

# Multidimensional Visualization

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## Introduction

- It is common to classify multidimensional visualization techniques into
  - **Point-based** techniques
  - **Line-based** techniques
  - **Region-based** techniques

The techniques discussed in this module can only be [used](#) for [lists and data tables/frames](#) that do not have spatial attribute(s)

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## Point-based Techniques

- Point-based techniques are **projections** of m-dimensional instances into a p-dimensional visual space ( $p=\{1,2,3\}$ )
  - And then, a **glyph** is associated to each point

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## Scatterplots and Scatterplot Matrices

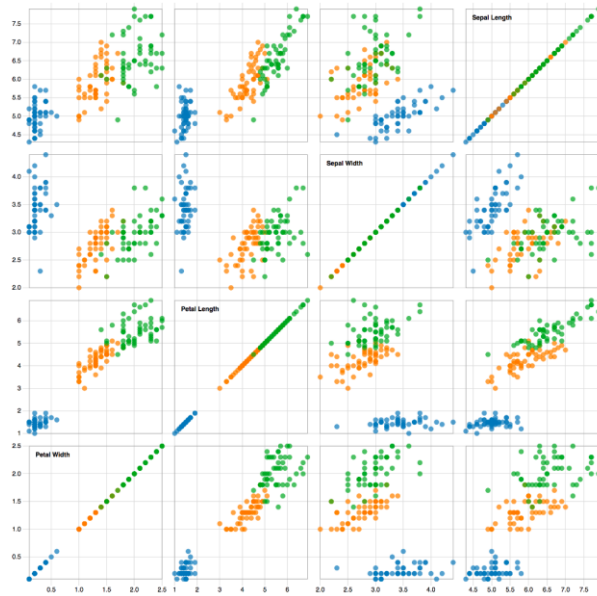
- **Scatterplots** are the **most common** visual representations, but as dimensionality increases new strategies have to be sought
  - **Dimension selection**: manually or using some algorithm
  - **Dimensionality reduction**: using techniques such as PCA
  - **Incorporate dimensions**: map other dimensions to graphical elements
  - **Multiple displays**: showing several displays side-by-side

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## Scatterplots and Scatterplot Matrices

Scatterplot matrices are grids of scatterplots that shows all combinations of the  $n$ -dimensions ( $n^2$ )



<https://bl.ocks.org/mbostock/4063663>

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## Scatterplots and Scatterplot Matrices

### • Papers to read

- T. N. Dang and L. Wilkinson, "ScagExplorer: Exploring Scatterplots by Their Scagnostics," 2014 IEEE Pacific Visualization Symposium, Yokohama, 2014, pp. 73-80.
- N. Elmqvist, P. Dragicevic and J. D. Fekete, "Rolling the Dice: Multidimensional Visual Exploration using Scatterplot Matrix Navigation," in IEEE Transactions on Visualization and Computer Graphics, vol. 14, no. 6, pp. 1539-1148, Nov.-Dec. 2008. (<https://www.youtube.com/watch?v=E1birsp9iYk>)

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## Multidimensional Projections

- Note that the goal of **projection** methods is to **keep**, as much as possible, the **relationships** of the  $m$ -dimensional space into the  $p$ -dimensional space of the visualization.
  - For instance, similarity relationships, neighborhood, etc.
- The result is a set of **points** (glyphs) on the **plane**
  - Close points indicate related instances, distant points indicate non-related objects

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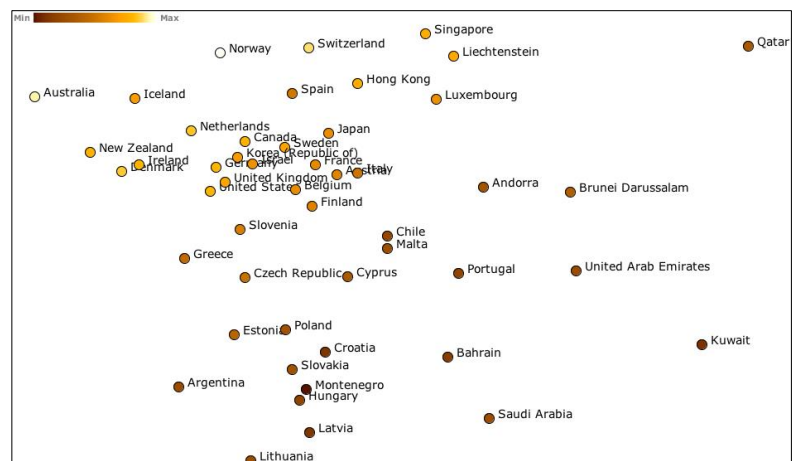
## Multidimensional Projections

- Example: Human Development Index of 2006

### Variables:

- Life expectancy at birth
- Expected years of schooling
- Mean years of schooling
- Gross national income (GNI) per capita

Can you identify the axis?

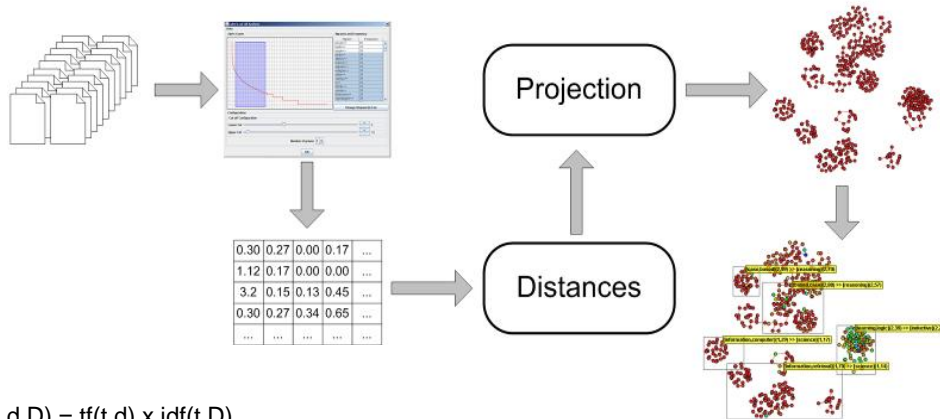


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## Multidimensional Projection for documents

- Example of Bag-of-words representation of document collections



$$\text{TF-IDF}(t,d,D) = \text{tf}(t,d) \times \text{idf}(t,D)$$

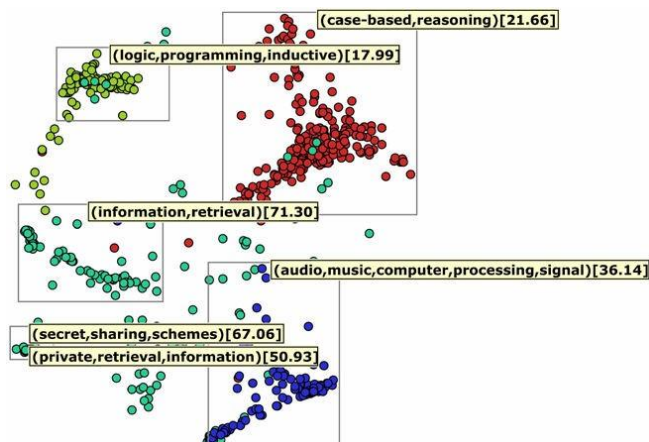
$$= (\#t \text{ in } d) \times \log (|D| / |\{d \text{ in } D: t \text{ in } d\}|)$$

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## Multidimensional Projection

- Example: collection of scientific papers



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## Point-based Techniques

- **Radviz** is another example of a point-based technique
  - For a  $m$ -dimensional dataset,  $m$  **anchors** are created and distributed over a **circumference**
- The position  $A_j$  of the  $j^{\text{th}}$  anchor ( $j=[0,m-1]$ ) is calculated as

$$A_j^x = r * \cos(j * 360/m) + cx$$

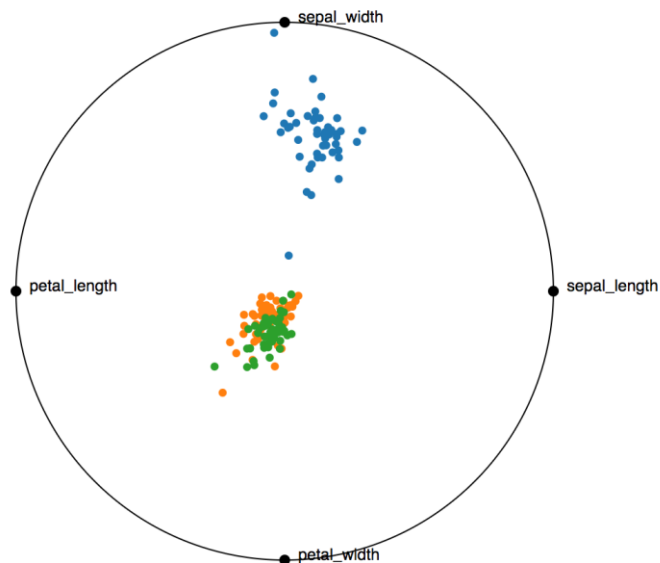
$$A_j^y = r * \sin(j * 360/m) + cy$$

- where  $r=1$  is the circumference radius and  $c=(cx,cy)$  its center

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## RadViz



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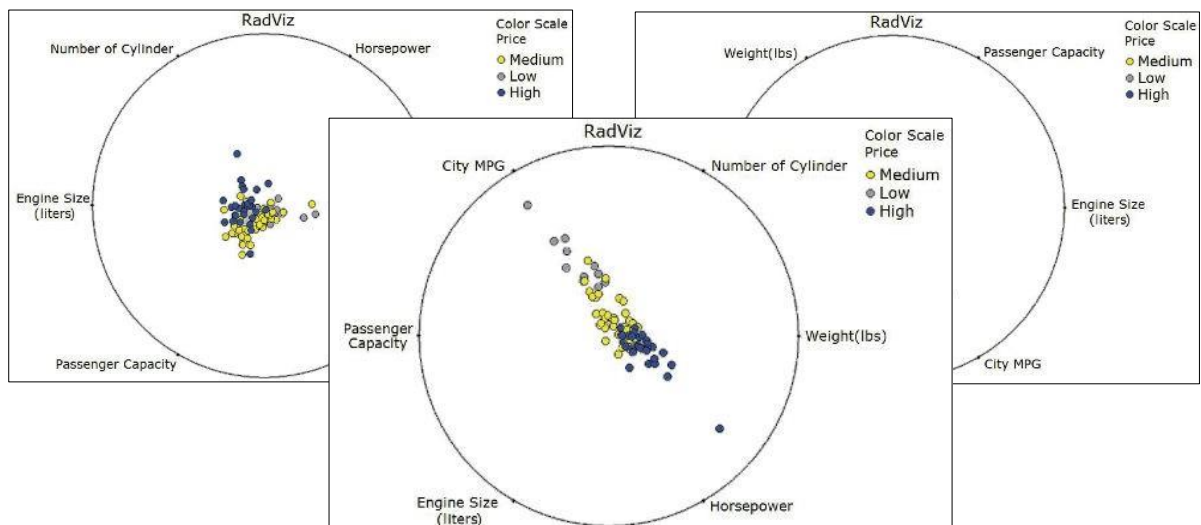
## RadViz

- Observe that the **order** of the **anchors** dictates the **final result**
  - **Interaction** can help users
  - There are techniques that **maximize** the points' **spread**

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## RadViz

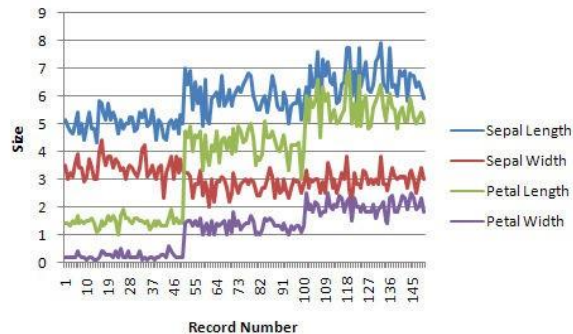


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## Line-based Techniques

- On **line-based** techniques, the data patterns are represented through **line crossing**, **curvatures**, etc.



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## Line-based Techniques

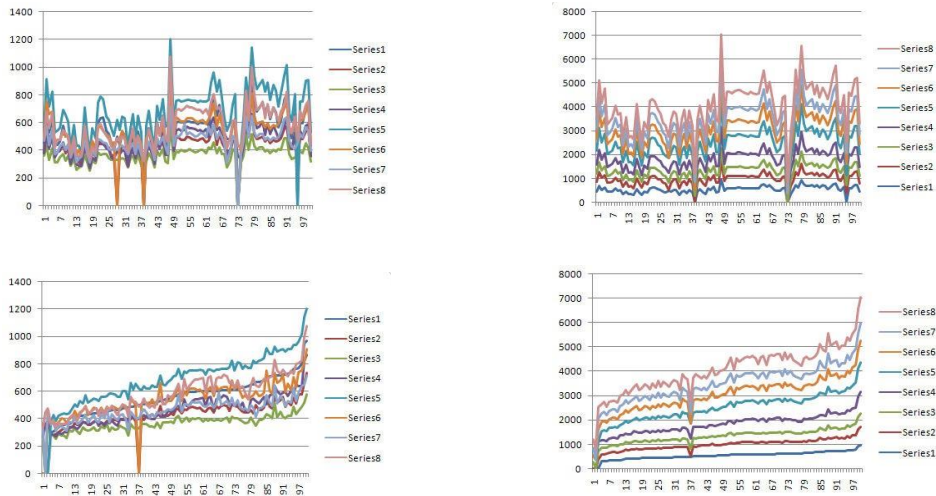
- **Superimposition** should not be used on datasets with many dimensions due to **occlusion problems**
- Potential solutions
  - **Stack up the lines** considering the previous dimension (difficult to evaluate the real value)
  - **Order** the instances considering one **dimension**

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## Line-based Techniques



Example: Professor salaries in 100 universities.

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## Line-based Techniques

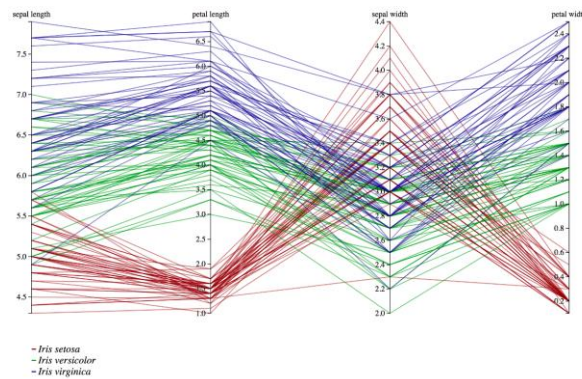
- These last techniques can only be used if the units of the **dimensions** are **related in all series**
- Solutions
  - Multiple coordinated stacked graphs
  - Multiple vertical axes can be used for different dimensions

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## Parallel Coordinates

- On the [parallel coordinates](#), the axes are parallel (non orthogonal) and the data instances are represented as [polylines](#) that cross the axes on positions proportional to the value on the dimension



<http://mbostock.github.io/d3/talk/20111116/iris-parallel.html>

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## Parallel Coordinates

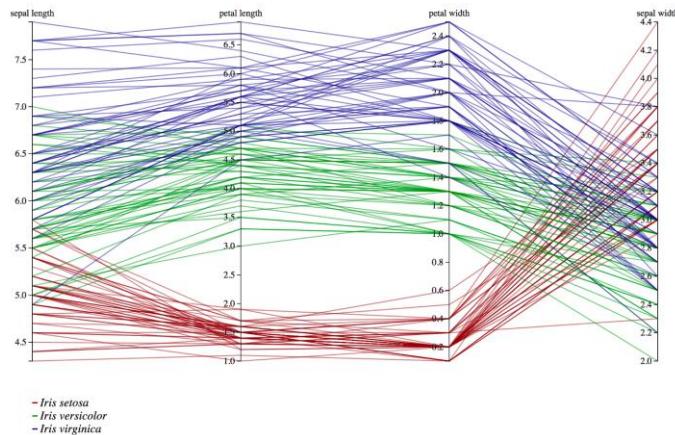
- With this visual representation it is possible to
  - Locate groups of [similar](#) polylines
  - Locate [crossing](#) points
  - Locate [different](#) polylines

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## Parallel Coordinates

- However, correlating **non-consecutive** dimensions is **difficult**
  - Interaction can be used to **reorder** the dimensions



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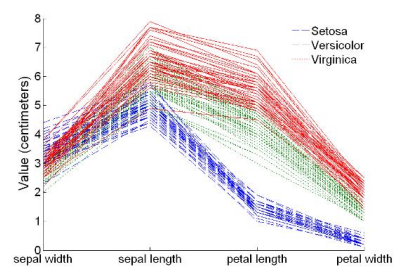
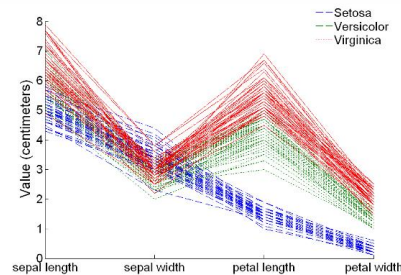
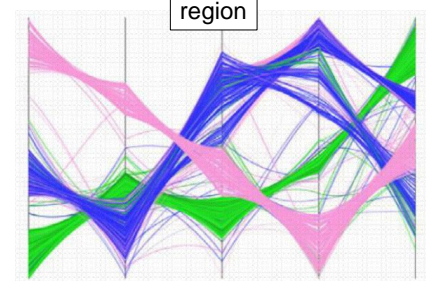
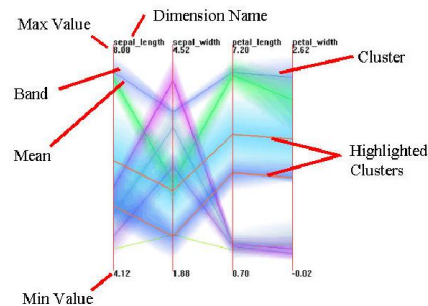
## Parallel Coordinates

- There are many **variants** of the original parallel coordinates technique
  - **Hierarchical parallel coordinates** show groups, not individual data instances
  - **Semi-transparency** can be used to show groups on large databases
  - Grouping, re-ordering, and different spacing based on **correlation**
  - Using **histograms** on the axes can help on interpreting the data distribution
  - **Curves** can be used on the crossing points to improve the **axes continuity**

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## Parallel Coordinates



Reordering the vertical axes

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## Parallel Coordinates

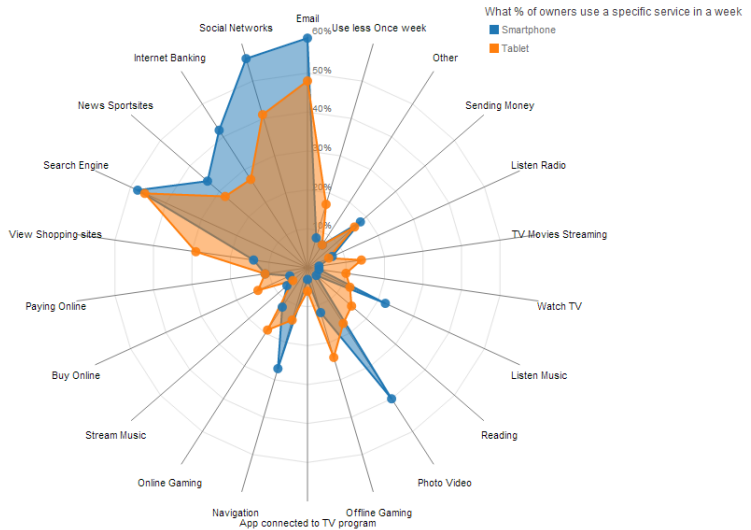
- For automatic [axes reordering](#) read
  - A.O. Artero, M.C.F.D. Oliveira, H. Levkowitz, **Enhanced high dimensional data visualization through dimension reduction and attribute arrangement**, in: Proceedings of the conference on Information Visualization, IEEE Computer Society, 2006, pp. 707–712.
  - Liang Fu Lu, Mao Lin Huang, Jinson Zhang, **Two axes re-ordering methods in parallel coordinates plots**, In Journal of Visual Languages & Computing, Volume 33, Pages 3-12, 2016.

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## Radial Axes Techniques

- For each technique with parallel axes there is a technique with radial axes

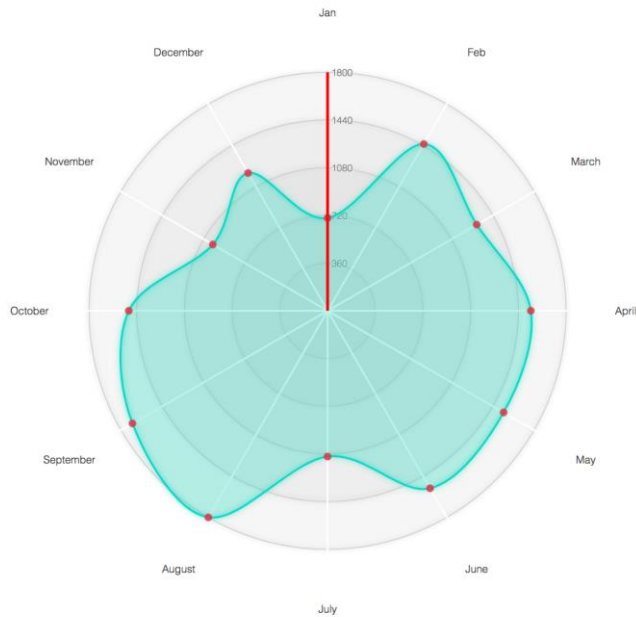


<http://bl.ocks.org/nbremer/6506614>

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## Radial Axes Techniques



The circular line graph is one example indicated for cyclic events

<http://bl.ocks.org/tezzutezzu/c9d8706587e8f5b5d72084b083b502f8>

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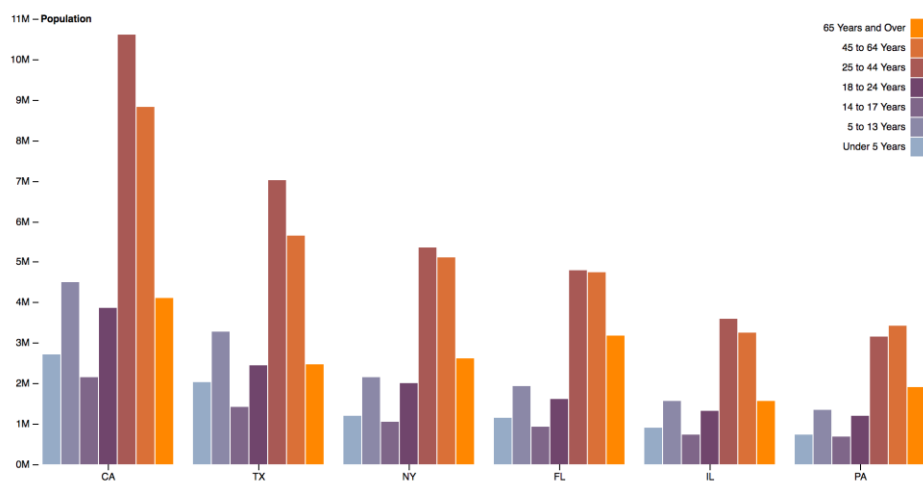
## Bar Graphs and Histograms

- One of the most **common** visual representation are the **bar graphs**
  - Stacked Bar Charts
  - Grouped Bar Charts
  - 3D Histogram

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## Grouped Bar Chart

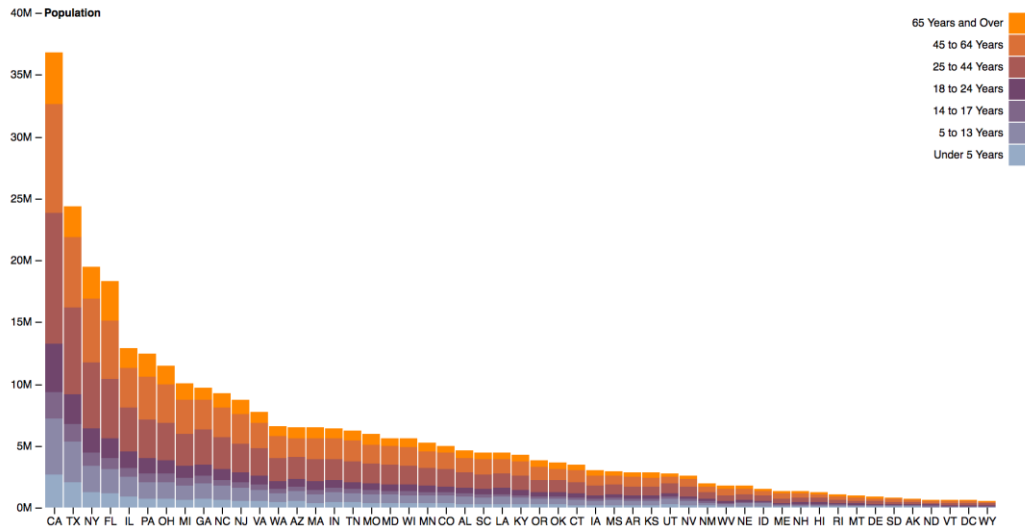


<https://bl.ocks.org/mbostock/3887051>

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## Stacked Bar Chart



<https://bl.ocks.org/mbostock/3886208>

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## Bar Graphs and Histograms

- If the task is to understand the [data distribution](#), then [histograms](#) can be used.
- It is [simple for nominal values](#) or few [integers](#) – the same number of bars and distinct values

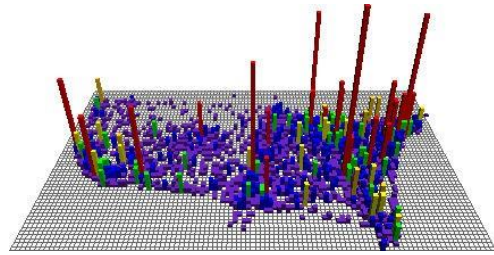
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## Bar Graphs and Histograms

- The 3D version of bar graphs is called **Cityscape**
  - It is also called **3D histogram**
  - Often used for **georeferenced** data



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## Tabular Displays

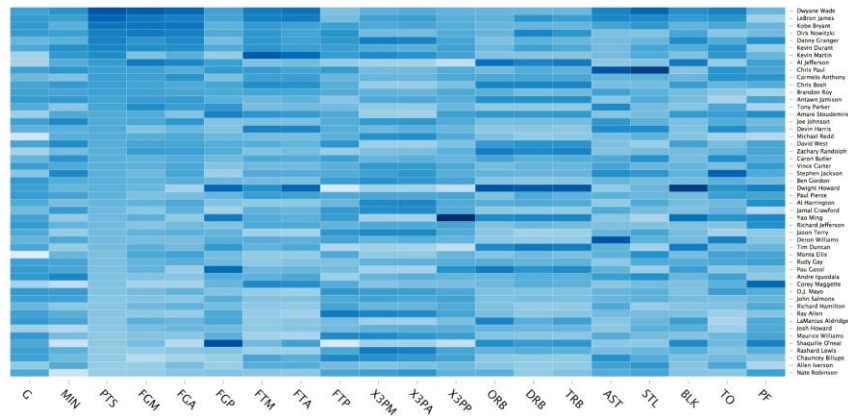
- It is **easy to generate** visual representations if the **data** is organized in a **table**

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## Tabular Displays

- **Heatmaps** map data values into rectangles filled with colors given by a color scale (colormap)



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## Tabular Displays

- **Permutation** and **reordering** can be used to **improve** the visual representation, changing the lines and columns positions

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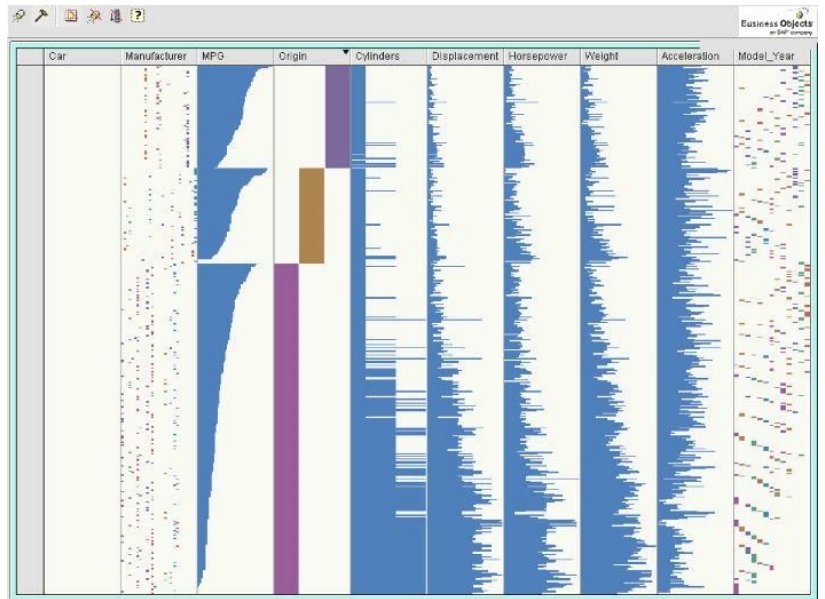
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## Tabular Displays

- **TableLens** combines these ideas and add mechanisms to visualize all the table while providing a detailed view (sorted by manufacturer and then by MPG)



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## Dense Pixel Displays

- **Dense pixel display** techniques map each **data value to individual pixels**, creating a filled polygon to represent each data dimension
  - Employ all visual space
  - Each value defines a **pixel color**

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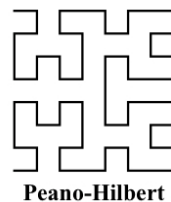
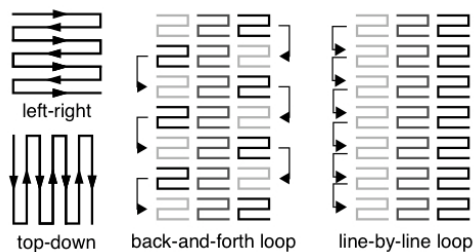
## Dense Pixel Displays

- The most basic approach create an **image per dimension**
- The elements are drawn so that **close values** are **place together** on the display
- An image can be created traversing the display from **left-to-right** (right-to-left)
- A **spiral traverse** can also be used
- **Space-filling recursive curves** can also be used (close elements in a list stay close on the display)

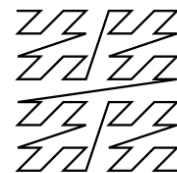
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## Dense Pixel Displays



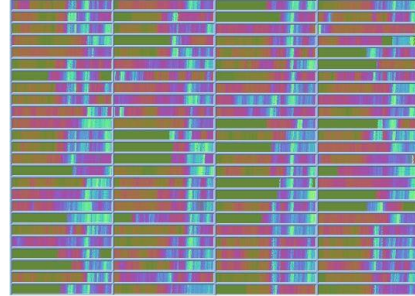
Peano-Hilbert



Morton

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## Dense Pixel Displays

- **Elements order** (whenever possible) can help on the identification of interesting patterns
  - Order **based on one dimension** help to reveal clusters in that dimension
  - Order **based on the distance to a vector** can help to reveal clusters on several dimensions

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## Combining Techniques

- Some techniques **combine the features** of two or more classes of techniques
  - **Glyphs** creation

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## Glyphs and Icons

- Different **types of mappings** can be used when creating glyphs
  - **One-to-one**: each data attribute is mapped to a distinct graphical attribute
  - **One-to-many**: redundant mapping is used to improve precision
  - **Many-to-one**: different attributes are mapped to a single graphical attribute using the space, orientation or other transformation to segregate them

Give examples!

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## Glyphs and Icons

- Some example of glyphs are
  - Profile – height and color of bars
  - Stars – size of rays emanating from the center
  - Anderson/metroglyphs – size of rays
  - Stick figures – size, angle, color
  - Trees – size, thickness, branch angles, etc.
  - Autoglyph – color
  - Boxes – height, width, depth, etc.
  - Hedgehogs – arrows in a vector field, varying thickness and orientation
  - Faces – size and position of eyes, nose, mouth, etc.
  - Arrows – size, length, color, etc.

See next slide

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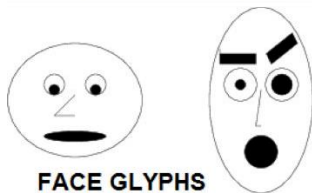
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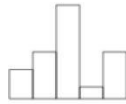
## Glyphs and Icons



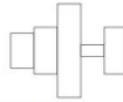
STARS AND  
METROGLYPHS



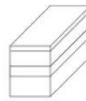
FACE GLYPHS



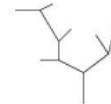
PROFILE GLYPHS



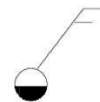
AUTOGLYPH/BOX GLYPH



STICKS AND TREES



ARROWS/WEATHERVANES



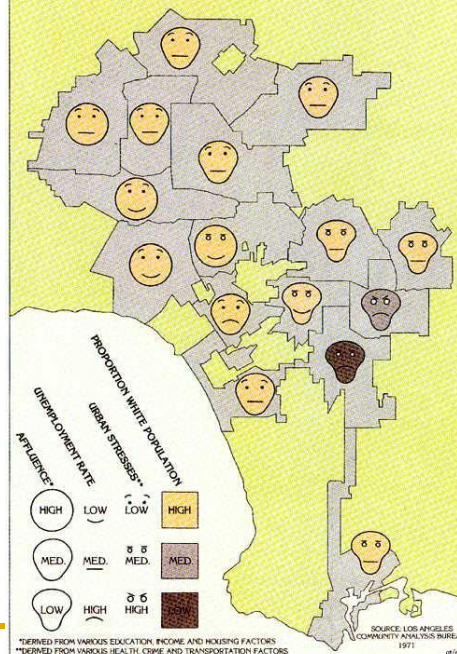
All convey information  
regarding several  
dimensions

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## Glyphs and Icons

### Life in Los Angeles



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## Glyphs and Icons

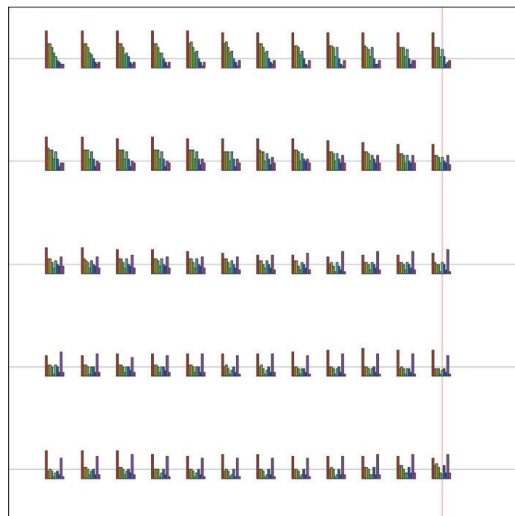
- There are three different strategies to [position the glyphs](#)
  - [Uniform](#): glyphs are scaled and positioned with equal spacing between them to fill the entire screen - avoid overlaps
  - [Data-driven](#): data values are used to determine the position of glyphs - can be positioned using scatterplots or multi-dimensional projections
  - [Structure-driven](#): If there is any implicit structure, such as cyclic or hierarchical, it can be used to position the glyphs

See next slides

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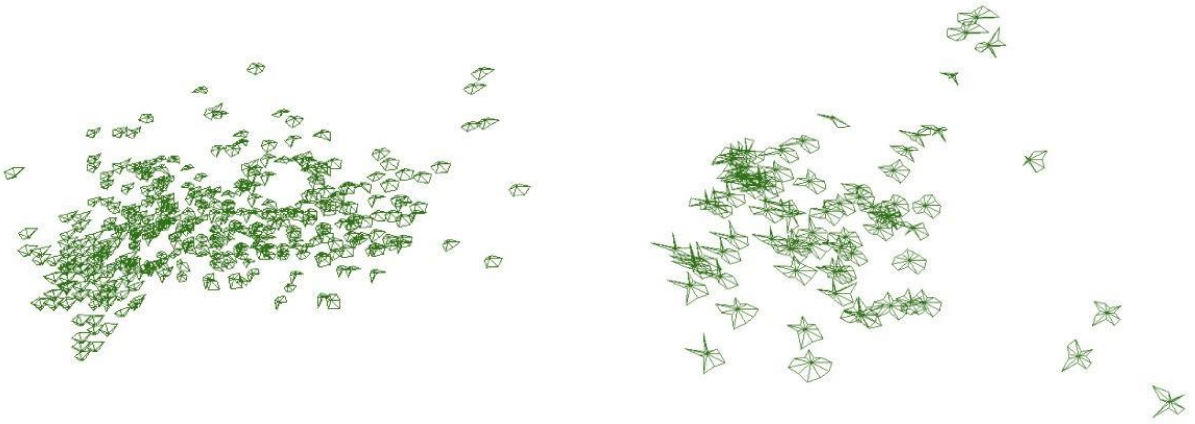
## Glyphs and Icons (uniform)



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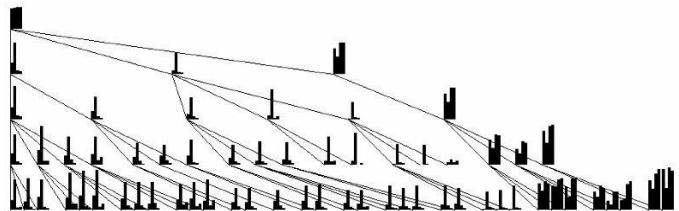
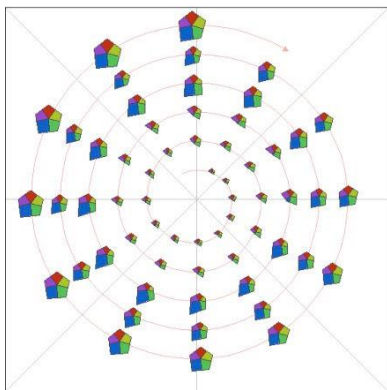
## Glyphs and Icons (data-driven)



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## Glyphs and Icons (structure-driven)



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## Reference

- Ward, M., Grinstein, G. G., Keim, D. **Interactive data visualization foundations, techniques, and applications**. Natick, Mass., A K Peters, 2010.