Sensors

June 16, 2023

[ ]:
```
import numpy as np
import pandas as pd
import random
from sklearn.preprocessing import PolynomialFeatures

#New! for Legendre
from scipy.special import legendre
from sklearn.metrics import r2_score #for r-squared


import matplotlib.dates as mdates
import matplotlib.pyplot as plt
from matplotlib import rc

from tqdm import tqdm

# Set default font to 'Times New Roman'
rc('font', family='Times New Roman')
```

1 Traffic Prediction

1.1 Functions

[ ]:
```
def generate_multi_legendre_design_matrix_(x_initial, polynomial_order):
    if np.isscalar(x_initial):
        x_initial = np.array([[x_initial]])

    # Ensure x_initial is a 2D numpy array
    x_initial = np.atleast_2d(x_initial)


    # N is the number of observations,      num_vars is the number of input␣
↪variables
    N, n_input_vars = x_initial.shape

    # Number of terms in the expansion for each variable (x1^0, x1^1, x1^2, ...)
```

```
    n_expansion_terms = polynomial_order + 1

    # Total number of terms in the expansion
    total_expansion_terms = n_expansion_terms ** n_input_vars

    # Initialize design matrix
    design_matrix = np.empty((N, total_expansion_terms), dtype='float64')

    # Loop over each data point
    for i in range(N):
        col_index = 0 # just a counter
        # Loop over all combinations of polynomial orders for each variable
        for terms in np.ndindex(*([n_expansion_terms]*n_input_vars)): ########␣
↪CHECK
            product = 1.0
            # Calculate the product of Legendre polynomials for this␣
↪combination of terms
            for var in range(n_input_vars):
                P_j = legendre(terms[var])
                product *= P_j(x_initial[i, var])
            # Store result in the design matrix
            design_matrix[i, col_index] = product
            col_index += 1


    return design_matrix
```

[ ]:
```
####                                                                        ␣
↪    ####
####                                                                        ␣
↪    ####
#### This one goes up until the parameters that do not exceed the polynomial␣
↪order ####
####                                                                        ␣
↪    ####
####                                                                        ␣
↪    ####

import numpy as np
from numpy.polynomial.legendre import Legendre
from sklearn.preprocessing import MinMaxScaler

def generate_multi_legendre_design_matrix(x_initial, polynomial_order):
    x_initial = np.atleast_2d(x_initial)
    n_input_vars = x_initial.shape[1]
```

```
    # Generate multi-index for which the sum of the indices is <=␣
↪polynomial_order
    indices = np.indices((polynomial_order + 1,) * n_input_vars).
↪reshape(n_input_vars, -1)
    indices = indices[:, np.sum(indices, axis=0) <= polynomial_order]

    # Initialize design matrix
    design_matrix = np.empty((x_initial.shape[0], indices.shape[1]),␣
↪dtype='float64')


    # Compute product of variables raised to the power of indices
    for row in range(x_initial.shape[0]):
        for col, idx in enumerate(indices.T):
            product = 1
            for var, power in enumerate(idx):
                if power != 0:
                    P_j = Legendre.basis(deg=power)
                    product *= P_j(x_initial[row, var])
                else:
                    product *= 1
            design_matrix[row, col] = product

    return design_matrix
```

[ ]:
```
xx = np.array([[1,2],[1,2],[1,2]])
print('xx: ', xx)

print('generate_multi_legendre_design_matrix_:\n',␣
↪generate_multi_legendre_design_matrix_(xx,2))
print('\ngenerate_multi_legendre_design_matrix:\n',␣
↪generate_multi_legendre_design_matrix(xx,2))

print('\ngenerate_multi_legendre_design_matrix_: ',␣
↪generate_multi_legendre_design_matrix_(xx,2).shape)
print('\ngenerate_multi_legendre_design_matrix: ',␣
↪generate_multi_legendre_design_matrix(xx,2).shape)
```

```
xx:  [[1 2]
 [1 2]
 [1 2]]
generate_multi_legendre_design_matrix_:
 [[1.  2.  5.5 1.  2.  5.5 1.  2.  5.5]
 [1.  2.  5.5 1.  2.  5.5 1.  2.  5.5]
 [1.  2.  5.5 1.  2.  5.5 1.  2.  5.5]]

generate_multi_legendre_design_matrix:
```

```
[[1.   2.   5.5 1.   2.   1. ]
 [1.   2.   5.5 1.   2.   1. ]
 [1.   2.   5.5 1.   2.   1. ]]
```

```
generate_multi_legendre_design_matrix_:  (3, 9)

generate_multi_legendre_design_matrix:  (3, 6)
```

1.1.1 Predictive Algorithm

[ ]:
```python
def online_kaczmarz_legendre_multiple(x_initial, target_values,␣
↪polynomial_order, weights=None):

    # Initialize MinMaxScaler to normalize to range [-1,1]
    #scaler = MinMaxScaler(feature_range=(-1, 1))
    # x_initial = ( x_initial - x_initial.min() ) / ( x_initial.max() -␣
↪x_initial.min() )

    design_matrix = generate_multi_legendre_design_matrix(x_initial,␣
↪polynomial_order)

    if weights is None:
        # initialize the weights to be the number of columns in the design␣
↪matrix
        weight_predictions = np.random.rand(design_matrix.shape[1])
    else:
        weight_predictions = weights

    beta_parameter = 0

    for i in range (design_matrix.shape[0]):
        a = design_matrix[i, :]
        #a = ( a_0 - a_0.min() ) / ( a_0.max() - a_0.min() )

        weight_predictions = weight_predictions + ((target_values[i] - np.
↪dot(a, weight_predictions)) / np.linalg.norm(a)**2) * a.T


        ################
        # weight_predictions =  (weight_predictions + a.min() ) * ( a.max() - a.
↪min() )
        ################


        # This is for beta
```

```
        # Note: you may want to uncomment this if you want to compute␣
↪beta_parameter
        # residual_errors = target_values - design_matrix @ weight_predictions
        # sse = residual_errors.T @ residual_errors
        # beta_parameter = sse / ( - polynomial_order)

    return weight_predictions, beta_parameter
```

1.2 1. Sensor 737433 (traffic increase after fire)

1.2.1 Load the data of the sensor of interest, and the m closest sensors.

[ ]:
```
## m=4

sensor_1_ = pd.read_csv('./Data/sensor_interest_1 737433.txt')
print('sensor_1: ', sensor_1_.shape)


sensor_1_m_1_ = pd.read_csv('./Data/sensor_1_m_8 772564.txt')
sensor_1_m_2_ = pd.read_csv('./Data/sensor_1_m_9 775975.txt')
sensor_1_m_3_ = pd.read_csv('./Data/sensor_1_m_11 775961.txt')
sensor_1_m_4_ = pd.read_csv('./Data/sensor_1_m_14 775949.txt')
```

```
sensor_1:  (9216, 38)
```

1.2.2 Filter out the measurements that are outside the time intervals of interest.

We want to see the measurements from ***7 am to 7 pm***

[ ]:
```
# Ensure 'Time' column is in datetime format
sensor_1_['Time'] = pd.to_datetime(sensor_1_['Time'])
sensor_1_m_1_['Time'] = pd.to_datetime(sensor_1_m_1_['Time'])
sensor_1_m_2_['Time'] = pd.to_datetime(sensor_1_m_2_['Time'])
sensor_1_m_3_['Time'] = pd.to_datetime(sensor_1_m_3_['Time'])
sensor_1_m_4_['Time'] = pd.to_datetime(sensor_1_m_4_['Time'])

# Filter out data outside of 7 AM to 7 PM
sensor_1 = sensor_1_[(sensor_1_['Time'].dt.hour >= 7) & (sensor_1_['Time'].dt.
↪hour < 19)]
sensor_1_m_1 = sensor_1_m_1_[(sensor_1_m_1_['Time'].dt.hour >= 7) &␣
↪(sensor_1_m_1_['Time'].dt.hour < 19)]
sensor_1_m_2 = sensor_1_m_2_[(sensor_1_m_2_['Time'].dt.hour >= 7) &␣
↪(sensor_1_m_2_['Time'].dt.hour < 19)]
sensor_1_m_3 = sensor_1_m_3_[(sensor_1_m_3_['Time'].dt.hour >= 7) &␣
↪(sensor_1_m_3_['Time'].dt.hour < 19)]
```

```
sensor_1_m_4 = sensor_1_m_4_[(sensor_1_m_4_['Time'].dt.hour >= 7) &
 ↪(sensor_1_m_4_['Time'].dt.hour < 19)]

# Reset the indices
sensor_1 = sensor_1.reset_index(drop=True)
sensor_1_m_1 = sensor_1_m_1.reset_index(drop=True)
sensor_1_m_2 = sensor_1_m_2.reset_index(drop=True)
sensor_1_m_3 = sensor_1_m_3.reset_index(drop=True)
sensor_1_m_4 = sensor_1_m_4.reset_index(drop=True)


print('sensor_1: ', sensor_1.shape)
print('sensor_1_m_1: ', sensor_1_m_1.shape)
print('sensor_1_m_2: ', sensor_1_m_2.shape)
print('sensor_1_m_3: ', sensor_1_m_3.shape)
print('sensor_1_m_4: ', sensor_1_m_4.shape)
```

```
sensor_1:  (4608, 38)
sensor_1_m_1:  (4607, 38)
sensor_1_m_2:  (4608, 38)
sensor_1_m_3:  (4608, 34)
sensor_1_m_4:  (4608, 34)
```

1.2.3 Generate the matrix as per the specifications in the paper.

Each row will have the measurements of traffic from the sensor of interest and the m-closest sensors. - The rows will include traffic information of the t-1, t-2,…, t-r observations. - The rows are organized by sensor, and by timestep: [sensor of interest @ t-1,…,sensor of interest @ t-r, … , m-closest sensor @ t-1,…,m-closest sensor @ t-r]

[ ]:
```
r = 3  # set r to any value

# Minimum number of rows across all dataframes
min_rows = min(sensor_1.shape[0], sensor_1_m_1.shape[0], sensor_1_m_2.shape[0],
 ↪sensor_1_m_3.shape[0], sensor_1_m_4.shape[0])

# Initialize an empty list to store all row vectors
row_vectors = []

# Iterate over each index from r to min_rows
for i in tqdm(range(r, min_rows)):
    # Generate a list of indexes you're interested in. In this case, it's [i-r,
 ↪i-r+1, ..., i]

    ind = list(range(i+1, i-r, -1))
    indexes = list(range(i, i-r, -1))
```

```python
    # Get the desired elements
    sensor_1_traffic = sensor_1.loc[ind, '10'].values
    sensor_1_m_1_traffic = sensor_1_m_1.loc[indexes, '10'].values
    sensor_1_m_2_traffic = sensor_1_m_2.loc[indexes, '10'].values
    sensor_1_m_3_traffic = sensor_1_m_3.loc[indexes, '10'].values
    sensor_1_m_4_traffic = sensor_1_m_4.loc[indexes, '10'].values

    # Concatenate them into a 1xr*5 row vector
    row_vector = np.concatenate([sensor_1_traffic, sensor_1_m_1_traffic, sensor_1_m_2_traffic, sensor_1_m_3_traffic, sensor_1_m_4_traffic])

    # Append the row vector to our list
    row_vectors.append(row_vector)

# Convert our list of row vectors into a 2D numpy array
traffic_737433 = pd.DataFrame(row_vectors)
traffic_737433
```

```
100%|      | 4604/4604 [00:09<00:00, 482.42it/s]
```

[ ]:

```
          0      1      2      3      4      5      6     7      8      9  \
0     535.0  496.0  532.0  496.0  445.0  406.0  452.0  86.0   74.0   38.0
1     523.0  535.0  496.0  532.0  459.0  445.0  406.0  60.0   86.0   74.0
2     505.0  523.0  535.0  496.0  437.0  459.0  445.0  80.0   60.0   86.0
3     498.0  505.0  523.0  535.0  497.0  437.0  459.0  49.0   80.0   60.0
4     489.0  498.0  505.0  523.0  464.0  497.0  437.0  55.0   49.0   80.0
…     …    …    …    …    …    …    …    …    …    …
4599  387.0  380.0  352.0  390.0  389.0  364.0  371.0  72.0  135.0  118.0
4600  366.0  387.0  380.0  352.0  381.0  389.0  364.0  90.0   72.0  135.0
4601  344.0  366.0  387.0  380.0  366.0  381.0  389.0  84.0   90.0   72.0
4602  339.0  344.0  366.0  387.0  348.0  366.0  381.0  84.0   84.0   90.0
4603  339.0  339.0  344.0  366.0  338.0  348.0  366.0  59.0   84.0   84.0

        10    11    12     13     14     15
0     68.0  50.0  32.0   88.0   86.0   93.0
1     50.0  68.0  50.0  108.0   88.0   86.0
2     64.0  50.0  68.0   90.0  108.0   88.0
3     54.0  64.0  50.0   85.0   90.0  108.0
4     47.0  54.0  64.0   65.0   85.0   90.0
…     …    …    …    …    …    …
4599  70.0  91.0  48.0   68.0   67.0   48.0
4600  60.0  70.0  91.0   48.0   68.0   67.0
4601  77.0  60.0  70.0   40.0   48.0   68.0
4602  70.0  77.0  60.0   30.0   40.0   48.0
4603  46.0  70.0  77.0   61.0   30.0   40.0
```

[4604 rows x 16 columns]

1.2.4 Tests - Sensor 737433

Estimation

```
[ ]: ####################################################################################
####    Definition of "Hyperparameters"
####################################################################################
days = 5  # Max=31.97 (available data)
t = int( (60/5)*12 * days)

#t = traffic_737433.shape[0] # Number of points to be tested on, and times the␣
 ↪coefficients will be updated.
        # The coefficients w are calculatd for each of these points, the idea␣
 ↪is to simulate an on-line stream of data.

X = traffic_737433.iloc[:t, 1:]

"""
    Update this if using more input variables, # the function is handling a␣
 ↪15-dimensional, second order polynomial.
"""

polynomial_degree = 2
"""
    USING Total (vs. Max) EXPANCTION TERMS
    Number of terms in the weights matrix:

"""



target_values = traffic_737433.iloc[:t, 0]
# Generate Y as target_values (real Y's)

# Initialize weights and estimations
weights_over_time = []
y_hat = []


noise = np.random.normal(scale= 10 , size=(t))

####################################################################################
####    Online estimation of coefficients
####################################################################################
```

```
for i in tqdm(range(t)):
    x_i = X.iloc[i, :]

    # Normalize and scale to -1,1 the input:
    x_i = ( x_i - x_i.min() ) / ( x_i.max() - x_i.min() )*2 - 1

    weight_predictions, _ = online_kaczmarz_legendre_multiple(x_i,␣
↪[target_values[i] + noise[i]], polynomial_degree)

    #De-normalize rescale the weights:
    weight_predictions = (((weight_predictions + 1) / 2)  * ( x_i.max() - x_i.
↪min() ) + x_i.min() )

    # Use predicted weights to compute y_hat
    y_predictions = generate_multi_legendre_design_matrix(x_i,␣
↪polynomial_degree) @ weight_predictions.T

    # Store the values of predicted y and estimated weights
    y_hat.append(y_predictions)
    weights_over_time.append(weight_predictions)

print("Done carajo")
```

```
71%|      | 510/720 [23:05<09:29,  2.71s/it]
```

[ ]:
```
weights_over_time = np.array(weights_over_time)
y_hat_ = np.array(y_hat) #########
print('y_hat: ', y_hat_.shape)

print('weights_over_time: ', weights_over_time.shape)

weight = weights_over_time[-1]
Weights = pd.DataFrame({
    'Estimated_Weights': weight
})

# Style DataFrame
# Weights.style.format("{:.4f}")

print('Weights: ', Weights.shape)
```

```
y_hat:  (720, 1)
weights_over_time:  (720, 136)
Weights:  (136, 1)
```

Results

[ ]:
```python
###########################################################################################
####    Results
###########################################################################################

Predicted_Values = y_hat_.flatten()
print('Predicted_Values: ', Predicted_Values.shape)
Real_Values = target_values
print('Real_Values: ', Real_Values.shape)
difference = (Real_Values-Predicted_Values).T


Y = pd.DataFrame({
    'Predicted_Values': Predicted_Values,
    'Real_Values': Real_Values,
    'Difference': difference
})

# Style DataFrame
Y[-10:].style.format("{:.4f}")
```

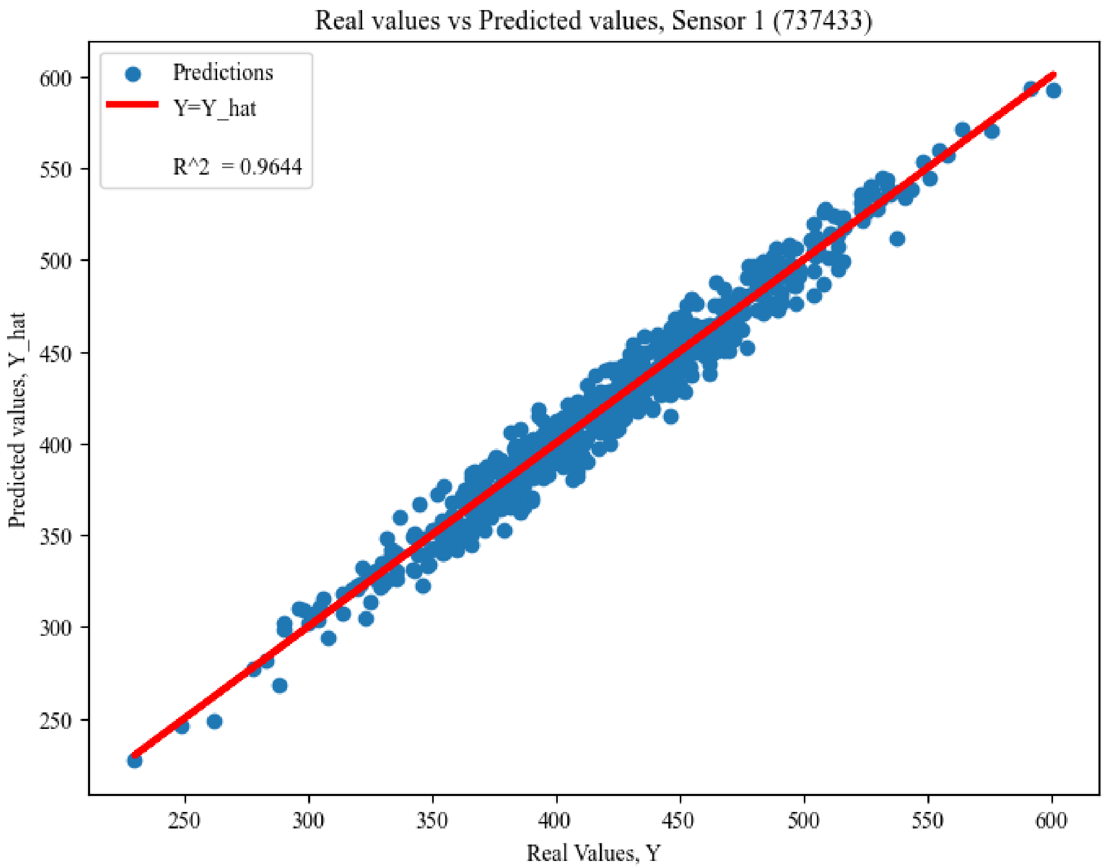
```
Predicted_Values:  (720,)
Real_Values:  (720,)
```

[ ]:
```
<pandas.io.formats.style.Styler at 0x1676685b0>
```

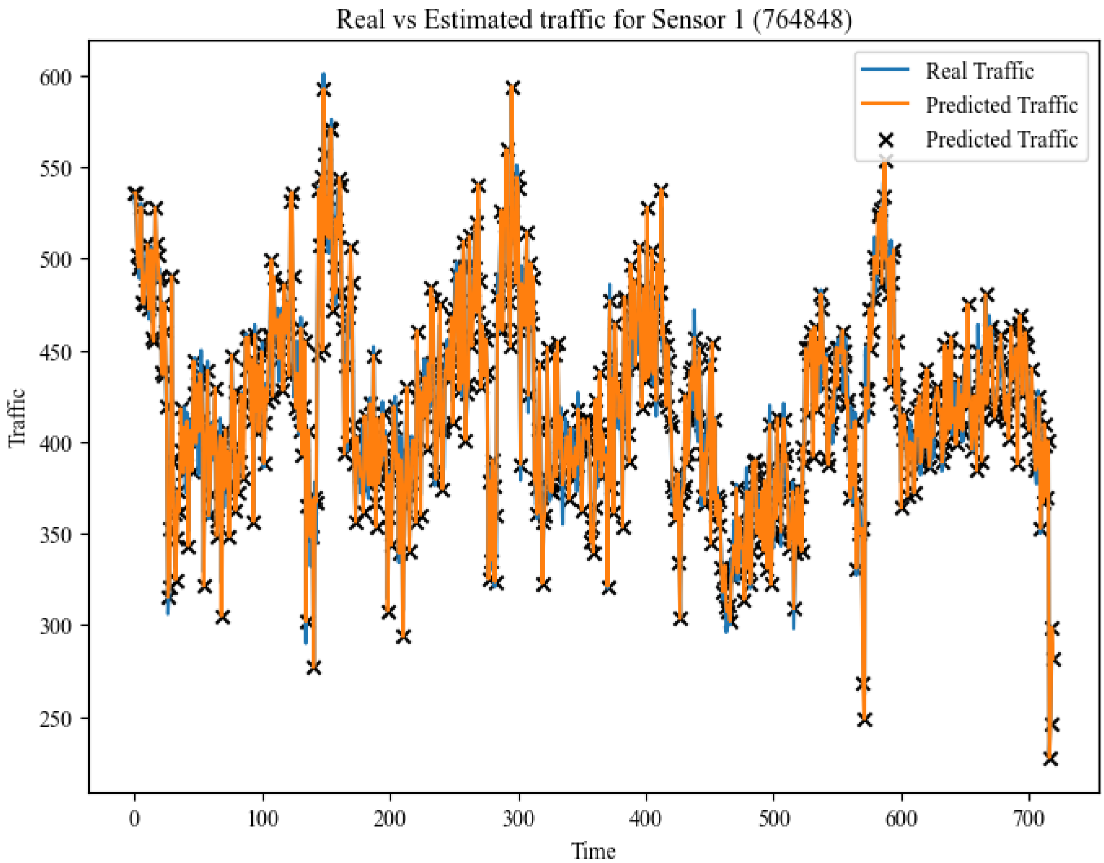
Plots

[ ]:
```python
# Plot the accuracy
plt.figure(figsize=(8, 6))

r2 = r2_score(target_values, y_hat_)
print('r2: ', r2)

plt.scatter(target_values, y_hat_, label='Predictions')
plt.plot(target_values, target_values, color='red', linewidth = 3, label=f'Y=Y_hat')
plt.scatter(target_values, target_values, marker='', label=f'\nR^2  = {r2:.4f}')

plt.xlabel('Real Values, Y')
plt.ylabel('Predicted values, Y_hat')
plt.title('Real values vs Predicted values, Sensor 1 (737433)')
plt.legend()

plt.savefig('./figures/traffic_sensor_1.png')

plt.show()
```

```
r2:  0.964373507355896
```




```
[ ]: # Plot the traffic
n = t

plt.figure(figsize=(8, 6))

plt.plot(Y['Real_Values'][-n:],label='Real Traffic')
plt.plot(Y['Predicted_Values'][-n:],label='Predicted Traffic')
plt.scatter(Y[-n:].index, Y['Predicted_Values'][-n:], marker='x', color =␣
↪'black',label='Predicted Traffic')

plt.xlabel('Time')
plt.ylabel('Traffic')
plt.title('Real vs Estimated traffic for Sensor 1 (737433)')
plt.legend()

plt.savefig('./Figures/traffic_sensor_1.png')

plt.show()
```



1.3 2. Sensor 764848 (traffic decrease after fire)

1.3.1 Load the data of the sensor of interest, and the m closest sensors.

[ ]:
```
## m=4

sensor_2_ = pd.read_csv('./Data/sensor_interest_2 764848.txt')
print('sensor_2: ', sensor_2_.shape)


sensor_2_m_1_ = pd.read_csv('./Data/sensor_2_m_2 771475.txt')
sensor_2_m_2_ = pd.read_csv('./Data/sensor_2_m_4 771410.txt')
sensor_2_m_3_ = pd.read_csv('./Data/sensor_2_m_7 771421.txt')
sensor_2_m_4_ = pd.read_csv('./Data/sensor_2_m_9 771463.txt')
```

```
sensor_2:  (9216, 38)
```

1.3.2 Filter out the measurements that are outside the time intervals of interest.

We want to see the measurements from ***7 am to 7 pm***

[ ]:
```
# Ensure 'Time' column is in datetime format
sensor_2_['Time'] = pd.to_datetime(sensor_2_['Time'])
sensor_2_m_1_['Time'] = pd.to_datetime(sensor_2_m_1_['Time'])
sensor_2_m_2_['Time'] = pd.to_datetime(sensor_2_m_2_['Time'])
sensor_2_m_3_['Time'] = pd.to_datetime(sensor_2_m_3_['Time'])
sensor_2_m_4_['Time'] = pd.to_datetime(sensor_2_m_4_['Time'])

# Filter out data outside of 7 AM to 7 PM
sensor_2 = sensor_2_[(sensor_2_['Time'].dt.hour >= 7) & (sensor_2_['Time'].dt.
 ↪hour < 19)]
sensor_2_m_1 = sensor_2_m_1_[(sensor_2_m_1_['Time'].dt.hour >= 7) &␣
 ↪(sensor_2_m_1_['Time'].dt.hour < 19)]
sensor_2_m_2 = sensor_2_m_2_[(sensor_2_m_2_['Time'].dt.hour >= 7) &␣
 ↪(sensor_2_m_2_['Time'].dt.hour < 19)]
sensor_2_m_3 = sensor_2_m_3_[(sensor_2_m_3_['Time'].dt.hour >= 7) &␣
 ↪(sensor_2_m_3_['Time'].dt.hour < 19)]
sensor_2_m_4 = sensor_2_m_4_[(sensor_2_m_4_['Time'].dt.hour >= 7) &␣
 ↪(sensor_2_m_4_['Time'].dt.hour < 19)]

# Reset the indices
sensor_2 = sensor_2.reset_index(drop=True)
sensor_2_m_1 = sensor_2_m_1.reset_index(drop=True)
sensor_2_m_2 = sensor_2_m_2.reset_index(drop=True)
sensor_2_m_3 = sensor_2_m_3.reset_index(drop=True)
sensor_2_m_4 = sensor_2_m_4.reset_index(drop=True)

print('sensor_2: ', sensor_2.shape)
print('sensor_2_m_1: ', sensor_2_m_1.shape)
print('sensor_2_m_2: ', sensor_2_m_2.shape)
print('sensor_2_m_3: ', sensor_2_m_3.shape)
print('sensor_2_m_4: ', sensor_2_m_4.shape)
```

```
sensor_2:  (4608, 38)
sensor_2_m_1:  (4608, 34)
sensor_2_m_2:  (4608, 34)
sensor_2_m_3:  (4608, 34)
sensor_2_m_4:  (4607, 34)
```

1.3.3 Generate the matrix as per the specifications in the paper.

Each row will have the measurements of traffic from the sensor of interest and the m-closest sensors. - The rows will include traffic information of the t-1, t-2,..., t-r observations. - The rows are organized by sensor, and by timestep: [sensor of interest @ t-1,…,sensor of interest @ t-r, … , m-closest sensor @ t-1,…,m-closest sensor @ t-r]

[ ]:
```
r = 3  # set r to any value

# Minimum number of rows across all dataframes
min_rows = min(sensor_2.shape[0], sensor_2_m_1.shape[0], sensor_2_m_2.shape[0],␣
↪sensor_2_m_3.shape[0], sensor_2_m_4.shape[0])

# Initialize an empty list to store all row vectors
row_vectors = []

# Iterate over each index from r to min_rows
for i in tqdm(range(r, min_rows)):
    # Generate a list of indexes you're interested in. In this case, it's [i-r,␣
↪i-r+1, ..., i]

    # ind = list(range(i-r, i+1))
    # indexes = list(range(i-r+1, i+1))

    ind = list(range(i+1, i-r, -1))
    indexes = list(range(i, i-r, -1))


    # Get the desired elements
    sensor_2_traffic = sensor_2.loc[ind, '10'].values
    sensor_2_m_1_traffic = sensor_2_m_1.loc[indexes, '10'].values
    sensor_2_m_2_traffic = sensor_2_m_2.loc[indexes, '10'].values
    sensor_2_m_3_traffic = sensor_2_m_3.loc[indexes, '10'].values
    sensor_2_m_4_traffic = sensor_2_m_4.loc[indexes, '10'].values

    # Concatenate them into a 1xr*5 row vector
    row_vector = np.concatenate([sensor_2_traffic, sensor_2_m_1_traffic,␣
↪sensor_2_m_2_traffic, sensor_2_m_3_traffic, sensor_2_m_4_traffic])

    # Append the row vector to our list
    row_vectors.append(row_vector)

# Convert our list of row vectors into a 2D numpy array
traffic_764848 = pd.DataFrame(row_vectors)
traffic_764848
```

```
100%|      | 4604/4604 [00:07<00:00, 625.17it/s]
```

[ ]:
```
       0      1      2      3      4      5      6      7      8      9  
0  529.0  500.0  415.0  399.0  322.0  295.0  308.0  159.0  158.0  179.0  \
1  542.0  529.0  500.0  415.0  344.0  322.0  295.0  202.0  159.0  158.0
```

```
2      526.0  542.0  529.0  500.0  326.0  344.0  322.0  188.0  202.0  159.0
3      507.0  526.0  542.0  529.0  333.0  326.0  344.0  183.0  188.0  202.0
4      514.0  507.0  526.0  542.0  410.0  333.0  326.0  218.0  183.0  188.0
…      …    …     …     …    …     …     …     …    …     …
4599   395.0  432.0  462.0  467.0  147.0  181.0  183.0  245.0  231.0  241.0
4600   434.0  395.0  432.0  462.0  185.0  147.0  181.0  255.0  245.0  231.0
4601   392.0  434.0  395.0  432.0  158.0  185.0  147.0  246.0  255.0  245.0
4602   397.0  392.0  434.0  395.0  179.0  158.0  185.0  196.0  246.0  255.0
4603   362.0  397.0  392.0  434.0  150.0  179.0  158.0  210.0  196.0  246.0

          10     11     12     13     14     15
0      203.0  149.0  182.0  292.0  276.0  268.0
1      185.0  203.0  149.0  296.0  292.0  276.0
2      196.0  185.0  203.0  288.0  296.0  292.0
3      185.0  196.0  185.0  286.0  288.0  296.0
4      235.0  185.0  196.0  357.0  286.0  288.0
…      …    …     …     …    …     …
4599   271.0  274.0  265.0  170.0  144.0  170.0
4600   263.0  271.0  274.0  147.0  170.0  144.0
4601   260.0  263.0  271.0  160.0  147.0  170.0
4602   241.0  260.0  263.0  145.0  160.0  147.0
4603   250.0  241.0  260.0  138.0  145.0  160.0

[4604 rows x 16 columns]
```

1.3.4 Tests - Sensor 764848

Estimation

[ ]:
```python
##########################################################################################
####    Definition of "Hyperparameters"
##########################################################################################
days = 5
t = int( (60/5)*12 * days)

#t = traffic_737433.shape[0] # Number of points to be tested on, and times the
↪coefficients will be updated.
        # The coefficients w are calculatd for each of these points, the idea
↪is to simulate an on-line stream of data.

X = traffic_764848.iloc[:t, 1:]

"""
    Update this if using more input variables, # the function is handling a
↪15-dimensional, second order polynomial.
"""
```

```python
polynomial_degree = 2
"""
    USING Total (vs. Max) EXPANCTION TERMS
    Number of terms in the weights matrix:

"""



target_values = traffic_764848.iloc[:t, 0]
# Generate Y as target_values (real Y's)

# Initialize weights and estimations
weights_over_time = []
y_hat = []


noise = np.random.normal(scale= 10 , size=(t))

##############################################################################################
####    Online estimation of coefficients
##############################################################################################


for i in tqdm(range(t)):
    x_i = X.iloc[i, :]

    # Normalize and scale to -1,1 the input:
    x_i = ( x_i - x_i.min() ) / ( x_i.max() - x_i.min() )*2 - 1

    weight_predictions, _ = online_kaczmarz_legendre_multiple(x_i,␣
 ↪[target_values[i] + noise[i]], polynomial_degree)

    #De-normalize rescale the weights:
    weight_predictions = (((weight_predictions + 1) / 2)  * ( x_i.max() - x_i.
 ↪min() ) + x_i.min() )

    # Use predicted weights to compute y_hat
    y_predictions = generate_multi_legendre_design_matrix(x_i,␣
 ↪polynomial_degree) @ weight_predictions.T

    # Store the values of predicted y and estimated weights
    y_hat.append(y_predictions)
    weights_over_time.append(weight_predictions)

print("Done carajo")
```

```
100%|     | 720/720 [30:32<00:00,  2.54s/it]
```

```
Done carajo
```

```
[ ]: weights_over_time = np.array(weights_over_time)
y_hat_ = np.array(y_hat) #########
print('y_hat: ', y_hat_.shape)

print('weights_over_time: ', weights_over_time.shape)

weight = weights_over_time[-1]
Weights = pd.DataFrame({
    'Estimated_Weights': weight
})

# Style DataFrame
# Weights.style.format("{:.4f}")

print('Weights: ', Weights.shape)
```

```
y_hat:  (720, 1)
weights_over_time:  (720, 136)
Weights:  (136, 1)
```

Results

```
[ ]: #############################################################################################
#### 	Results
#############################################################################################

Predicted_Values = y_hat_.flatten()
print('Predicted_Values: ', Predicted_Values.shape)
Real_Values = traffic_764848.iloc[:t, 0]
print('Real_Values: ', Real_Values.shape)
difference = (Real_Values-Predicted_Values).T


Y = pd.DataFrame({
    'Predicted_Values': Predicted_Values,
    'Real_Values': Real_Values,
    'Difference': difference
})

# Style DataFrame
Y[-10:].style.format("{:.4f}")
```

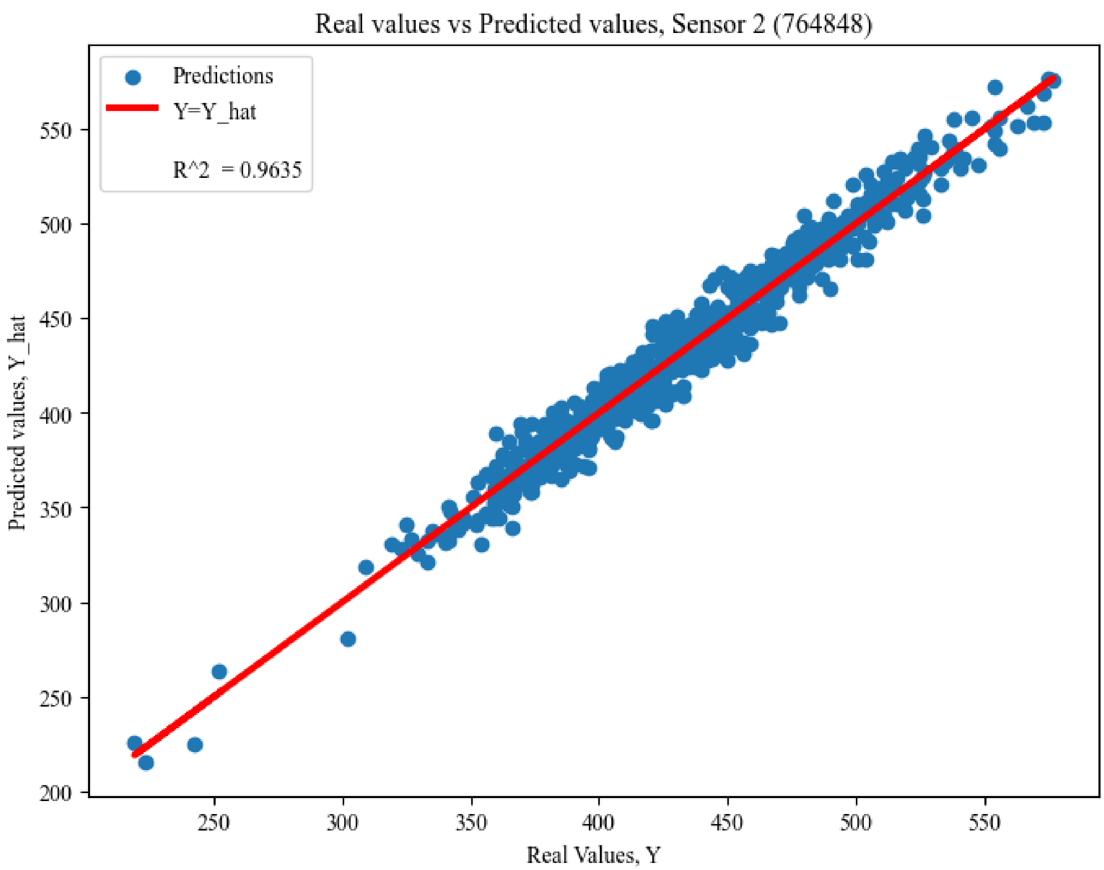
```
Predicted_Values:  (720,)
Real_Values:  (720,)
```

[ ]: <pandas.io.formats.style.Styler at 0x1620cab90>

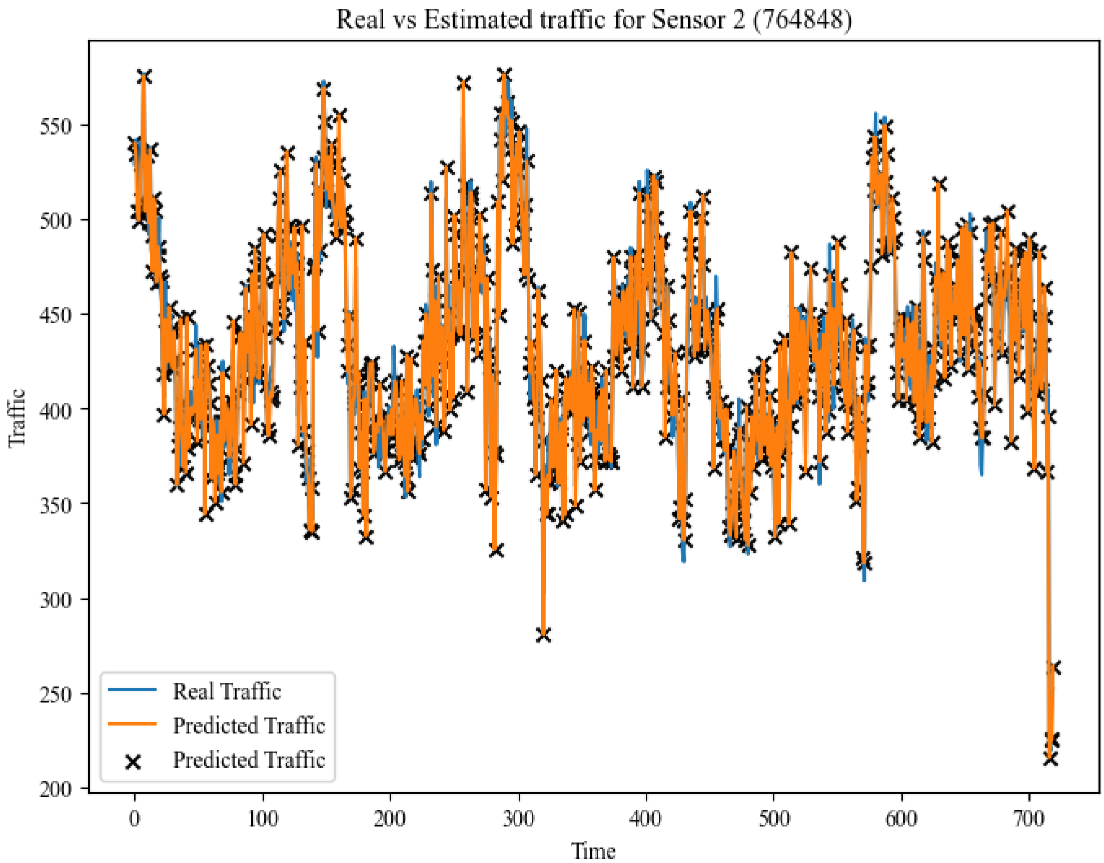
Plots

```
[ ]: # Plot the accuracy
plt.figure(figsize=(8, 6))

r2 = r2_score(target_values, y_hat_)
print('r2: ', r2)

plt.scatter(target_values, y_hat_, label='Predictions')
plt.plot(target_values, target_values, color='red', linewidth = 3,␣
  ↪label=f'Y=Y_hat')
plt.scatter(target_values, target_values, marker='', label=f'\nR^2  = {r2:.4f}')

plt.xlabel('Real Values, Y')
plt.ylabel('Predicted values, Y_hat')
plt.title('Real values vs Predicted values, Sensor 2 (764848)')
plt.legend()

plt.savefig('./figures/traffic_sensor_2.png')

plt.show()
```

```
r2:  0.9634552456533417
```



[ ]:
```
# Plot the traffic
n = t

plt.figure(figsize=(8, 6))

plt.plot(Y['Real_Values'][-n:],label='Real Traffic')
plt.plot(Y['Predicted_Values'][-n:],label='Predicted Traffic')
plt.scatter(Y[-n:].index, Y['Predicted_Values'][-n:], marker='x', color =␣
  ↪'black',label='Predicted Traffic')

plt.xlabel('Time')
plt.ylabel('Traffic')
plt.title('Real vs Estimated traffic for Sensor 2 (764848)')
plt.legend()

plt.savefig('./Figures/traffic_sensor_2.png')

plt.show()
```



1.4 3. Sensor 764632 (traffic changes after fire)

1.4.1 Load the data of the sensor of interest, and the m closest sensors.

[ ]:
```
## m=4

sensor_3_ = pd.read_csv('./Data/sensor_interest_3 764632.txt')
print('sensor_3: ', sensor_3_.shape)

sensor_3_m_1_ = pd.read_csv('./Data/sensor_3_m_3 764958.txt')
sensor_3_m_2_ = pd.read_csv('./Data/sensor_3_m_6 764181.txt')
sensor_3_m_3_ = pd.read_csv('./Data/sensor_3_m_8 765100.txt')
sensor_3_m_4_ = pd.read_csv('./Data/sensor_3_m_11 760892.txt')
```

```
sensor_3:  (9216, 38)
```

1.4.2 Filter out the measurements that are outside the time intervals of interest.

We want to see the measurements from ***7 am to 7 pm***

[ ]:
```
# Ensure 'Time' column is in datetime format
sensor_3_['Time'] = pd.to_datetime(sensor_3_['Time'])
sensor_3_m_1_['Time'] = pd.to_datetime(sensor_3_m_1_['Time'])
sensor_3_m_2_['Time'] = pd.to_datetime(sensor_3_m_2_['Time'])
sensor_3_m_3_['Time'] = pd.to_datetime(sensor_3_m_3_['Time'])
sensor_3_m_4_['Time'] = pd.to_datetime(sensor_3_m_4_['Time'])

# Filter out data outside of 7 AM to 7 PM
sensor_3 = sensor_3_[(sensor_3_['Time'].dt.hour >= 7) & (sensor_3_['Time'].dt.
 ↪hour < 19)]
sensor_3_m_1 = sensor_3_m_1_[(sensor_3_m_1_['Time'].dt.hour >= 7) &␣
 ↪(sensor_3_m_1_['Time'].dt.hour < 19)]
sensor_3_m_2 = sensor_3_m_2_[(sensor_3_m_2_['Time'].dt.hour >= 7) &␣
 ↪(sensor_3_m_2_['Time'].dt.hour < 19)]
sensor_3_m_3 = sensor_3_m_3_[(sensor_3_m_3_['Time'].dt.hour >= 7) &␣
 ↪(sensor_3_m_3_['Time'].dt.hour < 19)]
sensor_3_m_4 = sensor_3_m_4_[(sensor_3_m_4_['Time'].dt.hour >= 7) &␣
 ↪(sensor_3_m_4_['Time'].dt.hour < 19)]

# Reset the indices
sensor_3 = sensor_3.reset_index(drop=True)
sensor_3_m_1 = sensor_3_m_1.reset_index(drop=True)
sensor_3_m_2 = sensor_3_m_2.reset_index(drop=True)
sensor_3_m_3 = sensor_3_m_3.reset_index(drop=True)
sensor_3_m_4 = sensor_3_m_4.reset_index(drop=True)

print('sensor_3: ', sensor_3.shape)
print('sensor_3_m_1: ', sensor_3_m_1.shape)
print('sensor_3_m_2: ', sensor_3_m_2.shape)
print('sensor_3_m_3: ', sensor_3_m_3.shape)
print('sensor_3_m_4: ', sensor_3_m_4.shape)
```

```
sensor_3:  (4608, 38)
sensor_3_m_1:  (4608, 38)
sensor_3_m_2:  (4608, 38)
sensor_3_m_3:  (4608, 38)
sensor_3_m_4:  (4608, 38)
```

1.4.3 Generate the matrix as per the specifications in the paper.

Each row will have the measurements of traffic from the sensor of interest and the m-closest sensors. - The rows will include traffic information of the t-1, t-2,..., t-r observations. - The rows are organized by sensor, and by timestep: [sensor of interest @ t-1,…,sensor of interest @ t-r, … , m-closest sensor @ t-1,…,m-closest sensor @ t-r]

[ ]:
```
r = 3  # set r to any value

# Minimum number of rows across all dataframes
min_rows = min(sensor_3.shape[0], sensor_3_m_1.shape[0], sensor_3_m_2.shape[0],␣
↪sensor_3_m_3.shape[0], sensor_3_m_4.shape[0])

# Initialize an empty list to store all row vectors
row_vectors = []

# Iterate over each index from r to min_rows
for i in tqdm(range(r, min_rows-1)):
    # Generate a list of indexes you're interested in. In this case, it's [i-r,␣
↪i-r+1, ..., i]

    # ind = list(range(i-r, i+1))
    # indexes = list(range(i-r+1, i+1))

    ind = list(range(i+1, i-r, -1))
    indexes = list(range(i, i-r, -1))


    # Get the desired elements
    sensor_3_traffic = sensor_3.loc[ind, '10'].values
    sensor_3_m_1_traffic = sensor_3_m_1.loc[indexes, '10'].values
    sensor_3_m_2_traffic = sensor_3_m_2.loc[indexes, '10'].values
    sensor_3_m_3_traffic = sensor_3_m_3.loc[indexes, '10'].values
    sensor_3_m_4_traffic = sensor_3_m_4.loc[indexes, '10'].values

    # Concatenate them into a 1xr*5 row vector
    row_vector = np.concatenate([sensor_3_traffic, sensor_3_m_1_traffic,␣
↪sensor_3_m_2_traffic, sensor_3_m_3_traffic, sensor_3_m_4_traffic])

    # Append the row vector to our list
    row_vectors.append(row_vector)

# Convert our list of row vectors into a 2D numpy array
traffic_764632 = pd.DataFrame(row_vectors)
traffic_764632
```

```
100%|      | 4604/4604 [00:07<00:00, 622.11it/s]
```

[ ]:
```
       0      1      2      3      4      5      6      7      8      9  \
0  224.0  258.0  228.0  205.0  487.0  435.0  447.0  478.0  436.0  379.0
1  248.0  224.0  258.0  228.0  540.0  487.0  435.0  462.0  478.0  436.0
```

```
2     299.0  248.0  224.0  258.0  539.0  540.0  487.0  450.0  462.0  478.0
3     279.0  299.0  248.0  224.0  518.0  539.0  540.0  457.0  450.0  462.0
4     307.0  279.0  299.0  248.0  489.0  518.0  539.0  409.0  457.0  450.0
…     …    …    …    …    …    …    …    …    …    …
4599  403.0  346.0  419.0  379.0  417.0  444.0  491.0  464.0  412.0  420.0
4600  406.0  403.0  346.0  419.0  459.0  417.0  444.0  394.0  464.0  412.0
4601  348.0  406.0  403.0  346.0  397.0  459.0  417.0  425.0  394.0  464.0
4602  375.0  348.0  406.0  403.0  445.0  397.0  459.0  404.0  425.0  394.0
4603  343.0  375.0  348.0  406.0  404.0  445.0  397.0  426.0  404.0  425.0

         10     11     12     13     14     15
0     567.0  557.0  568.0  452.0  434.0  418.0
1     604.0  567.0  557.0  465.0  452.0  434.0
2     593.0  604.0  567.0  515.0  465.0  452.0
3     588.0  593.0  604.0  396.0  515.0  465.0
4     580.0  588.0  593.0  463.0  396.0  515.0
…     …    …    …    …    …    …
4599  439.0  482.0  496.0  338.0  384.0  407.0
4600  440.0  439.0  482.0  390.0  338.0  384.0
4601  453.0  440.0  439.0  332.0  390.0  338.0
4602  435.0  453.0  440.0  382.0  332.0  390.0
4603  395.0  435.0  453.0  324.0  382.0  332.0

[4604 rows x 16 columns]
```

1.4.4 Tests - Sensor 764632

Estimation

```
[ ]: ############################################################################################
####    Definition of "Hyperparameters"
############################################################################################
days = 5
t = int( (60/5)*12 * days)

#t = traffic_737433.shape[0] # Number of points to be tested on, and times the
 ↪coefficients will be updated.
        # The coefficients w are calculatd for each of these points, the idea
 ↪is to simulate an on-line stream of data.

X = traffic_764632.iloc[:t, 1:]

"""
    Update this if using more input variables, # the function is handling a
 ↪15-dimensional, second order polynomial.
"""
```

```
polynomial_degree = 2
"""
    USING Total (vs. Max) EXPANCTION TERMS
    Number of terms in the weights matrix:

"""



target_values = traffic_764632.iloc[:t, 0]
# Generate Y as target_values (real Y's)

# Initialize weights and estimations
weights_over_time = []
y_hat = []


noise = np.random.normal(scale= 10 , size=(t))

##########################################################################################
####    Online estimation of coefficients
##########################################################################################


for i in tqdm(range(t)):
    x_i = X.iloc[i, :]

    # Normalize and scale to -1,1 the input:
    x_i = ( x_i - x_i.min() ) / ( x_i.max() - x_i.min() )*2 - 1

    weight_predictions, _ = online_kaczmarz_legendre_multiple(x_i,␣
 ↪[target_values[i] + noise[i]], polynomial_degree)

    #De-normalize rescale the weights:
    weight_predictions = (((weight_predictions + 1) / 2)  * ( x_i.max() - x_i.
 ↪min() ) + x_i.min() )

    # Use predicted weights to compute y_hat
    y_predictions = generate_multi_legendre_design_matrix(x_i,␣
 ↪polynomial_degree) @ weight_predictions.T

    # Store the values of predicted y and estimated weights
    y_hat.append(y_predictions)
    weights_over_time.append(weight_predictions)

print("Done carajo")
```

```
100%|     | 720/720 [31:14<00:00,  2.60s/it]
```

```
Done carajo
```

[ ]:
```python
weights_over_time = np.array(weights_over_time)
y_hat_ = np.array(y_hat) #########
print('y_hat: ', y_hat_.shape)

print('weights_over_time: ', weights_over_time.shape)

weight = weights_over_time[-1]
Weights = pd.DataFrame({
    'Estimated_Weights': weight
})

# Style DataFrame
# Weights.style.format("{:.4f}")

print('Weights: ', Weights.shape)
```

```
y_hat:  (720, 1)
weights_over_time:  (720, 136)
Weights:  (136, 1)
```

Results

[ ]:
```python
###############################################################################################
####    Results
###############################################################################################

Predicted_Values = y_hat_.flatten()
print('Predicted_Values: ', Predicted_Values.shape)
Real_Values = target_values
print('Real_Values: ', Real_Values.shape)
difference = (Real_Values-Predicted_Values).T


Y = pd.DataFrame({
    'Predicted_Values': Predicted_Values,
    'Real_Values': Real_Values,
    'Difference': difference
})

# Style DataFrame
Y[-10:].style.format("{:.4f}")
```

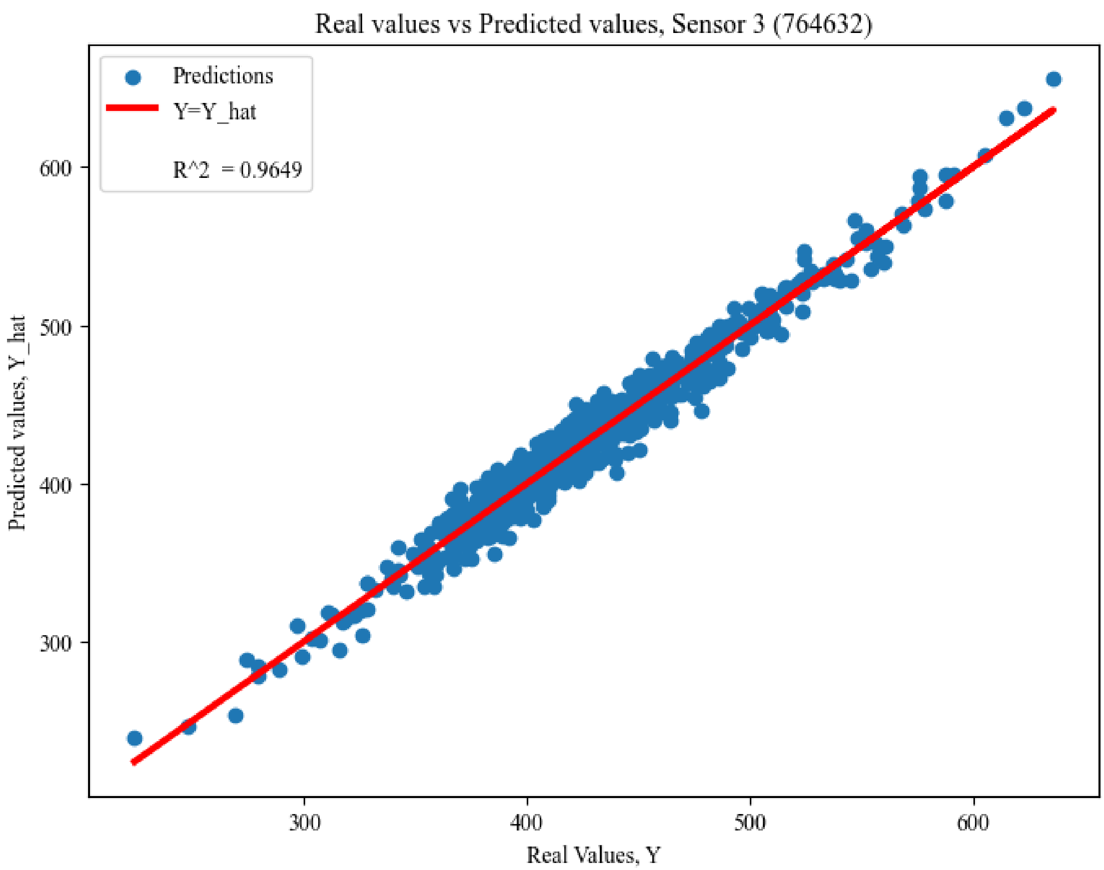
```
Predicted_Values:  (720,)
Real_Values:  (720,)
```

[ ]: <pandas.io.formats.style.Styler at 0x161b5b7f0>

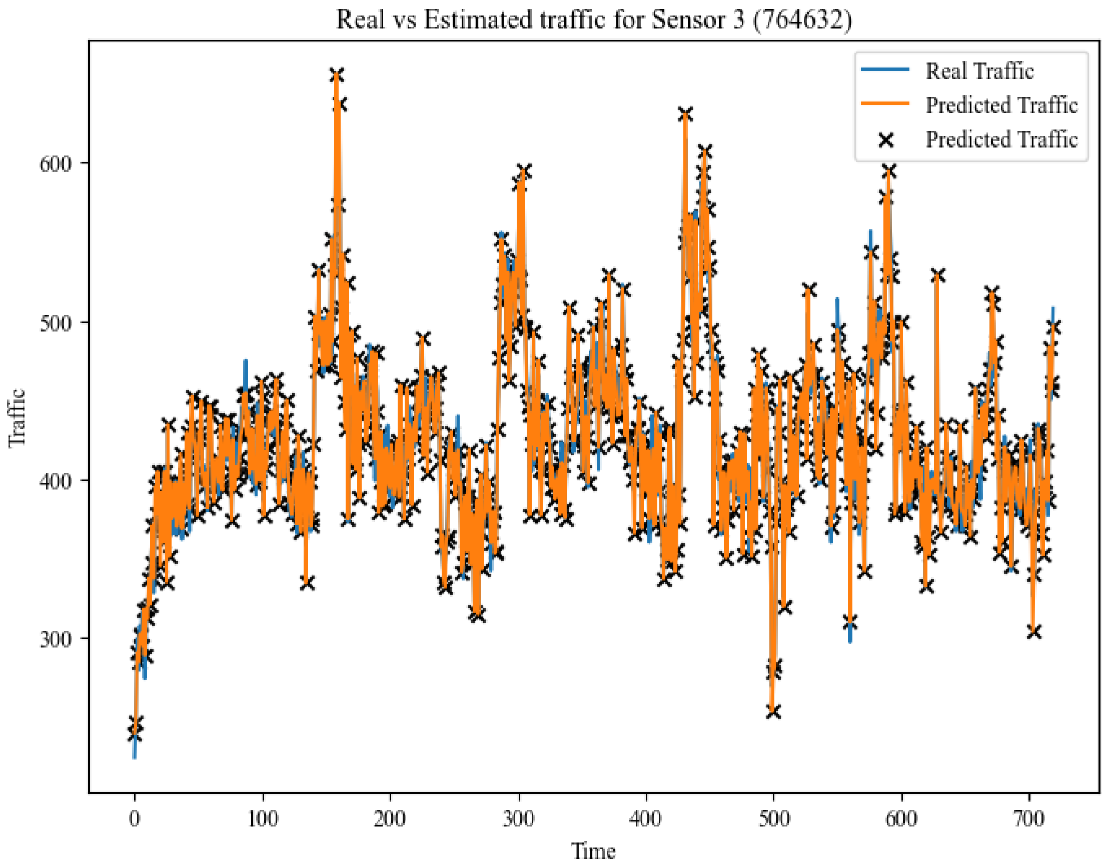
Plots

[ ]:
```python
# Plot the accuracy
plt.figure(figsize=(8, 6))

r2 = r2_score(target_values, y_hat_)
print('r2: ', r2)

plt.scatter(target_values, y_hat_, label='Predictions')
plt.plot(target_values, target_values, color='red', linewidth = 3,␣
 ↪label=f'Y=Y_hat')
plt.scatter(target_values, target_values, marker='', label=f'\nR^2  = {r2:.4f}')

plt.xlabel('Real Values, Y')
plt.ylabel('Predicted values, Y_hat')
plt.title('Real values vs Predicted values, Sensor 3 (764632)')
plt.legend()

plt.show()
```

```
r2:  0.9649484225792238
```



[ ]:
```
# Plot the traffic
n = t

plt.figure(figsize=(8, 6))

plt.plot(Y['Real_Values'][-n:],label='Real Traffic')
plt.plot(Y['Predicted_Values'][-n:],label='Predicted Traffic')
plt.scatter(Y[-n:].index, Y['Predicted_Values'][-n:], marker='x', color =␣
↪'black',label='Predicted Traffic')

plt.xlabel('Time')
plt.ylabel('Traffic')
plt.title('Real vs Estimated traffic for Sensor 3 (764632)')
plt.legend()

plt.savefig('./Figures/traffic_sensor_3.png')

plt.show()
```



[ ]:
```
s1 = pd.read_csv('./Data/sensor_interest_1 737433.txt')
s2 = pd.read_csv('./Data/sensor_interest_2 764848.txt')
```

[ ]:
```

```