation written in terms of marginal utility to obtain

$$e^{-\rho(\tau-t)}\frac{\lambda^C(\tau)}{\lambda^C(t)} = e^{\left(-\int_t^\tau r(u)\,du\right)}.$$

This shows that the planner and machine users discount future energy costs at the same rate.

B.4 Derivations and Calibration Neoclassical Growth Model

Cobb-Douglas in $K-E$. In the Cobb-Douglas case, I assume equal energy shares between low and high depreciation capital and modify the production function of $X_m(t)$ in equation (9) to

$$X_m(t) = \mathcal{E}_m(t)^{\alpha_{me}}\left((K_m(t)^{\alpha}L_m(t)^{1-\alpha}\right)^{1-\alpha_{me}}.$$

For the electricity sector, I assume that fossil fuel capital and energy are combined in Cobb-Douglas fashion. In particular, define $\tilde{K}_e(f,t) = \mathcal{E}_e(t)^{\alpha_{ee}}K_e(f,t)^{1-\alpha_{ee}}$ and replace $K_e(f,t)$ by $\tilde{K}_e(f,t)$ in equation (12). I calibrate $\{\alpha_{ee}, \alpha_{me}\} = \{0.605, 0.05\}$ to match the same sector level energy shares.

Leontief in $K-E$. In this case, we retain the same nested production structure as in the main model with energy demand equal to $\frac{K_i(t)}{A_i}$ where A_i is set equal to the steady state value implied by the calibration in Table 3.

Capital supply. Across both variants, I assume households own capital and savings are determined via the Euler equation (4). I allow for heterogeneous conversion rates from final output to capital to match the same capital stocks as in main model. Denoting by R_i the gross return to capital, no arbitrage requires

$$r(t) = \frac{R_i(t)}{c_i} - \delta_i.$$

Capital and energy demand. In the Cobb-Douglas case, capital demand is given by

$$\frac{\partial Y(t)}{\partial K_i(t)} = R_i(t).$$

Energy demand satisfies

$$\frac{\partial Y(t)}{\partial \mathcal{E}_i(t)} = p_e(t).$$

In the Leontief case, capital demand satisfies

$$\frac{\partial Y(t)}{\partial K_i(t)} = R_i(t) + x_i(t)$$

where $x_i(t) = \frac{p_e(t)}{A_i}$ for manufacturing capital and fossil fuel using electricity capital.

C Numerical Methods

In this appendix, I provide an overview of the numerical methods used to solve and calibrate the model.

Time dimension: Instead of choosing an equally spaced grid of time points and using finite differences to compute time-derivaties, I follow the spectral approach outlined in Trefethen (2000) and applied in economics by Hémous et al. (2023) and Schesch (2024). I guess all endogenous variables on a grid of 19 Chebyshev nodes and compute time derivatives using the dense differentiation matrix defined in Trefethen (2000). To compute transitional dynamics, I assume the economy has a time horizon of 1000 years and assume convergence to a new steady state after 500 years. I implement this idea by setting all time-derivatives after 500 years to zero. To account for the non-stationarity of the climate system, I hold damages fixed at the level of damages reached 400 years into the transition as is standard in the numerical computation of integrated assessment models..

Solution of value function. Given $\bar{k} = 1$, we choose an equally spaced grid points $k_{(i)} \in [0, 1]$ with 40 grid points. The drift V_k is approximated using finite differences. The drift in the time dimension is approximated by applying the Chebyshev differentiation matrix D to the value function at each point k. After these discretization steps, the value function (17) becomes a non-linear system of equations with unknows $V(k, t)$. The system can quickly be solved using a trust-region solver.

Solution of the distribution. I solve the transitional dynamics on a grid of Chebyshev nodes $t_1 \dots t_M$ and use the Chebyshev differentiation matrix D to compute time derivatives. This allows me to write the distribution $\mu(k, t)$ on the resulting two-dimensional grid as

$$D_j \mu(k_i, t) = \delta\mu(k_i, t_j) - \delta k_i \frac{\mu(k_i, t_j) + \mu(k_{i-1}, t_j)}{dk} - \lambda P^R(k_i, t_j)\mu(k_i, t_j)$$

where D_j is the jth row of the differentiation matrix D. This is a linear system in $\mu(k, t)$ that can be solved given an initial condition.

Numerical solution of the equilibrium. To solve the equilibrium, I take a guess of the following endogenous variables. The energy efficiency of the latest vintage in each sector $A_e(t)$, the discounted energy price $\tilde{p}_e(t)$, the price of capital in each sector $P^K(t)$, aggregate consumption $C(t)$ and the amount of labor in the electricity sector $L_e(t)$. The solution algorithm then proceeds as follows

1. Given the path of consumption $C(t)$ and the differentiation matrix D, we can compute the implied interest rate $r(t)$ from (4)
2. Given paths for $A_e(t)$ and $L_e(t)$, use the law of motion for research productivity (26) to compute labor demand for researchers in every period and then the labor market clearing constraint to compute labor outside of the research and the electricity sector
3. In each sector
   (a) Use guess of $\tilde{p}_e(t)$ and $A_e(t)$ to compute investment cost $P(t)$ in (25)
   (b) Use guess of $P^K(t)$ to compute profits in equation (22)
   (c) Given $r(t)$, $P(t)$, $\pi(k, A(k), t)$ solve the HJB (17) and the distribution $\mu(k, t)$ (19) numerically

(d) Compute $\tilde{p}_e(t)$ as implied by the replacement probability based on the HJB computed in the previous step

(e) Compute the implied price of capital $P^K(t)$, given the level of capital implied by $\mu(k,t)$ and guess for allocation of labor across sector

(f) Compute the implied level of energy efficiency $A_e(t)$ from equation (24)

4. Compute the implied level of consumption from the resource constraint (27)
5. Compute the implied level of $L_e(t)$ from equalization of wages across sectors

I define function residuals by comparing the guess of all endogenous variables to their implied values and feed these residuals to a numerical solver. To find a fixed-point of this system of equations, I iterate on the initial guess using Anderson Acceleration.

D Additional Quantitative Results

D.1 Larger Spillovers

To generate a larger long-run elasticity of substitution between capital and energy, I proceed as follows. Using equation (26) to solve for steady state productivity as $\phi = \frac{A_e}{\delta_A}$ and combining with equation (24), we can solve for equilibrium energy efficiency as

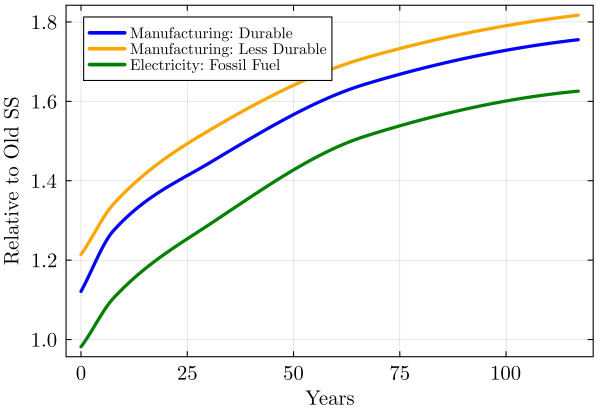
$$A_e = \left(\frac{\theta M \bar{k} \tilde{p}_e}{w\gamma} \right)^{\theta}.$$

The partial equilibrium elasticity with respect to a permanent change in energy prices is θ. To generate a larger elasticity of energy efficiency A_e with respect to energy prices or carbon taxes, I modify equation (26) so that it generates a partial equilibrium elasticity twice as large, i.e. 2θ:

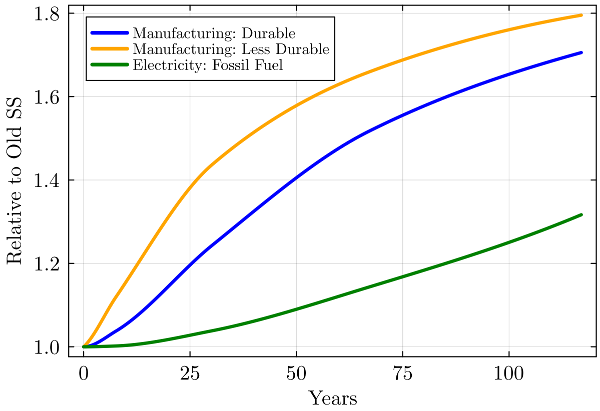
$$A_e = \left(\frac{\theta M \bar{k} \tilde{p}_e}{w\gamma} \right)^{2\theta}.$$

This requires changing the the law of motion to (26) $= -\delta_A + A_e^x$ where $x = 1 + \frac{1}{2\theta}$. I then recalibrate the model to the same target moments.

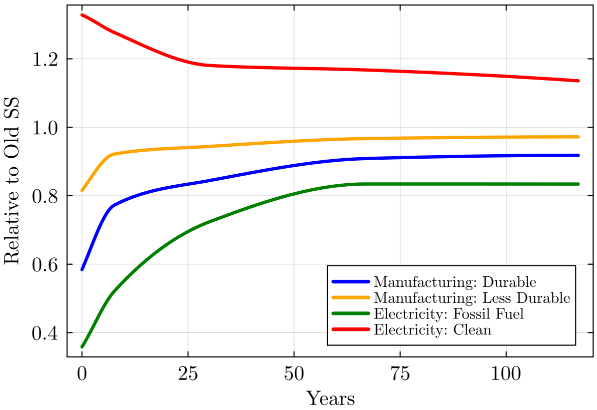
D.2 Further Results on Optimal Policy



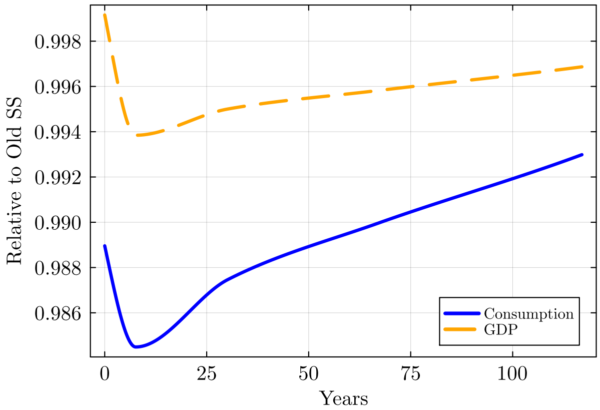
(a) Energy efficiency of new machine $A_e(t)$



(b) Average energy efficiency, $\bar{A}_e(t)$



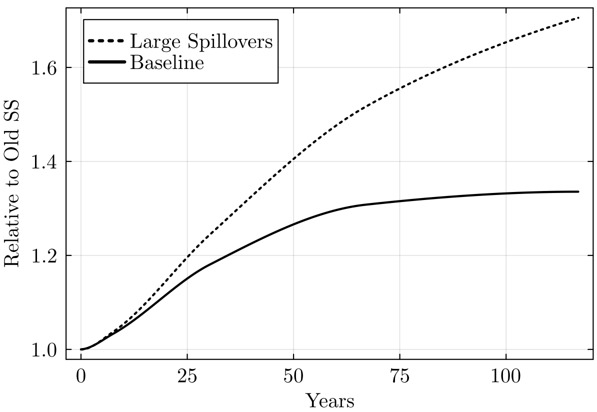
(c) Investment



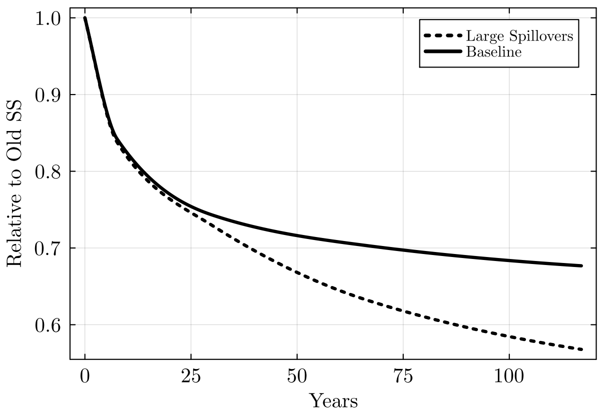
(d) Consumption and GDP

Figure D11: Transition Dynamics After Permanent 250 Percent Increase In The Price Of Energy.

Notes. This figure presents simulated responses of sectoral and macroeconomic aggregates in response to a permanent 250 percent increase in the exogenous price of energy $p_e(t)$. The y-axis measures responses relative to the baseline value of the variable prior to the shock. The simulations do not account for feedback from emissions to climate damages. Relative 6, this figure presents simulation results for larger spillovers.



(a) $\bar{A}_e(t)$ in durable manufacturing subsector



(b) Aggregate Energy Demand

Figure D12: Technology and Energy Demand Comparisons

Notes. This figure compares the response of average energy efficiency and aggregate energy demand across models. The dotted line corresponds to a model with with larger spillovers as described in Section D.

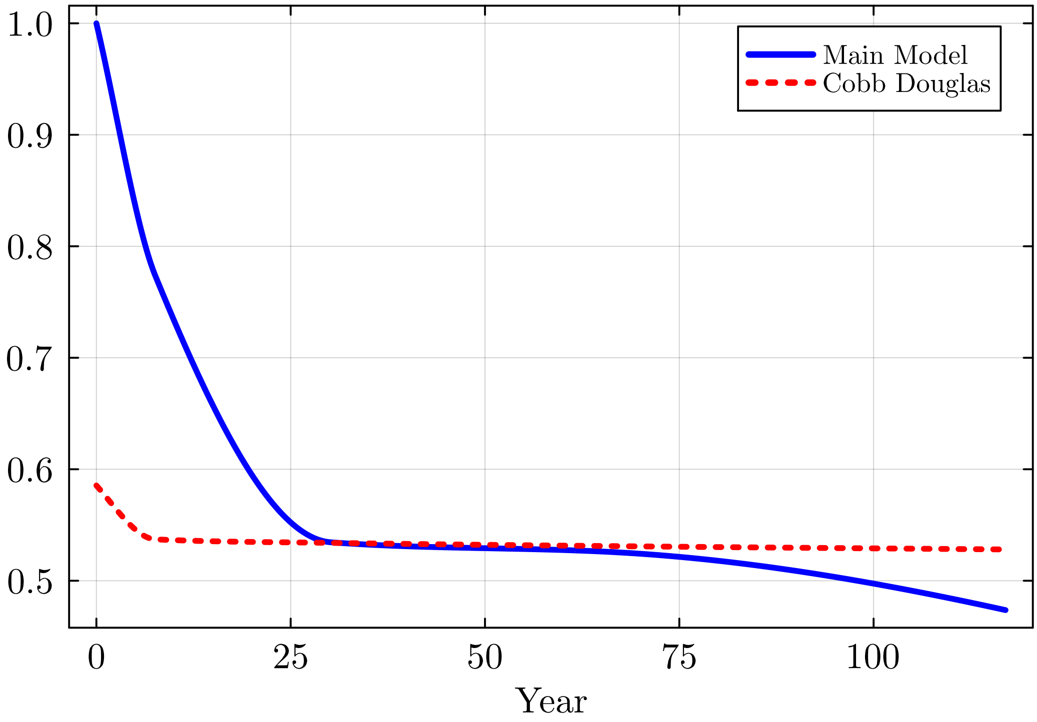
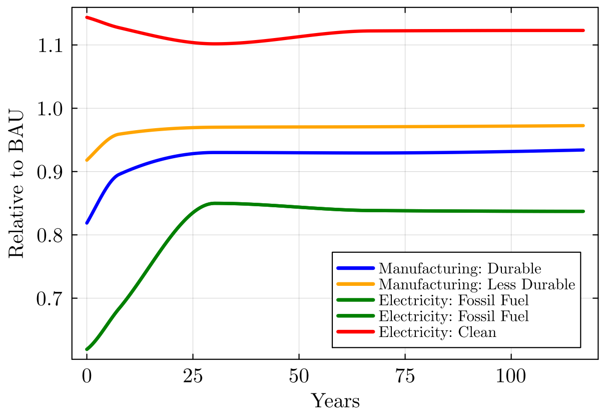
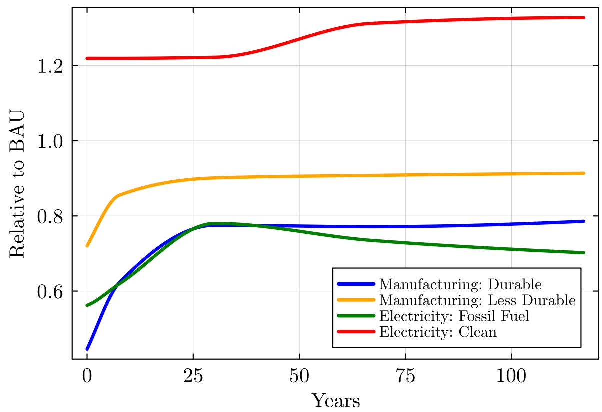


Figure D13: Energy Demand Across Models

Notes. This figure shows the response of energy demand in the Cobb-Douglas case in response to the optimal carbon tax as in Figure 9d and compares to energy demand in the main model under a constant carbon tax of $600 per ton of CO_2. The constant carbon tax is set so that the main model displays roughly the same decline in energy demand 25 years into the transition as the Cobb-Douglas case.



(a) Investment Response to Golosov et al. (2014) Tax



(b) Investment Response to $600 Tax

Figure D14: Investment esponse

Notes. The left hand side panel of this figure shows the response of investment to the optimal carbon tax for constant marginal damages of about $150 as shown by the black line in Figure 9c. The right hand side shows the response of investment to an ad-hoc tax of $600, chosen to achieve a reduction in emissions of about 45% within 25 years.

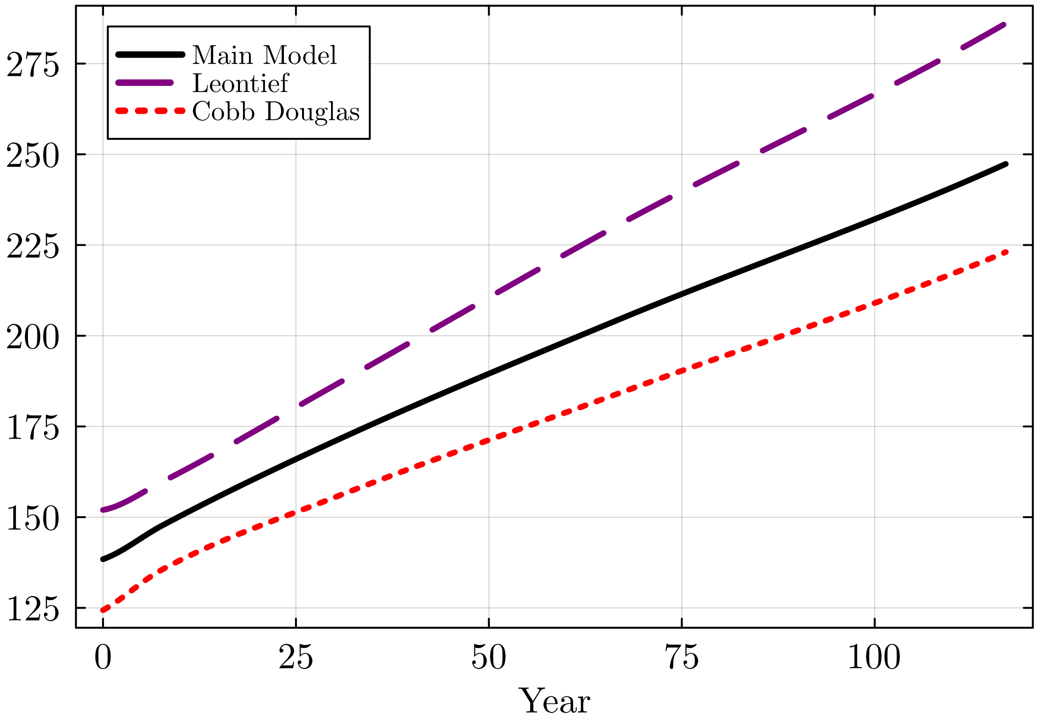


Figure D15: Carbon Taxes With Convex Damages

Notes. This figure shows optimal carbon taxes for a damage function $\exp\left(-\psi(S(t) - \bar{S})^2\right)$ where ψ is recalibrated so the SCC at $t = 0$ matches the SCC implied by my calibration of the baseline model.