

Lab 4 - Navigation with IMU and Magnetometer

Overview

In this lab, we mounted a GPS and IMU/magnetometer sensor in a car and drove around the Boston area. Our data collection path began with driving in circles in order to be able to calibrate the magnetometer in post processing, then we drove on the streets to gather more data for further analysis of behaviors and dead reckoning.

Analysis

1. Magnetometer Calibration

In order to correct the magnetometer readings for soft and hard iron effects, the data that was collected for the circular path. The expected output when plotting the magnetic field in the x direction vs the magnetic field in the y direction is a circle centered at the origin for a perfect magnetometer. Hard iron distortion is caused by materials that add a constant field to the earth's magnetic field, and therefore generate a bias in the magnetometer measurements. This causes the circle of magnetometer data to have a non-zero center. Soft iron distortion is caused by materials that influence or distort the magnetic field which vary with orientation. Soft iron distortion causes the desired circle of data to be more elliptical, and sometimes with a tilt angle.

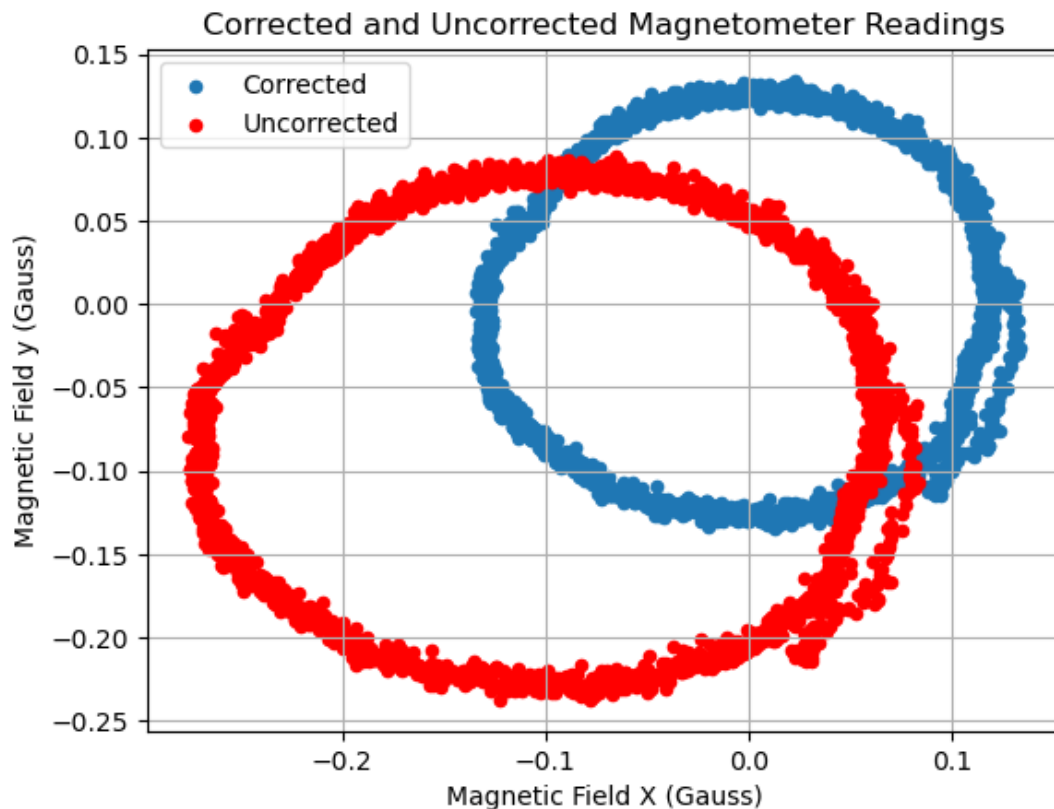


Figure 1: Uncorrected and Corrected Magnetometer Readings (Circles Path)

(Q1) The magnetometer was calibrated using a compensation model that accounts for both soft-iron and hard-iron effects. The hard-iron effects are counted for by removing the offset at which the magnetometer readings center around, and the soft iron effects map the biased ellipsoid into a circle. Very clearly from Figure 1, the hard iron distortions can be recognized as the offset is far from the origin, and slight soft iron effects can be seen as shape of the data appoints stretch outwards. When corrected we can see the difference in the magnetometer data points, as they appear to be more circular with the correction..

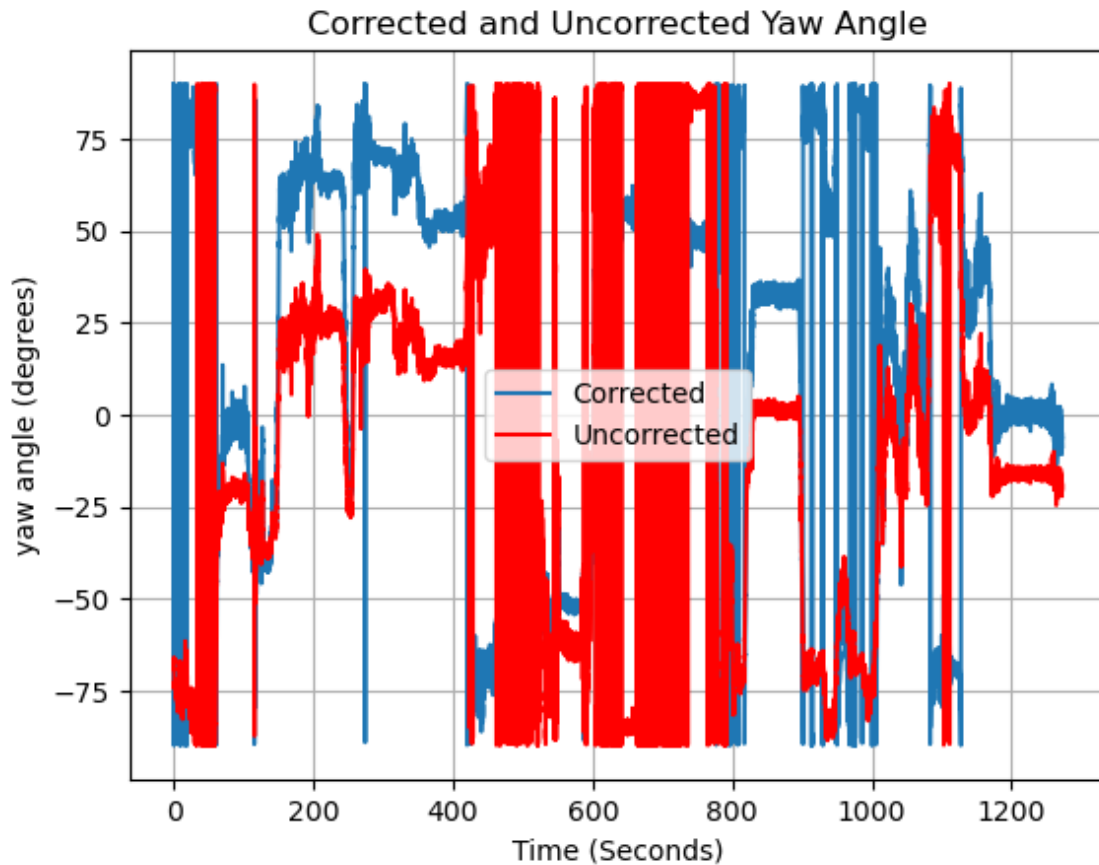


Figure 2: Magnetometer Yaw Estimate Corrected and Uncorrected

2. Sensor Fusion

In order to calculate the yaw angle from the IMU, we can integrate the gyroscope z-axis measurements using numerical integration methods. The results of integration to achieve the yaw angle can be seen compared to the magnetometer yaw in Figure 3. We can then apply the high pass filter because the IMU shows a slow drift in IMU readings over time. After filtering with a high pass filter, we preserve variations at higher frequencies and attenuate variations at lower frequencies. Then a low pass filter on the magnetometer readings to have the opposite effect.

(Q2)To make the complementary filter, we merge the outputs of two sensors with different time responses. The gyroscope has a good fast response, but long term drift. The magnetometer has good long term stability, but a poor time response. We combine the two yaw estimates by weighting each estimate. The output of the complementary filter is shown in Figure 4.

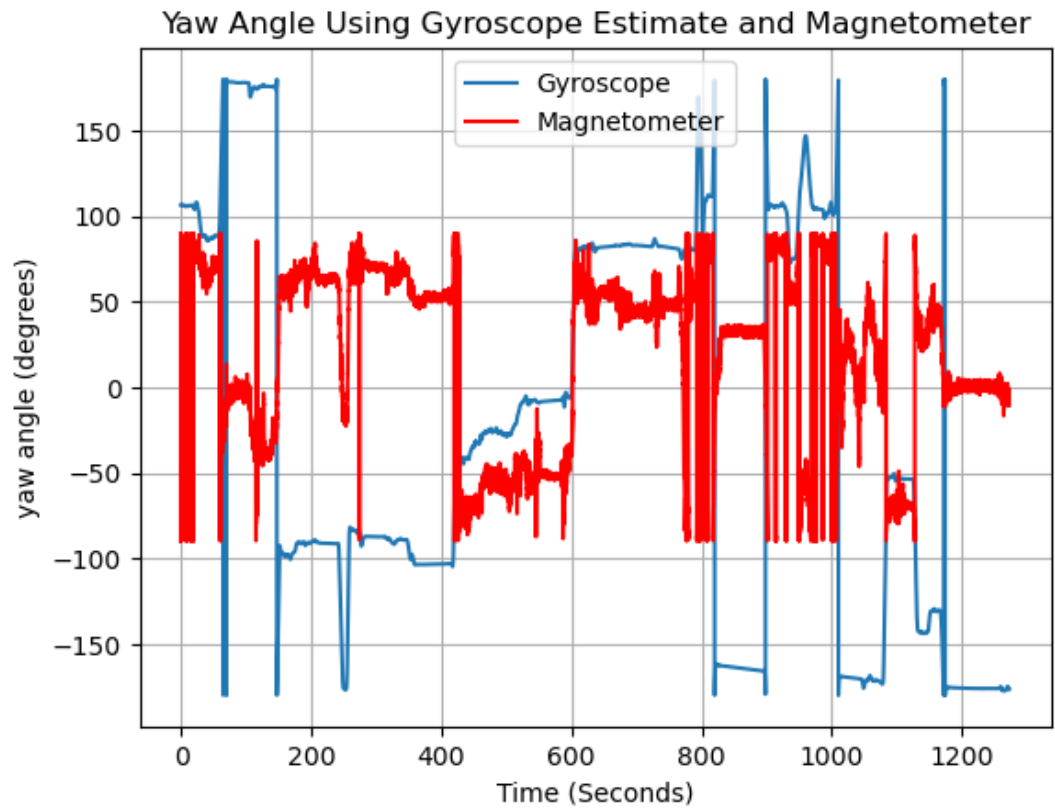


Figure 3: Yaw Angle Using Gyroscope Integration Estimate and Magnetometer Yaw

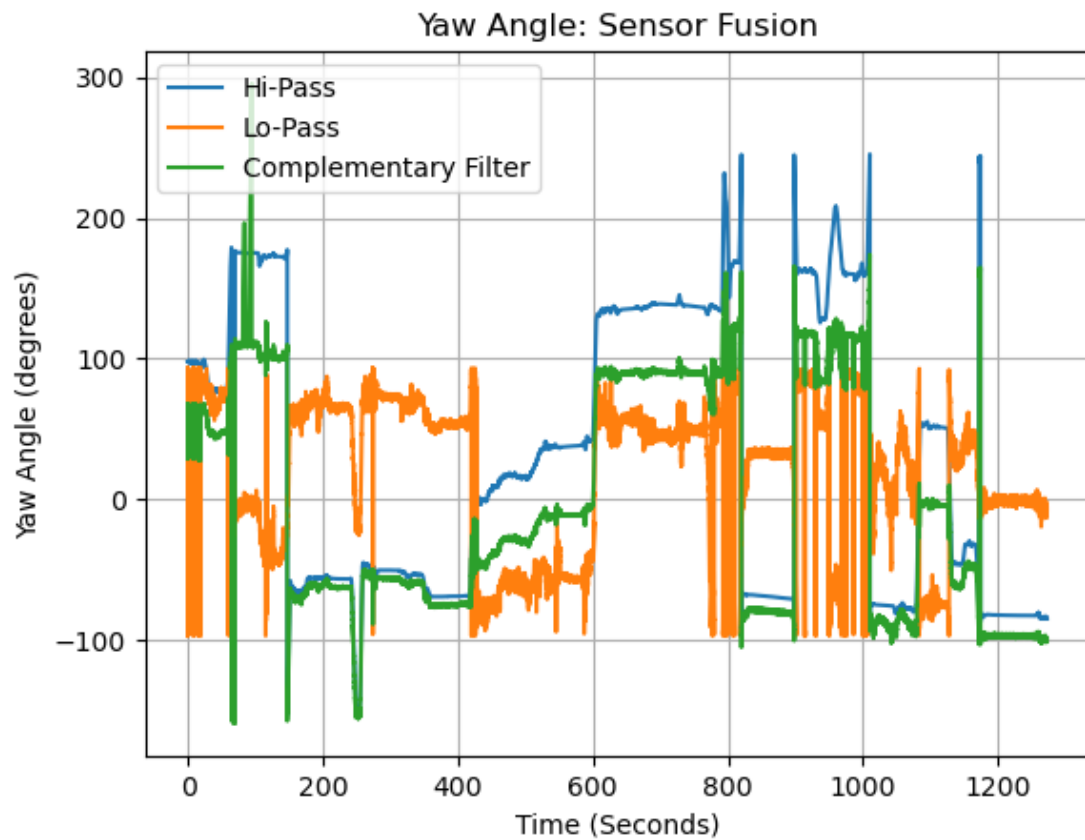


Figure 4: Hi-Pass vs. Low-Pass vs. Complementary Filter

(Q2.Cont) In Figure 4, we can see the effects of both high and low pass filters. We notice that the complementary filter smooths out some of the noise more due to the Hi-pass filter on the gyroscope, and we notice the low-pass filter from the magnetometer corrects for some of the outlying data seen on the hi-pass filter. For cutoff frequencies, 0.001 was used on the hi-pass filter, and 0.9 on the low-pass filter.

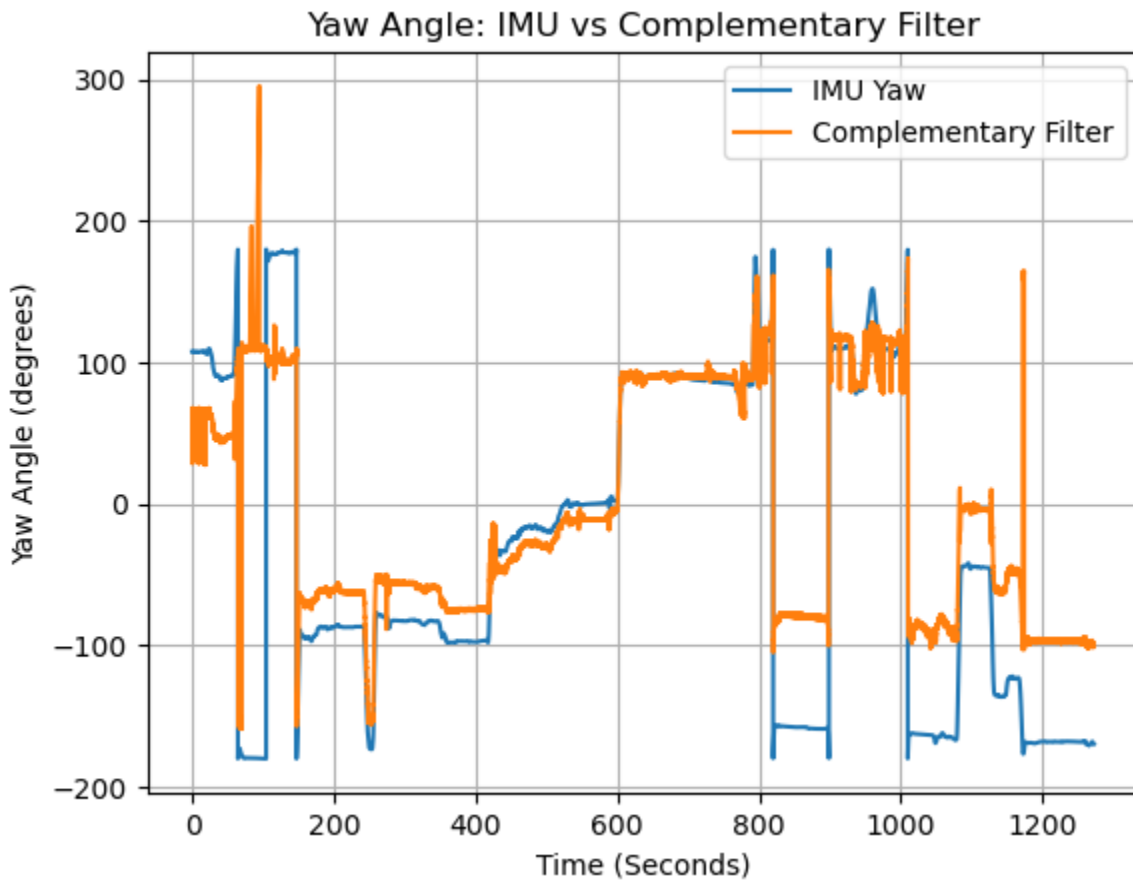


Figure 5: IMU vs Complementary filter Yaw Angles

(Q3) By looking through the data collected and plotted, I would trust the integration of the gyroscope for a more accurate estimate for navigation. This is because the magnetometer is very sensitive and sporadic, while the gyroscope tends to show less noisy data. In Figure 5, we can see that the IMU yaw angle is very close to that of the complementary filter. The complementary filter also more closely resembles the hi-pass filter done on the gyroscope integration of angular velocity. Thus, it shows that the gyroscope is far more trustworthy for navigation.

3. Forward Velocity Estimate

There are two methods we can use to calculate the forward velocity of the car during the latter part of the trajectory (excluding the circular motions). We can integrate the x-acceleration readings of the IMU, since the IMU was mounted such that the x-axis was aligned with the vehicle longitudinal axis. The second method of calculating forward velocity is by taking the time derivative of the GPS readings. The forward velocity estimates using each method are shown in Figure 6.

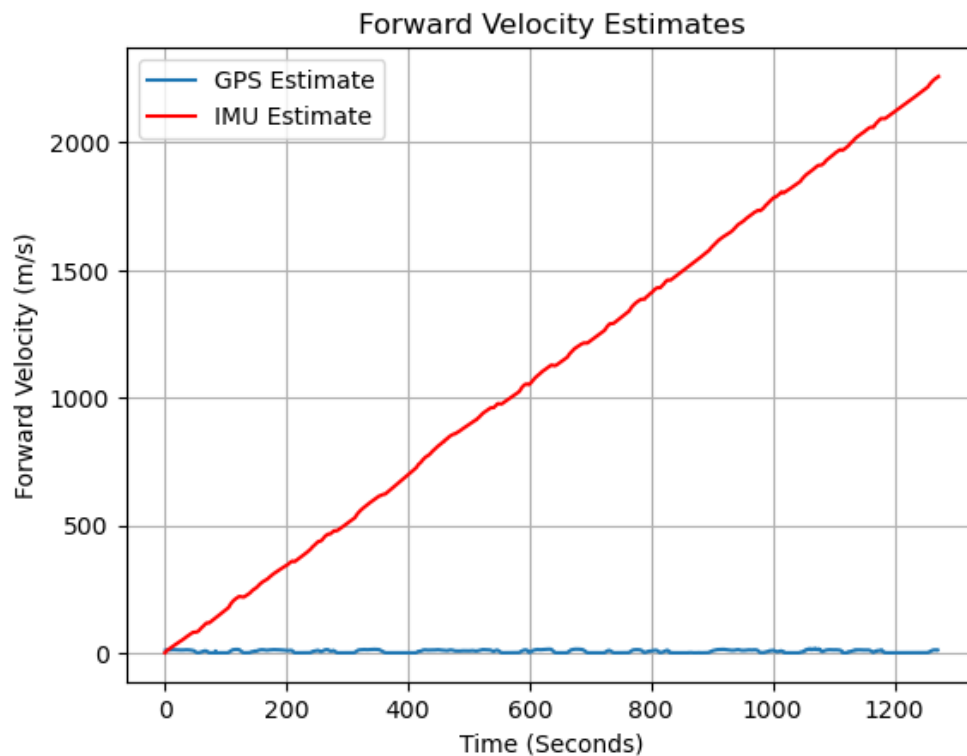


Figure 6: Forward Velocity Estimates From GPS and IMU

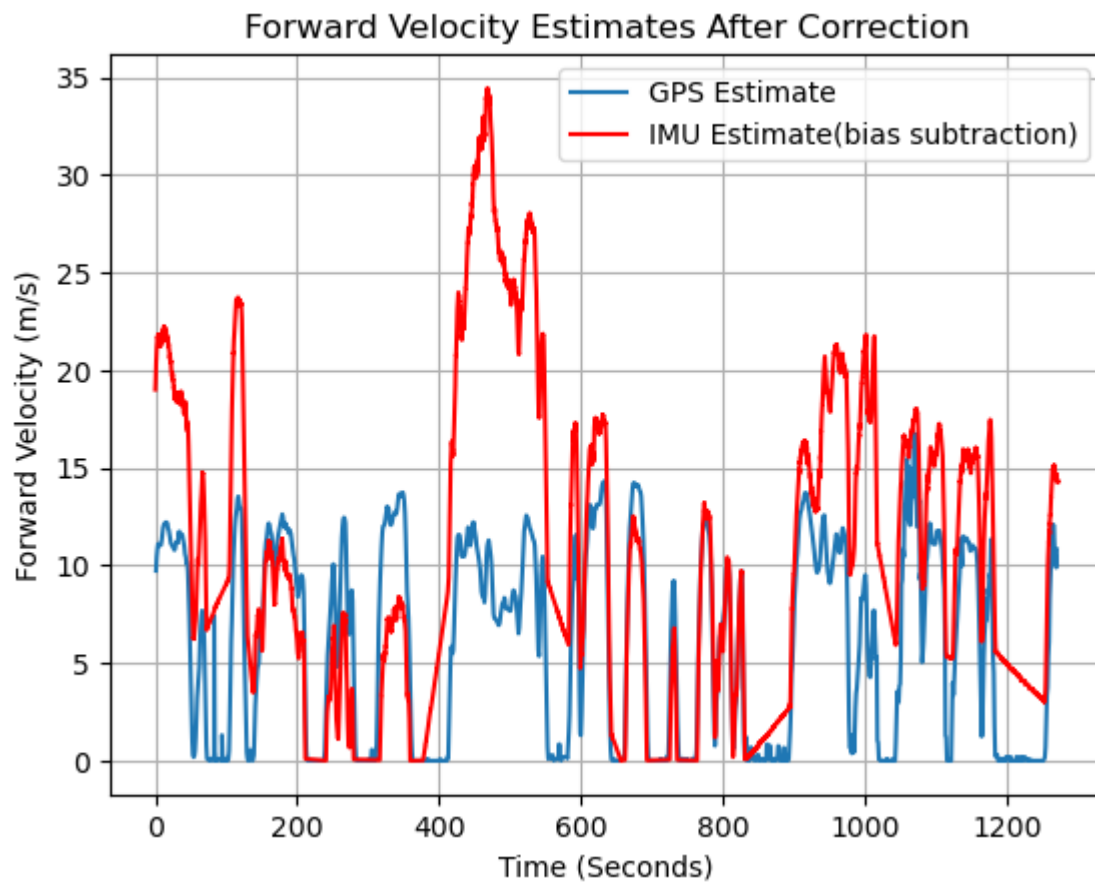


Figure 7: Forward Velocity Estimates From GPS and IMU After IMU Correction

However in Figure 6, it is hard to tell if there are any similarities within the GPS and IMU velocity estimates. Thus, the IMU data needs to be adjusted to be able to accurately determine the differences between the two plots. (Q4) Since there is clearly a long term drift in the acceleration measurements, as indicated by the positive slope over time. In order to correct for this, we can subtract a linearly increasing bias from the imu velocity estimate. The corrected IMU estimate is shown in Figure 7; here we can see that the IMU estimate for forward velocity much more clearly matches the GPS estimates, with a slight bias and amplitude difference between the two signals. (Q5) In Figure 7, we can see the discrepancies between both velocity estimates. We see that the plots show very similar patterns, however, the amplitudes are slightly offset. In some cases, the integrated acceleration from the IMU lines up very closely to that of the GPS velocity estimate, but in other places there is a large gap between the two. This is most likely due to some biases that could not be corrected from just subtracting the linear bias.

6. Compute and compare it to w_dotX and $yddot_obs$. How well do they agree? If there is a difference, what is it due to?

4. Dead Reckoning with IMU

In this analysis we post-process the IMU data to determine a position estimate of the sensor over the course of the trajectory, and compare this estimate to the trajectory mapped by the GPS. We simplify the description of motion of the vehicle by assuming the vehicle is moving in a two dimensional plane. Denote the position of the center-of-mass (CM) of the vehicle by $(X,Y,0)$ and its rotation rate about the CM by $(0,0,\omega)$. We denote the position of the inertial sensor in space by $(x,y,0)$ and its position in the vehicle frame by $(x_c,0,0)$. Then the acceleration measured by the inertial sensor (i.e. its acceleration as sensed in the vehicle frame) is:

$$\begin{aligned}\ddot{x}_{obs} &= \ddot{X} - \omega \dot{Y} - \omega^2 x_c \\ \ddot{y}_{obs} &= \ddot{Y} + \omega \dot{X} + \dot{\omega} x_c\end{aligned}$$

When doing analysis, two assumptions were made: “ $Y_ddot = 0$ ” because the vehicle is not skidding sideways, and the offset between the center of mass of the vehicle is not accounted for, so x_c is zero as the IMU is considered the center. In this part of the analysis, we will compare the y-acceleration from the IMU with the calculated acceleration by utilizing the equations above. We integrate the x acceleration readings to obtain X_dot and multiplying it by the x-angular velocity. Figure 8 shows these calculations compared to the observed y-linear acceleration from the IMU. (Q6) We can see by Figure 8, that both accelerations agree very closely. There is not lots of variation, except very few outliers where the calculated acceleration spikes outwards. These could be due to disturbances in which the IMU was not in optimal conditions, or there are some outliers in the angular velocity or in the integration of the x-acceleration.

(Q7) The dead reckoning of the trajectory was done by integrating the x-acceleration with time to get a velocity estimate. Next, the first point of the trajectory (as calculated by the IMU) was set to the first point in the trajectory produced by the GPS. The magnetometer heading was used to orient the dead reckoning trajectory such that it aligns with the GPS trajectory. Next, the IMU calculated velocity was integrated to produce position estimates, while applying rotations to the trajectory depending on the calculated yaw at that time step. This was done to map turns on the trajectory. Finally, the dead reckoned trajectory was scaled such that the magnitude of the IMU derived trajectory roughly matched the GPS trajectory. The resultant plot can be seen in Figure 9.

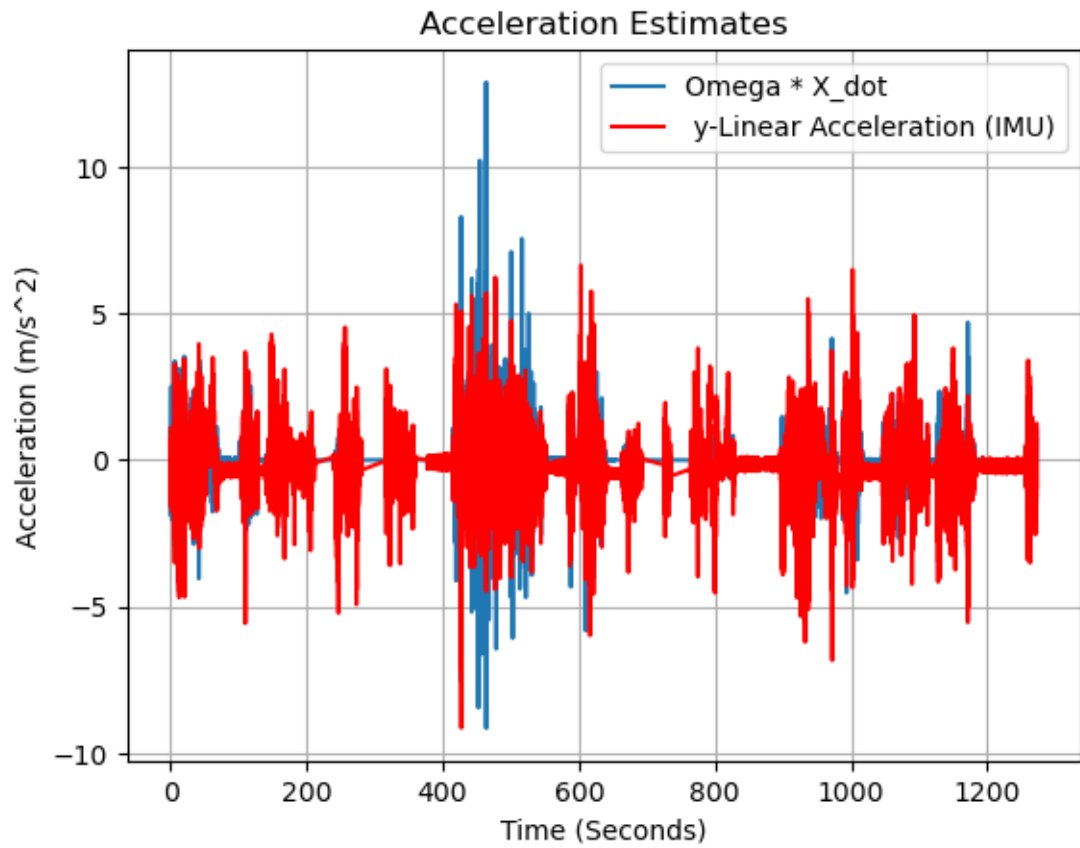


Figure 8: Acceleration Comparisons Between $\Omega * \dot{X}$ and observed Y acceleration



Figure 9: Comparison Between GPS Position and IMU Dead-Reckoning

In Figure 9, the IMU dead reckoning plot was originally much larger, and so the data points were all scaled down by 50%. Alongside with this, when orienting the trajectory, an extra 45 degrees was added to the calculation.

(Q8 Bonus) Since x_c was not accounted for, we can still calculate for it using the data we have.

$$\ddot{y}_{obs} = \ddot{Y} + \omega \dot{X} + \dot{\omega} x_c$$

Using the above equation, we can assume $\ddot{Y} = 0$ since the vehicle is not skidding sideways. Here we have \ddot{y}_{obs} and $\omega \dot{X}$. We can take the derivative to get $\dot{\omega}$ and then solve for x_c . Doing these calculations, we get an array for x_c , so taking the average will give us an accurate answer. **The estimate for x_c is about 2.38 meters** from the IMU. Which makes sense compared to where the IMU was placed and where the center of mass would be for the vehicle. .

(Q9) By looking at the specifications for the VN-100 IMU, we can see that the bias-stability for the gyroscope is $< 10^\circ/\text{hr}$ (5-7 $^\circ/\text{hr}$ typ.). Given these specifications and the plot we see in Figure 9, I would say that the IMU can go 10-15 minutes without a position fix. By looking at the timestamps for which the trajectories in Figure 9 match, it's about 9 minutes until the lines start to diverge from each other. So given the performance specifications, I think it matches decently well, since there was no position fix when calculating the IMU dead reckoning throughout the dataset except for the starting trajectory. The dead-reckoning was able to match the same shape as the GPS positioning.

Conclusion

In conclusion, we can employ many signal processing techniques to de-noise IMU and magnetometer data and correct for drifts over time. Complementary filters offer us the advantage of combining two responses for a more accurate signal. Dead reckoning with an IMU can be quite noisy and unreliable compared to a GPS signal, and it requires some kind of “truth” (in this case from the GPS) in order to correct for the orientation and location of the trajectory. The estimate of forward velocity from the IMU produces a reasonably accurate signal, but the double integration to get position introduces much more error, as seen in the dead reckoning output.