

# Extended Depth-of-Field with Speckle Based Super-resolution

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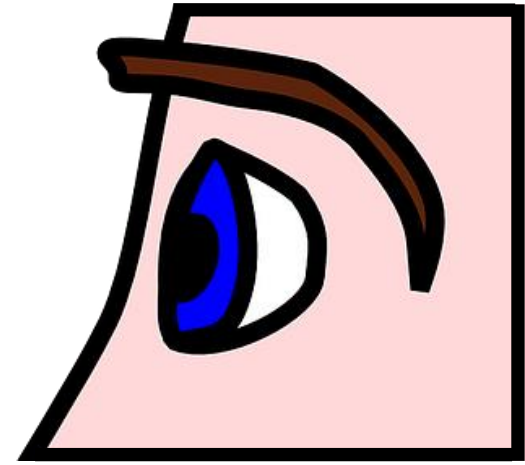
05/02/2019

# Challenge: HR-Biometrics with large DoF



$$\delta x \leq 20 \mu\text{m}$$

$$\delta t \leq 0.5 \text{ sec}$$



Depth-of-Field: 50 cm



f/16



f/11



f/8



f/5.6



f/4



f/2.8

Small aperture  
Stopped down

More Depth-of-Field  
Less Light

Less Depth-of-Field  
More Light

Large aperture  
Wide open



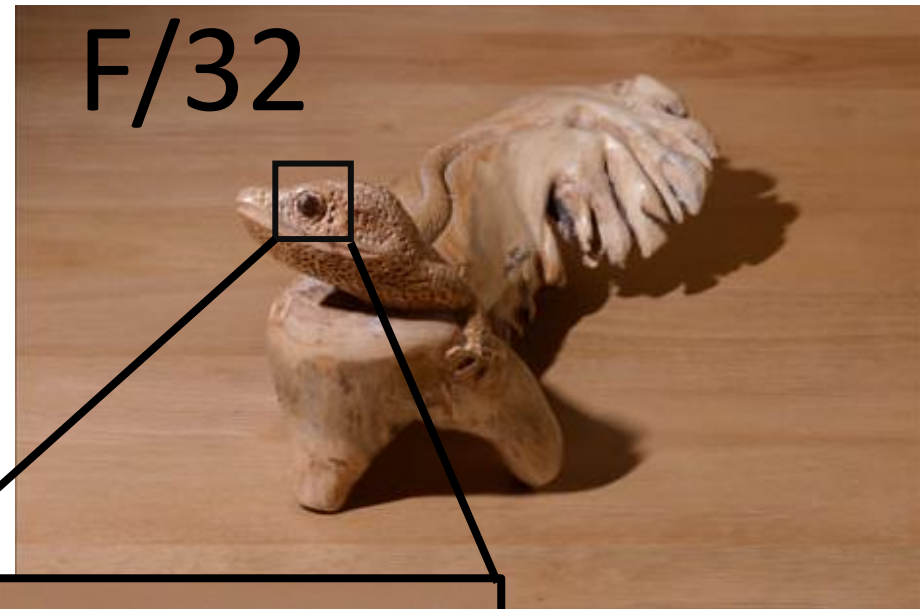
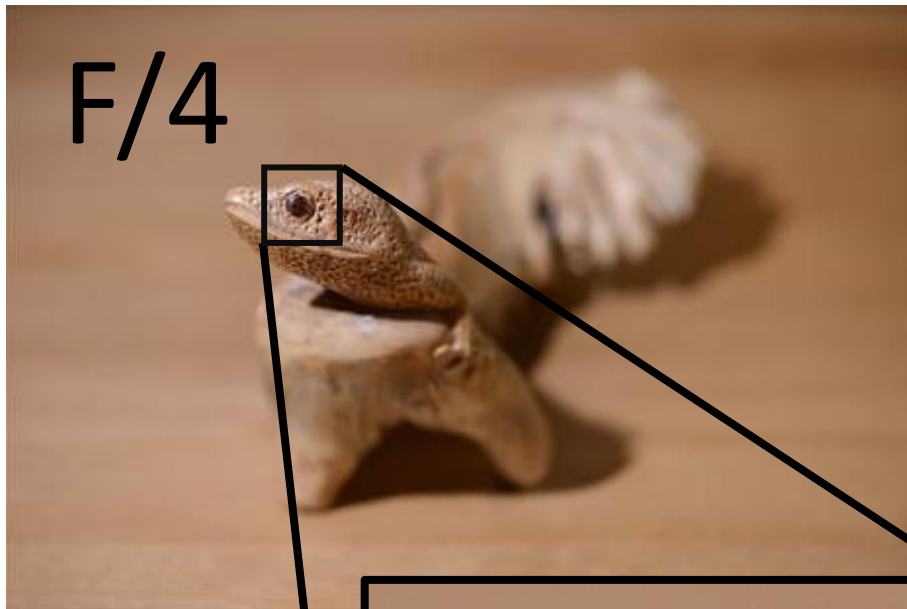
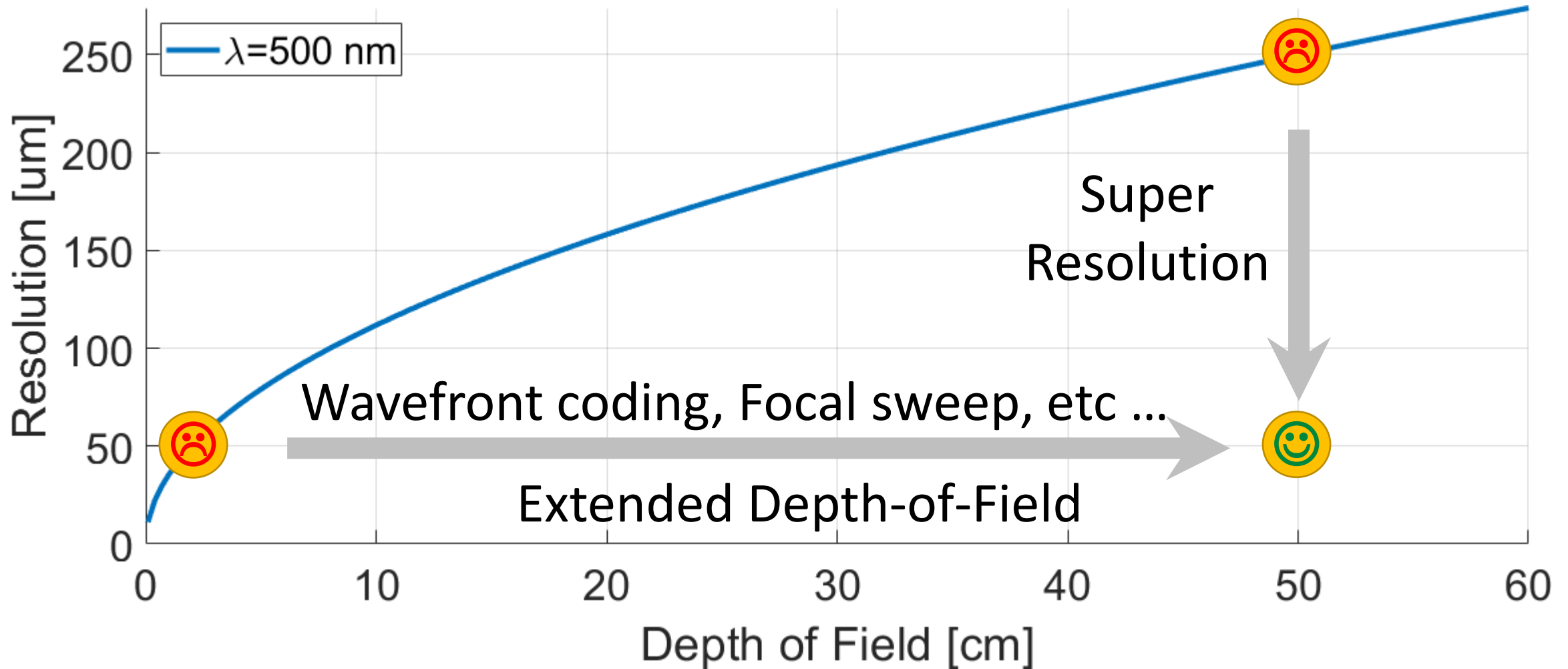


Image courtesy by Spencer Cox  
( <https://photographylife.com/what-is-diffraction-in-photography> )

Lateral Resolution

$$\delta x \sim \frac{\lambda}{\text{NA}}$$

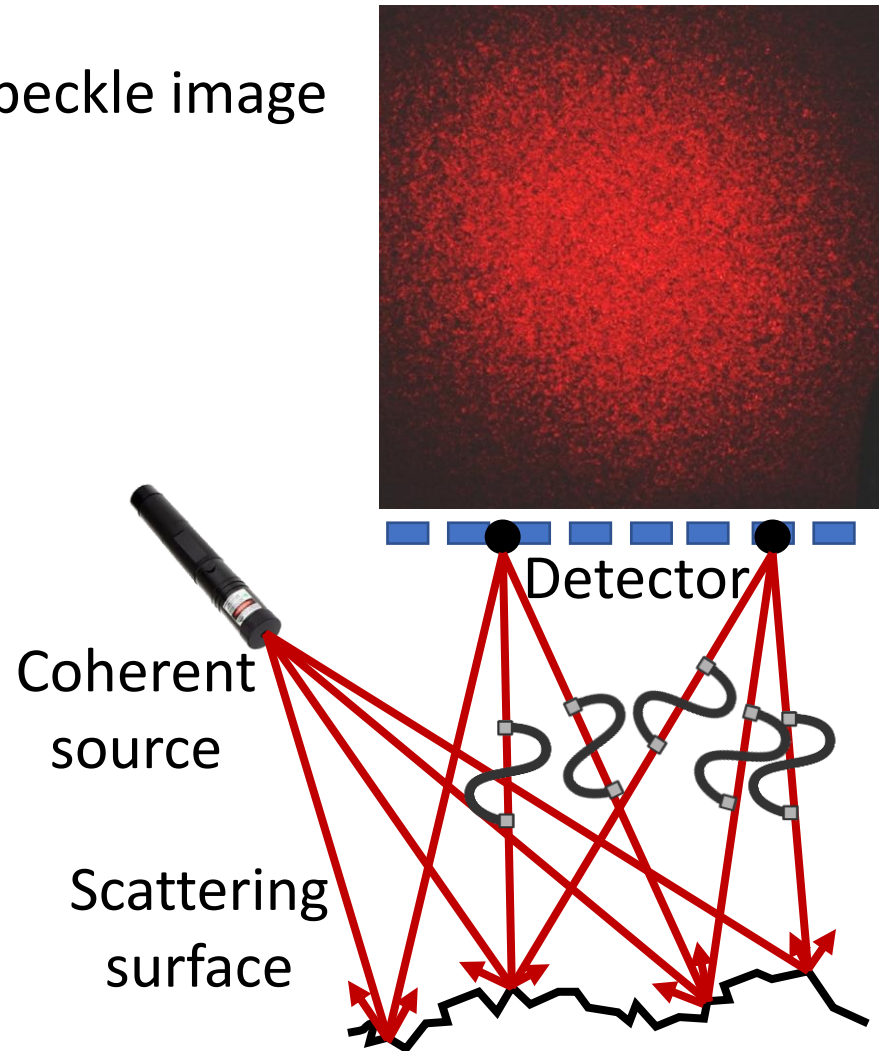
Depth-of-Field:  $\delta z \sim \frac{\lambda}{\text{NA}^2}$





# Speckle

Speckle image

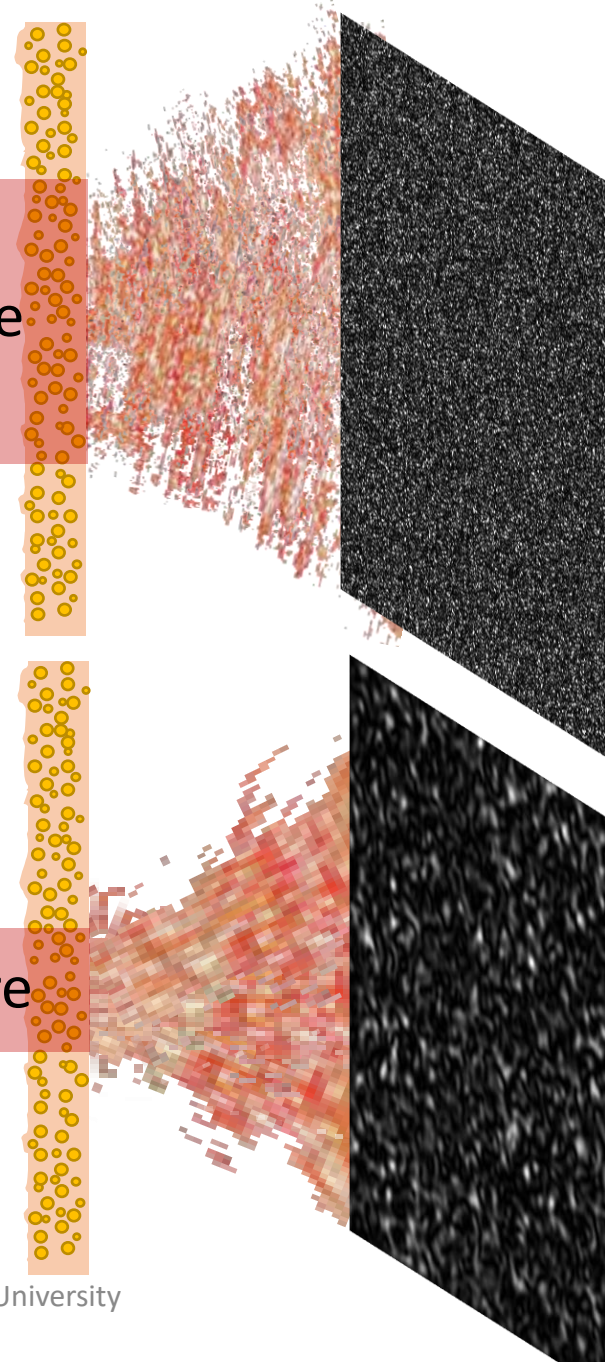


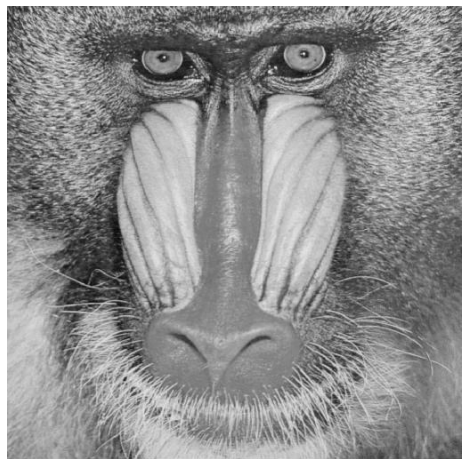
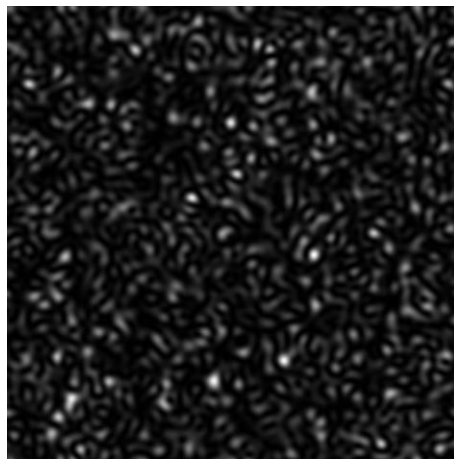
Large Aperture

$$\delta z \sim \frac{\lambda}{u^2}$$

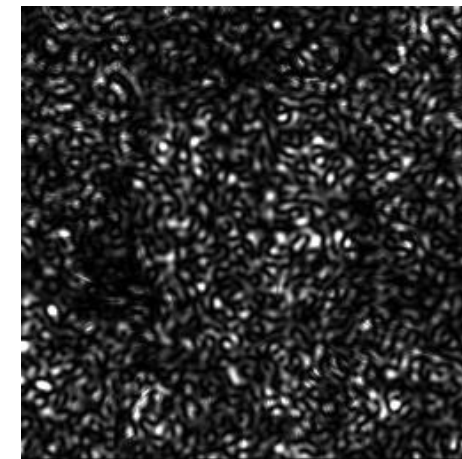
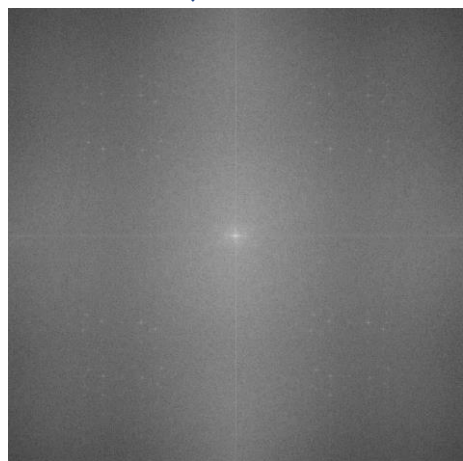
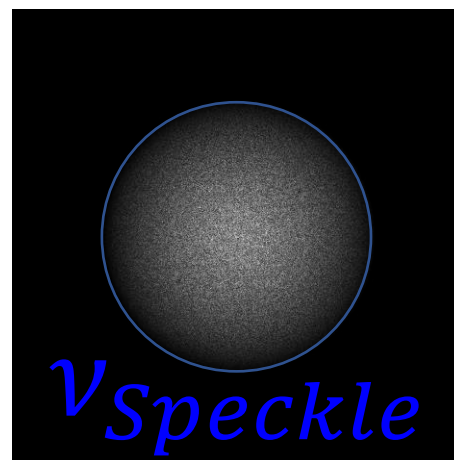
$$\delta x \sim \frac{\lambda}{u}$$

Small Aperture

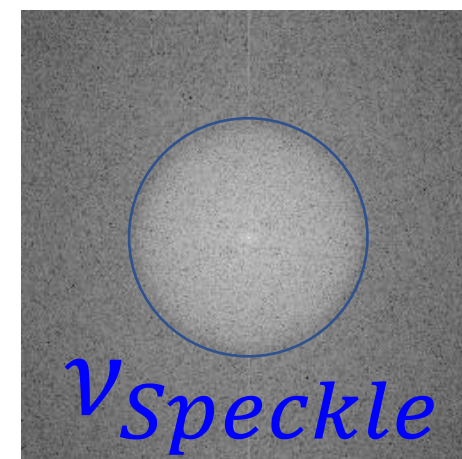


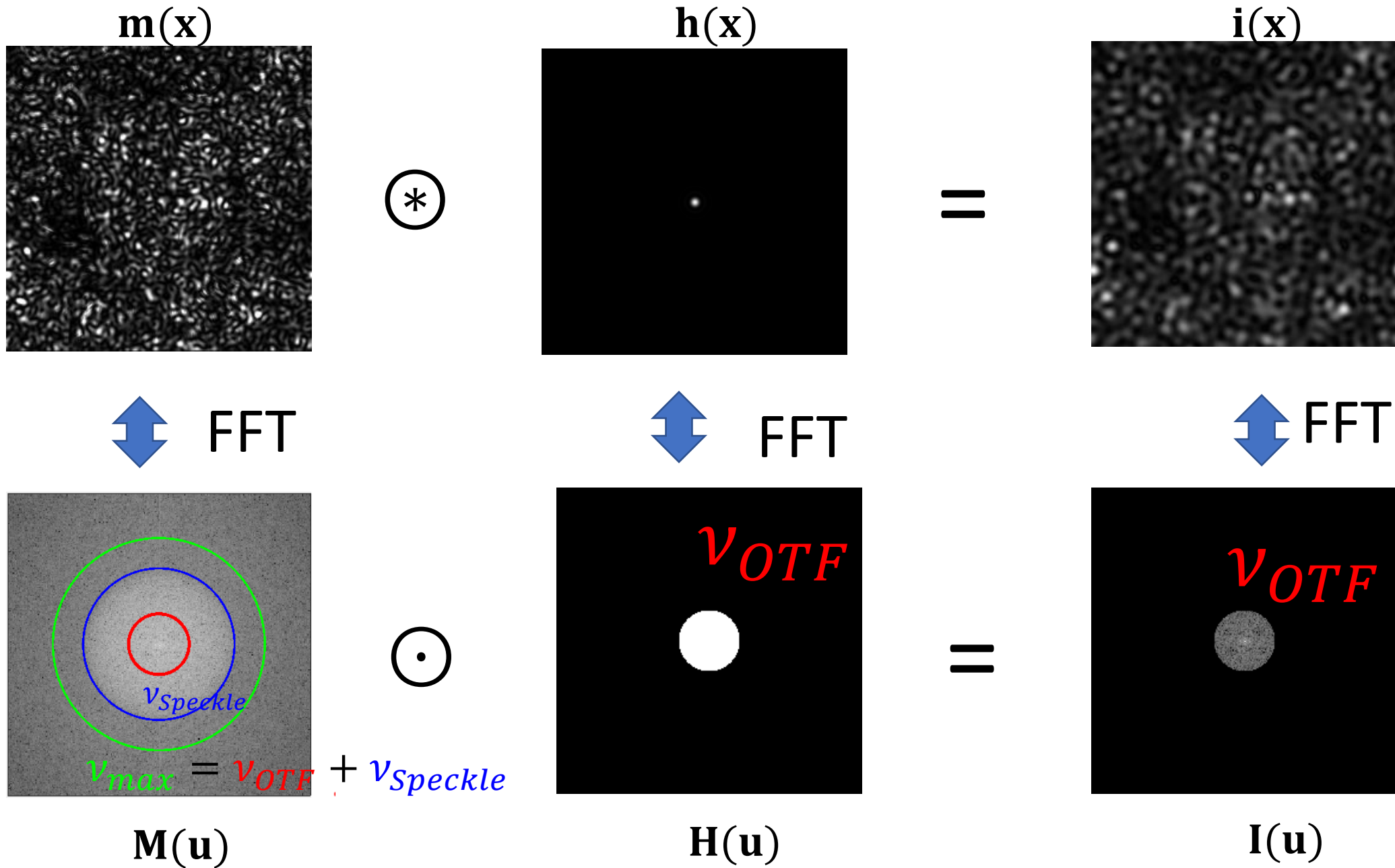
$\mathbf{o}(\mathbf{x})$  $\mathbf{p}(\mathbf{x})$ 

=

 $\mathbf{m}(\mathbf{x})$ 
 FFT

 FFT


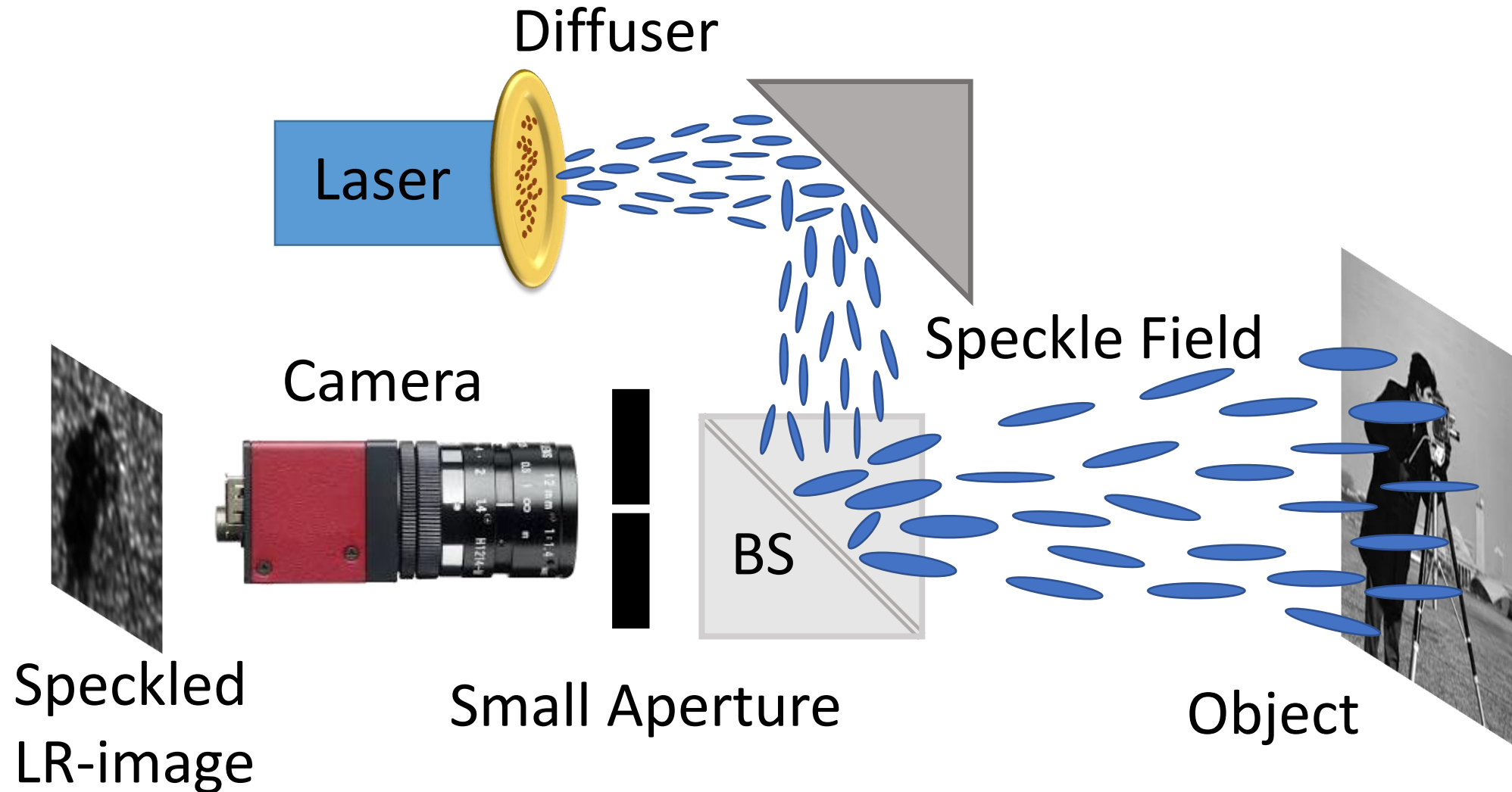
=

 FFT
 $\mathbf{O}(\mathbf{v})$  $\mathbf{P}(\mathbf{v})$  $\mathbf{M}(\mathbf{v})$

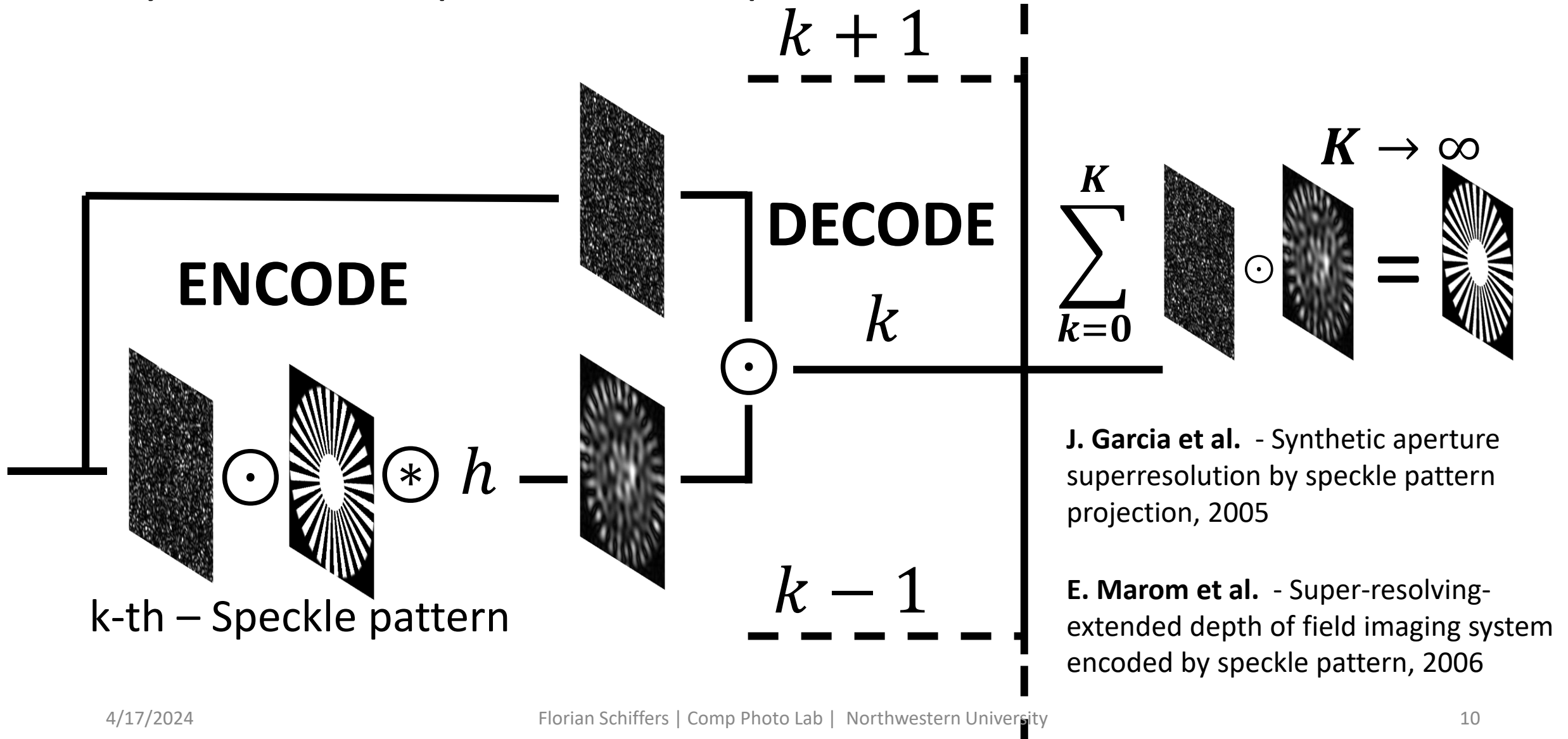




# Experimental Setup – 2D case

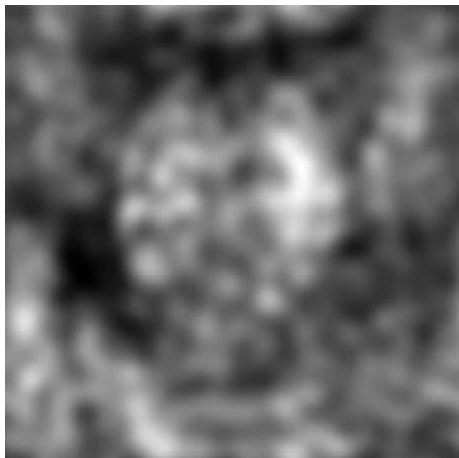


# Synthetic aperture super resolution

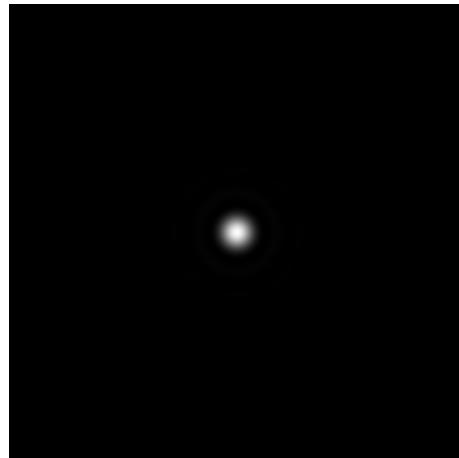


# Forward-Model

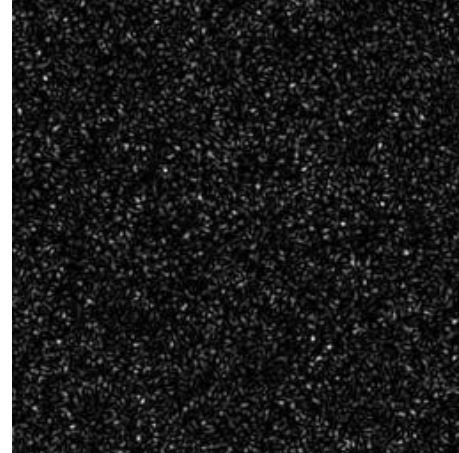
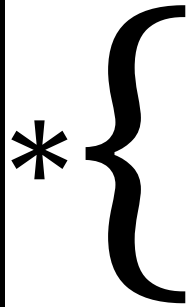
$$\mathbf{I}_k(\mathbf{x}) = \mathbf{h}(\mathbf{x}) * [\mathbf{p}_k(\mathbf{x}) \cdot \mathbf{o}(\mathbf{x})]$$



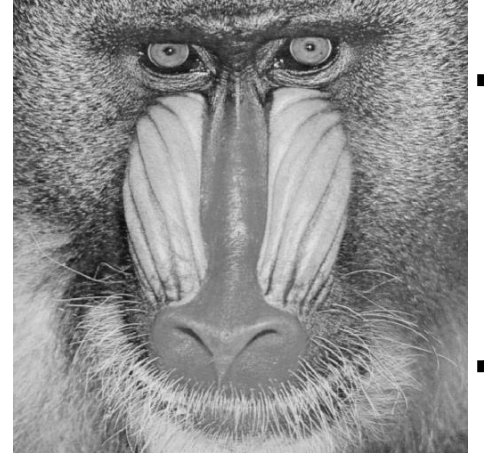
=



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Measurement

PSF

Speckle #K

Object

# Forward-Model – Inverse Problem

Continuous:  $\mathbf{I}_k(\mathbf{x}) = \mathbf{h}(\mathbf{x}) * [\mathbf{p}_k(\mathbf{x}) \cdot \mathbf{o}(\mathbf{x})]$

Discretized:  $\mathbf{I}_k = \mathbf{C} \cdot \mathbf{D}_k \cdot \mathbf{o}$

$$\begin{aligned} \mathbf{o}^* &= \arg \min_{\mathbf{o}} \sum_k \|\mathbf{I}_k - \mathbf{C} \mathbf{D}_k \mathbf{o}\|^2 \\ &= \left[ \sum_k \mathbf{D}_k^T \mathbf{C}^T \mathbf{C} \mathbf{D}_k \right]^{-1} \sum_k \mathbf{D}_k^T \mathbf{C}^T \mathbf{I}_k \end{aligned}$$



No Speckle:  $\mathbf{C}^T \mathbf{C} \longrightarrow$  Non-invertible

Speckle:  $\sum_{\mathbf{k}} \mathbf{D}_{\mathbf{k}}^T \mathbf{C}^T \mathbf{C} \mathbf{D}_{\mathbf{k}}$

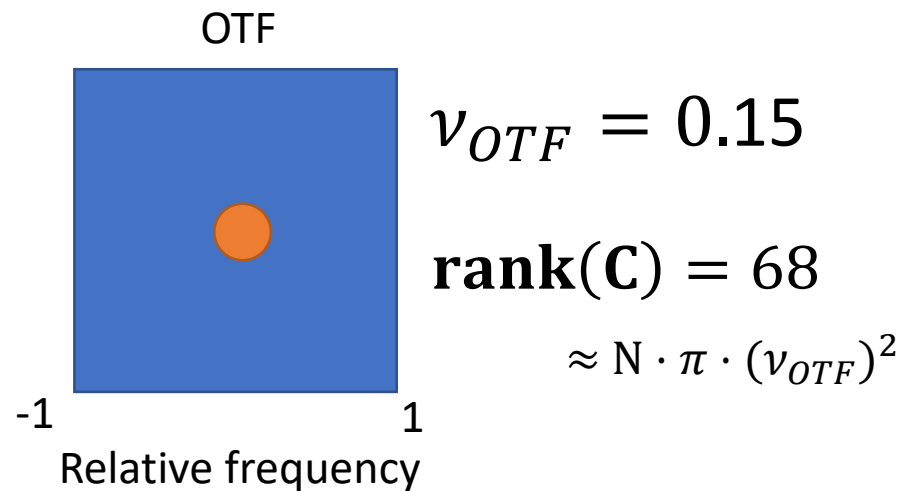
Full rank ?      How many speckle images?

# Numerical Analysis

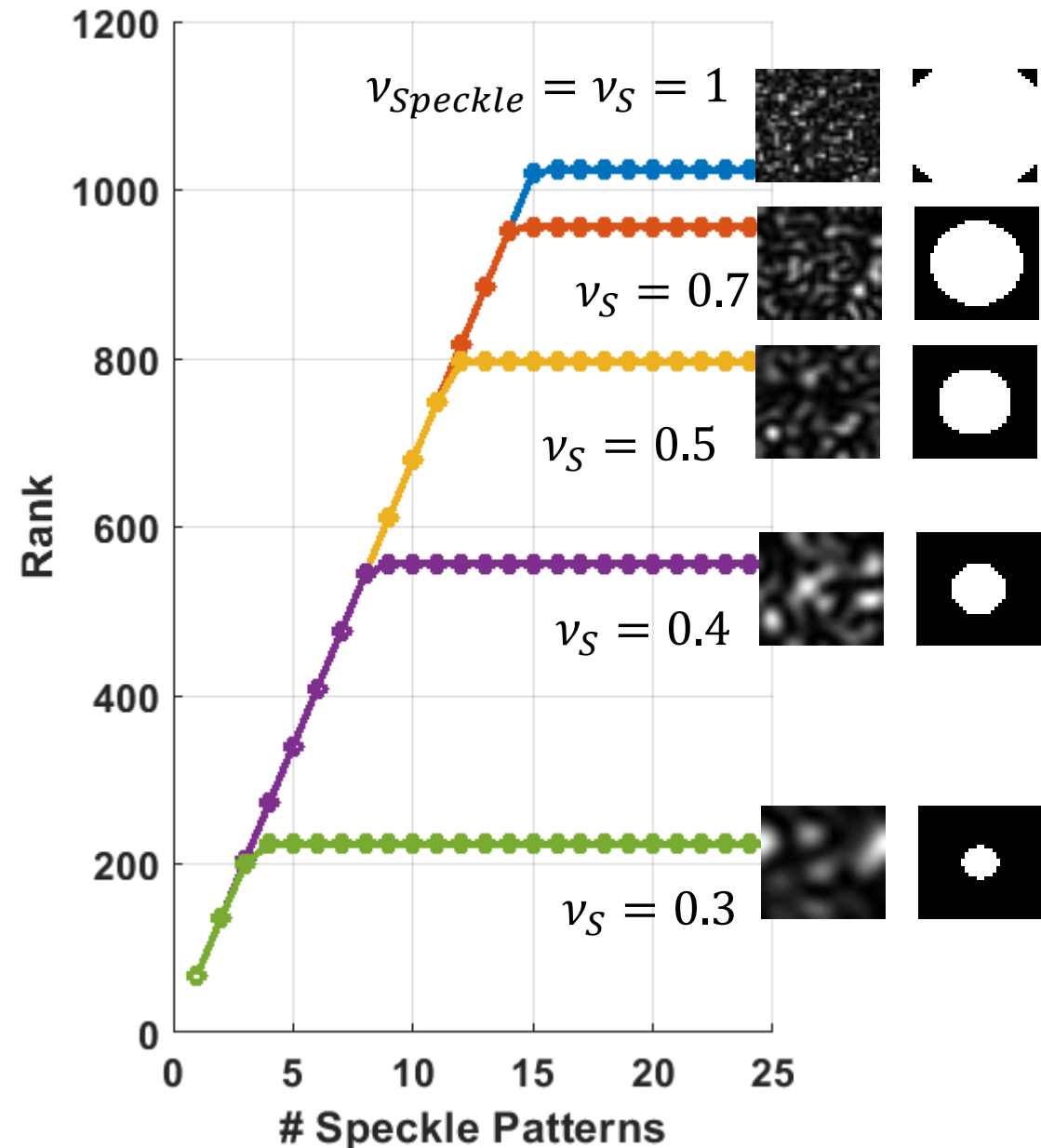
$$\mathbf{I}_k = \mathbf{C} \cdot \mathbf{D}_k \cdot \mathbf{S}$$

$$N = 32 \times 32 = 1024 \text{ Pixel}$$

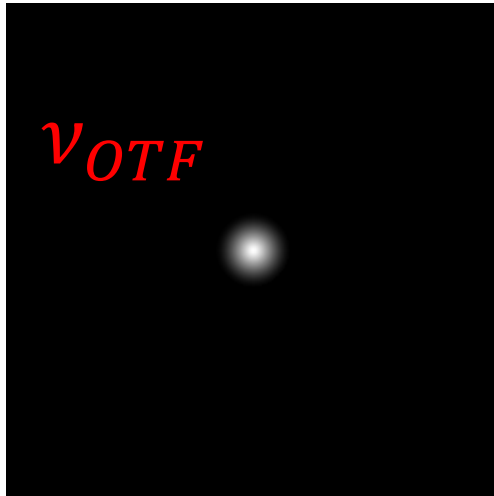
$$\mathbf{C} \in \mathbb{R}^{32^2 \times 32^2} = \mathbb{R}^{1024 \times 1024}$$



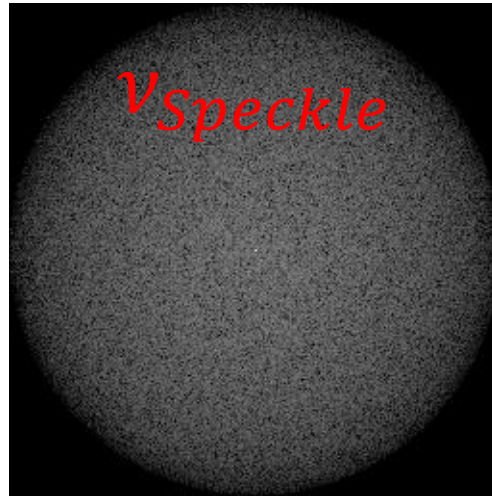
$$\#\mathbf{K}_{\min} = \frac{N}{\text{rank}(\mathbf{C})} = \frac{1024}{68} \approx 15$$



OTF

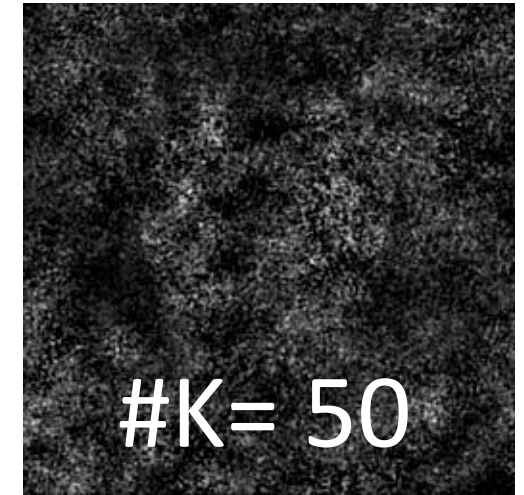
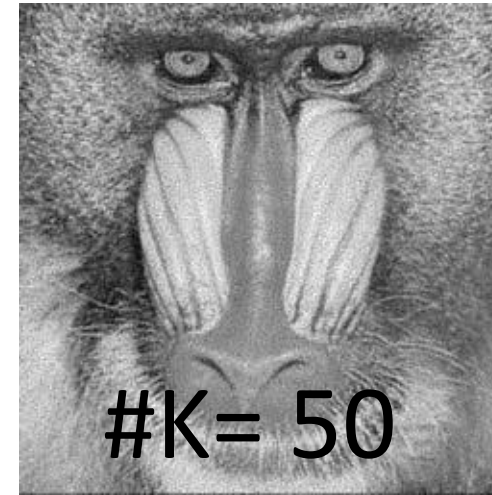
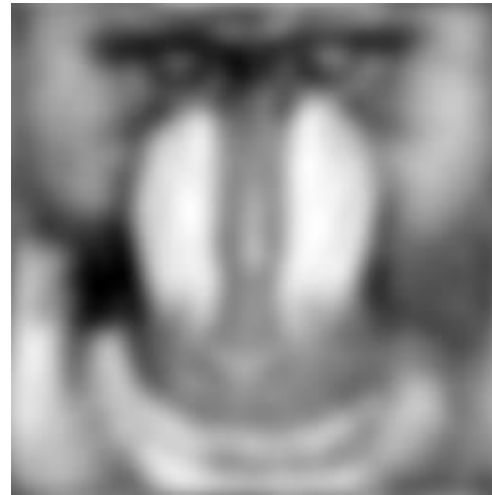
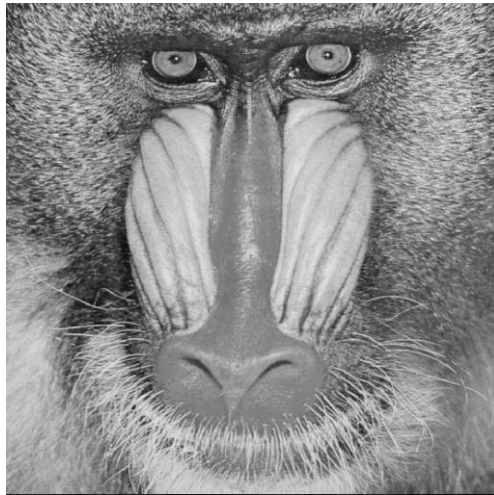
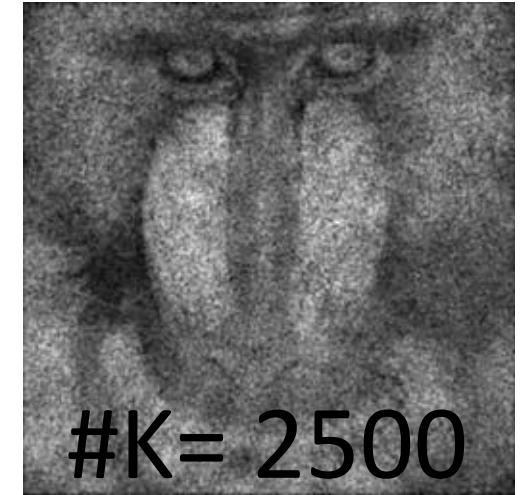
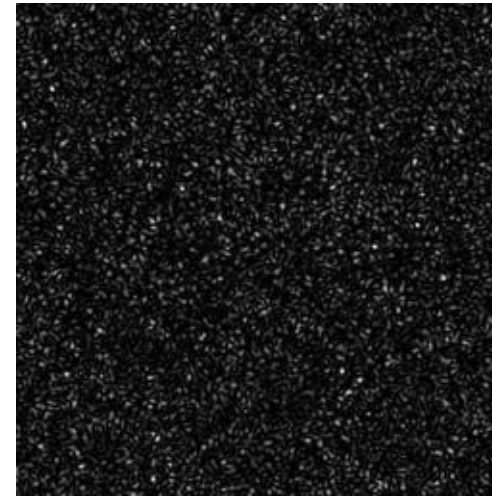


FFT(speckle)



FFT  
↔

speckle



ground truth

low resolution

proposed

Garcia et. al.

# Summary

- Small aperture:
  - ➔ Large Depth-of-Field
  - ➔ Low Optical Resolution
  - ➔ Super-resolution
- Simulation experiments
- **Prior knowledge** of projected speckle pattern

# Outlook

- **Coherent** projection but **incoherent** imaging ?
  - ➔ Subjective speckle ?
- **Joint estimation** of **pattern** and **object**
- **Comparison** to other eDoF methods